A comparative study of multifractal scaling in ^{28}Si-AgBr collisions

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Introduction

The concept of intermittency is in connection to the fractal geometry of the underlying physical process. Fractal geometry allows us to describe a system that is intrinsically irregular at all scales. A fractal structure has the property that if one magnifies a small portion of it that shows the same complexity as the system. The idea, therefore, is to construct a formalism that can describe systems with local properties of self-similarity. It has been suggested that the fluctuation in rapidity has a nontrivial multifractal structure. R.C. Hwa [1] was the first to provide mathematical formalism based on G_q moments for investigating the cascading mechanism in multiparticle production in the framework of the multifractal technique. This technique verifies the authenticity of basic scaling properties, which occur in multifractal theories when applied to particle production for the dynamical mechanism responsible for the hadronization process in nucleus-nucleus (AA) collisions. The explanation of cascading as a self-similar process in analogy with geometrical objects [5] such as fractals has been allowed by scaling law. In the present work, we have used the method of G_q moment to study the various interesting features of multifractality in $^{28}Si-AgBr$ collisions at 14.5A GeV/c. We have also generated HIJING events of similar characteristics and analyzed the data in order to compare the results of the study.

Theory

To examine the multifractal moments G_q, dependence on pseudorapidity, η, which is defined as; η = - $ln(tan\theta/2)$, where θ is the space the angle of a secondary particle with the mean direction of the primary, a given pseudorapidity range $\Delta\eta = \eta_{max} - \eta_{min}$ is divided into M bins of width $M = \Delta\eta/\delta\eta$. A multifractal average moment is defined as;

$$\langle G_q \rangle = \frac{1}{N} \sum_N \sum_{j=1}^{M} p_j^q, \qquad (1)$$

where $p_j^q = n_j/n$, such that $n = n_1 + n_2 + n_3 + ... + n_M$. M denotes the number of non-empty bins. q is a real number and may have both positive and negative values. If there is self-similarity in the production of particles, G_q moments can be written in the form of a power law;

$$\langle G_q \rangle = (\delta\eta)^{\tau_q} \text{ for } (\delta\eta) \rightarrow 0 \qquad (2)$$

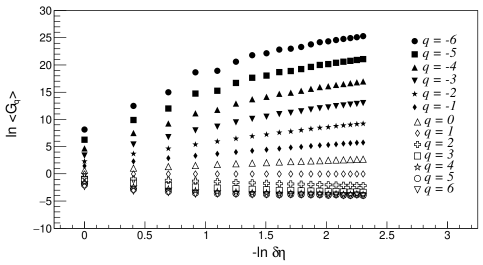
where τ_q stands for the mass exponent. The generalized dimensions may be defined as;

$$D_q = \frac{\tau_q}{q-1} \qquad (3)$$

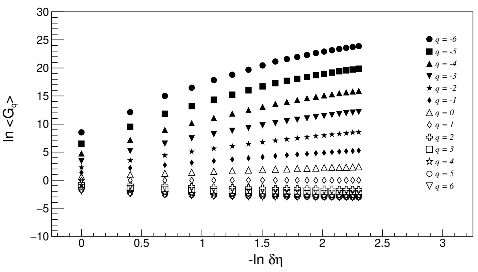
If we put q = 0, 1, 2 in equation 3, then we have
(1) D_0; Capacity dimension.
(2) D_1; Entropy dimension.
(3) D_2; Correlation dimension.

Results and discussions

In figure 1(a,b) we have plotted $ln\langle G_q \rangle$ versus $-ln\delta\eta$. From the figure, we notice that $ln\langle G_q \rangle$ increases linearly with decreasing bin width. This shows the presence of power-law behavior in considered collisions. The generalized dimension, D_q, is calculated by using the mass exponent, τ_q. The variation of D_q as a function of q is plotted in fig. 2. From the figure it may be pointed out that the generalized dimension, D_q, decreases with increasing the order of the moment, q. This exhibits the presence of multifractality in the considered sets of events. The observed results are nicely reproduced by the HIJING simulated events.



(a)



(b)

FIG. 1: Variations of $ln\langle G_q\rangle$ with -$ln\delta\eta$ for (a) HIJING and (b) Experimental data.

Conclusions

The above observations lead us to conclude that $\langle G_q\rangle$ shows a power-law dependence on the pseudo-rapidity bins. Also, the decreasing trend of D_q with an increasing value of q indicates the presence of multifractality in considered data. The observed results of the

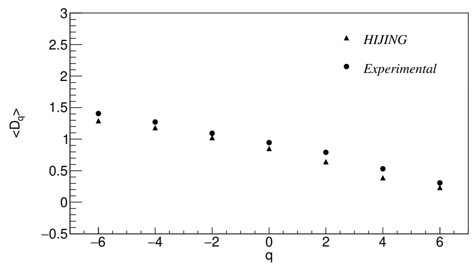


FIG. 2: Variations of $\langle D_q\rangle$ as a function of q in ^{28}Si-AgBr collisions.

present study are found to be in good agreement with those obtained for the simulated HIJING data.

References

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