

NON-STATIC PLANE SYMMETRIC DARK ENERGY WITH STRING COSMOLOGICAL MODEL IN SELF CREATION THEORY

T Vinutha¹, B Raja Shekar¹ and G Satyanarayana¹

¹Department of Applied Mathematics, Andhra University, Visakhapatnam-530003, India

E-mail: vinuthatummala@gmail.com, rajpydah@gmail.com,
satyanarayana075106@gmail.com

Abstract. In this paper we have investigated the non-static plane symmetric model in the presence of dark energy and one dimensional cosmic string in the frame work of Barber's second self creation theory by using hybrid scale factor. We have discussed some physical and geometrical properties of the obtained model such as spatial volume(V), deceleration parameter (q), expansion scalar(θ), Hubble parameter (H), shear scalar (σ), anisotropic parameter (A_h), EoS parameter (ω_{de}) and the state finder parameters (r, s). Here we have observed that our model is compatible with the present day cosmological observations.

1. Introduction

Recent observations of cosmology has indicated that our universe is currently experiencing a phase of accelerated expansion. Initially type Ia supernovae(SNe Ia) provide an evidence of this expansion (Riess et al. [1], Perlmutter et al. [2], Spergel et al. [3, 4], Bennett et al. [5], Wood-Vasey et al. [6], Tegmark et al. [7], Abazajian et al. [8]) and confirmed later by the cross checks from the Large Scale structures(LSS) [7]- [11] and Cosmic Microwave Background(CMB) [3]- [5]. At present, there is a considerable interest to explain the consequences of accelerated expansion of our universe through the dark energy(DE) cosmological models. Dark energy is composed of large negative pressure for matter distribution and yet the nature of the dark energy and dark matter is unknown. Astronomical observations reveal that our universe currently containing 70% of dark energy and 30% in the form of relativistic matter which includes both baryons and dark matter. Different dark energy models are distinguished by concentrating on the value of equation of state(EoS) parameter $\omega_{de} = \frac{p_{de}}{\rho_{de}}$ (p_{de} = pressure of dark energy and ρ_{de} = Density of dark energy) which is not necessarily constant. The cosmological constant(Λ) is one of the candidate of dark energy which is mathematically equal to vacuum energy($\omega_{de} = -1$) [12] [13]. In making contrast between constant and the variable ω_{de} , usually equation of state parameter considered as constant due to lack of observational evidence and ω_{de} has phase values -1, 0, $\frac{1}{3}$ for vacuum, dust and radiation dominated matter universe respectively [14] [15]. In general ω_{de} is a function of cosmic time or redshift [16] [17].

In recent years, many researchers have been inspired by the study of cosmological models with cosmic strings within the framework of general relativity and in alternative theories of gravitation. Letelier [18] has discussed the geometric strings with particles attached along their



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](#). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

extension and their gravitational effects. Subsequently, Letelier [19] discussed Bianchi type-I and Kantowski-Sachs geometric strings in general relativity. Later, Królik et al. [20] and Wang [21] have discussed the Bianchi type-II, VIII and IX string cosmological models in general relativity. Bali and Pradhan [22] have obtained various Bianchi type-I string cosmological models in general relativity. Many authors in the literature have studied various string cosmological models in alternative theories of gravitation [23]- [27].

The theory of Einstein's general relativity (GR) was only successful in describing the universe gravitational phenomena, but it failed to explain the mach's principle. In recent years there have been some interesting attempts to generalize the general theory of relativity by incorporating certain desired features which are lacking in the original theory. Barber(1982) [28] introduced two self creation theories in which the first theory is the modification of Brans-Dicke [29] theory and the second theory is a modification of general relativity to a variable G theory which includes the continuous creation with in the limits. In second self creation theory the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. Anisotropic non-static plane symmetric cosmological models of the universe have some interesting applications in cosmology. Various matter distributions in plane symmetric space-times have been discussed in GR owing to possible applications to astrophysics and cosmology. Venkateswarlu and Reddy [30], Vinutha and Rao [31], Katore et al. [32] are some of the authors who have worked in this theory.

Inspired by the above mentioned works in the present paper we have obtained a non-static plane symmetric dark energy string cosmological model with in the frame work of Barber's second Self-Creation theory. Here in the present value of deceleration parameter is calculated. We present the analysis of some cosmological parameters such as deceleration parameter, equation of state parameter, $r-s$ plane analysis and $\omega_{de}-\omega'_{de}$ analysis with respect to redshift(z). Cosmic strings are assigned along z-direction considered to comprise anisotropic effect including with dark energy fluid [33].

This paper is arranged as follows: In section 2, we formulated the basic field equations and the solutions of the field equations of the model. In section 3, we discuss some important properties like expansion scalar(θ), shear scalar(σ^2), average anisotropic parameter(\mathcal{A}_h) etc,. In section 4, we summarize the results of the obtained model.

2. Basic Field Equations and Solutions:

2.1. Basic Field Equations:

The field equations of Barber's second self creation theory are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi T_{ij}}{\phi}, \quad (1)$$

$$\phi'_{;k} = \frac{8\pi}{3}\eta(T + \bar{T}), \quad (2)$$

where R_{ij} is the Ricci tensor, R is the scalar curvature, g_{ij} is the metric tensor, η is coupling constant, T is the trace of the energy momentum tensor of matter, \bar{T} is the trace of the energy momentum tensor of dark energy fluid containing one dimensional string and semi colon represents the covariant differentiation of scalar function ϕ .

The energy momentum tensor of model is given as

$$T_{ij} = T_{ij}^{de} + T_{ij}^{\lambda}, \quad (3)$$

where T_{ij}^{de} is the stress energy tensor of the dark energy which is given as

$$T_{ij}^{de} = (\rho_{de} + p_{de})u_i u_j - p_{de}g_{ij}, \quad (4)$$

where p_{de} and ρ_{de} are pressure and density of the dark energy respectively. The energy momentum tensor for fluid containing one dimensional cosmic string is given by Mishra et al. (2017), Letelier (1980) and Stachel (1980)

$$T_{ij}^\lambda = (\rho_m + p_m)u_i u_j - p_m g_{ij} - \lambda x_i x_j, \quad (5)$$

here $u_i u_j = 1, x_i x_j = -1$ (along z-direction). In the co-moving coordinate system u^i is the four velocity vector and p_m is the isotropic pressure of the fluid. ρ_m is energy density due to massive particle and string tension density λ . In the absence of any string phase, the total contribution to the baryonic energy density comes from particles only. In contrast to isotropic pressure of usual cosmic fluid, we wish to incorporate some degree of anisotropy in the dark energy pressure.

Now we consider the non-static plane symmetric metric of the form

$$ds^2 = e^{2A} [dt^2 - dr^2 - r^2 d\theta^2 - B^2 dz^2], \quad (6)$$

where A and B are the metric potentials and functions of cosmic time 't' only. The field equations (1), (2), using (3)-(5) for the metric (6) can be written as

$$e^{-2A} [2\ddot{A} + \frac{\ddot{B}}{B} + \dot{A}^2 + 2\frac{\dot{A}\dot{B}}{B}] = \frac{8\pi}{\phi} (-p_{de} - p_m), \quad (7)$$

$$e^{-2A} [2\ddot{A} + \dot{A}^2] = \frac{8\pi}{\phi} (-p_{de} - p_m + \lambda), \quad (8)$$

$$e^{-2A} [3\dot{A}^2 + 2\frac{\dot{A}\dot{B}}{B}] = \frac{8\pi}{\phi} (\rho_{de} + \rho_m), \quad (9)$$

$$\ddot{\phi} + \dot{\phi} [4\dot{A} + \frac{\dot{B}}{B}] = \frac{8\pi}{3} \eta (T + \bar{T}). \quad (10)$$

2.2. Solutions of the field equations:

From the field equations (7) to (10) we have four independent equations with eight unknowns $A, B, \rho_m, \rho_{de}, p_{de}, p_m, \phi$ and λ . In order to find a deterministic solution we take the following four physically valid conditions.

(i) We consider the cosmological scale factor as a hybrid expansion law(Saha et al. [35])

$$a(t) = (t^n e^t)^{\frac{1}{m}}. \quad (11)$$

where $m > 0$ and $n > 0$ are constants.

(ii) The sum of the trace of the energy momentum tensor is equal to zero

$$i.e., \quad (T + \bar{T}) = 0. \quad (12)$$

(iii) We take the shear scalar σ in the model to be proportional to the expansion scalar θ , this condition leads to(Collins et al. [36])

$$e^A = B^l, \quad (13)$$

where $l \neq 1$ is an arbitrary constant, which preserves the anisotropic nature of the model. Using equation (12) we get,

$$\dot{A} = l \frac{\dot{B}}{B}. \quad (14)$$

The mean Hubble parameter H is given as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (15)$$

where $H_1 = H_2 = 2\dot{A}$ and $H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of r , θ and z respectively.

The Hubble parameter H is given by,

$$H = \frac{\dot{a}}{a}. \quad (16)$$

From equations (11) and (16) the Hubble parameter H is obtained as

$$H = \frac{n+t}{mt}, \quad (17)$$

then from equations (14), (15) and (16), we get

$$A = \frac{3l}{m(4l+1)} \log(t^n e^t), \quad (18)$$

$$B = (t^n e^t)^{\frac{3}{m(4l+1)}}. \quad (19)$$

Now the metric in equation(6) with the help of equations (18) and (19) can be written as

$$ds^2 = (t^n e^t)^{\frac{6l}{m(4l+1)}} \left[dt^2 - dr^2 - r^2 d\theta^2 - (t^n e^t)^{\frac{6}{m(4l+1)}} dz^2 \right]. \quad (20)$$

From equations (10), (14), (18) and (19) we obtain the scalar field ϕ as

$$\phi = \frac{-c_1 mt(4l+1)}{12l(n+t)} (t^n e^t)^{\frac{-3}{m}} + c_2, \quad (21)$$

where c_1 and c_2 are constants of integration.

From equations (7), (8), (18) and (19), we get the string tension density λ as

$$\lambda = \frac{-7 \left[(nm - \frac{3(n+t)^2}{2})l + \frac{nm}{4} - \frac{3(n+t)^2}{4} \right] \left[c_1 mt(l + \frac{1}{4})(t^n e^t)^{\frac{-18l-3}{4lm+m}} - 3lc_2(t^n e^t)^{\frac{-6l}{4lm+m}} (n+t) \right]}{44m^2(4l+1)^2t^2(n+t)l}. \quad (22)$$

We consider ,

$$\lambda = \alpha \rho_m, \quad p_m = \omega \rho_m, \quad (23)$$

where α and ω are assumed to be non evolving state parameters.

The proper density(ρ_m) from (22) and (23) is given by,

$$\left. \begin{aligned} \rho_m &= \frac{-7}{44m^2(4l+1)^2t^2(n+t)l\alpha} \left[(nm - \frac{3(n+t)^2}{2})l + \frac{nm}{4} - \frac{3(n+t)^2}{4} \right] \\ &\times \left[c_1 mt(l + \frac{1}{4})(t^n e^t)^{\frac{-18l-3}{4lm+m}} - 3lc_2(t^n e^t)^{\frac{-6l}{4lm+m}} (n+t) \right] \end{aligned} \right\} \quad (24)$$

From equations (22), (23) and (24), the isotropic pressure of the fluid p_m is given by,

$$\left. \begin{aligned} p_m &= \frac{-7\omega}{44m^2(4l+1)^2t^2(n+t)l\alpha} \left[(nm - \frac{3(n+t)^2}{2})l + \frac{nm}{4} - \frac{3(n+t)^2}{4} \right] \\ &\times \left[c_1 mt(l + \frac{1}{4})(t^n e^t)^{\frac{-18l-3}{4lm+m}} - 3lc_2(t^n e^t)^{\frac{-6l}{4lm+m}} (n+t) \right] \end{aligned} \right\} \quad (25)$$

From equations (9) and (24), we obtain the density of dark energy as

$$\left. \begin{aligned} \rho_{de} = & \frac{1}{11264l\alpha m^2 t^2 (n+t)(l+\frac{1}{4})^2} \left[504\alpha l^2 c_2 (n+t)^3 (t^n e^t)^{\frac{3-6l}{m(4l+1)}} \right. \\ & - 168\alpha l m t c_1 (n+t)^2 (l+\frac{1}{4}) (t^n e^t)^{\frac{-18l}{m(4l+1)}} \\ & - 252 \left[-3l c_2 (n+t) (t^n e^t)^{\frac{-6l}{m(4l+1)}} + c_1 m t (l+\frac{1}{4}) (t^n e^t)^{\frac{-18l-3}{m(4l+1)}} \right] \\ & \left[\alpha (n+t)^2 l^2 + \left(\frac{2t^2}{3} + \frac{4nt}{3} - \frac{4n(m-\frac{3n}{2})}{9} \right) l + \frac{t^2}{3} \right. \\ & \left. \left. + \frac{2nt}{3} - \frac{n(m-3n)}{9} \right] \right] \end{aligned} \right\} \quad (26)$$

From equations (8) , (22) and (25) we obtain the pressure of dark energy as

$$\left. \begin{aligned} p_{de} = & \frac{-1}{352\alpha m^2 t^2 l (n+t)(l+\frac{1}{4})^2} \left[7 \left(-3l (t^n e^t)^{\frac{-6l}{m(4l+1)}} c_2 (n+t) + c_1 m t (l+\frac{1}{4}) \right. \right. \\ & (t^n e^t)^{\frac{-18l-3}{m(4l+1)}} \left. \right) \left(\left(\frac{-3n^2}{8} + (m-\frac{3t}{4})n - \frac{-3t^2}{8} \right) \alpha l^2 \right. \\ & + \left(\left(\frac{-3\alpha}{4} + \frac{3\omega}{4} \right) n^2 + \left(\left(\frac{-3\alpha}{2} + \frac{3\omega}{2} \right) t + \frac{3(\alpha-\frac{2\omega}{3})m}{4} \right) n \right. \\ & \left. \left. - \frac{3t^2(\alpha-2\omega)}{4} \right) l + \frac{(\alpha-\omega)(3n^2 + (m-6t)n - 3t^2)}{8} \right) \right] \end{aligned} \right\} \quad (27)$$

From equations (26) and (27) we obtain the Equation of state parameter(EoS) of dark energy as

$$\omega_{de} = \frac{p_{de}}{\rho_{de}} = \frac{L}{M} \quad (28)$$

$$\text{where } L = \left(-3l^2\alpha + (-6\alpha + 6\omega)l - 3\alpha + 3\omega \right) n^2 + \\ \left(8\alpha(m - \frac{3t}{4})l^2 + ((6m - 12t)\alpha - 4\omega(m - 3t))l \right. \\ \left. + (\alpha - \omega)(m - 6t) \right) n - 3t^2 \left(l^2\alpha + (2\alpha - 2\omega)l + \alpha - \omega \right)$$

$$\text{and } M = 6l\alpha(n+t)^2(t^n e^t)^{\frac{3}{4lm+m}} + (9l^2\alpha + 6l + 3)n^2 + \\ \left(18l^2t\alpha + (-4m + 12t)l - m + 6t \right) n + \\ 9t^2 \left(l^2\alpha + \frac{2l}{3} + \frac{1}{3} \right)$$

3. Some other important properties of the model:

Using equation (11), the spatial volume of our model is given by,

$$V = a^3 = (t^n e^t)^{\frac{3}{m}}. \quad (29)$$

The expansion scalar θ of the model with equation (17) is

$$\theta = u_{;i}^i = 3H = 3 \left(\frac{n+t}{mt} \right). \quad (30)$$

We observe that the expansion scalar $\theta \rightarrow \infty$ as $t \rightarrow 0$ and this indicates the escalation scenario at early stages of the universe.

The shear scalar σ of our model with equations (15), (18), (19) and (30) is obtained as,

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{3}\sum_{i=1}^3 H_i^2 - \frac{1}{6}\theta^2 = 3\left(\frac{(n+t)^2(2l-1)^2}{m^2t^2(4l+1)^2}\right). \quad (31)$$

The average anisotropic parameter \mathcal{A}_h for our model with equations (15), (17), (18) and (19) is given by

$$\mathcal{A}_h = \frac{1}{3}\sum_{i=1}^3 \frac{(H_i - H)^2}{H^2} = \frac{10(2l-1)^2(n+t)^2}{3m^2t^2(4l+1)^2}. \quad (32)$$

In the discussions of graphical representation of our model, we constraint the constants as $n = 5.5$, $m = 18$, $l = 5.5$, $\alpha = 0.5$, $c_1 = -2.0625$, $c_2 = 1.5$, $\omega = 8.5$ and t is the cosmic time which is measured in billion years.

The deceleration parameter q indicates the rate of slowing down the expansion rate of our universe and for this model it is given as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{nm}{(n+t)^2}. \quad (33)$$

Figure 1 depicts the variation of deceleration parameter(q) with respect to redshift(z) which shows that the positive value of q represents the standard decelerating model and the negative value of q represents the inflationary scenario of the universe (Riess et al. [1]; Bennett et al. [5]). For our model we observed that $q < 0$ for $z < 1.4$ reveals the universe appears to be expanding in accelerating rate at present epoch and late time and $q > 0$ for $z > 1.4$ reveals the model was decelerating at early stage of the universe and also $q = 0$ at $z \approx 1.4$ reveals the transition form of the model.

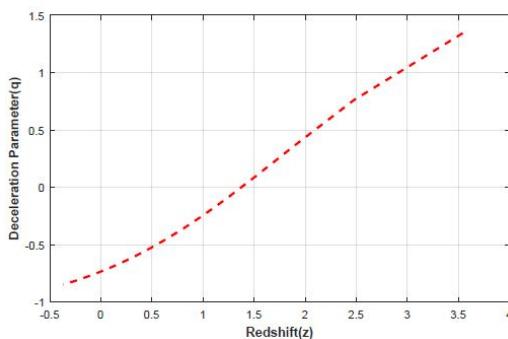


Figure 1. Plot of q versus redshift(z).

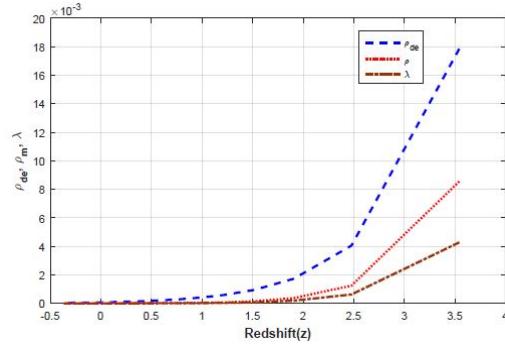
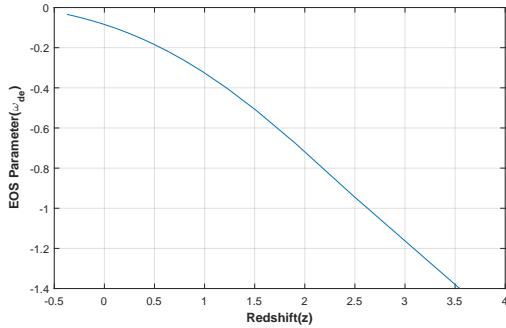
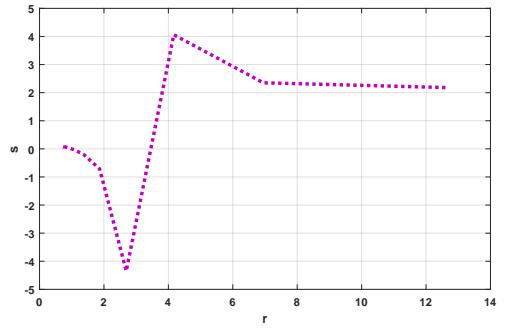


Figure 2. Plot of ρ_{de} , ρ_m and λ versus redshift(z).

The present value of deceleration parameter for the model is $q \approx -0.73$ which lies in the range $-1 \leq q < 0$ (SNe Ia) (Cunha et al [37]). From figure 2, we observed that the dark energy density is an increasing function of redshift(z) and remains positive through out. Also it is clear that the dark energy density dominates both the density of dark matter and string tension density through out the evolution of the universe.

**Figure 3.** Plot of ω_{de} versus redshift(z).**Figure 4.** Plot of r versus s .

3.1. Equation of State Parameter:

From equation (28), it can be seen that the equation of state parameter (ω_{de}) is a function of cosmic time t and in figure3, we plotted ω_{de} against redshift(z) and it crosses the phantom divide line ($\omega_{de} = -1$). So ω_{de} varying from quintessence to phantom and it has quintom like behavior.

3.2. State Finder Parameters:

The state finder parameters are constructed from a space time metric directly which are to analyse in depth the geometrical behavior of the universe. It is more homogeneous than the physical quantities which are describing the dark energy because the quantities of the model are dependent. Thus the higher order derivatives with respect to cosmic time of the scale factor $a(t)$ gives a better study of dark energy models [38] [39]. We introduced the state finder parameters $\{r, s\}$ which removes the degeneracy of the physical variables such as the Hubble parameter(H) and the deceleration parameter (q) of dark energy model at present epoch of the universe for ($z = 0$). In $r - s$ plane the evolution of universe varies from quintessence to phantom or vice-versa when co-ordinates of the point $\{r, s\} = \{1, 0\}$ is crossed(Wu and Yu 2010) [40]. The state finder parameters are defined as

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q - \frac{1}{2})} \quad (34)$$

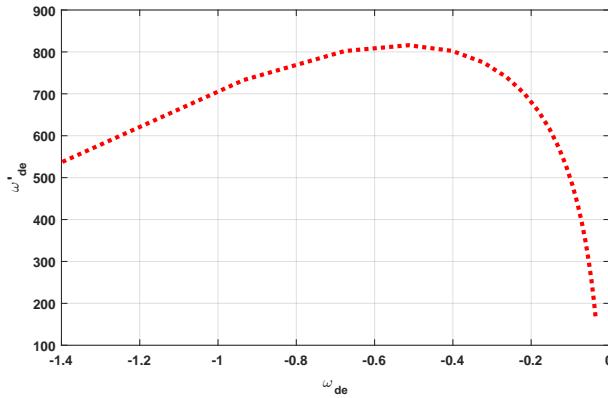


Figure 5. Plot of ω_{de} versus ω'_{de} .

Using equations (11) and (17) in (34) we get,

$$r = \frac{n^3 + (3t - 3m)n^2 + (2m^2 - 3tm + 3t^2)n + t^2}{(n + t)^3}. \quad (35)$$

From equations (33), (34) and (35) we get,

$$s = \frac{(2m - 3n - 3t)nm}{\left(\frac{-3n^2}{2} + (m - 3t)n - \frac{3t^2}{2}\right)(n + t)}. \quad (36)$$

From figure 4, we observed that $r=1$ and $s=0$ at late time and consistent with standard Λ CDM model (Huang et al. [41]).

3.3. $\omega_{de} - \omega'_{de}$ Plane Analysis:

The $\omega_{de} - \omega'_{de}$ plane analysis plays a vital role in present cosmological analysis (Caldwell and Linder [42]). Through the trajectories on its plane we can differentiate different dark energy

models. Initially, this method has been applied on quintessence DE model which leads to two classes of its plane the one with $\omega_{de} < 0$ and $\omega'_{de} < 0$ is called freezing region and the other with the property $\omega_{de} < 0$ and $\omega'_{de} > 0$ is known as thawing region.

We have

$$\omega'_{de} = \frac{\dot{p}_{de}\rho_{de} - p_{de}\dot{\rho}_{de}}{H\rho_{de}^2} = \frac{N}{D}, \quad (37)$$

where

$$\begin{aligned} N = & (-8l(tn(\frac{3l^2\alpha}{2} + (((t^ne^t)^{\frac{3}{4l^m+m}})\alpha - \frac{3\omega}{4} + \frac{3}{4})l + \frac{(t^ne^t)^{\frac{3}{4l^m+m}}\alpha}{2} - \frac{(t^ne^t)^{\frac{3}{4l^m+m}}\omega}{2} \\ & + \frac{1}{2})(l + \frac{1}{4})^2m^2 + \frac{3(n+t)^2(t^ne^t)^{\frac{3}{4l^m+m}}(l\alpha + \frac{\alpha}{2} - \frac{\omega}{2})n(l + \frac{1}{4})m}{8} \\ & - \frac{9(n+t)^4(t^ne^t)^{\frac{3}{4l^m+m}}(l^2\alpha + (2\alpha - 2\omega)l + \alpha - \omega)}{64})\alpha \end{aligned} \quad (38)$$

and

$$D = (3(\frac{-2n(l + \frac{1}{4})m}{3} + (n+t)^2(\frac{1}{2} + \frac{3l^2\alpha}{2} + ((t^ne^t)^{\frac{3}{4l^m+m}}\alpha + 1)l))(l + \frac{1}{4})). \quad (39)$$

From figure5, we observed that non-static plane symmetric dark energy model lies in thawing region.

4. Conclusion:

The present scenario of accelerated expansion of universe yet to be an open problem in modern cosmology. In this paper we have studied the non-static plane symmetric dark energy string cosmological model in Barber's second self creation theory of gravitation. The time dependent deceleration parameter (q) is positive at early age of the universe and becomes negative at present and late time, showing the model evolves from early decelerating phase to late time accelerating phase. We have found that the present value of deceleration parameter as $q_0 \approx -0.73$, which coincides with the observed value. Also we have observed that the dark energy density (ρ_{de}) is an increasing function with respect to redshift(z) and dominates both the proper density(ρ) of the matter and string tension density(λ) through out the evolution of the universe in the case of non-static plane symmetric cosmological model. The EoS parameter (ω_{de}) for the model crosses the phantom divide line $\omega_{de} = -1$, thus it has quintom-like behavior. We have observed that the values of state finder pair becomes $r = 1, s = 0$ at late time and consistent with standard Λ CDM model. From ω_{de} - ω'_{de} plane analysis it is clear that the non-static plane symmetric model lies in thawing region. The model obtained and presented here represents accelerating and expanding cosmological model of the universe.

- [1] Riess, A.G., et al.: *Astron. J.* **116**, 1009 (1998).
- [2] Perlmutter, S., et al.: *Astrophys. J.* **517**, 565 (1999).
- [3] Spergel, D.N., et al.: *Astrophys. J. Suppl.* **170**, 377 (2007).
- [4] Spergel, D.N.: *Astrophys. J. Suppl.* **148**, 175 (2003).
- [5] Bennett, C.L., et al.: *Astrophys. J. Suppl.* **148**, 1 (2003).
- [6] Wood-Vasey et al.: *Astrophys. J.*, **666**(2), 694715 (2007).
- [7] Tegmark, M., et al.: *Phys. Rev. D* **69**, 103501 (2004).
- [8] Abazajian K., et al.: *Astron. J.* **126**, 2081 (2003).
- [9] Abazajian K., et al.: *Astronom. J.*, **128**, 502 (2004).
- [10] Abazajian K., et al.: *Phys. Rev. D*, **69**, 103501 (2004).
- [11] Hawkins E. et al.: *Mon. Notices of the Royal Astronom. Soc.*, **346**(1), 7896 (2003).
- [12] V.Sahni, A.Shafielooa, and A.Starobinsky, : *Phys. Rev. D* **78**, 103502 (2008).
- [13] Vinutha .T, et al.: *Astrophys Space Sci* **363**:188(2018)
- [14] Bartelmann, M., et al.: *New Astron. Rev.* **49**, 199 (2005).

- [15] Kujat, J., et al.: *Astrophys. J.* **572**, 1 (2002).
- [16] Das, A., Gupta, S., Saini, T. D., Kar, S. : *Phys. Rev. D* **72**, 043528 (2005).
- [17] Jimenez, R.: *New Astron. Rev.* **47**, 761 (2003).
- [18] P. S. Letelier,: *Phys. Rev. D* **20** 12941302 (1979).
- [19] Letelier, P.S.: *Phys. Rev. D* **28** , 2414 (1980).
- [20] K. D. Krori, T. Chaudhury, C. R. Mahanta (Gauhati U.) and A. Mazumdar,: *Gen. Relativ. Gravit.* **22** 123130 (1990).
- [21] X. X. Wang, Chin.: *Phys. Lett.* **20** 615 617 (2003).
- [22] R. Bali and A. Pradhan, Chin.: *Phys. Lett.* **24** 585 (2007).
- [23] D. R. K. Reddy and M. V. S. Rao,: *Astrophys. Space Sci.* **302** 157160 (2006).
- [24] D. R. K. Reddy,: *Astrophys. Space Sci.* **286**, 397400 (2003).
- [25] T. Vinutha, et al.: *Astrophys. Space Sci.* **363** 188 (2018).
- [26] Rao V. U. M., M. Vijaya Santhi and T. Vinutha,: *Astrophys. Space Sci.* **317** 83 (2008).
- [27] Rao V.U.M., T. Vinutha, K.V.S. Sireesha,: *Astrophys. Space Sci.* **323** 83 (2009).
- [28] Barber, G.A.: *Gen. Relativ. Gravit.* **14**, 117 (1982).
- [29] Brans, C.H., and Dicke, R.H.: *Phys. Rev.* **124**, 925 (1961).
- [30] R.Venkateswarlu and D.R.K.Reddy,: *Astrophys. Space Sci.***68**(1990).
- [31] Vinutha T, V.U.M. Rao,: *Astrophys. Space Sci.***325** (2010).
- [32] Katore S.D , A.Y.Shaika and S.A.Bhaskar,: *Sci. Revs. Chem. Commun.***2(3)** 463-468 (2012).
- [33] Mishra B. Chin,: *Phys Lett* **21**:2359 (2004).
- [34] Stachel, J.: *Phys. Rev D* **21** 217 (1980).
- [35] Saha, B., Amirhashchi, H., Pradhan A. *Astrophys. Space Sci.* **342**, 257 (2012).
- [36] Collins, C.B., Glass, E.N., Wilkinson, D.A.: *Gen. Relativ. Gravit.* **12**, (1980).
- [37] Cunha, C. E., Lima, M., Ogaizu, H., Frieman, J., Lin, H.: *Mon. Not. Roy. Astron. Soc.* **396**, (2009).
- [38] Sahni, V., Saini, T. D., Starobinsky, A. A., and Alam, U.: *JETP Letters* **77**, (2003).
- [39] Alam, U., Sahni, V., Saini, T.D.: *Mon. Not. R. Astron. Soc.* **344**, 1057-1074 (2003).
- [40] Wu, P., Yu, H.: *Phys Lett. B* **693**, (2010).
- [41] Huang, Z.G., Song, X.M., Lu, H.Q., Fang, W.: *Astrophys. Space Sci.* **315**, (2008).
- [42] Caldwell, R.R. and Linder, E.V.: *Phys. Rev. Lett. B* **95**, (2005).