Fluid dynamics with a critical point

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Abstract: We present a chiral fluid dynamic model to study the dynamics of the QCD phase transition and of the critical point. The chiral fields are propagated out of equilibrium and interact with the fluid of quarks. Qualitatively the energy density for a discontinuous and a continuous phase transition are studied.

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1 Introduction

One of the most interesting features of the QCD phase diagram is the phase transition between the hadronic phase and the quark gluon plasma. As shown by lattice QCD calculations this phase transition at $\mu_B = 0$ is a crossover [1]. From model studies [2] it is assumed that at low temperature but large μ_B the phase transition is discontinuous. As a consequence the line of this discontinuous phase transition must end in a critical point. Near this critical point the behavior of the system is determined by long-range correlations and fluctuations.

The part of the QCD phase diagram where the critical point is supposed to be located can be scanned in heavy ion collisions. By varying the beam energy different trajectories mapped to the T-μ plane can be studied. For trajectories going through the critical point an increase of the fluctuations in observables should be visible [3, 4].

In this work the explicit non-equilibrium propagation of the order parameter of chiral symmetry breaking coupled to an ideal fluid of quarks is studied [5, 6].

2 Chiral fluid dynamics

The dynamics are governed by the linear sigma model [7] with quarks

$$\mathcal{L} = \overline{q}\left[\mathrm{i}\gamma\partial_\mu - \mathrm{g}\left(\sigma + \mathrm{i}\gamma_5\tau\boldsymbol{\pi}\right)\right]q + \frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\boldsymbol{\pi}\right)^2 - U\left(\sigma,\boldsymbol{\pi}\right), \tag{1}$$

with the chiral fields $\phi = (\sigma, \boldsymbol{\pi})$, the constituent quark field $q = (u, d)$ and the coupling g between the quarks. The potential for the chiral fields

$$U\left(\sigma,\boldsymbol{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0. \tag{2}$$

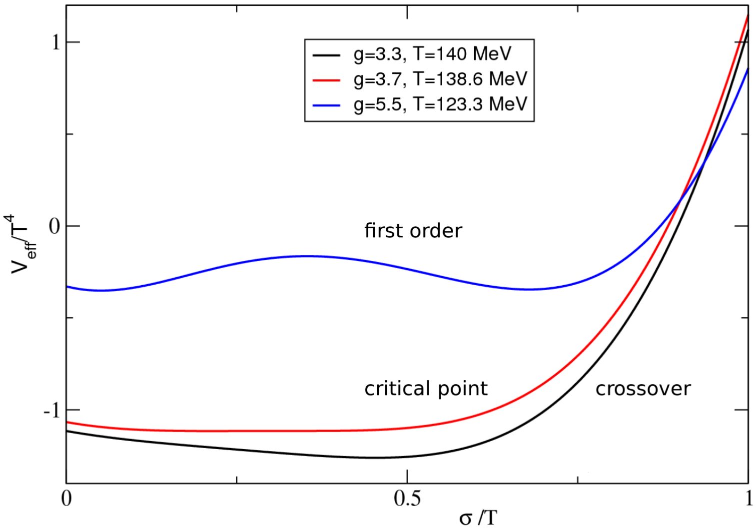

Figure 1: V_{eff} at $\mu_B = 0$ for different couplings g and temperatures T.

The chiral symmetry is spontaneously broken in the vacuum with the following choice of parameters. The explicit symmetry breaking term is $h_q\sigma = f_\pi m_\pi^2$ with $m_\pi = 138$ MeV. Thus $\nu^2 = f_\pi^2 - m\pi^2/\lambda^2$. Choosing $\lambda^2 = 20$ yields a sigma mass $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2 \approx 600$ MeV. In order to have zero potential energy in the ground state we choose $U_0 = m_\pi^4/(4\lambda^2) - f_\pi^2 m_\pi^2$.

The quarks are considered a heat bath in local thermal equilibrium and are integrated out by

$$\mathcal{Z} = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\boldsymbol{\pi} \exp\left[\int_0^{\frac{1}{T}} d(it)\right] \int_V \mathrm{d}^3x\mathcal{L}. \tag{3}$$

In a mean-field approximation $\mathcal{Z}$ can explicitly be calculated and the grand-canonical potential at $\mu_B = 0$ gives the effective potential for the chiral fields interacting with the quarks

$$V_{\text{eff}}(\phi, T) = -\frac{T}{V}\log\mathcal{Z} = -d_q T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\log(1+\mathrm{e}^{-\frac{E}{T}}) + U(\sigma,\boldsymbol{\pi}), \tag{4}$$

with the degeneracy factor $d_q = 24$ and the energy $E = \sqrt{p^2 + g^2\phi^2}$ of the quarks. Non-zero values of the chiral fields in the chirally broken phase dynamically generate a quark mass $m_q^2 = g^2\phi^2$. At $\mu_B = 0$ the strength of the phase transition can be varied by the coupling g as depicted in figure 1. We explicitly study a non-equilibrium propagation of the chiral field. With the scalar and pseudoscalar densities

$$\rho_{\text{S}} = g d_q \sigma \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{E}f_{\text{FD}}(p), \quad \boldsymbol{\rho}_{\text{PS}} = g d_q \boldsymbol{\pi}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{E}f_{\text{FD}}(p) \tag{5}$$

the classical equations of motion are

$$\partial_\mu\partial^\mu\sigma + \frac{\delta U(\phi)}{\delta\sigma} = -g\rho_{\rm S}, \quad \partial_\mu\partial^\mu\boldsymbol{\pi} + \frac{\delta U(\phi)}{\delta\boldsymbol{\pi}} = -g\boldsymbol{\rho}_{\rm PS}. \tag{6}$$

The evolution of the quark fluid is governed by fluid dynamics. For an ideal fluid the energy-momentum tensor reads

$$T^{\mu\nu}_{\rm fluid} = (e+p)u^\mu u^\nu - pg^{\mu\nu}, \tag{7}$$

where u^μ is the local four-velocity of the fluid and $g^{\mu\nu}$ the metric tensor. The local pressure and the local energy density are given by

$$p(\phi,T) = -V_{\rm eff}(\phi,T) + U(\phi), \quad e(\phi,T) = T\frac{\partial p(\phi,T)}{\partial T} - p(\phi,T). \tag{8}$$

The chiral fields locally interact with the quark fluid and exchange energy and momentum. A source term needs to be included

$$S^\nu = -\partial_\mu T^{\mu\nu}_\phi = g\rho_\sigma\partial^\nu\sigma + g\boldsymbol{\rho}_\pi\partial^\nu\pi, \tag{9}$$

where $T^{\mu\nu}_\phi$ is the energy-momentum tensor of the pure chiral field terms in (1). Within the fluid dynamic description of the quarks we have to solve

$$\partial_\mu(T^{\mu\nu}_{\rm fluid} + T^{\mu\nu}_\phi) = 0. \tag{10}$$

3 Numerical results

Equation (10) is solved by the full 3+1d SHASTA fluid dynamic code [8, 9, 10].

For an initial temperature $T_{\rm ini} = 160$ MeV $\sigma_{\rm eq}$ and $e_{\rm eq}$ are the equilibrium values of the sigma field and the energy density. The energy distribution is ellipsoidal in the x-y-plane

$$e(\boldsymbol{r}, t=0) = \left\{ \begin{array}{cl} e_{\rm eq} & \text{for } b^2x^2 + a^2y^2 < (ab)^2 \text{ and } |z| < l_z \\ 0 & \text{for } b^2x^2 + a^2y^2 > (ab)^2 \text{ or } |z| > l_z \end{array} \right. \tag{11}$$

Here $a = 3.5$ fm, $b \approx 5.8$ fm and $l = 6$ fm. The velocity is chosen $v_z(\boldsymbol{r}, t=0) = |z|/l \cdot v_{\rm max}$, where $v_{\rm max} = 0.2$. For a smooth distribution between the high energy phase, $\sigma_{\rm eq} \approx 0$, and the vacuum, $\sigma = f_\pi$, the sigma field is initiated with

$$\sigma(\boldsymbol{r}, t=0) = f_\pi + \frac{\sigma_{\rm eq} - f_\pi}{(1+\exp((\tilde{r}-\tilde{R})/\tilde{a}))(1+\exp\left((|z|-l_z)/\tilde{a}\right))} \tag{12}$$

where $\tilde{r} = \sqrt{x^2+y^2}$ and $\tilde{R} = ab\tilde{r}/\sqrt{b^2x^2+a^2y^2}$ for $\tilde{r} \neq 0$ and $\tilde{R} = a$ for $\tilde{r} = 0$.

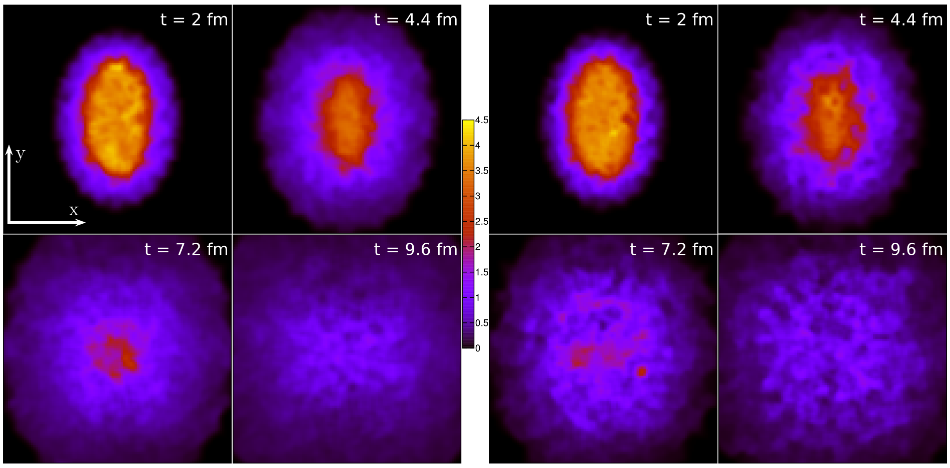

Figure 2: Energy density in units of $e_0 = 148$ MeV/fm^3 in the $x-y$-plane at $z=0$ for a continuous (left) and a discontinuous (right) phase transition.

The pion field is initially set to zero and neglected during the simulation. Initial fluctuations $\delta\sigma$ are put on top of (12). They are Gaussian distributed with a variance of 30 MeV and coarse grained in order to avoid an artificial correlation scale due to the grid spacing.

In figure 2 the energy density in the $x-y$-plane at $z=0$ is shown for four different times during the evolution through a critical point (left) and a discontinuous phase transition (right). Only right after the expansion started both systems behave similarly. As expected for a discontinuous phase transition the expansion is slower. Here we observe small patterns of high energy and of low energy clearly separated from each other. This indicates the nucleation and bubble formation. These inhomogeneities are clear non-equilibrium effects. For an evolution through the critical point this picture changes. The energy density is distributed more smoothly over the expansion volume.

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