Group contraction in physics: History and potential

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Abstract. This article serves as a review of the mathematical tool of group contraction with an emphasis on physical applications. It was implicitly understood that some theories contain others as a special case (e.g., the relation between special relativity and Newtonian mechanics), but until the 1950s there was no firm mathematical grounding for such relations. We discuss the historical development of these concepts, their application to symmetry groups of space and time, and their relevance to the phenomenon of spontaneous symmetry breaking. Finally, we close by proposing a speculative chronology of space-time symmetries in the early universe.

1. Introduction

Physical theories are, in essence, an attempt to explain observed physical phenomena by deducing them as a consequence of some smaller set of fundamental principles. Typically, they are framed as mathematical theories, as mathematics provides us with a good language for describing the relations of relevant entities.

A change in theory may be required as observational power increases (e.g., with the discovery of telescopes, microscopes, X-rays, etc.) or new situations are encountered. Sometimes the needed change is minor, such as the addition of a new element to the periodic table; other times, such as in the case of special relativity and Newtonian mechanics, the discovery necessitates a complete replacement of a theory.

In the second case, we are left with an "obsolete" theory; however, this theory is still useful, as after all, it was used to explain some phenomena. It is intuitively understood that the older theory is in some sense a special case of the newer theory. The question then arises: is it possible to describe the relationship between the two theories via some mathematical mechanism?

It turns out that the answer is "yes" in at least some cases. The case we are specifically interested in is that of SR and Newtonian mechanics, where the relation between the theories is described via group contraction of the underlying symmetry groups. If we have theories that are defined via their symmetry groups, we can relate them using this method, provided the groups involved satisfy certain relations.

2. History

Before delving into the mathematics involved and its applications, it is useful to review the history of the usage of groups in physics, specifically in the discussion of space-time symmetries.

The first implicit usage of symmetry groups when describing physics was by G. Galilei in 1632, when in his "*Discourses and Mathematical Demonstrations Relating to Two New Sciences*" [1] he stated his principle of relativity. This concept of relativity of motion arose from the observation that motions can be superimposed on one another. Effectively what he was proposing was invariance of physical laws under Galilean boosts, i.e. equivalency of inertial frames of reference, today this would be the group $\mathbb{R}^3 \rtimes SO(3)$. What was lacking in his description was any reference to or codification of absolute space and time.

That was done later by I. Newton in 1687 in his seminal work "*Philosophiæ Naturalis Principia Mathematica*" [2] where he elaborated on the ideas introduced by Galilei, introducing absolute space and time, and developed Newtonian mechanics, in a sense the first physical theory in "modern" understanding. The enlarged symmetry group of this theory was then $\mathbb{R}^4 \rtimes \left(\mathbb{R}^3 \rtimes SO(3)\right)$, today known as Galilean group.

This was the prevailing paradigm until 1905, when the tensions between then new theory of electromagnetism and classical mechanics were resolved by A. Einstein in his article "*On the Electrodynamics of Moving Bodies*" [3], which overturned and superseded Newtonian mechanics and replaced the Galilean boosts with relativistic ones, effectively changing the symmetry group from Galilean to Poincaré.

Very soon after publishing of the article it was intuitively understood that Newtonian mechanics represents low velocity limit of the new theory (or infinite speed of light limit, as the relevant quantity is $\frac{v}{c}$), however despite this understanding there was no proper mathematical method to relate these two theories. On the level of individual formulas, it was always possible to expand into a power series and then keep only the lowest relevant order terms; however, this was mathematically unsatisfying.

This changed in 1951 when I. Segal published an article "*A class of operator algebras which are determined by groups*" [4] where he first proposed the idea of a limiting procedure for groups and its application to symmetry groups of physical theories. These ideas were developed further by E. Inönü and E. P. Wigner who two years later, in 1953, published the now classical article "*On the contraction of groups and their representations*" [5], where they described a specific mathematical method for contracting one group into another. It was immediately clear that this mechanism can be seen as a way to link different theories and to describe theory change.

Further developments of this concept were made, e.g. by E. J. Saletan [6], however we will concentrate on the Inönü-Wigner mechanism as it is sufficient for our purposes.

3. Inönü-Wigner group contraction

In this section, we go over the necessary steps to describe and illustrate the group contraction procedure, we follow similar explanations in [7], [8] and [9].

The setting in which Inönü-Wigner contraction operates are continuous symmetries described by Lie groups G_i, with the easiest way to approach it being via studying transformation of the associated Lie algebras $\mathfrak{g}_i$.

Suppose then we have a Lie algebra $\mathfrak{g}$ associated with group G, with J_i as the chosen basis of the vector space of algebra. Commutation relations can be written in the given basis as

$$[J_i, J_k] = \sum_{k=1}^{n} f_{ijk} J_k \quad i, j = 1, \cdots, n \tag{1}$$

with f_{ijk} being the structure constants of the group. Additionally the basis must satisfy *Jacobi identity*, which puts constraint on the structure constants

$$\sum_{c=k}^{n} f_{jlk} f_{ikm} + f_{lik} f_{jkm} + f_{ijk} f_{lkm} = 0. \tag{2}$$

Suppose now we introduce a *contraction parameter* ϵ and redefine the basis elements $J_i \to J_i^{(\epsilon)}$ such that the following is satisfied:

- the infinite sequence $[J_i]^\epsilon$ and its corresponding structure constants $[f_{ijk}]^{(\epsilon)}$ are known
- the limit $\lim_{\epsilon\to 0}[f_{ijk}]^\epsilon = [f_{ijk}]^0$ exists for all i, j, k and is consistent under under Jacobi identity

If the above conditions are satisfied, then the structure constants $[f_{ijk}]^0$ generate a new Lie algebra $\mathfrak{g}'$ (and so also Lie group G') which is called the *contraction* of $\mathfrak{g}$.

It turns out that there are conditions which describe when/how can the contraction be performed for given Lie algebra. Specifically, if $\mathfrak{g}$ has a subalgebra $\mathfrak{h}$ such that

$$\mathfrak{g} = \mathfrak{h} + \mathfrak{p} \tag{3}$$

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \quad [\mathfrak{h}, \mathfrak{p}] \subset \mathfrak{p}, \quad [\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{h} + \mathfrak{p} \tag{4}$$

We can then explicitly re-parametrize generators from the subspace $\mathfrak{p}$ as $\mathfrak{p}' = \epsilon\mathfrak{p}$. This does not fundamentally change the algebra, as $\mathfrak{g}_\epsilon$ is isomorphic to $\mathfrak{g}$, since the commutators are

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \quad [\mathfrak{h}, \mathfrak{p}'] \subset \mathfrak{p}', \quad [\mathfrak{p}', \mathfrak{p}'] \subset \epsilon^2 (\mathfrak{h} + \mathfrak{p}) \tag{5}$$

However, if we now perform the limiting procedure $\epsilon \to 0$, the commutators in the singular limit become

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \quad [\mathfrak{h}, \mathfrak{p}'] \subset \mathfrak{p}', \quad [\mathfrak{p}', \mathfrak{p}'] = 0. \tag{6}$$

From this, it is clear that $\mathfrak{g}_0$ is no longer isomorphic to $\mathfrak{g}$, as the subspace $\mathfrak{p}'$ has become an abelian subalgebra after the limit. The algebra $\mathfrak{g}_0$ is clearly the algebra of the contracted group.The structure of the resulting group is that of semi-direct product, i.e. $G' = G_1 \rtimes G_2$ for some groups G_1, G_2.

It is good to notice two things about the procedure: one, the dimension of the symmetry group is preserved under the contraction. This means that we cannot, in this way, relate theories that have different numbers of symmetry generators. Two, the algebra of the contracted group contains an abelian subalgebra, and hence the presence of a subspace of commuting symmetry generators can be taken as a sign of possible group contraction. These observations will be important in the later discussion of the symmetries of space-time.

3.1. Example: SO(3) → ISO(2)

To finish this section, allow a demonstration of the procedure on a simple case. The most common example is the contraction of 3D rotation group $SO(3)$ to a group of isometries of a plane $ISO(2)$.

As the generators for the $SO(3)$ we will take the usual selection J_i

$$[J_1, J_2] = J_3, \quad [J_2, J_3] = J_1, \quad [J_3, J_1] = J_2. \tag{7}$$

Using the method described previously, we select for the subalgebra $\mathfrak{h}$ a one dimensional space generated by J_3, the complementary space $\mathfrak{p}$ is then generated by the remaining J_1, J_2. We now rescale $\mathfrak{p}$ via the contraction parameter ϵ

$$j_1 = \epsilon J_1, \quad j_2 = \epsilon J_2, \quad j_3 = J_3. \tag{8}$$

The commutators now change as

$$[j_1, j_2] = \epsilon^2 j_3, \quad [j_2, j_3] = j_1, \quad [j_3, j_1] = j_2, \tag{9}$$

which after the limit $\epsilon \to 0$ leads to a new Lie algebra

$$[j_1, j_2] = 0, \quad [j_2, j_3] = j_1, \quad [j_3, j_1] = j_2. \tag{10}$$

This Lie algebra characterizes the group $ISO\,(2) = SO\,(2) \rtimes \mathbb{R}^2$, with the generators j_1, j_2 being the translations and j_3 the rotation around the axis perpendicular to the plane.

We note that we can assign a geometric interpretation to the contraction parameter, and these can be interpreted physically as well. In this particularly simple case we can describe ϵ as the inverse radius of a sphere $1/R$. As the radius of this imagined sphere grows, the neighbourhood of any point starts to increasingly resemble a plane, and at the limit of an infinite radius, it becomes (geometrically) indistinguishable.

4. Symmetries of space-time

After exploring the basics of the historical context and the mathematical underpinnings of the group contraction, it is now time to apply the method to some physical theories.

Specifically, we will trace the evolution of the assumed symmetry group of space and time, through the lens of group contraction. From looking at the historical examples, we will also try to see if we can deduce something about potential future developments in this area.

As mentioned previously, the first proposal of a symmetry of space and time (even if not using these exact terms) came from Galilei in 1638 [1]. This was invariance under Galilean boosts and spatial rotations, $\mathbb{R}^3 \rtimes SO(3)$. To complement our understanding, we extend this by addition of space and time translations [2], to obtain full *Galilean group* $\mathbb{R}^4 \rtimes \left(\mathbb{R}^3 \rtimes SO(3)\right)$. The non-zero commutation relations are as follows:

$$\frac{1}{\mathrm{i}}\,[C_i, H] = P_i \tag{11}$$

$$\frac{1}{\mathrm{i}}\,[L_{ij}, C_k] = \delta_{ik}C_j - \delta_{jk}C_i, \quad \frac{1}{\mathrm{i}}\,[L_{ij}, P_k] = \delta_{ik}P_j - \delta_{jk}P_i \tag{12}$$

$$\frac{1}{\mathrm{i}}\,[L_{ij}, L_{jk}] = \delta_{ik}L_{jl} - \delta_{il}L_{jk} - \delta_{jk}L_{il} + \delta_{jl}L_{ik} \tag{13}$$

where the generators are

- H - time translation
- P_i - space translation
- L_ij - spatial rotations
- C_i - Galilean boost

We can immediately notice that the Galilean boosts form an abelian subalgebra, hinting that perhaps we can obtain this group from another via the contraction procedure. Indeed, if we take Poincaré group $\mathbb{R}^{3,1} \rtimes SO\,(3,1)$ and take the slowness parameter $1/c$ to zero, we do get (inhomogeneous) Galilean group.

However, we do need to point out that such an approach gets appropriate results only in the classical setting, once we move over to the quantum setting, there appear effects that leave imprints in the non-relativistic limit that are not described by the Galilean group [10]. It is then necessary to include additional generator M, and to take care in the choice of representation. For our discussion and arguments, however, it is sufficient to consider the Galilean group.

Explicitly, we can consider generators of the Lorentz group $SO(3,1)$ J_i, B_i that obey commutation relations

$$\frac{1}{\mathrm{i}}\,[J_i, J_j] = \epsilon_{ijk}J_k, \quad \frac{1}{\mathrm{i}}\,[J_i, B_j] = \epsilon_{ijk}B_k, \quad \frac{1}{\mathrm{i}}\,[B_i, B_j] = -\epsilon_{ijk}B_k \tag{14}$$

with J_i begin the spatial rotations, and B_i the Lorentz boosts. If we now introduce the inverse speed of light as the limit parameter (the "slowness"), we define

$$J_i^{(c)} = J_i, \quad B_i^{(c)} = \frac{B_i}{\mathrm{i}c}. \tag{15}$$

Taking now the limit $c \to \infty$ (or equivalently $1/c \to 0$) we obtain commutation relations

$$\frac{1}{\mathrm{i}}\left[J_i, J_j\right] = \epsilon_{ijk} J_k, \quad \frac{1}{\mathrm{i}}\left[J_i, B_j^{(0)}\right] = \epsilon_{ijk} B_k^{(0)}, \quad \frac{1}{\mathrm{i}}\left[B_i^{(0)}, B_j^{(0)}\right] = 0, \tag{16}$$

which are those of a homogeneous Galilean group.

We could have also speculated immediately from the form of the group that it can be obtained via contraction, as it has a semi-direct product structure, which does appear in groups resulting from contraction procedures.

Historically speaking, this relations of these groups, and hence also of Newtonian mechanics and special relativity was only discovered after the theory itself was. However, we can also look at it from another angle: if the mechanism was known prior to the formulation of special relativity, we could have speculated whether there is some physical parameter that is sufficiently large (or small), so that the effective symmetry group of low velocity mechanics is Galilean and the full symmetry group is different. Experimentally, of course, until there was observational evidence for the finite speed of light, there was no reason to consider such a parameter to play a role in the kinematical group of mechanics.

Taking this second viewpoint, we can ask if perhaps there are also other finite parameters that are currently outside of our observational bounds that we are not considering (e.g., a finite invariant length scale).

With this in mind, let us look at the Poincaré group in more detail. It is again a 10 dimensional group, which does have a semi-direct product structure, so it could be a contraction of another group. Its commutation relations are

$$\frac{1}{\mathrm{i}}\left[M_{\mu\nu}, P_\rho\right] = \eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu \tag{17}$$

$$\frac{1}{\mathrm{i}}\left[M_{\mu\nu}, M_{\rho\sigma}\right] = \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho} \tag{18}$$

with

- P_μ - space-time translations
- $M_{\mu\nu}$ - spatial rotations and boosts (spatio-temporal rotations)

Note that we can relate $M_{\mu\nu}$ to explicit generators of rotations and boosts as $J_i = \frac{1}{2}\epsilon_{ijk} M_{jk}$, $B_i = M_{0i}$. From this, we also see that spatial and temporal translations form an abelian subgroup, another hint that the group is a potential result of a contraction procedure. Specifically, Poincaré group can be obtained as a contraction from either the de Sitter group $SO(4,1)$ or the anti-de Sitter group $SO(3,2)$, via sending their scalar curvature Λ to zero.

Which of these two groups is more worthy of our attention? From the perspective of the author, it makes more sense to pick the de Sitter group and focus on it for the following reasons:

- positive cosmological constant (i.e. "dark energy")
- inflationary space-times are approximately de Sitter during inflation [11]
- de Sitter relativity is a mathematically appropriate version of doubly special relativity (in the sense that there is no explicit Lorentz symmetry breaking) [12]

The De Sitter group $SO(4,1)$ is again a 10-dimensional group, with commutation relations simplified to

$$\frac{1}{\mathrm{i}}\left[M_{AB}, M_{CD}\right] = \eta_{BC}M_{AD} - \eta_{BD}M_{AC} - \eta_{AC}M_{BD} + \eta_{AD}M_{BC}. \tag{19}$$

The contraction to the Poincaré group can be constructed as follows: let us define a new basis

$$\Pi_\mu = \frac{1}{l}M_{5\mu} \tag{20}$$

with the rest staying the same. Commutation relations can then be rewritten as

$$\frac{1}{\mathrm{i}}\left[\Pi_\mu, \Pi_\nu\right] = \frac{1}{l^2}M_{\mu\nu} \tag{21}$$

$$\frac{1}{\mathrm{i}}\left[M_{\mu\nu}, \Pi_\rho\right] = \eta_{\mu\rho}\Pi_\nu - \eta_{\nu\rho}\Pi_\mu \tag{22}$$

$$\frac{1}{\mathrm{i}}\left[M_{\mu\nu}, M_{\rho\sigma}\right] = \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}. \tag{23}$$

Sending now the pseudo-radius l to infinity, we obtain exactly the commutation relations of the Poincaré group, with $\Pi_\mu \to P_\mu$ becoming the translation generators.

From the commutation relations above, we can see that the de Sitter group has neither a semi-direct product structure nor a non-trivial abelian subgroup, so we could in principle end our discussion here.

However, we would like to take a step further by investigating transformations that can naturally extend the Sitter group, namely operations that act transitively on the Sitter space [13].

These de Sitter "translations" can be written as a combination of translations and special conformal transformations, i.e.

$$\partial_\mu - \frac{1}{4l^2}\left(2\eta_{\mu\tau}x^\tau x^\sigma - x^2\delta_\mu^\sigma\right)\partial_\sigma, \tag{24}$$

where $l2$ denotes the de Sitter pseudo-radius and $x_m u$ some specific coordinates. This object on its own is not a de Sitter generator, as its action on space-time leads to a conformal rescaling of the metric, transformation which is not from the de Sitter group. If we demand that the symmetry group of space-time include transformations under which it is transitive, we must include both the usual translations and the special conformal transformations.

This inclusion forces us to also include the dilation generator in order to close the commutation relations. This extensions then moves us to a group $ISO(4,1)$ a 15-dimensional group of isometries of $SO(4,1)$. This group has again the semi-direct product structure, and can be obtained as a contraction of the conformal group $SO(4,2)$, in a similar fashion to the preceding contractions.

Why should we be interested in the conformal group, if we are discussing symmetries of space-time? We see several reasons for this

- conformal group $SO(4,2)$ is the largest group of symmetries preserving causal ordering in 4D space-time
- other physical theories have natural scale symmetry (gauge theories, massless theories), and this can be promoted to full conformal symmetry under a broad set of conditions (in $d=2$ there is *Zamolodchikov-Polchinski theorem* [14])
- CMB is nearly scale-invariant, suggesting scale invariant (or conformal invariant) theories are suitable for description of very early universe [11]

Considering these arguments persuasive enough to explore this direction, let us take a closer look at conformal group. Its commutation relations can be compactly written as

$$\frac{1}{\mathrm{i}}\left[M_{AB}, M_{CD}\right] = \eta_{BC}M_{AD} - \eta_{BD}M_{AC} - \eta_{AC}M_{BD} + \eta_{AD}M_{BC} \tag{25}$$

in the exact same form as the de Sitter ones (which should not be surprising, as both are pseudo-rotation groups). We can also consider an alternative basis that is related to the usual generators as,

$$L_{\alpha\beta} = M_{\alpha\beta}, \quad D = M_{56} \tag{26}$$

$$P_\alpha = M_{\alpha 5} + M_{\alpha 6}, \quad K_\alpha = M_{\alpha 5} - M_{\alpha 6} \tag{27}$$

$$\alpha, \beta = 0, 1, 2, 3, \quad \alpha = 0 \equiv A = 4. \tag{28}$$

Again, we can see from the structure of the group and from the commutation relations that there is no obvious need for further extension. We could again consider transformations acting on the space as in the de Sitter case, but from a physical perspective, we are interested in 3+1 dimensional spacetimes, and any further extension would take us away from that (as $SO(4,2)$ is the conformal group of 3+1 dimensional space-time).

Additionally, now we are left with a problem. The present-day universe is well described by considering only 10 space-time symmetries (local), yet the conformal group has 15. How can we reduce this number? One answer is spontaneous symmetry breaking, and it turns out that group contraction has a relationship to it.

5. Group contraction and symmetry breaking

In this section we recall relation of group contraction to symmetry breaking patters, as developed in [15] and [16].

One of the important realizations of 20th century physics was that symmetry in a system need not be realized linearly, i.e., in the Wigner-Weyl realization, but instead can be realized non-linearly in what is known as the Nambu-Goldstone realization. This is also colloquially known as spontaneous symmetry breaking (SSB) [17], as a particular vacuum state is invariant only under a certain subgroup H of the symmetry group G, i.e., it breaks the symmetry, with the rest of the former symmetries now transforming it into a different vacuum state.

It is also a well known fact that spontaneous symmetry breaking leads to the appearance of massless Goldstone modes (which can be composite fields in the case of dynamical symmetry breaking). These fields are either scalar, spinorial, or vector, depending on whether the broken symmetry is internal, super, or space-time. The number of these Goldstone modes corresponds to the number of broken generators, i.e. to the difference between the dimensions of the symmetry group G and the symmetry group of the vacuum state H,

$$\#\, Goldstone\, modes = \dim G - \dim H \tag{29}$$

It turns out that the "remnant" symmetry group H combined with the abelian Goldstone modes (which generate field translations) must be a contraction of the original symmetry group G [15]. This then provides us with a mechanism to calculate the massless Goldstone modes. Also (and more importantly for us) it provides us with a mechanism to change the dimension of the contracted group, provided we ignore the "abelianized" field translations.

This relationship between symmetry breaking and group contraction provides us with a tool for describing the relationship of space-time symmetry groups in a unified manner. We can assume that the conformal group is spontaneously broken (e.g., by the introduction of scale, breaking the dilatation generator), and the left-over non-trivial part is the de Sitter group, with the remaining 5 generators being abelianized and representing field translations.

6. Hierarchy of space-time symmetry groups

Now that we have discussed the relation between symmetry breaking and group contraction, we have all the required ingredients ready. In section 4 we have progressed from the least to the most general symmetry group. However, in universe it would appear the other way around.

We would propose the following timeline of space-time symmetry groups:

$$\text{Conformal} \to \text{de Sitter} \to \text{Poincare} \to \text{Galilei}$$

In the pre-inflationary and inflationary universe we propose that the symmetry group of space-time was the conformal group $SO(4,2)$. Due to the fact that the present day universe is neither conformal nor scale invariant, this part of the symmetry was then spontaneously broken down to the de Sitter group $SO(4,1)$, with the symmetry breaking pattern determined by the contraction, as described in the previous sections.

We further suggest that the true "local" symmetry of space-time is the de Sitter, not the Poincaré. The transition from de Sitter to Poincaré (and from Poincaré to Galilei) are then not true transitions, but merely approximation artifacts due to limited observational capabilities.

As the shift to Poincaré happened when we had other theoretical signs and further experimental evidence of the finite speed of light, similarly, we propose that there is an invariant length scale that is currently out of observational bounds. If this invariant scale exists, the symmetry group of space-time would be naturally de Sitter, not Poincaré.

Another kind of theoretical argument in support of an invariant length (or energy) scale would be that the scale on which quantum gravity becomes relevant should play a fundamental role. An example could be that a photon with a wavelength of Planck length should collapse into a black hole (based on classical understanding), however in different reference frames its wavelength could be longer due to red-shift. As a result, the presence of a black hole would then be observer-dependent, leading to contradictions within the theory. Eventual quantum theory of gravity should be able to resolve this issue, and treating some length or energy scale as invariant would lead to resolution.

Additionally, we could argue that the structure of the Poincaré group combined with the knowledge and history of group contraction also points in the direction of further refinement being necessary.

7. Summary and conclusion

In this note, we have reviewed the historical context and mathematical footing of group contraction, with an emphasis on the physical applications and interpretation of the procedure.

We show that group contraction can be used to provide a natural way to link different theories, provided a link exists between their symmetry groups. Additionally, we show how this procedure is related to symmetry breaking pattern.

From a physical standpoint, we used this to investigate the relationship of historical space-time symmetry groups (Poincaré and Galilei) and then extended that to propose the de Sitter group for consideration. Additionally, we further extended this to the conformal group, for which discussion of the link between group contraction and symmetry breaking was necessary.

We used this to propose that the true symmetry group of space-time in the early universe was the conformal group, which was spontaneously broken down to the de Sitter group. The current understanding of the Poincaré group as the symmetry group is thus an observational artifact resulting from limited experimental and observational capabilities, similar to how the Galilean group was considered a symmetry of space-time until it was superseded by the Poincaré group due to advances in mathematical tools and other theories.

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