Chapter 15
The Making of AdA: Bruno Touschek's Journey from Widerøe's Betatron to Storage Rings

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Abstract In Italy, in 1960, Bruno Touschek conceived and built the first particle-antiparticle collider, called AdA. The different roads and pathways which led to the successful demonstration that it was possible to accumulate electrons and positrons in a single ring and make them collide, originated in different parts of Europe: Austria, Germany, Norway, UK, France and Italy, in parallel with similar developments in the US and USSR. AdA's success was due to Bruno Touschek's extraordinary formation as both an accelerator scientist and a theoretical physicist, and in the unique environment he found in the University of Rome and the Frascati Laboratories, where the post-war reconstruction of Italian physics was taking place, in parallel with the Europe-wide effort, that led to the creation of CERN.

15.1 Introduction

In 1972, Bruno Touschek was made a foreign member of the Accademia Nazionale dei Lincei, in recognition of his outstanding contribution to teaching and science. Three years before, ADONE, the beautiful machine in the Frascati National Laboratories, had successfully started operations. ADONE had started two beam operation in 1969, and new phenomena had appeared soon after electrons and positrons started circulating at a yet unsurpassed center of mass energy. Touschek had been envisioning and planning for it at least since November 1960, when AdA, the first ever electron-positron storage ring he had proposed to build in Frascati, could be seen to be well on its way. Since this first proposal, the world of particle physics had changed, and when Touschek was welcomed into the Academy, a number of particle-antiparticle colliders were in operation or in advanced planning and construction stage: ACO in France, VEPP-2 in the USSR, the ISR at CERN, SPEAR in the USA, DORIS in Germany. At the Academy, Touschek left one of his many legacies to Italian science and culture: the lectures he organized in the context of a project he called *Scienza*

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Fig. 15.1 Bruno Touschek at Accademia dei Lincei, on the occasion of his nomination, in 1972 (left panel), from family documents, courtesy of Francis Touschek, and (right panel) on April 15th, 1975, with (at center) Paul M. Dirac, the Accademia President Beniamino Segre and Marcello Conversi at right, courtesy of Sapienza University of Rome–Physics Department Archives, https://archivisapienzasmfn.archiui.com, documents provided for purposes of study and research, all rights reserved

vivente, living science in English, where lectures on modern science were given by renowned scientists, such as Paul Dirac, Marcello Conversi, Rolf Widerøe, Edoardo Amaldi, himself, and others, Fig. 15.1.[1]

In addition to his contribution to particle physics, teaching and science outreach efforts [1], Touschek also left an important legacy to the Frascati National Laboratories, when he created a theoretical physics group, aimed at future planning and exploitation of ADONE's expected physics results. This legacy is found in this volume in the contributions from Paolo Di Vecchia, Mario Greco, Giancarlo Rossi, and in the present one, as I had the great privilege to be one of the young people Touschek gathered in Frascati, in what Fernardo Amman, ADONE's director, called "the golden years of the Laboratory".[2]

I had graduated in physics from University of Rome in February 1966, with a thesis on a rather exotic process, *Coalescence and decay of photons on nuclei* under the supervision of Benedetto (Nino) De Tollis.

In 1966, returning to Rome after a vacation in the Dolomites which had followed my graduation, I learnt from Giancarlo Rossi and Paolo Di Vecchia that Touschek was starting a theoretical physics group in Frascati. After much hesitancy and fearful of rejection, I went to Touschek's office in the Physics Institute in Rome to inquire about the possibility of joining the new group. The answer which I received a fews days later was positive. Nino de Tollis had vouched on my behalf and in May I arrived in Frascati, with a post-graduation fellowship. This is when I started on the most important experience in my scientific life.

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[1] The Lincei lectures were video-taped by Francis Touschek.

[2] Amman's letter to Edoardo Amaldi, 1978, Sapienza University of Rome–Physics Department Archives.

15.2 Who Was Bruno Touschek?

In this contribution I will present some highlights in the story of Bruno Touschek and the accelerators he built, as well as other legacies he left.

Touschek was a protagonist in the development of particle physics in Europe in the second half of the XXth century, and his life journey, seen in Fig. 15.2, mirrors the tragedy of World War II and the hopes of postwar reconstruction.

Born in Vienna in 1921, Bruno Touschek left Austria for Germany, when his studies in theoretical physics were interrupted by racial discrimination, because of his Jewish origin from the maternal side, Sect. 15.3.[3] Improbable as it may appear, once in Germany he was able to survive and study through the war, while moving between Hamburg and Berlin, protected by Arnold Sommerfeld's former colleagues and students. This is known to have happened in a context where some German scientists would employ their Jewish, or half-Jewish, friends in technical and scientific projects of interests to the military, in order to protect them from deportation to forced labor and ultimately death in the concentration camps [2]. Such destiny would have been Touschek's, but he survived through extraordinary circumstances, Sect. 15.3.1. In Germany he participated in a project to build a 15 MeV betatron, directed by the Norwegian scientists Rolf Widerøe, and financed by the *Reichsluftfahrtministerium*, (RLM), the Reich Ministry of Aviation. The war finished, his knowledge of particle accelerators became of interest to the Western Alliance, and he was first taken to Göttingen, where he obtained his Diploma in Physics, and then to the University of Glasgow to participate in the construction of a 300 MeV synchrotron and continue his studies for a doctorate, Sect. 15.4.3. Moving to Italy in 1952, he was a protagonist of the reconstruction of physics in and around the University of Rome. In the Frascati National Laboratories and, later, in France, Touschek catalyzed the energy of the scientists around him toward the construction of an "unthinkable" machine, which was named AdA, for Anello di Accumulazione. In AdA, for the first time ever, "particles which are not found in the world which surrounds us, were kept and stored for a long time" (in Touschek's own words), Sects. 15.4.4 and 15.4.5.

After he prematurely passed away on May 25th, 1978, his life and work were described by his mentor and friend Edoardo Amaldi, who gathered recollections and documents in a biographical portrait of still unsurpassed emotional and historical impact [1]. Not long after, two young historians of physics at the University of Rome, started preparing a catalogue of all the papers which Touschek had left in his office. They were alerted to the ongoing trashing of these papers by one of Bruno's young collaborators, Amilcare Bietti. Awed by the still vivid memory of Bruno's extraordinary accomplishments and personality in the Rome Physics Institute, Battimelli and De Maria rushed to Bruno's former office and literally extracted his papers from the large trash bin already on its way out. Their rescue efforts were published in a report with reproduction of unpublished notes, a full listing of Touschek's papers and a detailed guide to Touschek's archives, offering a vivid portrait of his personality [3]. The saving of Touschek's office papers signed the beginning of the extensive

[3] See Luisa Bonolis' contribution to these Proceedings.

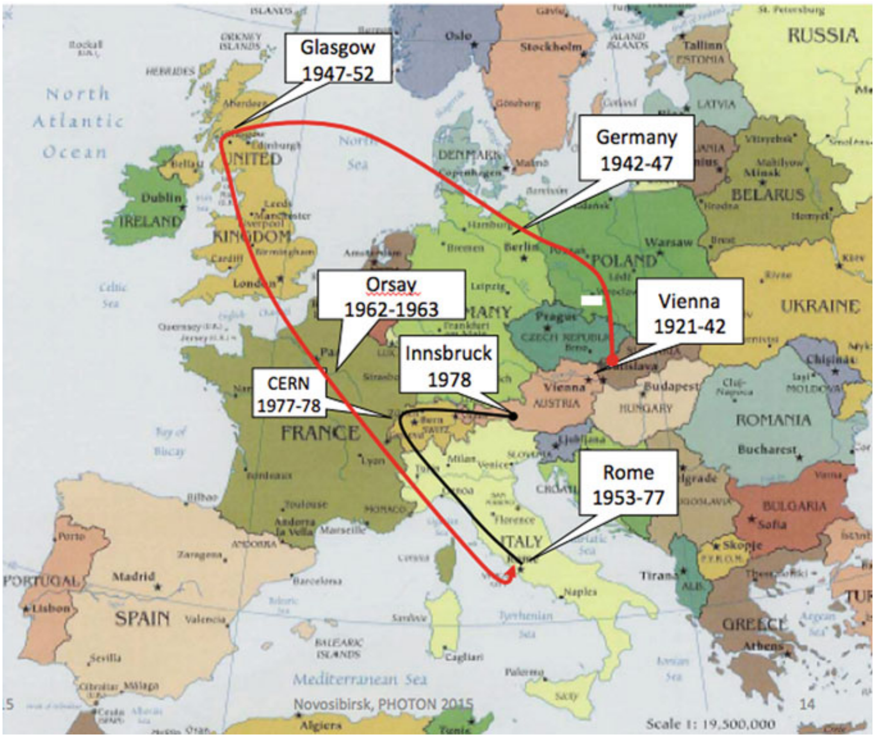

Fig. 15.2 A cartoon showing Touschek's life journey through Europe, from his birth in Vienna in 1921, to his death in Innsbruck in 1978

collection of documents from other physicists, now existing in the Sapienza University of Rome–Physics Department Archives, https://archivisapienzasmfn.archiui.com. Thus, to a large extent, these archives, which are continuously enriched as the Rome institute professors retire, represent one more legacy coming from Bruno to the institution which welcomed him in 1953.

In those years, tributes to Bruno Touschek appeared in the context of the birth of electron-positron colliders, as from Fernando Amman [4], a protagonist in the development of the particle accelerator science in Italy. In 1987, the Bruno Touschek Memorial Lectures brought to Frascati National Laboratories a roster of the scientists from the international particle community, with novel contributions to Bruno Touschek's memory [5].[4] In the years to follow, Bruno's memory was kept alive by his close friend and collaborator Carlo Bernardini [6] through writings and public talks, while interest in Touschek's life increased alongside planning for new particle colliders [7]. Much more has since been published about Bruno Touschek, as presented in [8]. The present status of the narration of Bruno's life includes three

[4] A series of circumstances delayed the publication of the Lectures contributions until the year 2004.

docu-films[5] and readings from unpublished letters sent by Bruno to his family [9]. These letters were kept and chronologically ordered by Bruno's father, Franz Xaver Touschek. After his death in 1971, they were sent to Bruno, and, later still, preserved by Touschek's widow, Elspeth Yonge Touschek. Thanks to her and these letters, large parts of Bruno's life, unknown at the time of his death, became accessible, in particular shedding light on Bruno's role in the making of Rolf Widerøe's betatron.

Now, a hundred years after Bruno's birth, from these different sources a coherent description arises of the extraordinary circumstances which led to AdA's proposal and what its construction brought to particle physics [10].

15.3 Touschek from Vienna to Hamburg and Berlin

Bruno was born in Vienna, on February 3rd 1921. His mother, Camilla Weltmann, came from a well to do assimilated Jewish family. His father, Franz Xaver Touschek, was an officer in the Austrian Army, who had fought on the Italian front during World War I [1].

His early family life, with strong artistic and literary interests, was soon disrupted by a number of tragic losses. His mother died when he was only 9 years old, and a much admired maternal uncle, a doctor and a painter, committed suicide in 1934, probably following Hitler's accession to power, as it happened to many Jewish intellectuals. A small portrait of Oskar Weltmann was found among Bruno's office papers, a testimony of profound attachment to his maternal uncle.

A further disruption in Bruno's life took place in March 1938, with the annexation of Austria to Germany. In 1931, Bruno had started his high school studies at the Piaristengymnasium, one of the best schools in Vienna, but after the annexation, a reallocation of Jewish students to different institutions took place. Bruno was a *mischling*, a mixed race person, and already a rebel, and had to leave the Piaristen before he could obtain his *Maturazeugnis*, certifying that the student had passed the required examination for university admission. He transferred to the Schottengymnasium, a Benedectine school of high renown, and from there obtained his graduation certificate from the State gymnasium, in February 1939. During such a difficult time, the impending war and the ongoing tragedy of racial discrimination against Jews matured in him a decision to emigrate. But when he tried to go to England and study chemistry at the University of Manchester, it was already too late. After passing the *Matura*, he had visited his maternal aunt, Adele, nicknamed Ada, who lived in Rome, attending some lectures at the University, with more passion than profit, and waiting for the visa for England to arrive.[6] It did not happen, or, perhaps, hesitation to leave

[5] All the three movies were authored by Enrico Agapito, *Bruno Touschek and the art of physics* with Luisa Bonolis, 2005 ©INFN-LNF, *Bruno Touschek with AdA in Orsay* with Luisa Bonolis and Giulia Pancheri, 2013 © INFN-LNF, *Soixante années d'exploration de la matière avec des accélérateurs de particules*, with Giulia Pancheri, 2017 © IN2P3.

[6] Letters to father from Rome, during March 1939.

his family and a lack of money made him return to Austria. In September, he enrolled to study physics at the University of Vienna, gaining recognition and consideration for his extraordinary intellectual capacities. He came to know the theoretical physicist Hans Thirring, who remained his mentor and friend through most of Bruno's life. Through him, Bruno would later become a friend and collaborator of his son Walter [11], in Glasgow in 1951.

Bruno's dream of becoming a physicist was shattered in June 1940, when, after brilliantly passing his university exams, Bruno was told he could no more attend classes nor frequent the library. Unwilling to give up, he spent the following academic year studying at home, or with young assistant professor Paul Urban.[7] In Fall 1941, he reapplied to be admitted to the university, but the reply was negative and, in December, he was definitively expelled. By this time, it was clear that his future studies and life in Vienna were in danger. Paul Urban, a former student of Hans Thirring, with antinazi ideas and barely tolerated by university authorities, took upon himself to find a way out. In November, anticipating the university negative decision, Touschek and Urban had visited Sommerfeld in Munich. By February 1942 a plan was laid out for Bruno to go to Germany, first to Munich and from there to Hamburg, where one of Sommerfeld's former students, Günther Jobst, could employ Bruno in his electronic firm. The plan included the possibility for Bruno to unofficially attend physics classes and Seminars at the University of Hamburg.

15.3.1 Hamburg Days and a Journey to Berlin

After leaving Vienna, Bruno spent nine months in Hamburg, before moving to Berlin at the end of the year. He would later return to Hamburg under different circumstances and renewed confidence, but his first period in Germany was very difficult.

In Hamburg, Bruno started working at the Studiengesellschaft für Elektronengeräte, but it did not take him long to be unsatisfied with the work at the laboratory, and the poor pay. News from Vienna about his grandmother's deportation to Theresienstadt during the summer of 1942 are the probable cause of a depression which gripped him at that time. After the summer, he impulsively resigned from the position at the firm, forcing himself to give up whatever he was doing and start on a new road.[8]

The move which changed his life course, took place in November 1942. After resigning from his job, with his resources at the lowest, he decided to go to Berlin and claim his compensation for the referee work he was doing for the *Chemischen Zentralblatt*. This was an odd job, for which his friends from Vienna had recom-

[7] Paul Urban 1905–1995 was later to become Professor at University of Graz and, in 1962, founded the Schladming Winter School on Theoretical Physics.

[8] From his letters home during the war, Bruno worked for the Studiengesellschaft für Elektronengeräte from March until November 1942, in partial contradiction with [1], where it is said that he worked there for a "long time", pag. 4.

mended him, knowing both of his capacity to do it as well as his need for money. Thus he left for Berlin, embarking on what he later called an *extraordinary journey*, not knowing whether he would have the money to pay for his return to Hamburg.[9] When he went back on the following day, he had collected his dues and, through some rather chancy encounter with a girl he knew from Vienna, had even secured a position in Berlin at the firm Löwe Opta, whose director, Karl Egerer, was a man in the confidence of the German military and editor of the scientific journal *Archiv für Elekrotechnik*. Two months later, assisting Egerer in his editorial task, he would come across an article which changed his life.

15.4 Bruno Touschek's Legacy to Accelerator Physics

In the brief span of his life, Bruno Touschek built, or contributed to build, three particle accelerators; first came Widerøe's 15 MeV betatron during World War II (WWII), then AdA, an electron-positron storage ring, first in the world to observe collisions in 1964, and ADONE, an electron-positron collider which reached the world highest c.m. energy when it went into operation in 1969. Of these, the least known is his contribution to Widerøe's betatron [12].

15.4.1 Touschek and His First Accelerator: Widerøe's Betatron

Rolf Widerøe had proposed the betatron principle during his university years. Later, he had tried to built an electron accelerator built on this principle, but had not succeeded. He then turned to something easier to attain, and, for his Ph.D. thesis at University of Karlsruhe in 1928, built the first linear accelerator [13]. The article describing the construction of the linear accelerator included also the equation on which a betatron would work, and had an impact on the subsequent development of particle accelerators in the US. As a matter of fact, soon after its publication, the article reached Berkeley, catching the attention of Ernest Lawrence, who always acknowledged to have been inspired by Widerøe's article to conceive and build his cyclotron, the first circular electron accelerator, in 1933. The difficulties inherent in the cyclotron reaching higher energies, were overcome by the 1941 successful operation of the first betatron at University of Illinois, by Donald Kerst, who announced to have been able to accelerate electrons up to 2.3 MeV, using Widerøe's principle. Such energy had never yet been reached in the laboratory. When Widerøe came to know of this success, he saw the possibility to construct both a 15 MeV betatron and, in a later stage, to go up to 100 MeV in electron's energy and reaffirm his priority,

[9] Touschek's letter to his friend Peter on November 29th, 1942, from Franz Touschek's collection of his son's letters.

and, in September 1942, he submitted an article with this proposal to the *Archiv für Elekrotechnik*. This article caught Touschek's attention and started his long life interest in particle accelerators.

He discussed it with his boss Egerer, and, through him, Widerøe's proposal became of interest to the military, in search of a miracle weapon. The betatron, capable to produce high energy X-rays, would then be built in the context of on-going *death ray* projects [2]. Touschek had initially been critical of some theoretical aspects in Widerøe's proposal. After his objections had been taken in full consideration, he was asked to join Widerøe's project. Given his status as half-Jewish in Nazi Germany, he had no choice but to accept, and, in fall 1943, he was hired to participate in the construction of Widerøe's 15 MeV betatron, at the C.H. Müller factory in Hamburg. It is during these early times of the project, that Widerøe shared with Bruno Touschek a novel concept for increasing the collision center of mass energy, i.e. head-on-collisions [12]. As the story goes, Touschek was not impressed at the time, but later he would when he wrote "The first time I heard of head-on-collisions" was from Widerøe.[10]

In Hamburg, Touschek learnt the art of making an electron accelerator, under the leadership of Rolf Widerøe. Although this machine is usually referred to as Widerøe's betatron, a careful reading of sources, including Bruno's letters home during the years 1943–45, shows that Bruno was involved in the planning and construction of this betatron from its very beginning until its transfer to Wrist in March 1945, ordered by the German authorities in order to save it from the arriving Allied forces.

Proof of Touschek's impact on the functioning of the 15 MeV betatron comes from many sources. Apart from being hired in the project, the first evidence is that Touschek was able to avoid being drafted to forced labor, the first step to deportation and concentration camps. Touschek's letters to his family in 1944 and 1945 mention three such summons, all avoided through appeals to General Milch and Minister Speer, on the part of his co-workers in the betatron project. In all three cases, the only successful motivation could have been that his work was important for a project of interest to the war. This could have been a ruse invented by his friends, but the situation in Germany in 1944–45 was dire enough that friendship alone could not save many lives. Instead, in my opinion, this was exactly the truth, namely that without Touschek's theoretical knowledge and exceptional physics intuition, Widerøe's betatron would not have worked. The same statement, about Bruno's indispensable contribution, i.e. that the miracle device would not function without him, is found to be the reason of the more human treatment Bruno enjoyed during his imprisonment between March 17th and mid April 1945 [1, 9].

Touschek was well aware of how important his role was in Widerøe's project. In fall 1944, he wrote to his parents that he was finally engaged in a project he could call "his own", one that could establish a world record. It should also be noticed that during these months,1944–45, Widerøe was often in Oslo, and the team went on

[10] Undated manuscript, Sapienza University of Rome–Physics Department Archives, https://archivisapienzasmfn.archiui.com.

without him. Touschek's contribution is clearly acknowledged by Rolf Widerøe, who wrote to Amaldi that Bruno did very many important calculations for the betatron group.

15.4.2 A Death-Ray Project in Hamburg and Touschek's Imprisonment

Of interest is also a mention of death-rays as 'decisive weapons' in the memories of Albert Speer, the Minister of Armament during the war, in Chap. 31 of [14]. In his reminiscences, Speer comments on wild notions flourishing as the enemy approached in the early days of Aril 1945, and a reference is made to the inventor's appeal having been rejected by the Ministry. If this sentence refers to Touschek, who was in prison in Fuhlsbüttel at the time, it would be a confirmation of his importance in the betatron project. No other death ray projects existed anymore, except for Widerøe's, and no other scientist working on a betatron project is known to have been applying for clemency or work, except for Bruno Touschek.

The appreciation of Touschek's contribution is present in the reports prepared by the US and UK occupation forces after the war [9, 15]. A direct consequence of such high opinion of Touschek's work, is that Bruno's future education was taken in charge by the British, interested in developing an accelerator program. As for the betatron, it was requisitioned and brought to Woolwich Arsenal near London for inspection and studies, its later whereabouts are unknown.

At the end of the war, Bruno was one of the few scientists in continental Europe who had a working knowledge of building a particle accelerator of an advanced type such as the betatron. Another one was clearly Widerøe, but he had gone back to Norway and was also entangled in an inquiry for collaborationism with the Germans [16]. By this time, the synchrotron principle had been discovered and new accelerators based on it were planned in the UK and, mostly, in the US. Touschek, who had been part of Widerøe's team, became of particular interest to the British in their postwar effort to develop particle accelerators. After earning his Diploma in Physics from the University of Göttingen in June 1946, and a six month period as Werner Heisenberg's assistant, Bruno was brought to the UK and enrolled in the PhD program at the University of Glasgow [17]. He was very actively participating in the British post-war accelerator program, which included building a number of synchrotrons, among them a 300 MeV in Glasgow itself, but not only. His contribution is glimpsed through his research report of the years 1947–48 and acknowledged through correspondence with Frank Goward,[11] and other notable scientists of the time.[12]

[11] F. Goward together with D.E. Barnes had demonstrated synchrotron acceleration for the first time in August 1946.

[12] See Touschek's papers in Sapienza University of Rome–Physics Department Archives, https://archivisapienzasmfn.archiui.com.

15.4.3 Bruno's Dream: To Become a Physicist

Touschek's success in proposing, and carrying through, the construction of the first electron-positron collider is not only due to the experience gained with Widerøe's betatron, but just as well on his theoretical physics capacities and insight.

In one of his letters home from Göttingen, Bruno, pressed by his father to return to Vienna, writes:

`I want to become a physicist.`[13]

and so he did. Touschek's mentors in his formation as a theoretical physicist include some of the most illustrious theoreticians of last century physics: Hans Thirring, during Bruno's first—and only—year at the University of Vienna and through both the war and post-war years, Arnold Sommerfeld, who suggested and sponsored Bruno's moving to Germany, Werner Heisenberg who had Bruno as his Assistant in post-war Göttingen, Max Born during Touschek's years in Glasgow, and Wolfgang Pauli until his passing in 1958. Many other scientists influenced Bruno's thinking and were, in return, influenced by him, and the list can be glimpsed in [1].

After leaving Vienna, where Hans Thirring had been one of Bruno's teachers, Bruno remained in close contact with his former professor. Until 1945, Bruno travelled regularly from Berlin to Vienna, and in at least one occasion, his letters home mention physics discussion with him or other Vienna physicists. It is after one such discussion, probably on the occasion of a trip to Vienna to celebrate various family birthdays (his own, his father's, and his stepmother's) all occurring between January and February, that Touschek started thinking about the working of cyclotrons and the need to apply corrections when the electron's energy became close to be relativistic [9]. The last year of the war interrupted travel across Austria and Germany, and Bruno's visits to Austria resumed only during Bruno's Glasgow years. After his Ph.D. and becoming a Nuffield Lecturer, he could finally take a real vacation to his family favourite places, in Tyrol, Fig. 15.3. He visited the Thirring family in Kitzbühel, and became friends with Hans' son Walter, with whom he would write a paper which played an important role in Bruno's thinking about infrared radiative corrections [11, 18].

While still in Vienna, Bruno had also established a connection with Arnold Sommerfeld, which remained close until Sommerfeld's death in 1951. Thanks to Bruno's having approached him about some corrections to the second volume of his famous treaty *Atombau und Spektrallinien*, Bruno's high intellectual qualities and passion for physics became known to the great scientist. A scientific correspondence ensued, a rather surprising show of intellectual courage on the part of a twenty years old physics students and the father of atomic physics. Thanks to Sommerfeld, Bruno found a way out of Vienna where marginalization and discrimination were engulfing his hopes to become a physicist, threatening his life as well.

As Bruno moved into Germany, Sommerfeld's former friends or pupils befriended him, and he could attend, unofficially, lectures and seminars by Max von Laue and

[13] Ich will ein physiker warden, January 1947, letter to father.

Fig. 15.3 At left, a sketch found among Bruno Touschek papers, probably a memory of Bruno's walking with his father in Tirolean gear, during a summer vacation, courtesy of Sapienza University of Rome–Physics Department Archives, documents provided for purposes of study and research, all rights reserved. At right, a photograph of Bruno, seated with his father and stepmother, included in one of Bruno's letters to his father, probably from a summer 1950 vacation, Family Documents, © Francis Touschek, all rights reserved

Werner Heisenberg. A close relationship with Heisenberg was developed after the war, later in Göttingen, in 1946, during the first year of the reconstruction of German science under Heisenberg's leadership. Bruno was strongly influenced by Heisenberg's theoretical work on the observer as guiding principle in physics investigations, mired in a statistical approach. Such influence is present through the correspondence between Bruno and Heisenberg, in particular about ongoing questions of analyticity of the S-matrix, which continued through the years Bruno was in Glasgow.

Max Born, then Tait professor of Mathematical Physics at University of Edinburgh, was also to have influence on Bruno's interests. Bruno was introduced to him in May 1947 by Ian Sneddon, the only other theorist in Glasgow at the time, and soon became a regular attendee of the bi-weekly Seminar Born held in Edinburgh.[14] They exchanged letters and ideas about quantum mechanics, and Touschek collaborated to prepare the appendix on weak interactions of the second edition of Born's famous book *Atomic Physics*.

In 1952, when Bruno left the UK to accept a position with INFN at the University of Rome, he had developed into a brilliant theoretical physicist, aware of his genius and ready to take his own road and to exchange ideas, at level with no other than Wolfang Pauli, visiting Rome at the time of Bruno's arrival. Through the 1950s, Pauli became Bruno's intellectual companion, sparring ideas over a glass of wine in many occasions, often meeting at conference sites, inspiring Bruno's interest in the CPT theorem [19, 20]. After Pauli's death, in December 1958, the way was open for

[14] Letter to parents, May 3rd, 1947.

Bruno to be completely on his own, and start his greatest adventure, to explore the unknown with a new type of experiment, colliding matter against anti-matter. AdA, the first electron-positron collider in the world, was to be conceived in just over one year, and came to life not long after.

15.4.4 The Making of AdA

The official date of AdA's birth is March 7th, 1960, when the scientific council of the Frascati National Laboratories (LNF) approved its construction. Once approved by the laboratories, the project ran its course towards the national agencies, and, by the end of the month, orders had started to be placed, with Touschek in charge of the project. However, the March 7th meeting, where AdA's construction was approved, had not sprung out of nowhere.

15.4.4.1 Between Rome and Frascati

According to Nicola Cabibbo [21], Touschek's first proposal for an experiment to study electron-positron collisions came up in the discussion which followed a seminar held in Rome by Wolfgang Panofsky in late '59. Records kept in the Frascati National Laboratories (LNF) also show that, on October 26 1959, Panofsky held a seminar in Frascati, entitled *Sull'acceleratore lineare da due miglia*, 'About the two mile linear accelerator' in English. The seminar probably included what Panofsky had presented at the Kiev conference in July, in particular the ongoing electron-electron project at Stanford [22]. At the end of Panofsky's seminar, whether the one in Frascati or a similar one in Rome, Touschek launched the idea to make electrons collide against positrons.

From these records, late October 1959 may be considered the starting date of AdA's conception. A confirmation comes from the official permission for Touschek to enter the Frascati laboratories, dated as October 30th, 1960 [23]. Shortly after, a group of scientists from Frascati and the Rome physics institute started working on Touschek's idea. The interesting possibilities created by electron-positron annihilation in the study of the pion form factor were explored. The discussion involved some more senior theorists such as Raoul Gatto, Marcello Cini and the American visitor Laurie Brown, and younger ones such as Nicola Cabibbo and Francesco Calogero, who had graduated with Bruno Touschek in 1958.

In the months to follow, while the Rome theorists were calculating, Bruno Touschek's attention turned to the newly built electron synchrotron in the Frascati National Laboratories, and to its potential for physics experiments. The construction of a national laboratory had been approved by INFN in 1953, under Giorgio Salvini's direction, and the construction of the synchrotron had officially started in Frascati in 1957, beginning its operation on April 4th, 1959 [24]. This is how, by February 1960, everything was in place for Touschek's idea to become reality.

Fig. 15.4 From left: Carlo Bernardini, Giorgio Ghigo and Bruno Touschek, from [27]

Cabbibbo, Gatto, Brown and Calogero had finished and submitted their work in two separate articles to *The Physical Review Letters* [25, 26] while Touschek, pressed to become head of a future theoretical physics group in Frascati, remembered his years with Widerøe's betatron, and looked at the possibility of making the synchrotron into an electron-positron collider. A meeting of the laboratory council was held in Frascati on February 17th, 1960. Two conclusions were reached: the idea of using the synchrotron to make an electron-positron experiment was rejected, but, at the same time, a proposal to build a smaller, dedicated machine to study the feasibility of such an experiment was accepted and a mandate was given to the supporters of the idea, such as Carlo Bernardini, Giorgio Ghigo, and Bruno Touschek, to prepare a proposal, Fig. 15.4. On March 7th, the proposal was accepted and AdA's construction started in April.

15.4.4.2 AdA's Adventure in Orsay

To prove the feasibility of an electron-positron collider was not an easy task. It took almost four years, during which important effects affecting the operation of particle colliders were discovered and studied. The final measurements took place at the Laboratoire de l'Accélérateur Linéare, in Orsay. This is where AdA had been taken in 1962, to take advantage of the higher injection rate obtainable with the linear accelerator, which had started functioning around the same time as the Frascati synchrotron [27]. The idea to take AdA to Orsay had been put forward by Bernardini and Touschek during a visit by Pierre Marin and Georges Charpak to Frascati in July 1961 [28]. Following the exchange of letters ad visits between the two laboratories, the transportation from Frascati to Orsay was agreed. An almost epic trip took AdA and all its support system of vacuum pumps and power batteries, to the Laboratoire de l'Accélérateur Linéare, Fig. 15.5. The French team included a young doctoral student, Jacques Haïssiski, whose *Thèse d'État* gives the best description of how AdA reached

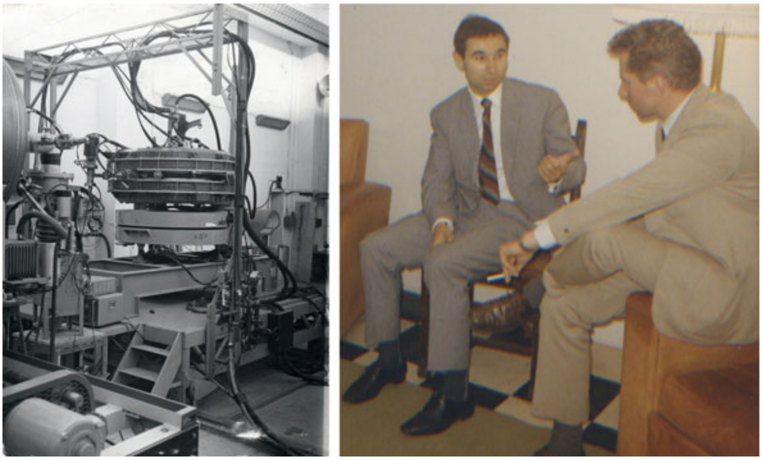

Fig. 15.5 At left, AdA in Orsay, placed in the 500 MeV hall, near the linear accelerator, and, at right, a 1967 picture of Pierre Marin and Jacques Haïssinki (in gray suit), whose 1965 thesis is the most complete description of AdA's operation in Orsay, photographs courtesy of Jacques Haïssinki

its success as proof-of-principle for electron-positron colliders to become the main tool in the exploration of high energy particle physics.

By February 1964, when the final runs were recorded in Orsay, the Touschek effect had been discovered [29] and new theoretical physics calculations had explored e^+e^- physics [30–32]. All over the world, in France with ACO, in the USSR with VEPP-2, at CERN with the ISR, in the USA with SPEAR, new particle-antiparticle accelerators had been designed and their construction approved.

15.4.5 ADONE: Touschek's Last Accelerator

ADONE, the better and more beautiful version of AdA, was Touschek's last accelerator. There exist a posthumous note by Touschek about stochastic cooling in proton-antiproton colliders, a contribution to hadron accelerators, discussed by C.Rubbia in these proceedings. It was of limited practical impact, but it highlights Touschek's ever lasting interest in matter-antimatter accelerators, [33].

Testimonies abound of Touschek's deep involvement in ADONE's beginnings, its construction and dedication to extract meaningful physics from it. Touschek had proposed the construction of ADONE in an handwritten note in November 1960, basically as soon as AdA's construction was sufficiently advanced that he could seriously suggest to build a machine with beam energy six times higher, 3 GeV in the c.m., and an eight times bigger radius. This preliminary note became a full fledged proposal, authored by Fernando Amman, Carlo Bernardini, Raoul Gatto, Giorgio Ghigo and Touschek [34].

According to Fernando Amman, director of the ADONE project, Touschek was integral part of ADONE's success. Still ADONE was a much bigger enterprise that AdA had been, and, as such, the responsibility, both merits and failures, belonged to many people, including political events on which Touschek had no control. Among the latter, between 1963, when ADONE's construction was formally approved by INFN, and 1969, when two beams circulated in ADONE, two interruptions occurred, both to leave long lasting consequences. The first is the so called Ippolito affaire, *il caso Ippolito*, taking place in 1964, and signalling a stop in the road to nuclear energy independence in Italy. Felice Ippolito was General Director of CNEN, the national agency in charge of funding nuclear energy related activities, in particular the Frascati laboratories. His arrest and inquiry on accusation of mismanagement and corruption almost stopped ongoing work in Frascati. The second interruption was the 1968 strike in the laboratory, following student and workers unrest, in the Italian universities and in the industrial sector. In the University of Rome, confrontations between students and the faculty led to slow down of scientific exchanges and affected Bruno's person as well,[15] as reflected in his many drawings of the time.

In Frascati, activities at ADONE restarted in 1969, and the many years work was rewarded by the first observation of abundant hadronic particle production, whose impact on particle physics is described by other contributions to these proceedings, by Mario Greco and Giorgio Parisi, in particular.

Unfortunately, the various problems which had marred ADONE's progress since 1967, delayed its operation. As the 1970s rolled in, other colliders had come into operation and made some of the discoveries which ADONE had been conceived to do. Still, ADONE's operation brought many interesting results in addition to first observing multihadron production: a new resonance, $\rho(1600)$, was studied and discovered, photon-photon collisions were observed just after similar observations with VEPP-2, and, in November 1974, ADONE gave an immediate confirmation to the American discovery of the J/Ψ resonance, with results published together with those from Brookhaven and Stanford [35]. Defying the odds of a late start, ADONE made Italy a member of the international particle accelerator community.

An indirect consequence of ADONE's construction is the rise of an important research field in theoretical particle physics, infrared resummation, which occupied Touschek's attention soon after ADONE's approval. Touschek saw that extraction of information for particle physics experiments at high energies such as those proposed for ADONE, needed to disentangle the process of interest from the radiation effects which always surround charged particle collisions. Thus, as early as 1963, he embarked on the problem of the "administration of the infrared radiative corrections" to ADONE's future experiments. His ideas influenced the young researchers in his Frascati group with a lasting effect on theoretical work in Frascati and Rome, and his approach to the problem may still be of interest.

[15] See also Ugo Amaldi's contribution to these proceedings.

15.5 Touschek's Way to the Infrared Catastrophe

Just like AdA's proposal had its roots in Bruno's past, his treatment of infrared corrections to ADONE's experiments can be traced back to his 1951 work with Walter Thirring, Heisenberg's influence on the role of the observer, and his own interest in radiation damping effects in electron's accelerators.

The problem of a divergence when photons of infinitely small energy are emitted by charged particles during their acceleration, was well known to Bruno Touschek, who had studied Bloch and Nordsieck [36] and formulated their work in covariant formalism during a 1950 visit by Walter Thirring [11]. Its solution had been found by Schwinger, who showed that the divergence arising from real photon emission was cured by cancelling a corresponding one from virtual photon exchanges, and hypothesized the exponentiation of a finite correction term. Further elaborated by Brown and Feynman, Lomon, Erikson, and fully treated to all orders in perturbation theory by Jauch and Rohrlich, Yennie, Frautschi and Suura, Schwinger's guess was confirmed, showing the exponentiation of a factor, which modifies the observed cross-section [37].

As always, Touschek's own way to the problem was very original. Two physics ideas join in Touschek's proposal to deal with the flood of soft photons which accompany a charged particle process, one is the Bloch and Nordsieck's result that the process of emission follows a Poisson distribution, the other that the observable quantity in reactions between charged particles is the energy-momentum loss due to soft photon emission. The emphasis on observable quantities shows the influence of Heisenberg's thinking about the concept of the observer as protagonist in physical observations. After his Diploma in June 1946 and until March 1947, Touschek had been Heisenberg's assistant in Göttingen for six months, and had continued to work on problems of Heisenberg's interest, such as analyticity properties of the S-matrix, after joining the University of Glasgow on April 1st 1947.[16]

Bloch and Nordsieck's fundamental theorem about the quantum theory of the emission of soft photons from a classical source had shown that the distribution of the number of soft photons is given by a Poisson distribution. Assuming a discrete momentum spectrum for the photons, corresponding to the quantization of the electromagnetic field in a finite conducting box, the probability that a scattering process among charged particles gives $n_{\mathbf{k_1}}$ photons with momentum $\mathbf{k_1}$, $n_{\mathbf{k_2}}$ photons with momentum $\mathbf{k_2}$, is given as

$$P(\{n_{\mathbf{k}}\}) = \Pi_{\mathbf{k}} \frac{[\bar{n}_{\mathbf{k}}]^{n_{\mathbf{k}}}}{n_{\mathbf{k}}!} e^{-\bar{n}_{\mathbf{k}}} \qquad (15.1)$$

where $\bar{n}_{\mathbf{k}}$ the average value of number of photons of momentum $\mathbf{k}$.

To this result, with which he was utterly familiar, Touschek added the constraint of energy momentum conservation, via a four dimensional $\delta-$ function which would

[16] See Heisenberg-Touschek correspondence at Deutsches Museum Archives, in Munich, and Sapienza University of Rome–Physics Department Archives.

select the distributions $\{n_{\mathbf{k}}\}$ with the right energy-momentum loss K. With these two starting points, the probability of soft photon emission with overall energy-momentum K_μ in the infinitesimal interval between K_μ and $K_\mu + d^4K$ could be written as [18]:

$$d^4P(K) = \sum P(\{n_{\mathbf{k}}\})d^4K\ \delta_4\left(K - \sum_{\mathbf{k}} kn_{\mathbf{k}}\right) = \sum \Pi_{\mathbf{k}} \frac{[\bar{n}_{\mathbf{k}}]^{n_{\mathbf{k}}}}{n_{\mathbf{k}}!} e^{-\bar{n}_{\mathbf{k}}} d^4K\ \delta_4\left(K - \sum_{\mathbf{k}} kn_{\mathbf{k}}\right) \tag{15.2}$$

where the sum $\sum$ is carried out over all the values of all the $n_{\mathbf{k}}$. By virtue of the δ-function, the sum and the product can be exchanged, and the result leads to the exponentiation of a regularized photon spectrum, namely

$$d^4P(K) = \frac{d^4K}{(2\pi)^4}\int d^4x\ e^{-iK\cdot x} exp\left\{-\sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1 - e^{ik\cdot x}]\right\} \tag{15.3}$$

If the boundary conditions allow, as is the case in QED, one can take the continuum limit of (15.3). After integration over the unobserved variables, one can follow the calculation outlined in [18] and obtain the probability distribution for observing a total energy loss $K_0 = \omega$ as

$$N(\beta)dP(\omega) = \beta\frac{d\omega}{\omega}\left(\frac{\omega}{E}\right)^\beta \quad \text{with} \quad \mathrm{N}(\beta) = \frac{\int_0^\infty \mathrm{dP}(\omega)}{\int_0^{\mathrm{E}} \mathrm{dP}|(\omega)} = \gamma^\beta\Gamma(1+\beta) \tag{15.4}$$

Touschek's derivation of (15.4) was based on positivity and analyticity of the energy distribution, starting from semi-classical considerations and statistical mechanics formalism, and confirmed results already well known since the 1950s. But Touschek's elegant treatment made it physically transparent. Then he added one of his jokes to the problem, calling *Bond factor* the quantity $\beta(E)$, whose numerical value at ADONE's energy was 0.07. This results was also obtained by Mario Greco and Giancarlo Rossi, using a coherent state approach, later extended to gauge theories.[17]

Interest in (15.3) did not stop at the energy distribution. Throughout 1966, Touschek, Etim and myself spent many months in trying to derive a closed form for the momentum distributions, obtainable from (15.3) after integration over the energy variable. We finally had to resort to an approximation, but this brought a long life to Touschek's thinking about the infrared problem. Indeed, his insistence to go beyond the energy distribution arose the interest in the Frascati group. The transverse momentum distribution, obtained from (15.3) as

$$d^2P(\mathbf{K}_\perp) = \frac{d^2\mathbf{K}_\perp}{(2\pi)^2}\int d^2\mathbf{b}\ e^{i\mathbf{K}_\perp\cdot\mathbf{b} - \int d^3\bar{n}(\mathbf{k})[\mathbf{1}-\mathbf{e}^{-\mathbf{i}\mathbf{k}_\mathbf{t}\cdot\mathbf{b}}]} \tag{15.5}$$

was applied to study hadronic processes, as in the case of a constant coupling in the infrared limit [38]. In 1978, a landmark calculation by Giorgio Parisi and Roberto

[17] See Greco and Rossi's contributions to this volume.

Petronzio [39] obtained the transverse momentum distribution of Drell-Yan pairs arising from soft gluon emission using perturbative QCD and the asymptotic freedom expression for the strong coupling constant.[18] Other studies by members of the Frascati group, which had expanded to include theorists Fabrizio Palumbo and Calogero Natoli, followed, as did a calculation of the W-boson transverse momentum [40].

The potency of Touschek's way does not only rely on phenomenological applications. Standing mainly on its applicability to different types of interactions, it also has the possibility to extend it to the calculation of the zero energy mode in some theories [41]. The calculation addresses what happens in Abelian gauge theories in passing from the discrete to the continuum limit in (15.3) in the energy distribution case. The limit can be taken by first separating the zero energy mode from all the others, and then examining the zero mode in light of the boundary conditions in the theory under consideration. This leads to the overall energy distribution to be written as

$$dP(\omega) = \frac{d\omega}{2\pi} \int dt\, e^{i\omega t - h(t)}, \quad h(t) = h_0(t) + \bar{h}(t) \tag{15.6}$$

with $\bar{h}(t)$ having the usual expression

$$\bar{h}(t) = \int d^3\bar{n}_k[1 - e^{-ik\cdot t}] \tag{15.7}$$

and

$$h_0(t) = \bar{n}_0[1 - e^{-i\omega_0 t}] \approx i(\eta\tilde{\omega})t \tag{15.8}$$

where η is a dimensionless parameter proportional to the coupling constant, while $\tilde{\omega}$ is energy and mass dependent (of the emitting particles), i.e.

$$\eta = \frac{4\pi e^2}{L^3\mu^2 m} \qquad \tilde{\omega} = \frac{1}{2}m\left|\sum_i \epsilon_i \mathbf{v}_i\right|^2 \tag{15.9}$$

In this equation, m is rhe mass of the emitting particles, μ is a fictious photon mass used for the regularization procedure. The quantities L and μ depend on the way one passes from the discrete limit (in which one obtained the original resummation expression with classical statistical mechanics formalism) to the continuum, namely how the limits $L \to \infty$ (size L of the lattice), and $\mu \to 0$ are taken. In QED with the usual vanishing boundary conditions, the zero mode is killed by the measure of the integral, but it cannot be excluded that this treatment, directly derived from Touschek's approach to the infrared region, can be of relevance in other theories or in cosmology [42].

[18] Giorgio Parisi, 2021 Nobel Prize in Physics, was a member of the Frascati theory group from 1971 to 1981.

15.6 Conclusions

I have outlined Touschek's contribution to XXth century physics, through the three accelerators he built or helped building: the 15 MeV 1945 German betatron, the electron-positron colliders AdA, the first such machine ever in the world, and ADONE, where multihadron particle production first appeared. The extension of his legacy to theoretical physics in dealing with infrared phenomena was also outlined.

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