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PAPER

Maximum acceleration and quantum clock: on the existence of a new universal constant

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Abstract

In the pseudo-Euclidean Minkowski space, the four-dimensional volume element is invariant under Lorentz transformations. By hypothesising that in this space there is a minimum volume, it is possible to demonstrate the existence of a maximum acceleration. The volume element cannot be derived from the theory and must be obtained through direct measurement, thus it assumes the role of a bona fide universal constant. Two different estimates of the elementary volume are given, which differ by several orders of magnitude: the first is obtained in a pseudo-Euclidean space for particles with mass, and the second represents an absolute minimum volume, independent of the mass.

1. Introduction

As far as observations suggest, the physical laws appear to be the same throughout the Universe. The fact that we can recognize the atomic species emitting electromagnetic radiation in distant galaxies and quasars, along with the recently detected gravitational waves, serves as evidence for this assertion. At the foundation of all mathematical formulations of these laws, there are parameters whose values, almost by definition, cannot be derived from the theory itself and must be inferred from measurements [1]. These are known as physical constants.

The constants present in the modern vision of physics are listed in [2]. However it is useful to recall that not all of them hold the same level of importance and can be classified according to their role in the theory. Currently, it is generally agreed that only the speed of light $c \cong 3 \times 10^{10}$ cm sec^{-1}), the reduced Planck's constant ($\hbar \cong 1.05 \times 10^{-27}$ erg sec) and Newton's gravitational constant ($G \cong 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}) can be considered *fundamental* (throughout the Paper we adopt cgs units and, according, all numerical values and expressions are given in this system of units) . Of these constants, two are critical quantities: c is the natural unit of speed, and all angular momenta, including the total angular momentum of the Universe (if it exists), must be an integer multiple of $\hbar/2$. Other constants, such as the elementary charge ($e = 4.80 \times 10^{-10}$ esu), can also be associated with critical quantities; for instance, all free charges must be an integer multiple of e. The value of e enters into the definition of the dimensionless fine structure constant $\alpha = e^2/(\hbar c) \cong 1/137$, which characterizes the strength of electromagnetic interactions. However, in Quantum Field Theory, the value of coupling parameters in renormalizable theories depends on the energy scale, thereby making α not strictly constant.

There are two related topics concerning the considerations above. Indeed, it is possible to combine physical constants to create new units of measurement that could replace the usual textbook units commonly used in different physical domains. Thus, in Atomic Physics, it is customary to use the Bohr radius $a_0 = \hbar^2/(m_e e^2) \cong 5.29 \times 10^{-9}$ cm (related to the average electron-proton distance in the hydrogen ground state), the electron mass $m_e \cong 9.11 \times 10^{-28}$ g, and $\tau = \hbar^3/(me^4) \cong 2.42 \times 10^{-17}$ sec, respectively, as units of length, mass, and time. In General Relativity and Cosmology, the so-called Planck units are widely used; these are $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \cong 1.62 \times 10^{-33}$ cm, $m_P = \sqrt{\frac{\hbar c}{G}} \cong 2.18 \times 10^{-5}$ g, and $\tau_P = \sqrt{\frac{\hbar G}{c^5}} \cong 5.39 \times 10^{-44}$ sec for length, mass, and time, respectively; atomic units naturally emerge from the Schrödinger theory of the hydrogen atom, while Planck units originate from an algebraic combination of constants; they carry a somewhat self-referential flavor: since they can be defined, they must have a physical meaning that supports their existence. In any case, Planck units are defined in terms of the three universal constants and seem to be the natural candidates for a unified theory of Physics for these qualities, they have been jocularly dubbed *God*'s units. However recently the fact that Planck units can be written without relying on G and $\hbar$ has been pointed out [3, 4].

One related issue raises the question of whether Nature is trying to suggest something by assigning the *relative* values to physical constants. This question was fundamental to Dirac's large number hypothesis [5] and led to the study of the time dependence of physical parameters. Indeed, some experimental evidence suggested that α was smaller in the early Universe. However, additional experimental evidence, along with the realization that observations could have been biased by assumptions and systematic errors, cast shadows over these findings and necessitate new ad hoc focused and well-planned experiments [1, 2, 6–12].

The quest for a consistent framework that integrates quantum theory and gravity has been a central challenge in the development of theoretical physics. After many years of discussions and research, it appears increasingly likely that such a theory would entail modifications to the fundamental concepts of physics. In this context, it becomes clear that the value of the Planck length is dependent upon the reference frame due to Lorentz transformations, and therefore, loses the aura of universality that a God given system of units should possess. In response, modifications to the Lorentz transformation have been proposed that incorporate ℓ_P with c as an invariant minimum length [13] or introduce an invariant energy parameter at the Planck scale [14, 15]. However, these modifications come at the cost of non-linearity in the new transformations. See [16] for a discussion of the difficulties encountered. At the root of this conundrum is the fact that the Lorentz transformations impose a geometrical constraint on a four-dimensional space with the metric $(+, -, -, -)$. In this framework, Planck length and energy are alien objects of physical origin; a similar situation occurred with the photon, which was not introduced into the theory by modifying Maxwell's equations but through second quantization. In fact, within the theory of matter interacting with the electromagnetic field, the so-called second quantization promotes the classical electromagnetic field, appearing in the Hamiltonian, to the rôle of operator and, accordingly, quantizes it. In this way the field parameters, such as intensity and energy density, become a function of the number of quanta present in the field mode and the particles in this description are identified with the normal modes (photons) of the field. At this stage of the theory the quantum concept of photon is defined in a coherent way without any need of changing Maxwell equations. In passing we note that the procedure is not specific to the electromagnetic case but common to all fields and particles [17].

During the last decades, there has been significant interest in the hypothesis of a maximal acceleration in Nature. This refers to the general idea that the proper accelerations of test particles are bounded with respect to a given space-time structure. The origin of this concept can be traced back to the foundational work of E. Caianiello [18] and Brandt [19].

In 1984, Caianiello [20] provided a direct proof that, under appropriate conditions, the Heisenberg uncertainty relations impose an upper limit, $a_C = \frac{2mc^3}{\hbar}$, on the acceleration that can be achieved along a particle's worldline. This limit, referred to as maximum acceleration (MA), is determined by the particle's own mass.

The main result of the Paper is that the hypothesis of space-time having a granular structure naturally leads to the existence of a MA but the value of the elementary volume cannot be derived from the theory and assumes the rôle of a new universal constant. However, we will make conjectures about its value.

Classical and quantum arguments supporting the existence of MA have been frequently discussed in the literature [21, 22]. Existence of a MA would eliminate divergence difficulties affecting the mathematical foundations of Quantum Field Theory [23] and it would also prevent ultraviolet divergences in the calculation of the black hole entropy (see e.g. [24]).

2. Caianiello's maximum acceleration: review

Caianiello's MA can be derived from the uncertainty principle of Quantum Mechanics combined with some assumptions through the following argument. Let $\hat{A}$ and $\hat{B}$ be two generally non-commuting operators. We call $A = \langle \hat{A} \rangle$ the expectation value of $\hat{A}$ (over a generic quantum state) and $\Delta A = \sqrt{\langle (\hat{A} - A)^2 \rangle}$ the associated quantum uncertainty. Analogous definitions hold for operator $\hat{B}$.

Then according to the Robertson uncertainty principle (generalized uncertainty principle) [25], ΔA and ΔB fulfill

$$(\Delta A)(\Delta B) \geqslant \frac{1}{2}|C|. \tag{1}$$

with

$$\hat{C} = [\hat{A}, \hat{B}] \tag{2}$$

(consistently with the previous notation, $C = \langle C \rangle$).

For $\hat{A} = \hat{x}$ and $\hat{B} = \hat{p}_x$ (respectively position and momentum of a one-dimensional particle) the commutator is $[\hat{x}, \hat{p}_x] = i\hbar$ and (1) reduces to the standard Heisenberg uncertainty principle

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2}. \tag{3}$$

We next define the acceleration operator $\hat{a}$ from the equation of motion of the velocity operator $\hat{v}$ in the Heisenberg picture, which reads

$$\hat{a} = \frac{d\hat{v}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{v}] \tag{4}$$

where $\hat{H}$ is the Hamiltonian of the system. Combining this with (1) for $\hat{A} = \hat{H}$ and $\hat{B} = \hat{v}$ then gives

$$\Delta E \Delta v \geqslant \frac{\hbar}{2}|a| \tag{5}$$

where we replaced $\Delta E = \Delta H$ (to comply with the standard notation for the energy uncertainty).

By making the following assumptions

$$\Delta E = mc^2 \quad \Delta v = c, \tag{6}$$

where mc^2 is the particle's rest energy, and next replacing in (5) we end up with

$$a \leqslant a_C = \frac{2mc^3}{\hbar}, \tag{7}$$

which expresses the Caianiello's proposal for the MA.

Note that $a_C = 2mc^3/\hbar$ does depend on the mass particle m. Hence, this theory does not produce a universal (particle-independent) value for the MA. Also note that in the case of an electron we would predict $a_C \simeq 4.7 \times 10^{31}$ cm s^{-2}. This bound is so high that it challenges experimental observation. Nonetheless, the existence of an upper bound for acceleration is conceptually important, thus motivating the hope to find phenomena where it plays a role.

3. Space granularity and maximum acceleration

In Special Theory of Relativity the volume of the four-space element is invariant; therefore we make the ansatz that a minimum space-time volume Ω exists which is independent of the reference frame:

$$\Delta\ell \Delta T \geqslant \Omega \tag{8}$$

that can be considered as a geometrical constraint and, as such, universal. We stress that $\Delta\ell$ and ΔT are dimensionful quantities (space and time) and individually obey the traditional dilation and contraction Lorentz rules. To proceed further we make a connection to Quantum Mechanics. Since the Heisenberg uncertainty principle is always valid, the relation (3) must in particular hold when the equality is taken, that is

$$\Omega \leqslant \frac{\hbar}{2\Delta p}\Delta T = \Delta T \frac{\hbar}{2m\Delta v} \Rightarrow \Omega \leqslant \frac{\hbar}{2ma} \tag{9}$$

which yields the maximum acceleration

$$a \leqslant \frac{\hbar}{2m\Omega}. \tag{10}$$

Here Ω is a fundamental constant as yet undefined. A possible conjecture on its value is that it is related to the electron classical radius as

$$\Omega = \varpi \frac{r_e^2}{c} \tag{11}$$

with ϖ a constant and r_e the electron classical radius:

$$r_e = \frac{e^2}{m_e c^2} \cong 2.82 \times 10^{-13} \text{ cm}. \tag{12}$$

In this way we obtain

$$\Omega = \varpi \frac{\alpha^2 \hbar^2}{m_e^2 c^3} \tag{13}$$

and, for the MA,

$$a_1 \leqslant \frac{1}{4\varpi\alpha^2} \frac{2m_e c^3}{\hbar} \tag{14}$$

with α the constant of fine structure. The constant ϖ acts as a proxy for Ω and cannot be derived from the theory outlined above; its role is to incorporate known constants such as e and m_e into the equation. Assuming this represents the maximum acceleration for the electron in Caianiello's theory, we can derive $\varpi = \frac{1}{4\alpha^2} \approx 5 \times 10^3$ and $\Omega = \frac{\hbar^2}{4m_e^2 c^3}$, which in this context depends on the electron mass.

4. The quantum clock and the maximal acceleration

Based on a gedanken experiment, an uncertainty principle involving space and time has been proposed in [26]. The argument can be summarized as follows: to measure and define time, we need a clock. Any clock requires a certain amount of energy to function and measures time at its position. To make this measurement as precise as possible at a given position, we need to increase the energy of the clock and confine it in the smallest volume possible. However, there exists a limit to the size of the clock, which is defined by its Schwarzschild radius. In fact, if the size of the clock is smaller than its Schwarzschild radius, then, according to the Hoop Conjecture [27], it will collapse into a black hole and will not be able to communicate any time measurement anywhere in the Universe. Using a clock based on particles decaying randomly, the authors derive the following relation between the size of the clock Δr and the minimum time interval Δt measurable by the clock:

$$\Delta r \Delta t \geqslant \frac{G\hbar}{c^4}. \tag{15}$$

In this case, therefore, the minimum possible four-volume of equation (8) is given by the quantity

$$\Omega_{\min} = \varpi' \frac{G\hbar}{c^4}, \tag{16}$$

independent of the specific particle considered. Again the parameter ϖ' has been introduced similarly to ϖ to acts as a proxy of Ω.

By setting $\varpi' = 1$ equation (10) would give:

$$a_2 \leqslant a_{\max} = \frac{c^4}{2Gm} = \frac{c^2}{r_{\rm EH}} \tag{17}$$

where $r_{\rm EH}$ is the radius of the event horizon of the particle. In agreement with [26], we also assume that the event horizon of any particle cannot be smaller than the minimum measurable length, that is the Planck length, ℓ_P. In this way, we can derive the MA:

$$a_2 \leqslant a_{\max} = \frac{c^2}{\ell_P} = \sqrt{\frac{c^7}{\hbar G}}. \tag{18}$$

Remarkably, this value of MA depends only on three universal constants. The corresponding value of MA is then $\simeq 5.6 \times 10^{55}$ cm s^{-2} and should be considered as an upper bound for the acceleration of any object or particle, regardless of its mass.

Equation (15) has been obtained within the limits of the Schwarzschild solution of the Einstein field equations; this metric describes the gravitational field of a spherical, non-rotating mass collapsed into a black hole. Other metrics have been introduced for black holes with different characteristics. For example Kerr metric describes an uncharged black hole endowed of an angular momentum and the Reissner–Nordström metric describes an electrically charged rotating black hole. The choice of the particular metric may change the size of the black hole of a factor two (for a short outline on the topics see [28]) and may be essential for obtaining the size of a particular black hole and the minimum time interval as given by (15), however it is not dramatic for the value of the minimum volume Ω. In fact the two values given in this Paper must be seen as mere estimations based on conjecture and reasonless but that can be shown wrong by actual measurement effort.

5. Discussion

The starting point of this paper is the hypothesis that space-time has a granular structure with a minimum volume Ω, which, however, cannot be derived from the theory itself and must be obtained from experiments. For this characteristic, Ω should be considered as a fundamental physical constant. Nevertheless, two estimations of Ω can be given using two different models. The first model, derived from the properties of a pseudo-Euclidean space, yields $\Omega = \varpi\alpha^2\frac{\hbar^2}{m_e^2c^3}$, while the second, obtained from the operational definition of a quantum clock at the Planck scale, gives $\Omega = \varpi'\frac{G\hbar}{c^4}$. In both expressions, the undetermined parameters ϖ and ϖ' appear, and their presence is necessary because the value of the minimum volume cannot, by any means, be theoretically derived and must be measured. By setting $\varpi = \varpi' = 1$, the two different values of Ω differ by almost 40 orders of magnitude; this should not be considered a disadvantage because an eventual experiment detecting Ω would also distinguish between the two models.

As a consequence of the hypothesis of the existence of a minimum volume Ω, a maximal acceleration (MA) exists, for which two different expressions can be provided. The expression a_2 incorporates those three constants considered truly fundamental and should therefore be preferred to the expression for a_1. Our expression of the maximal acceleration, given by equation (18), is in agreement with the maximum physical value of acceleration as indicated in [29], and in turn, gives a minimum value for the horizon distance, $l_{\min} \sim \sqrt{8\pi G\hbar/c^3}$, which can also be viewed as an intrinsic uncertainty in the horizon position.

One might wonder why the gravitational constant G appears in the volume of the space-time granule. In fact, the structure of space-time has a purely geometrical origin and should, theoretically, be independent of the presence of a gravitational field and thus independent of G. However, in the Theory of General Relativity, the presence of a gravitational field distorts the metric of space-time. Let us assume a coordinate system (x^0, x^1, x^2, x^3) that becomes the Galilean system (X^0, X^1, X^2, X^3) in the absence of gravity. The relationship between the metrics in the two regions of space is given by

$$dx^0dx^1dx^2dx^3 = \left|\frac{\partial(x^0, x^1, x^2, x^3)}{\partial(X^0, X^1, X^2, X^3)}\right| dX^0dX^1dX^2dX^3 \tag{19}$$

where $\frac{\partial(x^0, x^1, x^2, x^3)}{\partial(X^0, X^1, X^2, X^3)}$ is the Jacobian determinant. Thus, the gravitational field alters the volume of the space-time granule, and we assume that it becomes smaller as the gravitational energy density w increases.

If our conjecture is correct, then the value of ϖ' would depend on the gravitational energy density w, implying that

$$\Omega_{\min} = \kappa(w)\Omega_\infty \tag{20}$$

where $\kappa(w)$ is a monotonically decreasing function with $\kappa(0) = 1$, and Ω_∞ represents the asymptotic volume of the granule in intergalactic space where $w \to 0$. Consequently, we are led to the conclusion that our universal constant Ω is, in reality, dependent upon position. A similar conjecture has recently been proposed for $\hbar$ [30, 31] and the fine structure constant [32]. This unusual perspective suggests that the distribution of matter and its state of motion determine all the properties of the known Universe, including the values of physical constants. Recent studies indicate that galaxies are organized into very long and slender filaments that are spinning about their axes [33], and such a distribution of mass and angular momentum may affect the local values of physical constants, which perhaps should be described as propagating fields themselves.

We conclude this discussion by observing that the acceleration of a particle is maximum in the reference frame where it is at rest (see appendix A). This consideration strengthens the analysis developed in this paper, which primarily refers to the rest reference frame. It allows us to extend our conclusions to any reference frame.

The concept of a maximal acceleration, supported by various dynamical theories [29, 34, 35], plays a crucial role in preventing the collapse of large gravitational bodies into singular points. This limitation on acceleration directly constrains the curvature and energy density, as exemplified in the context of black holes. Furthermore, the presence of singularities in the solutions to the field equations of general relativity highlights the constraints of classical theory, particularly its inability to account for quantum effects.

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Data availability statement

No new data were created or analysed in this study.

Conflict of interest statement

The Authors have no conflict of interests related to this publication.

Ethics statement

In the present study no activity has involved biological organisms.

Appendix A. Relativistic acceleration

According to Special Relativity, the acceleration observed in the rest frame is the maximum possible compared to all other reference frames. This property will be reviewed in the following section, beginning with an explanation of four-acceleration as defined in special relativity.

We denote by $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ the standard (classical) position, velocity and acceleration of a particle. Let $x^\mu = (ct, \boldsymbol{r})$ and $u^\mu = \gamma(c, \boldsymbol{v})$ be the four-position and four-velocity vector in the usual four-dimensional Minkowski space, where

$$\gamma = \left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{v}}{c^2}\right)^{-1/2} \tag{21}$$

is the Lorentz factor. The four-acceleration is defined as

$$a^\mu = \gamma\frac{du^\mu}{dt} = \left(\gamma^4\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c},\ \gamma^2\boldsymbol{a} + \gamma^4\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c^2}\boldsymbol{v}\right), \tag{22}$$

where we used the identity

$$\frac{d\gamma}{dt} = \gamma^3\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c^2}. \tag{23}$$

Only when $\boldsymbol{a}$ is parallel to $\boldsymbol{v}$ does the spatial part of a^μ align with the acceleration vector. However, in the rest frame, a^μ simplifies to $a_0^\mu = (0,\ \boldsymbol{a}_0)$. The four-acceleration of a particle, with acceleration $\boldsymbol{a}_0$ in the rest frame, can be derived by applying the Lorentz transformation matrix to $a_0^\mu = (0,\ \boldsymbol{a}_0)$. It is necessary to reverse the sign of the velocity components because if the acceleration in the rest frame is directed along the positive x-axis, then at a later time t, the rest frame exhibits a negative velocity relative to the particle. Consequently, in the moving frame, a_0^μ transforms as follows ($\beta_k = v_k/c$):

$$a_0^\mu = \begin{pmatrix} \gamma & \gamma\beta_1 & \gamma\beta_2 & \gamma\beta_3 \\ \gamma\beta_1 & 1 + \frac{\gamma-1}{\beta^2}\beta_1^2 & \frac{\gamma-1}{\beta^2}\beta_1\beta_2 & \frac{\gamma-1}{\beta^2}\beta_1\beta_3 \\ \gamma\beta_2 & \frac{\gamma-1}{\beta^2}\beta_1\beta_2 & 1 + \frac{\gamma-1}{\beta^2}\beta_2^2 & \frac{\gamma-1}{\beta^2}\beta_2\beta_3 \\ \gamma\beta_3 & \frac{\gamma-1}{\beta^2}\beta_1\beta_3 & \frac{\gamma-1}{\beta^2}\beta_2\beta_3 & 1 + \frac{\gamma-1}{\beta^2}\beta_3^2 \end{pmatrix}\begin{pmatrix} 0 \\ a_0^1 \\ a_0^2 \\ a_0^3 \end{pmatrix} = $$

$$= \left(\gamma\frac{\boldsymbol{v}\cdot\boldsymbol{a}_0}{c},\ \boldsymbol{a}_0 + \frac{\gamma-1}{\boldsymbol{v}\cdot\boldsymbol{v}}(\boldsymbol{v}\cdot\boldsymbol{a}_0)\boldsymbol{v}\right). \tag{24}$$

Equating this with equation (22) we get:

$$\begin{cases} \gamma^4\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c} = \gamma\frac{\boldsymbol{v}\cdot\boldsymbol{a}_0}{c} \\ \gamma^2\boldsymbol{a} + \gamma^4\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c^2}\boldsymbol{v} = \boldsymbol{a}_0 + \frac{\gamma-1}{\boldsymbol{v}\cdot\boldsymbol{v}}(\boldsymbol{v}\cdot\boldsymbol{a}_0)\boldsymbol{v} \end{cases} \tag{25}$$

and then:

$$\boldsymbol{a} = \frac{1}{\gamma^2}\left\{\boldsymbol{a}_0 + \frac{\gamma-1}{\boldsymbol{v}\cdot\boldsymbol{v}}(\boldsymbol{v}\cdot\boldsymbol{a}_0)\boldsymbol{v} - \gamma^4\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{c^2}\boldsymbol{v}\right\} \tag{26}$$

which gives the instantaneous acceleration $\boldsymbol{a}$ as a function of the acceleration in the rest frame and of the speed $\boldsymbol{a}$.

By using the expression above the module of the acceleration is

$$\boldsymbol{a} \cdot \boldsymbol{a} = \frac{1}{\gamma^4}\left\{\boldsymbol{a}_0 \cdot \boldsymbol{a}_0 - \frac{(\boldsymbol{v} \cdot \boldsymbol{a}_0)^2}{c^2}\right\} \Rightarrow \boldsymbol{a} \cdot \boldsymbol{a} \leqslant \boldsymbol{a}_0 \cdot \boldsymbol{a}_0; \tag{27}$$

Thus, the acceleration in the rest frame is the largest, and consequently, the maximal acceleration (MA) in the rest frame will exceed the acceleration observed in any other frame.

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