SPIN, CONFINEMENT, LOCALIZATION AND SUPERSYMMETRY

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New mechanism of localization of fields in a particular sector of a special supermanifold is proposed and described, based on previous works of the authors. Different aspects of this new mechanism are discussed as an alternative to confinement in elementary particle physics and to the Randall-Sundrum scenarios in warped manifolds.

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1. INTRODUCTION AND MOTIVATION

The study of symmetries plays a fundamental role in modern physics. The geometrical interpretation of the physical phenomena takes as basic object the action, where all the dynamics of the theory is derived. The idea of associating an underlying geometrical structure to these physical phenomena coming from a fundamental idea of unification of all the interactions into the natural world and not from an heuristic thought. The interrelation between physical and mathematical definitions and concepts (i.e. geometry, groups, topology↔space-time, internal structure, fields) turns more and more concrete and basic in the physics of the XX and XXI centuries. If there are well elegant formulations of the physical problems of interest from the mathematical point of view, there exists a lack of uniqueness in the geometrical definition of the Lagrangian density.

Great difficulties appear (or almost are evidently explicit) at the quantum level where the geometrical objects playing the role of Lagrangian or Hamiltonian pass to play the role of (super) operators. These troubles carry inexorably to the utilization of diverse methods or prescriptions that change the original form of the action (or Hamiltonian). This distortion of the original form of these fundamental operators at the classical level generally does not produce changes into the dynamical equations of the theory but quantically introduces several changes, because the spectrum of physical states is closely related with the form of the Hamiltonian. This fact was pointed out by the author in the previous paper [1]. Clearly, in order to construct the Lagrangian and other fundamental invariants of the theory, the introduction of a manifold as the important ingredient is the relevant thing. In particular it can be very interesting to introduce a super-manifold (in the sense of [1] and references therein) in order to include the fermionic fields in a natural manner.

It is therefore of interest to study the geometry not only of the simplest superspaces, but also the more unusual or non-standard ones and elucidate all the gauge degrees of freedom that they possess. This fact will clarify and expand the possibilities to construct more realistic physical models and new mathematically consistent theories of supergravity. On the other hand, the appearance of supergroups must draw attention to the study of the geometries of the homogeneous superspaces whose groups of motions they are. Another motivation of the study of these Riemannian superspaces is the establishment of some degree of uniqueness in the obtained supersymmetric solutions.

Motivated by the above, we comment and discuss our previous paper [1] reviewing, studying and analyzing from the point of view of the possible vacuum solutions, the simplest non trivial supermetric given by Volkov and Pashnev in [2] that was the "starting point" toy model of the first part of this work:

$$ds^2 = \omega^{\mu}\omega_{\mu} + \mathbf{a}\omega^{\alpha}\omega_{\alpha} - \mathbf{a}^{*}\omega^{\dot{\alpha}}\omega_{\dot{\alpha}}. \qquad (1)$$

This particular non-degenerate supermetric contains the complex parameters $\mathbf{a}$ and $\mathbf{a}^*$ that make it different from other more standard supermetrics. As we showed in [1, 3], the degenerate supermetrics are not consistent into a well theoretically formulated supergeometry. Then, our main task is to find the meaning and the role played by these complex parameters from the geometrical and physical points of view.

Our goal is to show that, from the point of view of the obtained solutions, the complex parameters $\mathbf{a}$ *localize* the fields in a specific region of the bosonic part of this special superspace, that they explicitly

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break down the chiral symmetry when some conditions are required and that all these very important properties *remain* although the supersymmetry of the model was completely broken. Also, besides all these highlights, we also show that the obtained vacuum states from the extended supermetric are very well defined in any Hilbert space.

2. "WARPED" GRAVITY MODELS, CONFINEMENT AND THE SUPERMETRIC

It is well known that large extra dimensions offer an opportunity for a new solution to the hierarchy problem [4]. Field theoretical localization mechanisms for scalar and fermions [5] as well as for gauge bosons [6] were found. The crucial ingredient of this scenario is a brane on which standard model particles are localized. In string theory, fields can naturally be localized on D-branes due to the open strings ending on them [7]. Up until recently, extra dimensions had to be compactified, since the localization mechanism for gravity was not known. It was suggested in Ref. [8] that gravitational interactions between particles on a brane in uncompactified five dimensional space could have the correct four dimensional Newtonian behavior, provided that the bulk cosmological constant and the brane tension are related. Recently, it was found by Randall and Sundrum that gravitons can be localized on a brane which separates two patches of AdS_5 space-time [9]. The necessary requirement for the four-dimensional brane Universe to be static is that the tension of the brane is fine-tuned to the bulk cosmological constant [8,9]. On the other hand, recent papers present an interesting model in which the extra dimensions are used only as a mathematical tool taking advantage of the AdS/CFT correspondence that claims that the 5D warped dimension is related with a strongly coupled 4D theory [10].

A remarkable property of the vacuum solution of (1) given by the expression

$$g_{ab}(t) = e^{-\left(\frac{m}{|\mathbf{a}|}\right)^2 t^2 + c_1 t + c_2} e^{\xi\varrho(t)} g_{ab}(0) \qquad (2)$$

with

$$\varrho(t) = \overset{\circ}{\phi}_\alpha \left[\left(\alpha e^{i\omega t/2} + \beta e^{-i\omega t/2}\right)\right.$$
$$\left. - \left(\sigma^0\right)^\alpha_{\ \dot{\alpha}} \left(\alpha e^{i\omega t/2} - \beta e^{-i\omega t/2}\right)\right]$$
$$+ \frac{2i}{\omega}\left[\left(\sigma^0\right)_\alpha^{\ \dot{\beta}}\ \overline{Z}_{\dot{\beta}} + \left(\sigma^0\right)^\alpha_{\ \dot{\alpha}}\ Z_\alpha\right]$$

and

$$g_{ab}(0) = \langle\Psi(0)|\, L_{ab}\, |\Psi(0)\rangle$$

is that the physical state $g_{ab}(x)$ is localized in a particular position of the space-time. The supermetric coefficients $\boldsymbol{a}$ and $\boldsymbol{a}^*$ play the important role of localizing the fields in the bosonic part of the superspace in similar and suggestive form as the well known "warp factors" in multidimensional gravity [11] for a positive (or negative) tension brane. But the essential difference is, because the $\mathbb{C}$-constants $\mathbf{a}$ and $\mathbf{a}^*$ coming from the $B_{L,0}$ (even) fermionic part of the superspace under consideration, no additional and/or topological structures that break the symmetries of the model (i.e. reflection Z_2-symmetry) are required: the natural structure of the superspace produces this effect.

Also it is interesting to remark here that the Gaussian type solution (2) is a very well defined physical state in a Hilbert space [10, 12] from the mathematical point of view, contrarily to the case $u(y) = ce^{-H|y|}$ given in [11] that, although it was possible to find a manner to include it in any Hilbert space, it is strongly needed to take special mathematical and physical particular assumptions whose meaning is obscure. The comparison with the case of 5-dimensional gravity plus cosmological constant [11] is given in the following table:

Comparison of Superspace $(1, d \mid 1)$ *with the case of 5-dimensional gravity plus cosmological constant [11]*

Space-time	5D gravity + Λ	Superspace $(1, d \mid 1)$
Interval	$ds^2 = A(y)\, dx^2_{3+1} - dy$	$ds^2 = \omega^\mu \omega_\mu + \mathbf{a}\omega^\alpha\omega_\alpha - \mathbf{a}^*\omega^{\dot{\alpha}}\omega_{\dot{\alpha}}$
Equation	$\left[-\partial_y^2 - m^2 e^{H\|y\|} + H^2\right.$ $\left.-2H\delta(y)\right] u(y) = 0$	$\left[\|a\|^2\left(\partial_0^2 - \partial_i^2\right) + \frac{1}{4}\left(\partial_\eta - \partial_\xi + i\ \partial_\mu\left(\sigma^\mu\right)\xi\right)^2 - \right.$ $\left.-\frac{1}{4}\left(\partial_\eta + \partial_\xi + i\ \partial_\mu\left(\sigma^\mu\right)\xi\right)^2 + m^2\right]^{ab}_{cd} g_{ab} = 0$
Solution	$u(y) = ce^{-H\|y\|}, \quad H \equiv \sqrt{-\frac{2\Lambda}{3}} = \frac{\|T\|}{M^3}$	$g_{ab}(x) = e^{-\left(\frac{m}{\|\mathbf{a}\|}\right)^2 x^2 + c_1' x + c_2'} e^{\xi\varrho(x)} \|f(\xi)\|^2 \begin{pmatrix}\alpha \\ \alpha^*\end{pmatrix}_{ab}$

Here, in order to make our comparison consistent, the proposed superspace has $d = n + 4$ bosonic coordinates and the extended superspace solution for $n = 0$ can depend, in principle, on any or all the 4-dimensional coordinates: $x \equiv (t, \overline{x})$, $c_1' x \equiv c_{1\mu}' x^\mu$ and c_2' scalar (e.g.: the t coordinate in expression (2)); for $n \neq 0$ it depends on the n-additional coordinates.

Notice the following important observations:

i) The solution in the 5-dimensional gravity plus Λ case, the explicit presence of the cosmological term is necessary for the consistency of the model: the "fine-tuning" $H \equiv \sqrt{-\frac{2\Lambda}{3}} = \frac{|T|}{M^3}$, where T is the tension of the brane and M^3 is the constant of the Einstein-Hilbert $+\Lambda$ action.

ii) about the localization of the fields given by the particular superspace treated here, the Z_2 symmetry is non-compatible with the solution that clearly is not chiral or antichiral. This fact is consistent with the analysis given for a similar superspace that the considered here in Ref. [1,12] where the solutions are superprojected in a sector of the physical states that is not chiral or antichiral.

iii) because for $n = 0$ our solution (2) is attached to the 3+1 space-time but the localization occurs on the time coordinate (in any of the remanent 3 space coordinates) the physics seems to be very different with respect to the warped gravity model where the field equation in final form for the 5-dimensional gravity depends on the extra dimension[1]. This $n = 0$ case can give some hints for the theoretical treatment of the confinement mechanism with natural breaking of the chiral symmetry in high energy physics (e.g., instanton liquid models, etc);

iv) for $n = 1$, the situation in our model with the solution depending on the extra coordinate changes favorably: the localization of the field is in the additional bosonic coordinate (as the graviton in the RS type model) but with all the good properties of the solution (2) already mentioned in the beginning of this paragraph.

From the points discussed above and the "state of the art" of the problem, we have seen the importance of the proposition of new mechanisms and alternative models that can help us to understand and to handle the problem. Also it is clearly important that the supermetric (1), cornerstone of this simple super-model, is non-degenerate in order to solve in a simultaneous manner the localization-confinement of the fields involved and the breaking of the chiral symmetry. Then, it is not difficult to think to promote the particular supermetric under study towards to build a strongly coupled 4D model, using this particular N=1 toy superspace.

3. DISCUSSION

The proposal for the choice of a model with underlying basic structure starts from the very early times. Today, a large effective group given by the standard model (multiplicity in the representations and the different coupling constants) stimulates from time ago the search of such models. As we saw in the first part of this work and other references, starting from the most simplest non-degenerate supermetric where the supercoordinates are the fields of the theory and retaining the original form of the fundamental geometrical operators (namely Lagrangian or Hamiltonian), the physical states obtained are constructed from the basic ones by means of operators that characterize the most fundamental symmetries of the space-time.

The situation is more or less clear: although the supersymmetry is broken, the physical states are localized in the "even" part of the manifold due to the metric coefficients of a non-degenerate supermetric. The physical states are composed by most fundamental (non-observable) basic states. Operators belonging to the metaplectic group (the most fundamental covering group of the SL(2C)) lead, due to a map produced by the basic CS, to the observable spectrum of physical states. This fact is clearly important as the "cornerstone" of a new realistic composite model of particles based on coherent states where the space-time symmetry is directly connected with the physical spectrum.

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СПИН, КОНФАЙНМЕНТ, ЛОКАЛИЗАЦИЯ И СУПЕРСИММЕТРИЯ

Д.Х. Сирило-Ломбардо

Предложен и описан новый возможный механизм локализации поля в определенном секторе специального супермногообразия. Различные аспекты этого нового механизма рассматриваются в качестве альтернативы конфайнменту в физике элементарных частиц и сценариям Рэндалл-Сандрама в искривленных многообразиях.

СПІН, КОНФАЙНМЕНТ, ЛОКАЛІЗАЦІЯ І СУПЕРСИМЕТРІЯ

Д.Х. Сіріло-Ломбардо

Запропоновано та описано новий можливий механізм локалізації поля у певному секторі спеціальної супермногостатності. Різні аспекти цього нового механізму розглядаються в якості альтернативи конфайнменту в фізиці елементарних частинок і сценаріям Рендалл-Сандрама у викривлених многостатностях.