High order fluctuations of conserved charges in the continuum limit

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Abstract. We calculate the grand canonical fluctuations of the net baryon number and net strangeness in the region of chiral crossover from lattice QCD. We continuum extrapolate the results in a finite volume using the new 4HEX discretization scheme up to χ_6^B and also for χ_8^B at a single temperature. Our results contradict recent estimates at a finite lattice spacing. See Ref. [1] for the full version of this work.

1 Introduction

Fluctuations of conserved charges are popular observables to study the phase structure of QCD both in experiment, such as the RHIC Beam Energy Scan program, and in QCD and its various effective models [2–5]. The appeal of higher order fluctuations as experimental observables was boosted by their sensitivity to near-criticality [6]. Lower orders, in turn, were used to extract freeze-out parameters in the cross-over region of the QCD chiral transition [7]. Under ideal circumstances, such as the instantaneous hadronization of a non-fluctuating, sufficiently large volume of equilibrated plasma, the fluctuation data could be predicted by non-perturbative equilibrium methods, such as lattice QCD.

In a grand canonical ensemble fluctuations are also the expansion coefficients in chemical potential. Their use is ubiquitous in finite density lattice QCD studies, where direct simulations at finite μ_B are extremely costly, and a workaround with Taylor expansion or other analytical techniques is sought for. For example, the baryon fluctuations, including variance and kurtosis, allow for a next-to-leading order extension of the QCD equation of state to finite chemical potential.

To study the analytic structure of equilibrium QCD lattice simulations offer valuable insight. The radius of convergence and other analytic estimators in QCD [8, 9] and effective models [10] can hint for the phase structure in a parameter range, not accessible with direct methods. Recent work [11] finds hints for a critical end-point based on existing lattice data on fluctuations. In this work, we compute and extrapolate these same fluctuations, and find disagreement with some results available to the community [12].

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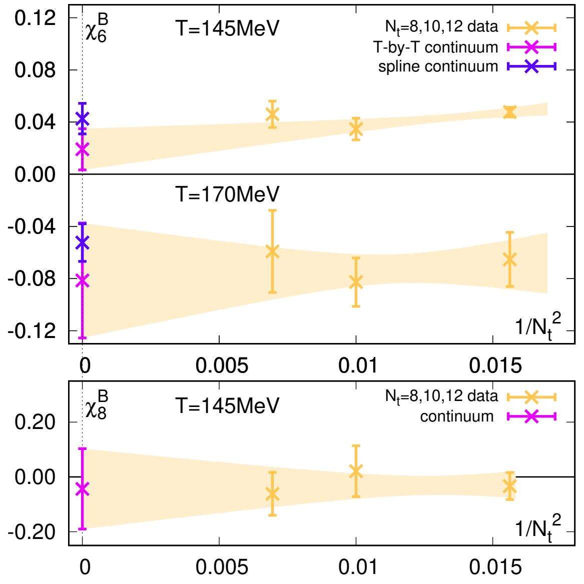

Figure 1. Examples for the continuum extrapolation for high order fluctuations based on lattices $16^3\times8$, $20^3\times10$ and $24^3\times12$. The magenta error bars are the direct extrapolation of the results shown, the blue error bar is the result of the full analysis that takes into account the variations in the scale setting as well as the effect of spline interpolation.

2 Methods

We simulate QCD on the lattice with the new 4HEX staggered action. Detailed information on the definition, tuning and scale setting is found in Ref. [1].

In this work we generated ensembles at vanishing chemical potential in a temperature range between 130 and 200 MeV on three lattice spacings $N_t = 8$, 10 and 12. The continuum extrapolations based on these resolutions show linear behaviour with $1/N_t^2$, as shown in Fig. 1. The simulation volume was selected to be $L^3 = (2/T)^3$. This is smaller than the volumes used in other works e.g. Refs. [12, 13] and we will discuss finite size effects in the next section.

Fluctuations are derivatives of the grand canonical thermodynamic potential with respect to the chemical potential, which can conveniently be expressed as derivatives of the pressure:

$$\chi_n^B = \frac{\partial^n p/T^4}{(\partial\mu_B/T)^n}\,, \qquad \chi_n^S = \frac{\partial^n p/T^4}{(\partial\mu_S/T)^n}\,, \qquad \chi_{nk}^{BS} = \frac{\partial^{n+k} p/T^4}{(\partial\mu_B/T)^n(\partial\mu_S/T)^k} \tag{1}$$

Here μ_S stands for the chemical potential associated with strangeness, and μ_B is coupled to the net baryon number. We will refer to the total baryon derivatives under the constraint $\chi_1^S(\mu_B,\mu_S) \equiv 0$ as p_n. These are the Taylor coefficients of the QCD pressure under the strangeness neutrality condition.

The chemical potential is defined on a discretized space-time using the exponential definition [14] that guarantees the absence of renormalization on the fluctuation observables. This is different from the approach in other works, e.g. [12]. The exponential definition is also suitable to connect fluctuation observables to those at imaginary chemical potential, though we do not exploit that possibility in this work.

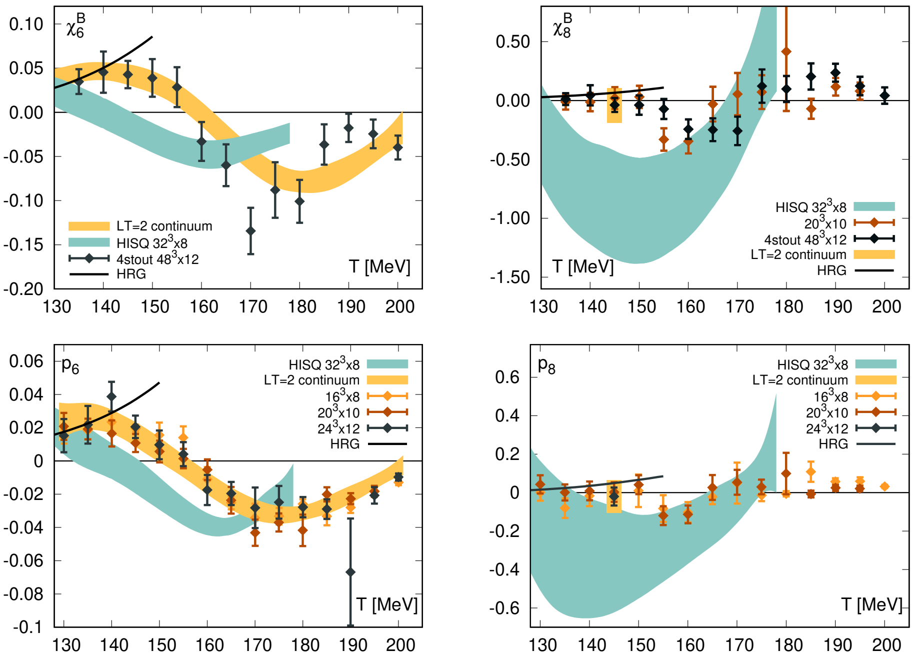

Figure 2. The sixth and eight order fluctuation coefficients and their continuum extrapolation (top row). The bottom row shows the strangeness-neutral expansion coefficients of the QCD pressure. The plots also show two earlier result form the literature: the label `4stout`-$48^3 \times 12$ refers to our earlier large volume study at a finite lattice spacing using imaginary chemical potentials [13]. The blue bands are the spline interpolations of the HISQ result on a $32^3 \times 8$ lattice [12].

To obtain the fluctuations on the finest lattice we followed the standard procedure. For the two coarser lattices we made a shortcut: We computed all eigenvalues of the fermion matrix in the reduced matrix formalism [8, 15]. These eigenvalues give direct access to all μ-derivatives of both the light and strange quark flavors. We have checked that this shortcut gives identical result to the standard approach, provided the standard approach uses a sufficient number of stochastic sources. For the considered volume the eigenvalue method is also more efficient in terms of computer time.

3 Results

We simulated three ensembles ($16^3 \times 8$, $20^3 \times 10$ and $24^3 \times 12$) for a set of temperatures. The temperatures are aligned according to the f_π scale setting. The continuum extrapolation is performed with a different, gradient-flow base scale as well. For this the result is interpolated using cubic splines. These splines combine the statistics from nearby data points at each temperature and result in a lesser error bar at the cost of correlations between temperatures. We show the continuum extrapolations based on these interpolators as a yellow band in Fig. 2.

There is a striking incompatibility between the results in Fig. 2. Our earlier large volume result [13] shows agreement with this work, hinting for a good control of volume effects for $T \leq 145$ MeV. There are visible differences due to smaller volume at higher temperatures. The finite lattice spacing HISQ result is in disagreement beyond the observed finite volume effects, both for χ_6^B and χ_8^B. This is especially relevant if we consider the slopes in temperature and the qualitative agreement vs. disagreement with the Hadron Resonance Gas prediction. While we cannot comment on the HISQ result, we stress that there is a difference in the definition of the chemical potential: while we use exactly conserved charges, the HISQ result is less understood both in terms of renormalization and lattice spacing effects.

In summary, we have shown the first continuum extrapolated results on fluctuations beyond the fourth order. This was possible through the introduction of a new staggered action that reproduces the QCD meson spectrum better than any other staggered schemes. Further details and observables can be found in Ref. [1].

Acknowledgements. The project was supported by the BMBF Grant No. 05P21PXFCA. This work is also supported by the MKW NRW under the funding code NW21-024-A. Further funding was received from the DFG under the Project No. 496127839. This work was also supported by the Hungarian National Research, Development and Innovation Office, NKFIH Grant No. KKP126769. This work was also supported by the NKFIH excellence grant TKP2021_NKTA_64. D.P. is supported by the UNKP-23-3 New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund. The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. (www.gauss-centre.eu) for funding this project by providing computing time on the GCS Supercomputers Juwels-Booster at Juelich Supercomputer Centre and HAWK at Höchstleistungsrechenzentrum Stuttgart.

References

[1] S. Borsányi, Z. Fodor, J.N. Guenther, S.D. Katz, P. Parotto, A. Pásztor, D. Pesznyák, K.K. Szabó, C.H. Wong (2023), 2312.07528
[2] P. Isserstedt, M. Buballa, C.S. Fischer, P.J. Gunkel, Phys. Rev. **D100**, 074011 (2019), 1906.11644
[3] R. Poberezhnyuk, V. Vovchenko, A. Motornenko, M.I. Gorenstein, H. Stoecker, Phys. Rev. **C100**, 054904 (2019), 1906.01954
[4] W.j. Fu, X. Luo, J.M. Pawlowski, F. Rennecke, R. Wen, S. Yin, Phys. Rev. D **104**, 094047 (2021), 2101.06035
[5] V. Koch, M. Marczenko, K. Redlich, C. Sasaki (2023), 2308.15794
[6] M. Stephanov, Phys.Rev.Lett. **102**, 032301 (2009), 0809.3450
[7] P. Alba, W. Alberico, R. Bellwied, M. Bluhm, V. Mantovani Sarti et al., Phys.Lett. **B738**, 305 (2014), 1403.4903
[8] M. Giordano, K. Kapas, S.D. Katz, D. Nogradi, A. Pasztor, Phys. Rev. D **101**, 074511 (2020), 1911.00043
[9] P. Dimopoulos, L. Dini, F. Di Renzo, J. Goswami, G. Nicotra, C. Schmidt, S. Singh, K. Zambello, F. Ziesché, Phys. Rev. D **105**, 034513 (2022), 2110.15933
[10] O. Savchuk, V. Vovchenko, R.V. Poberezhnyuk, M.I. Gorenstein, H. Stoecker, Phys. Rev. C **101**, 035205 (2020), 1909.04461
[11] G. Basar (2023), 2312.06952
[12] D. Bollweg, J. Goswami, O. Kaczmarek, F. Karsch, S. Mukherjee, P. Petreczky, C. Schmidt, P. Scior (HotQCD), Phys. Rev. D **105**, 074511 (2022), 2202.09184
[13] S. Borsanyi, Z. Fodor, J.N. Guenther, S.K. Katz, K.K. Szabo, A. Pasztor, I. Portillo, C. Ratti, JHEP **10**, 205 (2018), 1805.04445
[14] P. Hasenfratz, F. Karsch, Phys.Lett. **B125**, 308 (1983)
[15] A. Hasenfratz, D. Toussaint, Nucl. Phys. **B371**, 539 (1992)