Explaining mass and spin in the visible matter: the next challenge

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Abstract. Understanding in depth *...the origin of the nucleon mass, the nucleon spin, and the emergent properties of a dense gluon system...* (Electron-ion collider Yellow Report, arXiv:2103.05419v) represents the frontier of modern hadron physics, both on the experimental and theoretical side. In this contribution, after briefly outlining the above challenges in view of the various experimental facilities planned in the near future, some insights into the non-perturbative regime of QCD will be provided. In particular, the most recent achievements in the description of the pion, playing a fundamental role in the hadron dynamics, will be illustrated within a genuinely relativistic quantum-field theoretical framework, based on actual solutions of the homogeneous Bethe-Salpeter equation.

1. Introduction

Quantum Chromodynamics (QCD) is the recognized, successful non-Abelian gauge theory of the strong interaction in the Standard Model ($SU(3)_C \times SU(2)_L \times U(1)_Y$), and hence it is the basis of any reliable investigations of the hadron dynamics. It has fermionic and bosonic degrees of freedom: six quarks and eight gluons, that mediate the color interaction. The QCD Lagrangian has a structure similar to the QED one, namely it is given by the following expression

$$\mathcal{L} = \bar{\psi}_f(x) \; \Big[i\not{D} - M_f \Big] \psi_f(x) - \frac{1}{2} Tr \Big\{ G_{\mu\nu}(x) G_{\mu\nu}(x) \Big\} \tag{1}$$

where i) the covariant derivative is given by

$$D_\mu = \partial_\mu - ig \frac{\lambda_a}{2} A_\mu(x) \quad , \tag{2}$$

with $A_\mu(x)$ the linear combination of Gell-Mann matrices λ_a and gauge fields $A^a_\mu(x)$ (gluons)

$$A_\mu(x) = \sum_{a=1}^{8} \frac{\lambda_i}{2} \; A^a_\mu(x) \quad ; \tag{3}$$

ii) $\psi_f(x)$ is a column vector containing the quark fields with different flavors: $u(x)$, $d(x)$, $s(x)$, $c(x)$, $b(x)$ and $t(x)$; iii) the matrix M_f is a diagonal one, with the elements along the diagonal given by the current quark masses, generated by the coupling with the Higgs-boson; and iv) the gluon-field tensor reads

$$G_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x), A_\nu(x)] \quad . \tag{4}$$

In spite of its formal simplicity, the non-Abelian nature of QCD generates a high degree of non linearity, since the vector-bosons mediating the interaction are charged (each gluon has a color and an anti-color charge) unlike the case of the photon, and therefore interact with each other, already at the leading order.

The striking non linear behavior of QCD leads to fundamental questions that are still open, and just to mention a few, the following can be listed (see, e.g., the Electron Ion Collider Yellow Report [1] for a wide review, and also the proposal for an Electron-ion collider in China [2]):

- how the nucleon spin can be fully accounted for in terms of the inner degrees of freedom (quarks and gluons);
- how the mass of the proton emerges from the highly non linear dynamics of QCD, even with tiny current quark masses in the light sector;
- how the pion (and the pseudoscalar octet) shares the double nature of Nambu-Goldstone boson and eigenstate of a $SU(3)_F$ multiplet, in coherence with the mechanism generating the proton mass;
- how partons are distributed inside hadrons, both in momentum and coordinate spaces;
- how both color confinement and hadronization are established, from the dynamical point of view;
- how the quark-gluon interactions generate the nuclear forces.

These questions and many others have motivated a huge number of experimental efforts, as well as plans for future facilities. In particular, the facility at the most advanced stage is the Electron-ion collider (EIC) to be built at Brookhaven Natl. Lab. [1], but one should also mention an analogous project in China [2]. The BNL Electron-ion collider is an innovative, large-scale particle accelerator, with the capability of colliding beams of polarized electrons with polarized beams of light ions (and also unpolarizec heavy ions), at high intensities. For the sake of concreteness, the main design requirements of the EIC@BNL, are the following

- Highly polarized electron ($\sim 70\%$) and proton ($\sim 70\%$) beams.
- Ion beams from deuteron to heavy nuclei such as Gold, Lead, or Uranium.
- Variable e+p center-of-mass energies from $20 - 100$ GeV, upgradable to 140 GeV (notably $Q^2 \leq 10^4$ Gev^2, where Q^2 is the square of the momentum transfer between the electron and proton/nuclei and is inversely proportional to the resolution).
- High collision electron-nucleon luminosity $10^{33} - 10^{34}$ cm^{-2}s^{-1}.

2. Understanding spin and mass of the nucleon: a challenge

Let us briefly illustrate the questions still open regarding the spin degree of freedom. In particular at the end of the 80's, a major surprise came from the measurements of the European Muon Collab. (EMC), that measured the proton spin-dependent structure function, $g_1(x)$, down to the Bjorken variable x = 0.01 [3]. From this data, it seemed that spin of the nucleon could not be explained in terms of only quark degrees of freedom, leading to what was popularly known as the *spin crisis*. In particular the nucleon spin can be decomposed into three contributions as follows (see, e.g., Refs. [4, 5, 6, 7, 8] for an introduction)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\zeta) + \Delta G(\zeta) + \Delta\mathcal{L}(\zeta) \tag{5}$$

where $\Delta\Sigma(\zeta)$ is the quark spin contribution, from both valence and sea quarks, $\Delta G(\zeta)$ the gluonic term and $\Delta\mathcal{L}(\zeta)$ the orbital-angular momentum contribution of both quark and gluon. The variable ζ is the renormalization scale.

In the last three decades, understanding the nucleon spin in terms of the above decomposition (see Ref. [9] for the seminal analysis of the gauge-invariant decomposition of the nucleon spin) has been a compelling puzzle for nuclear physicists, and it is still a key issue that attracts interest and efforts from both experimental and theoretical side (see below).

The first attempt to experimentally measure the partonic contribution to the nucleon spin, performed by the European Muon Collab. [3], yielded a quark contribution $\Delta\Sigma(\zeta)$ almost vanishing. Further experimental efforts, carried out at CERN, DESY, JLab, RHIC, and SLAC, have improved our knowledge revealing that only about $25\% - 30\%$ of the nucleon spin budget stems from the quarks. Presently, from the RHIC spin program, we argue that the gluon contribution, $\Delta G(\zeta)$, is not vanishing [10]. Unfortunately, large uncertainties affect the two contributions $\Delta\Sigma(\zeta)$ and $\Delta G(\zeta)$, and not to mention that very little is known about the orbital angular-momentum content $\Delta\mathcal{L}(\zeta)$. It is important to recall that EIC@BNL has the potentiality to provide significantly more inputs to constrain $\Delta G(\zeta)$ and to reduce the error bars of $\Delta\Sigma(\zeta)$, as well as to start a wide experimental campaign for extracting information on $\Delta\mathcal{L}(\zeta)$. However, it should be pointed out that in order to take under control the model dependence in extracting the above mentioned quantities from the data it is necessary to undertake non trivial theoretical efforts. In view of this, it is useful to mention recent, high-precision lattice QCD (LQCD) calculations [11], at the physical pion mass. The Collaboration was able to determine the gluon and quark contributions to the spin of the proton, obtaining i) the total quark spin contribution $\Delta\Sigma(\zeta) = 0.191 \pm 0.015$, ii) the quark orbital-angular momentum $\Delta\mathcal{L}_q(\zeta) = 0.094 \pm 0.052$ and the total gluon contribution $\Delta G(\zeta) + \Delta\mathcal{L}_g(\zeta) = 0.187 \pm 0.048$. Hence, with those values the proton spin amounts to 0.473 ± 0.072.

To gather more information about the poorly-known orbital angular moments of the partons, it is crucial to be able to implement a 3D map of the partonic distributions in both coordinate and momentum spaces. To achieve the goal of a 3D imaging of the nucleon, beside the electromagnetic form factors, it is more helpful to exploit the so-called *Generalized Parton Distributions* (GPDs), measured via deeply-virtual Compton scattering (see, e.g., Refs. [12, 13] for a general introduction). The GPDs describe the correlations in a hadron between the (light-cone)longitudinal momentum of quarks and gluons and their position in the *transverse spatial plane*, also providing a connection between ordinary parton-distribution functions and electromagnetic form factors.

Another source of information on 3D imaging, but in *momentum space*, is the measurements of the *Transverse Momentum Dependent parton distributions* (TMDs) (see, e.g., Ref. [14] for a primer), through the semi-inclusive deep-inelastic scattering. The TMDs involve the correlation between the spin degrees of freedom, the longitudinal and transverse momenta of the partons, and naturally lead to a 3D parton structure of hadrons. .

3. Where does the mass of the nucleon come from?

Answering to this apparently naive, simple question has an amazing impact, since more than 99% of the mass of the visible universe is made up of protons and neutrons. In spite of the simplicity, it is a paramount challenge to find a complete answer, since one needs to face with the highly non linear behavior of QCD, in full.

The nucleon mass is not even approximately given by summing up the quark current masses, that are generated by the coupling with the Higgs boson ($2m_u + m_d \sim 10\ MeV$). Such an observation immediately points to the active role of gluons, that manifests itself particularly in the so-called *trace anomaly* of the QCD energy-momentum tensor. To give just an idea of the issue, let us consider the chiral limit of the QCD Lagrangian given in Eq. (1), i.e. putting the quark current masses equal to zero, viz

$$\mathcal{L}^{\chi} = \bar{\psi}_f(x)\ i\not{D}\ \psi_f(x) - \frac{1}{2} Tr\Big\{G_{\mu\nu}(x) G_{\mu\nu}(x)\Big\} \quad . \tag{6}$$

Remarkably, in the chiral limit QCD becomes scale-invariant, Such an approximation is particularly valid in the light-quark sector, namely, inter alia, for pion and nucleon where the u and d quarks generate the main flavor content. To understand the implication of this invariance, let us first recall what happens in the classical field theory. The invariance for the global space-time translations, i.e. $x'_\mu = x_\mu + \epsilon_\mu$, entails the conservation of the (necessarily) symmetric energy-momentum tensor $T^{\mu\nu} = P^\mu P^\nu$, i.e. $\partial_\mu T^{\mu\nu} = 0$, via the Noether theorem (see, e.g., Ref. [15] for a wide discussion). In addition, the invariance for scale transformations (dilatations), i.e. $x'_\mu = \lambda x_\mu$, leads to require the following constraint on the dilatation or scaling current $\mathcal{D}^\mu = T^{\mu\nu} x_\nu$, viz.

$$\partial_\mu \mathcal{D}^\mu = 0 = T^\mu_\mu \quad .$$

Inserting the definition of $T^{\mu\nu}$ one gets the result that a scale-invariant Lagrangian does not generate masses, since $T^\mu_\mu = M^2 = 0$. Then the question arises: how is it possible to establish the dynamics inside the system for generating the mass of the hadrons, when the constituents can be tacitly assumed massless? To be also recalled that the mass of the lightest meson, i.e. the pion, yields the typical nuclear scale $\approx$ 1 fm. Hence, elucidating the mechanism of the emergent hadron mass is crucial for understanding the generation of the mass of the visible matter.

Unlike the classic case, within a quantum-field theory framework the singular behavior at short and large distances of the Green's function, for a given theory, produces divergences, that have to be cured by a quantum-loop renormalization. This automatically introduces a scale. Hence, the scaling current $\mathcal{D}^\mu$ is not more conserved and $T^\mu_\mu \neq 0$, i.e. a trace anomaly occurs [16, 17]. For instance, the dilatation-invariance is spontaneously broken in QCD via the introduction of the constant Λ_{QCD} that imposes a momentum cutoff in the infrared regions. In general, the phenomenon of the spontaneously breaking of the scale-invariance is referred to as *dimensional transmutation* (see Refs. [18, 19, 20] for the case of QCD, as well as Refs. [21, 22] for a pedagogical introduction).

To be specific, in the 90's, X.D. Ji [18, 19] proposed to decompose the mass of the nucleon (as well as of any other hadron) in terms of quark and gluon contributions through the *traceless and tracefull* components of the QCD energy-momentum tensor. In particular, the trace part relevant for the mass reads (see, e.g., Ref. [21] for details)

$$\langle P|\widehat{T}^{\mu\mu}|P\rangle \overset{\zeta>\geq m_N}{=} \langle P|H_m + \underbrace{\gamma_m(\zeta^2)\ H_m + \beta(\zeta^2)\frac{1}{2}Tr\Big\{G^{\mu\nu}G_{\mu\nu}\Big\}}_{anomalous\ term}|P\rangle \qquad (7)$$

where $\langle P|T^\mu_\mu|P\rangle/\langle P|P\rangle = M$, $H_m = \sum_f m_f(\zeta^2)\bar{q}_f\ q_f$ is the quark contribution with q_f the field of the quark with flavor f, $\gamma_m(\zeta^2)$ the anomalous dimension of the quark mass operator and $\beta(\zeta^2)$ the QCD renormalization-group beta-function (recall that at the leading order $\beta = -\beta_0\, g^3/(4\pi)^2$ and $\beta_0 = 11 - 2n_f/3$). Recent LQCD calculations confirm that the trace anomaly is the largest source of the proton mass, $\approx 92\%$ [23, 24]. Hence, the bulk of the nucleon mass originates from the QCD dynamics.

Immediately a corollary, though fundamental, question arises: is the same mechanism also sufficient to explain the small mass of the pion ($\approx$ 1/7 of the nucleon mass), that sets the typical size in nuclear physics? It is well-known that in the chiral limit the pion mass vanishes, and we expect that the trace anomaly should be able to return the result of massive pion. As argued by C. Roberts and collaborators in Ref. [21], the existence of a pion as bound system requests a non trivial role of gluons. The quark-masses and gluon contributions in the trace anomaly have to largely cancel each other in order to lead to a vanishing pion mass, via a balance between the residual mass effects (proportional to $\gamma(\zeta)$) and the attractive gluonic term. Hence, a possible answer could be: the effect of the dynamical chiral-symmetry breaking, that make massive the quarks, is still alive to some extent also in the chiral limit, producing the cancellation with the gluonic term. Indeed, a residual mass is generated through a subtle dynamical combination between the renormalization constant and the mass, i.e. $Z_m\, m \neq 0$, as shown by the preliminary result in χCollab. [25, 24].

In conclusion, the pion has a pivotal role in elucidating fundamental aspects of the QCD, and its investigation, possibly by inferring information from suitable experimental quantities, is highly desirable. In particular, a detailed study of the quark running mass in both nucleon and pion could shed light on the dynamical issue $M_N/m_\pi \sim 7$. Interestingly, it has been shown [26] that in the chiral limit the scalar part of the self-energy of the quark in the pion, given by

$$\Sigma = \not{p}\; A(p,\zeta) + B(p,\zeta) \quad , \tag{8}$$

is related to the main component of the pion-quark vertex function $\Gamma_\pi(k; P_\pi, \eta)$ (with η all the needed quantum numbers)

$$\text{main part of } \Big\{\Gamma_\pi\Big\}_\chi \approx\; B_\chi(p)$$

In turn, $\Gamma_\pi(k; P_\pi, \alpha)$ is related to the Bethe-Salpeter (BS) amplitude, Φ_{BS}, that describes the pion state within the genuinely relativistic quantum-field theory framework based on the BS equation [27, 28]. This suggests that already from a close study of the BS amplitude, linked to the vertex function through , $\Phi_{BS} = S_q\; \Gamma_\pi\; S_{\bar{q}}$ (with $S_{q(\bar{q})}$ the proper fermionic propagator), one can address in depth the issue of the hadron dynamics.

4. The pion as a bed-test for non perturbative QFT

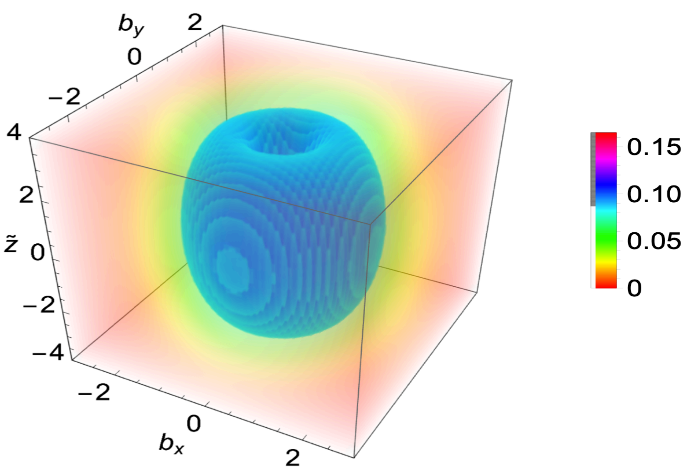

Figure 1. (Color online). Pion density plot of $|\mathbf{b}_\perp|^2\; |\psi(\tilde{z}, b_x, b_y)|^2$, with $\psi(\tilde{z}, b_x, b_y)$ obtained from our solutions of the ladder Bethe-Salpeter equation [29]. In the figure, $\tilde{z} \equiv$ is the Ioffe-time, and $\{b_x, b_y\} \equiv$ the transverse coordinates.

The BS amplitude of the pion can be obtained by solving the homogeneous BS equation, i.e. the integral equation that describes non perturbatively a bound state (as well as a scattering state in its inhomogeneous version), within a relativistic quantum-field framework. For a quark-antiquark pair, the BS equation schematically reads

$$\Phi_{BS}(k,P) = S_q(k+P/2)\; \int d^4p\; \mathcal{K}(k,p,P)\; \Phi_{BS}(p,P)\; S_{\bar{q}}(-k+P/2) \tag{9}$$

where P is the pion four-momentum, k the relative one, and $\mathcal{K}(k, p, P)$ is the interaction kernel that is constructed in terms of all the possible two-particle-irreducible diagrams [27]. In principle, both fermion and gluon propagators has to be dressed through a proper self-energies (obtained by solving the corresponding gap-equations), as well as the quark-gluon vertex, fulfilling the suitable Slavnov-Taylor identities (see e.g. Ref. [30]).

A huge amount of work has been carried out with the aim of solving BS equation, gap-equations and interaction-vertex integral equation, but in the Euclidean space (see, e.g., Refs. [31, 21, 32] for reviews), thus establishing what is called the continuous QCD approach. The long-established Brazil-Italy collaboration (ITA-Rome-Pisa) has been pursuing the goal to develop an analogous framework for a realistic description of hadron phenomenology directly in Minkowski space, i.e. the space where the physical processes actually take place. This means that one has to address the non trivial analytical behavior of the amplitudes involved in the problem, still remaining in the physical space. The main ingredients of the approach are i) the Nakanishi integral representation (NIR)[33] of the BS amplitudes and ii) the so-called light-front projection, that amounts to formally project the BS equation onto the hyperplane $x^+ = x^0 - x^3 = 0$ (see, e.g., Ref. [34, 35], for an introduction to the approach). In this way, without any approximation thanks to NIR, it is possible to get rid of all the difficulties related to the complicate analytic structure of the BS amplitude, that depends upon the Minkowskian variables, and eventually obtain the so-called Nakanishi weight functions, that are real functions. Noteworthy, the last quantities allow a full reconstruction of the BS amplitude in Minkowski space.

The application to the pion has been performed by using an interaction kernel in ladder approximation and free propagators, as thoroughly illustrated in Refs. [36, 37, 29, 38, 39] (see Refs. [34, 35, 40, 41] for the two-boson homogeneous and inhomogeneous case, and Ref. [42] for the fermion-boson bound system). It is planned to substantially improve the description by dressing the fermionic propagator, solving the corresponding Dyson-Schwinger equation in Minkowski space (see Ref. [43] for an approach based on a backward Wick rotation for solving the quark gap-equation and Ref. [44] for a general investigation of the gap-equations with the NIR). The effectiveness of the method can be shown by Fig. 1, where the probability distribution of the quarks inside the pion, sitting on the the hyperplane $x^+ = 0$ that is tangent to the light-cone, is presented in the space given by the Cartesian product of the *Ioffe-time* and the plane spanned by the transverse coordinates $\mathbf{b}_\perp$[29]. To understand the relevance of this 3D space, it should be pointed out that in addition to the familiar infinite-momentum frame, one can study the deep-inelastic scattering processes in the target frame, adopting the configuration space, so that a more detailed investigation of the space-time structure of the hadrons can be performed. In particular, the *Ioffe-time* is useful for studying the relative importance of short and long light-like distances, and its covariant definition is (see Ref. [45])

$$\tilde{z} = x \cdot P_{target} = x^- P^+_{target}/2 \quad \text{on the hyperplane } \mathrm{x}^+ = 0 \tag{10}$$

The above mentioned probability distribution is obtained by properly Fourier-transforming the pion BS amplitude, so that an iconic view of the pion observed from the light-cone can be build. In the introduced 3D space the pion has a toroidal shape, where the central hole is given by the presence of a Jacobian factor $|\mathbf{b}_\perp|^2$.

More interestingly, the study of the BS amplitude has testable consequences that can be investigated experimentally. For instance, we have compared the spacelike electromagnetic form factor of the pion, calculated from the pion BS amplitude, with the available data. In Fig. 2, following Ref. [38], the theoretical results are shown along with the data and also the asymptotic form factor predicted within the perturbative QCD [46]. It has to be pointed out that only three inputs parameters are used: the quark mass $m_q = 255$ MeV, the gluon mass $m_g = 637.5$ MeV and the parameter $\Lambda = 306$ MeV, that controls the extended quark-gluon vertex. Those values, inspired by LQCD calculations, are tuned to reproduce the experimental value of the decay constant $f_\pi^{PDG} = 130.50(1)(3)(13)$ MeV, and are able to return the pion charge radius of the $r_{ch}^{BSE} = 0.663$ in very good agreement with the experimental one $r_{ch}^{PDG} = 0.659\pm0.004$. Recently, the parton distribution function has been also calculated [39], and very encouraging comparisons with both other theoretical calculations and re-analyzed experimental data have been obtained.

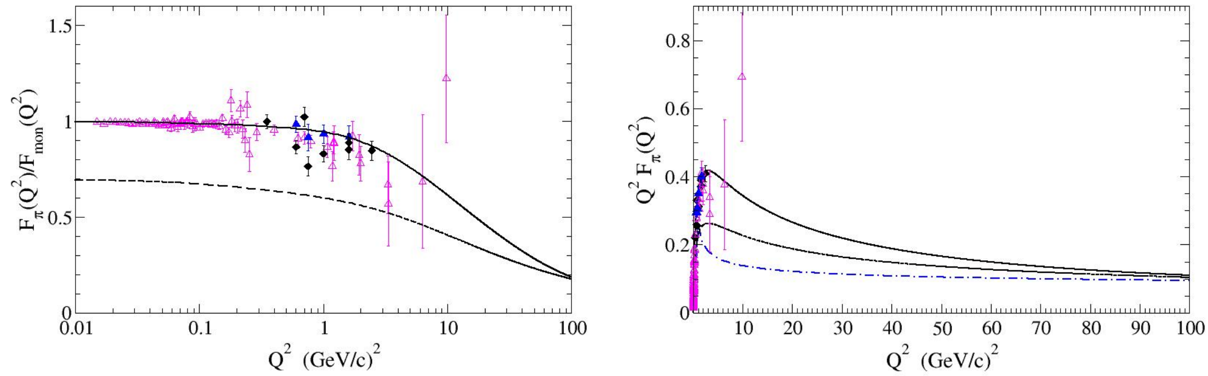

Figure 2. (Color online). Spacelike electromagnetic form factor of the pion. Left panel.Solid curve: pion form factor, obtained from the solution of the BS equation in ladder approximation. Dashed line: the valence contribution. Right panel. The same as in the left panel, but with the perturbative QCD result[46] represented by the dash-dotted line (see Ref. [38] for more details). The monopole form factor is $1/(1 + Q^2/m_\rho^2)$, with $m_\rho = 770.$ MeV.

5. Conclusion& Perspectives

The near future will offer an innovative view of the dynamics inside the hadrons, thanks to planned experimental activities at the Electron-ion colliders. It will be possible to gather a wealth of experimental information on fundamental issues, like the spin content and the origin of the nucleon mass, but also the pion will be investigated in detail, given its peculiar dual nature of both quark-antiquark bound system and Goldstone meson, so important for understanding the strong interaction. Similarly to the impressive experimental efforts, also the theoretical community is going to face with great challenges, since i) extracting relevant information from experimental measurements as well as ii) proposing new processes for addressing the intriguing features of QCD are really demanding. Finally Minkowski space investigations, once the approach composed by BS equation and gap-equations will become fully available, will allow to intensify the cooperation with the well-established lattice and continuous QCD communities, in order to shed light in full on the ultimate mechanisms of the hadron dynamics.

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