More on beta-deformed matrix model

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Despite the importance of M-theory, its formulation is still not established. The best candidate, the matrix model, has unsolved fundamental problems such as the $N \to \infty$ limit. The interpretation of the matrix model as the regularised supermembrane theory provide a promising approach to them. However, one seems to be still far from the resolution of them, and needs better understanding of the physics of the matrix model and membranes.

Here we report two properties of the stable solutions of a deformation of the matrix model proposed by the author. The original potential term, $-\frac{1}{4}\mathrm{Tr}[X^\alpha, X^\beta]^2$, is deformed according to the rules, $[Z, W] \to = e^{i\pi\beta}ZW - e^{-i\pi\beta}WZ$, $[Z, W^\dagger] \to = e^{-i\pi\beta}ZW^\dagger - e^{i\pi\beta}W^\dagger Z$, where X^α ($\alpha = 1, \ldots, 9$) are $N \times N$ matrices and $Z = X^1 + iX^2$, $W = X^3 + iX^4$. This form is motivated by an analogy to a deformation of $D = 4, \mathcal{N} = 4$ super-Yang-Mills preserving the conformal symmetry. The deformed model admits an eleven-dimensional interpretation: it is a regularised supermembrane theory on a supergravity background, which is of the pp-wave type with a certain non-constant four-form flux.

If the deformed commutator vanishes for a configuration, it is automatically a (marginally) stable solution, due to the positivity of the potential. These solutions exist when βN is an integer, and are written by the so-called shift, clock matrices $h_1 h_2 = e^{-i2\pi/N} h_2 h_1$. In the simplest case, $\beta N = 1$, the solution is $Z = ah_1, W = bh_2$, with arbitrary parameters a, b. From known results about the regularisation, it follows that the matrices correspond to a membrane with torus topology, $z = a' e^{i\sigma^1}, w = b' e^{i\sigma^2}$, where σ^1, σ^2 (ranging from 0 to 2π) parametrise the torus.

An interesting phenomenon occurs for general βN. We focus on the case where $\beta N = 2$ and N is odd. The vanishing of the deformed commutator is achieved by (a) $Z = ah_1^2, W = bh_2$ and (b) $Z = ah_1, W = bh_2$. They respectively correspond to membranes (a) $z = a' e^{i2\sigma^1}, w = b' e^{i\sigma^2}$ and (b) $z = a' e^{i\sigma^1}, w = b' e^{i2\sigma^2}$. They are physically distinguishable as they have different winding numbers. We found, however, that the matrix model soultions are related by an unitary transformation and therefore should be considered physically equivalent. An analogy to D-branes suggests that this remarkable degeneracy stemms from non-Abelian nature of membranes.

Second interesting property concerns the generalised backgrounds where the four-form flux is given by the sum of a constant term and the above-mentioned non-constant term. For constant flux, it is known that the stable solutions are those corresponding to spherical membranes. We have found a class of deformed models associated with the generalised backgrounds such that one can explicitly constuct stable solutions which corresponds to membranes interpolating the sphere and the torus.