Violation of Third Law of Black Hole Thermodynamics

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Abstract

We derive the black hole solutions in higher curvature gravitational theories and discuss their properties. In this talk Lovelock theory is mainly investigated, which includes cosmological constant, Einstein-Hilbert action, and Gauss-Bonnet term as its lower order terms. Among the solutions, there are solutions which may become extreme solutions with zero temperature through physical processes. This may be a counterexample of the third law of black hole thermodynamics. We also discuss whether an extreme black hole is formed from a regular spacetime by considering collapse of a shell.

1 Introduction

Black holes are characteristic objects to general theory of relativity. Recent observational data show the existence of one or more huge black holes in the central region of a number of galaxies. While over the past decades much concerning the nature of black hole spacetimes has been clarified, a good many unsolved problems remain. One of the most important ones is what the final state of black-hole evaporation through quantum effects is. The mid-galaxy supermassive black holes are certainly not related to this problem; however, it has been suggested that tiny black holes, whose quantum effect should not be neglected, could be formed in the early universe by the gravitational collapse of the primordial density fluctuations. Black holes may become small enough in the final stage of evaporation enough for quantum aspects of gravity to become noticeable. In other words, such tiny black holes may provide a good opportunity for learning not only about strong gravitational fields but also about of the quantum aspects of gravity.

Up to now many quantum theories of gravity have been proposed. Among them superstring/M-theory formulated in the higher dimensional spacetime is the most promising candidate. So far, however, no much is known about the non-perturbative aspects of the theory have not been To take string effects perturbatively into classical gravity is one approach to the study of the quantum effects of gravity.

We focus on the n-dimensional action with the Gauss-Bonnet terms for gravity as the higher curvature corrections to general relativity. The Gauss-Bonnet terms naturally arise as the next leading order of the α'-expansion of superstring theory, where α' is inverse string tension [1], and are ghost-free combinations [2]. The black hole solutions in Gauss-Bonnet gravity were first discovered by Boulware and Deser [3]. Since then many types of black hole solutions have been intensively studied.

In this paper, we investigate the third law of the black hole thermodynamics. In general relativity it is shown that the the 3rd law hold under the following conditions [4]; the energy-momentum tensor of infalling matters is finite, and the weak energy condition is hold in the neighborhood of outer apparent horizon. In Gauss-Bonnet gravity, however, there are exotic types of black hole solutions [5], they may be the first counter examples to the third law. As the first step, we consider the collapse of a thin dust shell and formation of the extreme black hole solution with a degenerate horizon.

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2 Black Hole Solutions

We start with the following n-dimensional action

$$S = \int d^n x \sqrt{-g} \left[\frac{1}{2\kappa_n^2} (\mathcal{R} - 2\Lambda + \alpha \mathcal{L}_{GB}) \right], \tag{1}$$

where $\mathcal{R}$ and ${}^{(n)}\Lambda$ are the n-dimensional Ricci scalar and the cosmological constant, respectively. $\kappa_n := \sqrt{8\pi G_n}$, where G_n is the n-dimensional gravitational constant. $\mathcal{L}_{GB} := \mathcal{R}^2 - 4\mathcal{R}_{AB}\mathcal{R}^{AB} + \mathcal{R}_{ABCD}\mathcal{R}^{ABCD}$ is Gauss-Bonnet Lagrangian, and $\alpha \geq 0$ is the coupling constant of the Gauss-Bonnet term. This type of action is derived from superstring theory in the low-energy limit. We assume the static spacetime with the following line element:

$$ds^2 = -f(r)e^{-2\delta(r)}dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{n-2}^2, \tag{2}$$

where $d\Omega_{n-2}^2 = \gamma_{ij}dx^i dx^j$ is the metric of the $(n-2)$-dimensional Einstein space with the volume Σ_{n-2}^k.

The solution of the gravitational equations is obtained [3, 5] as

$$f = k + \frac{r^2}{2\tilde{\alpha}}(1 + \epsilon x), \qquad \delta \equiv 0, \tag{3}$$

where we have defined $\tilde{\alpha} := (n-3)(n-4)\alpha$, $\Lambda = -(n-1)(n-2)/2\ell^2$,

$$x := \sqrt{1 + 4\tilde{\alpha}\left(\frac{\tilde{M}}{r^{n-1}} - \frac{1}{\ell^2}\right)}, \qquad \tilde{M} := \frac{2\kappa_n^2 M}{(n-2)\Sigma_{n-2}^k}. \tag{4}$$

The integration constant M is the mass of the black hole. There are two families of solutions which correspond to $\epsilon = \pm 1$.

We focus on the family of solutions with the following parameters: $n \geq 6$, $\ell^2 = 1$, $k = -1$ and $\epsilon = +1$. Fig. 1 shows the M-r_h diagram of the solution. The curve of horizon is obtained by the condition $f(r_h) = 0$. When the mass parameter vanishes $\tilde{M} = 0$, the spacetime is pure vacuum expressed by Eq. (3) with $x = x_0 := \sqrt{1 - 4\tilde{\alpha}/\ell^2}$. For a well-defined theory, the condition $4\tilde{\alpha} \leq \ell^2$ should be satisfied. The pure vacuum solution has a black hole event horizon. However, the center is not singular but regular and spacelike. For $0 < \tilde{M} < \tilde{M}_{ex}$, the solution has a black hole and an inner horizons. The positive-mass solutions have a timelike central singularity. For $\tilde{M} = \tilde{M}_{ex}$, the solution has a degenerate horizon and represents the extreme black hole spacetime. For $\tilde{M} > \tilde{M}_{ex}$, the solution has no horizon and represents the spacetime with a globally naked singularity.

3 Motion of the Thin Dust Shell

We define the trajectory of the $(n-1)$-dimensional dust shell as $t = t(\tau)$ and $r = R(\tau)$, where τ is the proper time on the shell. The induced metric is

$$ds^2 = -d\tau^2 + R(\tau)^2 d\Omega_{n-2}^2. \tag{5}$$

Since it is shown that there is the generalized Birkhoff's theorem in Gauss-Bonnet graivty [6], we can employ the generalized thin shell formalism [7, 8]. The junction condition at the shell is

$$[K_{\mu\nu}]_\pm - h_{\mu\nu}[K]_\pm + 2\alpha\Big(3[J_{\mu\nu}]_\pm - h_{\mu\nu}[J]_\pm - 2P_{\mu\rho\nu\sigma}[K^{\rho\sigma}]_\pm\Big) = -\kappa_5^2 \tau_{\mu\nu}, \tag{6}$$

where

$$J_{\mu\nu} = \frac{1}{3}\big(2KK_{\mu\rho}K^\rho{}_\nu + K_{\rho\sigma}K^{\rho\sigma}K_{\mu\nu}, -2K_{\mu\rho}K^{\rho\sigma}K_{\sigma\nu} - K^2 K_{\mu\nu}\big), \tag{7}$$

$$P_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + 2h_{\mu[\sigma}R_{\rho]\nu} + 2h_{\nu[\rho}R_{\sigma]\mu} + Rh_{\mu[\rho}h_{\sigma]\nu}, \tag{8}$$

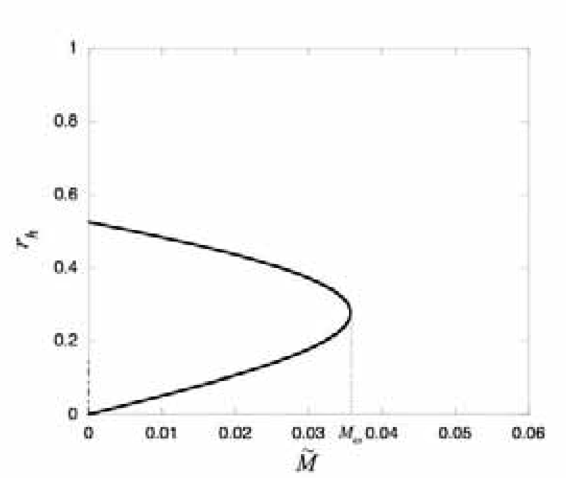

Figure 1: The $\tilde{M}$-r diagrams of the static solutions in the six-dimensional Einstein-Gauss-Bonnet-Λ system with $1/\ell^2 = 1$ (negative cosmological constant), $k = -1$, and $\epsilon = +1$. We set $\tilde{\alpha} = 0.2$. The $\tilde{M}$-r diagrams of the higher-dimensional solutions where $n > 6$ have qualitatively similar configurations.

and $\tau_{\mu\nu}$ is the energy-momentum tensor on the brane. We have introduced $[X]_\pm := X_+ - X_-$, where $X\pm$ are X's evaluated either on the plus or minus side of the shell. The shell is assumed to be dust with the surface energy density ρ. Since the surface energy density of the dust shell is conserved [8], it behaves as

$$\frac{d}{d\tau}(\rho R^{n-2}) = 0. \tag{9}$$

The proper mass of the dust shell defined as $M_s = \Sigma^k_{n-2} R^{n-2}\rho$ remains constant.

In 6-dimensional spacetime the Equation of the shell can be written in a simple way as

$$\left[D\sqrt{f + \dot{R}^2}\right]_\pm = -C, \tag{10}$$

where

$$C := \frac{\tilde{M}_s}{2R^3}, \qquad D_\pm := 1 + \frac{4\alpha k}{R^2} - \frac{2\tilde{\alpha}}{3R^2} f_\pm. \tag{11}$$

This equation can be solved with respect to $\dot{R}^2$ as

$$\dot{R}^2_\pm = \frac{1}{(D^2_+ - D^2_-)^2}\Big\{C^2(D^2_+ + D^2_-) - (D^2_+ - D^2_-)(f_+ D^2_+ - f_- D^2_-)$$
$$\pm 2C\sqrt{D^2_+ D^2_-\left[C^2 - (f_+ - f_-)(D^2_+ - D^2_-)\right]}\Big\}. \tag{12}$$

It is noted that the $\pm$ of the $\dot{R}^2_\pm$ does not mean the inner and outer value of $\dot{R}^2$ but two roots in Eq. (12).

Here we set $\tilde{\alpha} = 0.02$. Fig. 2 shows square of the velocity of the shell. The positive (negative) sign of the square root $\dot{R}_\pm = \sqrt{\dot{R}^2_\pm}$ $\left(\dot{R}_\pm = -\sqrt{\dot{R}^2_\pm}\right)$ is the speed of the expanding (collapsing) shell.

For the case with $\tilde{M}_s = 0.1\tilde{M}_{ex}$, $\dot{R}^2$ behaves as R^4 for large R. The collapsing shell from the infinity bounces at $r = 5.1648$ and expands to infinity. There is another region where the shell can move. As the shell moves inward below $r = 0.7772$, its speed decreases, and the solution curve is terminated at $r = 0.3652$. Below this radius $\sqrt{f_- + \dot{R}^2}$ takes imaginary value. This means that the shell moves spacelike. Since the radius of the extreme horizon is $r_{ex} = 0.2764$, the degenerate horizon is not formed.

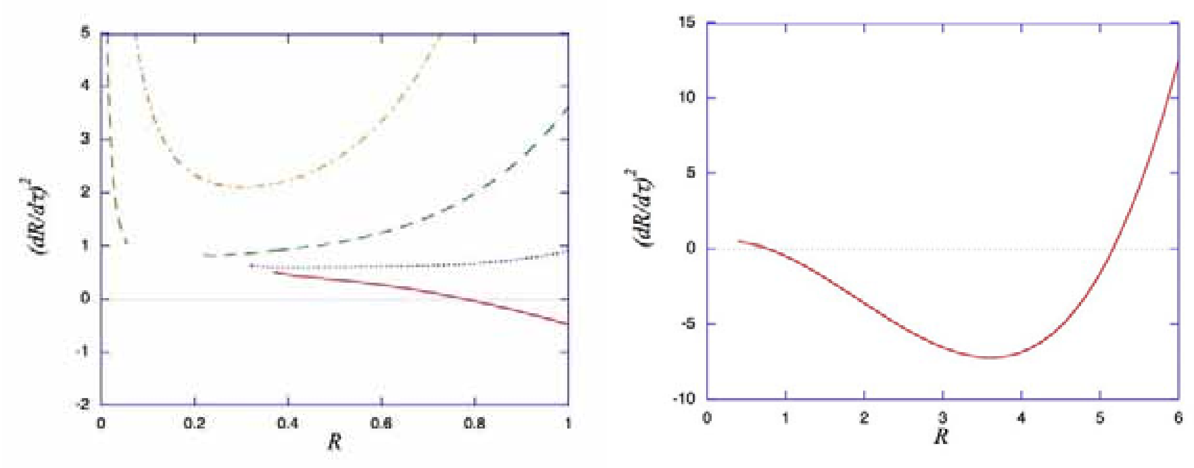

Figure 2: The motion of the thin dust shell. We set the parameters as $\tilde{\alpha} = 0.2$, $\ell^2 = 1$, $k = -1$ and $\epsilon = 1$. The interior region is pure vacuum solution with the black hole horizon at $r_h = 0.525731$, and the exterior of the shell is extreme black hole solution with the degenerate horizon at $r_{ex} = 0.276393$. The mass of the black hole is $\tilde{M}_{ex} = 0.035771$. The curves shows the velocity of the shell with $\tilde{M}_s = 0.1\tilde{M}_{ex}$ (solid curve), $\tilde{M}_s = 0.5\tilde{M}_{ex}$ (dotted curve), $\tilde{M}_s = \tilde{M}_{ex}$ (dashed curve), and $\tilde{M}_s = 2\tilde{M}_{ex}$ (dot-dashed curve).

For the case with $\tilde{M}_s = 0.5\tilde{M}_{ex}$, the region where $\dot{R}^2$ is negative disappears. The collapsing shell from the infinity does not bounce but continues to collapse to $r = 0.3154$ where the $\sqrt{f_- + \dot{R}^2}$ takes imaginary value.

For the case with $\tilde{M}_s = \tilde{M}_{ex}$, the collapsing shell from the infinity continues to collapse to $r = 0.2099$. This is inside of r_{ex}. By the generalized Birkhoff's theorem, the exterior spacetime is static extreme black hole solution. This means that the degenerate horizon is formed. However, below $r = 0.2099$, the shell moves spacelike. There is another region where the shell can move below $r = 0.05922$. However, if the shell moves in this region, the degenerate horizon exist from the beginning. This does not mean the formation of the degenerate horizon.

For the case with $\tilde{M}_s = 2\tilde{M}_{ex}$, the shell moves timelike in all the region. The shell from the infinity continues to collapse to the center and would form the central singularity. In this case the degenerate horizon is formed without any irrelevant phenomena. This can be the counter example to the third law of the black hole thermodynamics.

References

[1] D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987).

[2] B. Zwieback, Phys. Lett. B **156**, 315 (1985).

[3] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).

[4] W. Israel, Phys. Rev. Lett. **28**, 397 (1986).

[5] T. Torii and H. Maeda, Phys. Rev. D **71**, 124002 (2005).

[6] B. Whitt, Phys. Rev. **D38**, 3000 (1988).

[7] S. C. Davis, Phys. Rev. **D67**, 024030 (2003).

[8] K. Maeda and T. Torii, Phys. Rev. D **69**, 024002 (2004).