SINGLE PASS COLLIDER MEMO CN-326

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TITLE: CENTRIFUGAL SPACE–CHARGE FORCES IN SLC*

1. Introduction

The purpose of this note is to make a preliminary estimate of the effect of the centrifugal space charge forces on SLC – assuming that the results of Piwinski [1] are correct.

2. Estimate of the chromaticity

According to Piwinski the new space–charge effect is a highly nonlinear radial force whose important term is the second derivative at the beam center. He evaluates this term numerically but gives his results in terms of the horizontal chromaticity for various storage rings. I use here a scaling law to relate his results to the arcs of SLC.

The horizontal chromaticity ξ is given by

$$\xi = -\frac{R}{2E}\left\langle \beta\eta\left(\frac{d^2F_r}{dx^2}\right)_\circ \right\rangle \tag{1}$$

where E is the energy, R the radius of the ring, β and η are the radial betatron and off–energy functions, and F_r is the radial space–charge force. From Piwinski's calculations we can expect the second derivative of F_r to be proportional to I/σ_x^2R, where I is the local, instantaneous current density in a bunch and σ_x is the r.m.s. bunch width. The worst case is at the center of the bunch where $I = \hat{I}$, the peak bunch current, so I will use this value. This scaling assumption presumes that the transverse aspect ratio of the bunch σ_z/σ_x is a constant. I ignore this problem for now.

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We may then expect the chromaticity to scale in proportion to a scaling parameter S given by

$$S = \frac{\beta \eta \hat{I}}{\sigma_x^2 E} \tag{2}$$

Actually, since β, η, and σ_x^2 are correlated, I should take $\langle \beta\eta/\sigma_x^2 \rangle$, but I will not make this refinement here.

Piwinski gives average values of the factors in Eq. (2) for three rings, CESR, PETRA, and LEP, together with a calculated chromaticity. In the following table I list these values together with a calculated S and the ratio ξ/S. You see that, for these rings at least, S is a proper scaling parameter.

TABLE 1

Parameter	CESR	PETRA	LEP	SLC
$\hat{I}$ (amp)	152	460	1350	1000
β (m)	13	18	80	4
η (m)	1.0	1.1	1.64	0.035
E (GeV)	5	7	20	50
σ_x (mm)	1.0	0.9	0.87	0.035
S (10^8 A/GeV)	4.0	16.8	117	23
ξ	8.6	39	300	{58 }
ξ/S	2.2	2.3	2.5	{2.5 }

The table gives also the value of S obtained for SLC. Using this S together with the factor $\xi/S = 2.5$ obtained from the LEP column, we get the value of 58 for the space–charge chromaticity expected for SLC. In the table I have taken for σ_x only the betatron oscillation width ($= \sqrt{\epsilon\beta_x}$). More about this later.

The biggest extrapolation is in the value of σ_x – which enters as the square. One might worry because all of the rings considered have about the same σ_x while SLC is twenty to thirty times smaller. I believe that ξ must vary as σ_x^{-2} from quite general arguments; and the results of Piwinski (his Fig. 10), which gives ξ for various σ_x agree very well with a σ_x^{-2} dependence.

$$\sigma_x^2 \approx \sigma_{x\beta}^2 + (\eta \Delta E/E)^2$$

$$\approx (35^2 + 70^2)(\mu m)^2,$$

or

$$\sigma_x \approx 78 \mu m.$$

And since the chromaticity goes as σ_x^{-2}, it is reduced by the factor $(0.35/78)^2 = 0.2$. The appropriate $\Delta\phi$ is now 0.09 which would seem to be quite benign.

Notice that any larger ΔE would give an even lower $\Delta\phi$, because then ξ would be proportional to ΔE^{-2} and $\Delta\phi$ would vary as ΔE^{-1}.

Now let me show that the above estimates are not quite correct, because I have assumed that ΔE represent the energy spread at the bunch center. In reality we expect that there will be a strong correlation between the mean energy deviation ΔE at each longitudinal location within the bunch and the longitudinal coordinate, say τ. It is this ΔE which is expected to vary between $\pm 2 \times 10^{-3}$ of the mean energy. In addition, there will be an energy spread $\pm\delta E$ at each longitudinal position. This spread is related to the energy spread injected into the linac from the damping ring and compressor. I am told that the expected spread gives $\delta E/E = \pm 4 \times 10^{-4}$ at the arcs. In other words as the bunch enters an arc the energy varies from one τ to another within the bunch (with a total range $\pm\Delta E/E \approx 2 \times 10^{-3}$) which at each τ there is a local energy spread of $\pm\delta E/E \approx 4 \times 10^{-4}$.

In the arcs then the bunch will have a snake-like shape because the axis of the bunch is dispaced laterally (because of the η-function) proportional to ΔE, which is varying with τ. At each τ, however, the bunch has an energy spread $\pm\delta E$, and a width σ_x which is given almost completely by the betatron emittance – because η times the "inner" energy spread δE is quite small. Now since the space-charge nonlinearity is centered at the displaced bunch axis, it will produce a phase displacement $\Delta\phi$ given by the chromaticity times the inner energy spread δE.

The phase spread $\Delta\phi$ produced by an energy spread δE of 2×10^{-4} is only 0.04 which is even smaller than the $\Delta\phi$ estimated above.

There is an additional complication in the arcs. As the bunch goes along it gets shorter and its energy spread increases. Both of these effects will increase