REPORT ON THE PRESENT SITUATION IN THE NON-LINEAR SPINOR THEORY OF ELEMENTARY PARTICLES

W. Heisenberg

Max-Planck-Institut für Physik und Astrophysik, München, Germany.

The non-linear spinor theory, as an attempt for a more fundamental theory of elementary particles, rests on the conviction that the complicated spectrum of elementary particles—as, for example, in old times the optical spectrum of the iron atom—must finally be deducible from an underlying natural law; and we hope it to be a simple law, whatever its mathematical form may be. From the experimental material available ten years ago, when Pauli and I worked on this problem, it looked as if the spinor equation

$$i\sigma_\nu \frac{\partial \chi}{\partial x_\nu} + \sigma_\nu : \chi (\chi^* \sigma^\nu \chi) : = 0 \tag{1}$$

(this is the form given to the equation later by Dürr) could possibly be a sufficient frame, a suitable 'master equation' for such a theory. In the meantime, much new information has been collected by the experiments, and the theory has been developed in many details. Therefore a survey of the present situation may be useful.

The first part of my talk will be devoted to the mathematical structure of the theory, the second to the symmetry properties of the equation and their consequences with regard to the experiments; in the third part I will try to compare the methods and results of this theory with those of more conventional schemes.

1. The mathematical scheme

If one wants to give a mathematical meaning to a field equation like (1), the obvious example is quantum electro-dynamics. This latter theory is undoubtedly a working theory, even if its mathematical structure is not completely known; it gives very accurate results, e.g. for the Lamb shift. During the development of this theory in the early thirties we learnt that the postulates of quantum theory and those of special relativity cannot easily be reconciled. The requirement of local causality in special relativity together with the uncertainty relations of quantum theory may cause divergencies, and the long range of the electromagnetic field (rest mass zero of the photon) introduces new problems. A price has to be paid, and it may be paid at different points. First the process of renormalization seems to be an essential part of the formalism. Furthermore one can either introduce an indefinite metric in Hilbert space, as Bleuler and Gupta have done or one can give up the manifest Lorentz covariance of the scheme and introduce the non local Coulomb forces, as in Dirac's theory of radiation; or one may introduce limiting processes as suggested by Källén. All these forms are equivalent. In any case, when the price has been paid, one can construct an approximation scheme, which in every step gives well defined, finite results. It is a characteristic feature of this perturbation theory that in every step the number of variables in the wave-functions is limited, but it increases indefinitely by going to higher and higher approximations. In principle a hydrogen atom may not only consist of proton and electron, it may also be composed of a proton, 2 electrons and 1 positron, or generally a proton, n electrons and $n - 1$ positrons. If all these infinite possibilities were to be included from the beginning, the equations of quantum electro-dynamics would probably not define a mathematical problem. But every step in the approximation scheme does define a mathematical problem, and the complete theory has a meaning, if this approximation scheme converges. A proof of convergence has not yet been given.

Taking this mathematical interpretation of quantum electro-dynamics as a model, one can try to give a mathematical meaning to Eqn. (1) in a similar fashion. If one studies the behaviour of the 2-point function

$$F(x - y) = \langle 0| \chi(x)\chi^*(y) |0\rangle \tag{2}$$

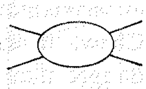
and the rôle played by it in every step of the approximation scheme, one sees that it cannot contain δ- or δ'-functions at the light cone—contrary to the conventional Schwinger functions—otherwise the occurring integrals could diverge. This cancelling of the δ- and δ'-functions can only be achieved by introducing an indefinite metric in Hilbert space. After doing this one may represent (2) by

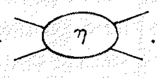
$$\langle 0| \chi(x)\chi^*(y) |0\rangle = (2\pi)^{-4} \int \rho(\kappa^2)\, d(\kappa^2) \int d^4p e^{ip(x-y)} \frac{p_\nu \bar{\sigma}^\nu \cdot \kappa^4}{(p^2)^2(p^2 - \kappa^2)}. \tag{3}$$

For a general spectrum $\rho(\kappa^2)$ this representation is actually not less general than the usual representation given by Umezawa, Kamefuchi, Källén, or Lehmann. But for a mass spectrum consisting only of a few lines, or rapidly converging at high masses, it will generally state the existence of a regularizing dipole ghost at mass zero. Therefore in low approximations, when only a few masses can be considered, this representation (3) contains a significant statement concerning the behaviour of the mass spectrum at very small masses. This statement was, as a trial, suggested by the experimental situation and by an argument discussed at this conference by Chew. If one starts from a 'master equation', like (1), there is always the danger that some particles could appear as 'real elementary particles', while from the experiments we have good reason to believe, that 'every elementary particle consists of all other particles', i.e. that actually all particles are compound systems, are 'dressed up' by interacting with the others. In this respect I would agree completely with the philosophy outlined by Chew in the first part of his talk. In case of Eqn. (1) there is obviously the danger that the neutrino could appear as 'really elementary'. Therefore this has at once been excluded by putting a dipole ghost at mass zero, thereby giving the neutrino solution the norm zero. Hence there is in this approximation no real particle of mass zero which could interact with the others. This dipole ghost could therefore, in the lowest approximation, represent the lepton part of the spectrum, which does not take part in the strong interactions; and the hope would be, that in higher approximations the dipole ghost at mass zero would gradually develop into the more complicated real spectrum of leptons, which then interact electromagnetically or weakly with other particles. The consequences of a dipole ghost have been studied in detail with the help of the Lee model.

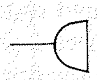
The actual approximation method can be constructed from different schemes, which have in common the fundamental assumption that, as in quantum electro-dynamics, in every finite approximation the number of variables in the wavefunctions is limited, that this number however increases indefinitely with going to higher and higher approximations. Every single step in the scheme gives well defined, finite results, and if the scheme converges it defines a solution of the problem. The convergence is expected to be much slower than in quantum electro-dynamics since in Eqn. (1) there is no weak perturbation term; again no proof of convergence has yet been given. In the early papers mostly the new Tamm–Dancoff method had been used. Recently considerable help has been obtained from the methods of many body physics, which have proved successful e.g. in the theory of solid bodies. Actually the problems of solid state physics are frequently similar to those of elementary particle physics, since 'excitons' and 'polarons' etc. are also particles 'dressed up' by interaction, and the spontaneous breakdown of symmetries by the ground state may happen equally well in both cases. It is especially the method of the Green's functions, developed by Schwinger and others, which can be used with success in both theories.

I might just mention two examples, one using the Tamm–Dancoff method the other one that of the Green's functions, in order to illustrate the practical applications. The following notation will be used:

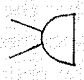
$\langle 0|\, \chi(x)\chi^*(y)\, |0\rangle =$  ; $\langle 0|\, \chi(x)\chi(y)\chi^*(z)\chi^*(u)\, |0\rangle =$ 

The irreducible part of  will be called  .

Wave functions:

$\langle 0|\, \chi(x)\, |\text{Fermion}\rangle =$ 

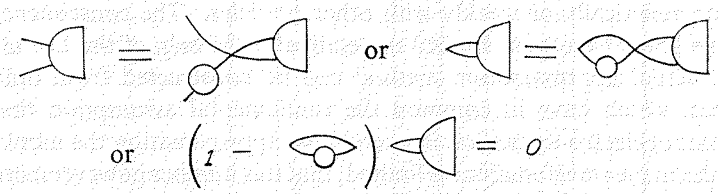
(4)

$\langle 0|\, \chi(x)\chi^*(y)\, |\text{Boson}\rangle =$ 

Green's function of mass zero:

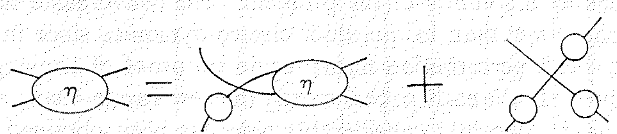
$$(2\pi)^{-4}\int d^4p\, e^{ip(x-y)}\, \frac{p_\nu \sigma^\nu}{p^2} = \text{——} .$$

The lowest Tamm-Dancoff approximation for the boson eigenvalue equation is



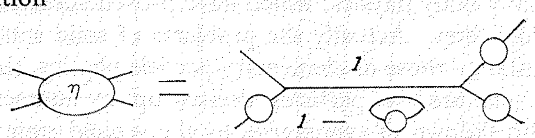
(5)

The lowest approximation for the 4-point function in the method of Green's function is



(6)

with the solution



(7)

Equation (5) gives the masses of π- and η-meson in a fair approximation. The low mass of the pion, which had been mentioned as a major problem by Chew, comes out naturally from (5); and this result rests essentially on the dipole ghost at mass zero in (3). Equation (7) shows clearly the poles of the 4-point-function at the masses of the bosons and it gives, according to Dhar and Katayama, a value for the coupling of the bosons to the nucleons in reasonable agreement with the experiments.

The method of the Green's functions allows, at least in principle, the determination of the 2-point function (3) and the spectrum $\rho(\kappa^2)$ from Eqn. (1). Unfortunately already the lowest order approximation is too complicated for an explicit solution. Still one can see from the equation, that there cannot be δ- or δ'-functions at the light cone of (3), and that the asymptotic behaviour defined by the dipole ghost may go well together with the requirements of the equation.

The flexibility of quantum electro-dynamics with respect to the point where the price has to be paid for the reconciliation of quantum theory and relativity suggests the existence of other mathematical schemes interpreting Eqn. (1), which do not introduce an indefinite metric in Hilbert space, but replace it by the concept of non-local forces. A simple scheme of this type has been suggested by Dürr. Instead of the field operator $\chi(x)$ one may introduce another field operator $\psi(x)$ connected with $\chi(x)$ by the relation:

$$\psi(x) = \Box\chi(x) \quad \text{or} \quad \chi(x) = \Box^{-1}\psi(x) + \chi_0(x). \tag{8}$$

For the 2-point function $\langle 0|\, \psi(x)\psi^*(y)\, |0\rangle$ one gets from (3)

$$\langle 0|\, \psi(x)\psi^*(y)\, |0\rangle = (2\pi)^{-4} \int \rho(\kappa^2)\, d(\kappa^2) \int d^4p\, e^{ip(x-y)} \frac{p_\nu \bar{\sigma}^\nu \cdot \kappa^4}{p^2 - \kappa^2}. \tag{9}$$

The dipole ghost at mass zero has disappeared and the spectrum $\rho(\kappa^2)$ could well be positive definite, since now the wave Eqn. (1), written in terms of $\psi(x)$, contains only a non local interaction ($\Box^{-1}$ is a non local operator), which allows the integrals in the approximation scheme to be finite, even if the 2-point function (9) contains δ- or δ'-functions at the light cone. It is true that the non local forces in the wave-equation would give rise to rather complicated problems concerning the boundary conditions, i.e. the in- and out-fields connected with (8), but there may well be a one-to-one correspondence between the solutions written in terms of an indefinite metric and those starting from the non local forces. Therefore the theory established by Eqn. (1) has probably the same kind of flexibility as quantum electro-dynamics.

2. The symmetry properties connected with Eqn. (1)

Equation (1) is invariant under the proper Lorentz group, including the two discrete operations PC or PCT, the isospin group SU_2, a gauge group which may represent the conservation of the baryonic number and finally the dilatation group. It is not invariant under SU_3 or higher groups of this type, and it does not immediately represent parity P, strangeness (or hypercharge) and lepton conservation. Furthermore the invariance of (1) under the dilatation group cannot lead to an invariance of the complete theory under this group, since an invariant 2-point function or commutator would imply a δ-function in (3), which is impossible. Therefore a mass scale must be introduced; then as a kind of compensation one can define parity, as has been pointed out by Dürr, since for a massive particle one may always define 'left-hand' and 'right-hand' wavefunctions. But in order to represent the empirical spectrum of elementary particles with regard to strangeness and SU_3 it will certainly be necessary to introduce new degrees of freedom.

The most natural way of doing this seems to be the assumption of a spontaneous breakdown of symmetry, the introduction of an asymmetrical groundstate, like in solid state physics; such a breakdown could on the one hand lead to an understanding of the electromagnetic violation of SU_2; on the other hand it would give an isospin property to the vacuum; it would therefore, without changing Eqn. (1), introduce new degrees of freedom from the vacuum, which could be just sufficient to account for hypercharge (or strangeness) and, in a very rough approximation, for SU_3 and the higher groups.

This general scheme has been followed up to some extent in recent years. The most important step was the application of the theorem of Goldstone in a somewhat generalized form (in a paper by Dürr, Yamamoto, Yamazaki and myself). In the special form given to the theorem in the first papers of Goldstone, Salam, Weinberg, Nambu and others, it states that the degeneracy of the vacuum will automatically lead to the existence of particles of mass zero, and thereby indirectly to long range forces. In more recent investigations of Higgs, Kibble and others, which have been reported by Dürr at this conference, the problem has been reversed by asking: if long range forces are assumed from the beginning, what are the consequences of a degeneracy of the vacuum? I need not discuss the answers again. In any case the investigations have revealed an intimate connexion between an asymmetry of the groundstate and long range forces. Therefore in the non linear spinor theory the problem could be formulated by asking: Is it possible that the equations for the Green's functions, which are themselves symmetrical under the isospin group SU_2, could have asymmetrical solutions, and that this asymmetry in isospace could be connected with the appearance of a boson-pole at mass zero, corresponding to the photon in its transformation properties?

This seems in fact to be the case. It turns out that a long range field of this type must be defined with regard to its transformation properties in isospace by a projection operator $\frac{1}{2}(Y + \tau_3)$, in accordance with the rule of Gell-Mann and Nishijima; this projection operator then distinguishes between neutral and charged particles. If the dipole of the charged leptons in (3) is moved from zero to finite masses—thereby indicating the asymmetry in isospace and an electromagnetic mass of the leptons—a boson-pole at mass zero and spin 1 can actually be established from (5) or (7). In that approximation, in which the baryon octet is represented by just one pole in (3), the average lepton mass is determined to be $\sim$40 MeV, and the coupling, i.e. the fine-structure constant has, according to (7), a value around $\frac{1}{120}$. Therefore it seems that the actual behaviour of nature in this field can be well imitated in the mathematical scheme of the non linear spinor theory.

At the same time the interaction between the particles and the groundstate—which may be studied e.g. by the model of a ferromagnet—can lead to the formation of strange particles i.e. particles, to which some isospin from the groundstate (a 'spurion') has been attached as has been pointed out by Biritz. In this way the hypercharge Y can be established by a gauge transformation in isospace of the groundstate. Finally the two lowest octets in the baryon and the boson spectrum seem to be a natural consequence of the symmetry in the interaction between particles and groundstate.

If this picture is correct, the three main interactions can be described in the following way. In the strong interaction the groundstate never takes up any property of the particles and vice versa; therefore in collision processes pairs of spurions and antispurions with isospin zero may be created or annihilated, but this makes no change in the groundstate. In electromagnetic processes spurion–antispurion pairs of the electromagnetic (photon) type may be transferred between the particles and the groundstate, and these pairs have with equal probability isospin 1 and 0; therefore in such processes isospin is transferred, but no hypercharge. Finally in weak interactions even single spurions (hence isospin and hypercharge) may be transferred. This would look like a process contradicting causality; but it may (according to Dürr) be compared with the Mössbauer effect where a momentum seems to be transferred at once on the whole crystal. In the latter case Weisskopf has demonstrated, that a deviation from causality cannot be observed.

The idea of using the interaction between particles and the groundstate for producing the strange particles (in a first approximation the two lowest octets) makes SU_3 appear as a secondary symmetry of dynamical origin.

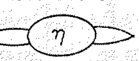
In fact, if SU_3 were considered as a fundamental (i.e. exact) symmetry of the underlying natural law disturbed by an SU_3-asymmetrical vacuum, one would either expect Goldstone particles of mass zero and of undefined symmetry in SU_3, which interact strongly with the other particles, or one would have to assume a spectrum of the type studied by Higgs and Kibble, which both seem not to agree with the actual spectrum. Therefore it looks more natural to compare the SU_3-multiplets with the optical multiplets in the atomic spectra. These optical multiplets can be considered as the result of the group $O_3 \times O_3$ (independent rotation of the orbits and the spins of the electrons), which obviously is not a fundamental group; yet it may result approximately from dynamics in some parts of the spectrum.

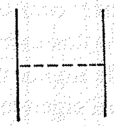
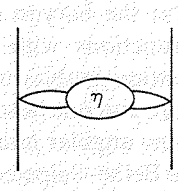
On the other hand the classification of SU_3 as a secondary dynamical symmetry has consequences of considerable importance, which can be checked by the experiments. The most important one is the non-existence of quark-particles or of non integer electromagnetic charges. Only such (approximate) representations of SU_3, which can be obtained by multiplying the octet representation with another octet representation etc. could be expected as parts of the spectrum. Furthermore the algebra of the currents should represent very accurately $SU_2 \times SU_2$, but only with considerably minor accuracy $SU_3 \times SU_3$. All these results seem to be compatible with existing experimental evidence; but one may doubt whether or not future experiments could reveal the existence of quark-particles, which then would rule out Eqn. (1).

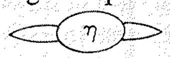
In any case the classification of SU_3 as secondary dynamical symmetry may be a controversial problem, and if there should be good arguments in favour of the opposite classification of SU_3 as fundamental symmetry, I hope that these arguments will be brought forward in the discussion.

Two more remarks should be added concerning the leptons and electrodynamics. In an approximation where the baryon octet is represented by one pole in (3), only the average lepton mass can be determined. If however the mass splitting in the octet is taken into account, there must be at least two different charged leptons corresponding in their symmetry to proton and Ξ. The particle connected with the proton gets a very small mass and should be identified with the electron, the other one gets a mass not very much smaller than the pion mass and corresponds to the muon. The muon mass has this rather high value in spite of being of purely electromagnetic origin. From this point of view one may call the leptons not 'real matter', but—as one might have done some hundred years ago—rather 'pure electricity' or 'electromagnetic singularities'.

Equation (1) has been interpreted mathematically along the lines of quantum electro-dynamics. Therefore it is not surprising that the mathematical scheme of quantum electro-dynamics seems to be contained completely in the more general scheme defined by (1). This can be seen in two ways. The operator $\chi^*(x)\sigma_\nu\chi(x)$ in (1) contains matrix elements connected with the creation or annihilation of photons (or more generally of electromagnetic field). If this part of the operator is called A_ν and treated separately from the rest of the operator, one recognizes in (1) the fundamental field equation of quantum electro-dynamics [the Dirac equation or more correctly Weyl equation for a (bare) electron without mass interacting with the electromagnetic field A_ν].

The other possibility is the translation of every special Feynman graph in quantum electro-dynamics into a corresponding graph in the formalism defined by (1). Any photon line in the electromagnetic formalism could in fact be replaced by the expression  of the non linear spinor theory. Hence e.g. the graph for electron–electron or proton–proton scattering

 should be replaced by  (9)

The latter graph contains, besides the electromagnetic forces, also short range forces which may be present besides the electromagnetic interactions, e.g. the proton–proton interaction by means of pions. Still the pole of  at mass zero guarantees the equivalence of the two graphs with respect to electromagnetic forces in that approximation in which the coupling constants on both sides are equal. Hence e.g. the well known formula for the Lamb shift expressed in terms of the charge and mass of the electron should be a consequence of (1) as well as of quantum electro-dynamics.

Almost nothing has been done in the non linear spinor theory so far with respect to the weak interactions. The violation of parity and the weakness of the interaction in β-decay obviously suggest that this phenomenon should, like the electromagnetic forces, be considered under the viewpoint of an asymmetry of the groundstate. The same should be true for the still smaller PC-violating interaction in K_2-decay and finally for gravitation. But no serious attempt has yet been made in this direction. As one argument in favour of an incorporation of weak interactions into a theory given by the Eqns. (1) and (3), one should mention the fact that the interaction term in (1) has just the same form as the interaction term in β-decay (especially with respect to the $(1 + \gamma_5)$-factor corresponding to a 2-component theory), and the universality of weak interaction.

3. Comparison with methods and results of other theories

Equation (1) contains a contact interaction which in the 'Feynman graphs' of the practical applications (5)–(7) appears as a vertex point, where four fermion lines meet. This fundamental interaction will in higher approximations lead to more indirect interactions which are produced by the exchange of other particles. In this way the formal scheme will in higher approximations gradually approach that kind of scheme which one would use in order to describe the assumptions of the bootstrap mechanism or other more phenomenological methods: Any particle may be considered as a compound system of other particles, held together by forces, which again are particles taken in the 'crossed channel'.

This is clearly seen in a recent paper by Dürr and F. Wagner, which investigates the baryon eigenvalue equation in an approximation where all wavefunctions with more than three variables are omitted. This approximation enables us already to consider the baryon as composed of one fermion and one boson, and should therefore include higher resonance states of any angular momentum. Actually it leads to an eigenvalue equation of the Bethe–Salpeter type, where the two particles, boson and fermion, are held together by an exchange force produced by the exchange of a fermion. Calculations of this type have been carried out on a phenomenological basis many years ago in a low energy approximation by Chew and Low, for the Bethe–Salpeter equations for scalar particles in case of mass zero exchange particles by Wick, Cutkowsky and others. The recent investigations of Dürr and Wagner show how the Tamm–Dancoff approximations gradually become similar to the more conventional schemes. Actually the Bethe–Salpeter equation seems to lead to a series of resonance states, where in each series the angular momenta increase by two units from one member to the next, in other words to Regge trajectories. The only difference against the more conventional schemes can be seen in the existence of the contact interaction and in the fact that some baryons can be expressed by the fundamental operator $\chi(x)$ alone, others only by the combination of at least three such operators. Whether this fact will finally have some influence on the shape of the Regge-trajectories is still an open question. The existence of the contact interaction may be connected in the bootstrap method with the problem of the behaviour of the dispersion integrals at very high energies.

The general dynamical aspects and the problems of convergence in the non linear spinor theory have recently been investigated in papers by Stumpf, Wagner, Wahl, Rampacher and others; in these papers the methods described in the first part of my talk have been applied to old problems of quantum mechanics, especially the anharmonic oscillator, in order to compare the various methods.

The numerical evaluation of a single step in the approximation procedure of the field theory has been simplified in a paper by Dürr and Wagner by the elaboration of the 'Feynman diagrams' to precise mathematical formulae. In a further development of this technique one may hope that an equation written thereby in the form of a 'Feynman diagram' can be directly translated into a programme for a computer. For the single integrals the computers have already been used by Géhéniau, Mitter and Biritz with considerable success. Hence a further extension of these methods seems quite feasible. It should be emphasized that a development in this direction will be necessary, independently of what finally the theoretical formulation of elementary particle physics will be. The degree of complication in the eigenvalue equations of this part of physics cannot possibly be smaller than that in the theory of complicated atoms or molecules, since in both cases we have typical many body problems. Therefore one will finally have to rely on computers in order to cope with this degree of complication. The adaptation of any method that has proved successful in many body physics will certainly also be useful.

The general tendencies in the bootstrap method as reported by Chew and in the non linear spinor theory are very closely related. However, at some points a theory starting from a master equation like Eqn. (1) will contain information which is not too easily added to the fundamental assumptions of the bootstrap formalism; yet they seem to be necessary to form a complete theory. The first point is that Eqn. (1) states the fundamental groups, which are—except for the Lorentz group—not stated explicitly in the bootstrap method. I cannot believe that the constraints in this formalism should suffice to determine the groups.

Then the analytical behaviour of the S-matrix elements is defined precisely by the differential Eqn. (1), while only general concepts like 'maximum analyticity' are available in the bootstrap scheme. Finally the concept of a degenerate vacuum, which is natural for a theory defined by (1), cannot without additive complications be incorporated into a pure *S*-matrix theory. Still, the general tendency of considering all particles as compound systems is common for both theories and is immediately suggested by the experiments.

Coming back to the general situation in the non linear spinor theory at the present time, it is clear that we are still very far from a complete theory on this basis. However, the outcome of the experiments during the last ten years may allow the conclusion that, when looking for a master equation for elementary particle physics, one need not look for anything more complicated than Eqn. (1).

May I make one final remark concerning a criticism which has often been expressed: that it is a too ambitious programme to try the formulation of a theory for the complete spectrum of elementary particles. Looking back on the development of atomic physics 40 years ago, it is clear that it would have been much more difficult to formulate a theory for only some part, say the triplet part, of the iron spectrum, than a theory for the complete spectrum. Therefore I feel that it would be more ambitious in our time to construct a theory only for the hadrons, or only for the leptons etc. than a theory for the complete spectrum.

Discussion on the report of W. Heisenberg

R. E. Marshak. As someone who is sympathetic to your programme to develop a unified theory of hadrons and leptons, I feel that your master equation in terms of one four-component field ('urmaterie') does not do full justice to the suggestiveness of the lepton triplet (ν_e and $\bar{\nu}_\mu$ can be represented by one 4-component Dirac spinor and the other two are e and μ). Since the leptons are weakly interacting, they seem to be reflecting the attractiveness of starting with a master equation in terms of three fields (quark model). In your theory, you must work very hard to simulate broken SU_3 symmetry results (e.g. I am not sure that you can reproduce the Gell-Mann–Okubo formula for the baryon decuplet, the condition $2I + y = 0$ (mod. 2) etc.). Also, it would be difficult to understand the universality of the weak interactions between leptons and hadrons. Finally, if you do not believe one will observe 'urmaterie', I do not see why quarks should be found.

So much by way of comment. My question is whether in your theory, the muon mass has an electromagnetic origin. In the Baker–Johnson–Willey theory, the scale is not fixed for the electron mass but they hope to determine the (m_μ/m_e) ratio. So far they have not succeeded. I wonder how you can obtain such a large (m_μ/m_e) ratio by means of electromagnetism? My own inclination would be to connect m_μ with the hypercharge effect on mass for hadrons.

W. Heisenberg. First I would prefer to speak about 4 leptons (e, μ, ν_e, ν_μ) corresponding somehow to one-half of an octet; I cannot see any strong argument for speaking of a triplet and therefore in this sense referring to SU_3. But with respect to the question whether the muon mass is purely electromagnetic the situation in our theory is as follows: in that approximation, where the vacuum is isosymmetric (and when we therefore have no electromagnetic field), all lepton masses are zero. Hence they may afterwards all be called electromagnetic. When the vacuum is considered as asymmetric and when therefore the photons appear, those leptons that interact with the photon acquire their mass. But this mass is related to the baryon masses in the sense that each baryon needs supplementation (or you may call it compensation) by one (or possibly several) lepton dipoles, to make the photon of mass zero possible. The electron is the supplement of the photon, the muon that of the Ξ. In so far these masses are determined by the baryon masses, and the mass difference Ξ-proton has its counterpart in the different masses of e and μ.

With regard to the universality of weak interactions I would like to emphasize that it is at least a very natural basis for an understanding of this universality that baryons and leptons are created by the same field operator. The universality of electromagnetism (proton and positron have exactly the same charge) is due just to this feature of the theory. On account of the Ward identity the coupling constant (the charge) cannot depend on the amount of creation operator for a special kind of fermions, which is present in the universal field-operator. Therefore their charges are exactly equal. A similar situation could occur for the weak interactions, but this problem has not been worked out yet.

H. P. Dürr. I think we should not talk too much about leptons and weak interactions in connexion with the non-linear spinor theory at the moment. Too little has been done, up to now. Perhaps in ten years from now we will know a little more, and then we can come back to these points.

I want to comment on the possibility to consider $SU(3)$ as a fundamental symmetry broken only by the vacuum state. The first impression was that on the basis of the Goldstone theorem we would have to expect the occurrence of four scalar, strange massless particles if we only consider the violation of strong interactions. These scalar massless particles should interact strongly with hadrons and two of them should be charged. These particles we do not know experimentally, and we can be pretty certain that they do not exist. However, Higgs, Kibble, Brout, Englert and others have shown how the Goldstone theorem can be invalidated to some extent by coupling in gauge fields. In this case the strong breaking of $SU(3)$ would produce four massive strange vector mesons, so to say, half the octet. Generally speaking the spontaneous breakdown of a symmetry either produces mass zero particles or incomplete multiplets. Usually the four know strange vector mesons K^* are considered to be part of a vector–meson octet, including in addition the ρ and the ω. Hence one would have to find some other candidates for the incomplete multiplet to make such an interpretation acceptable.

I want to comment on another point. Heisenberg has compared the indefinite metric in the nonlinear spinor theory with the indefinite metric which occurs in the Gupta–Bleuler formalism of quantum electrodynamics. Now, it is rather clear that the indefinite metric in q.e.d. is of a much less dangerous type than the one used in the spinor theory, in particular, we can give a straightforward prescription how it can be avoided altogether. Nevertheless, there seems to be at least some formal similarity which I may express in the following way: in q.e.d. in the Bleuler–Gupta description we introduce two redundant fields, a longitudinal field connected with quanta of positive norm and a scalar field connected with quanta of negative norm. Taking the plus and minus combinations we obtain what I will shortly call a ghost couple, i.e. two ghost fields connected with quanta of zero norm, which, however, are not orthogonal to each other. One ghost, a 'good' ghost, obeys the Lorentz condition, the other, the 'bad' ghost, however, does not. We arrive at the physical theory by projecting out the 'bad ghost' by a subsidiary condition on the infields. Then the 'good ghosts' cannot do any harm because of their vanishing norm; they do not give contributions to any matrix elements and just reflect the gauge independence of the matrix elements. One can repeat the same construction for a massless spin two field couple to a conserved source function. Here one has to introduce 10 field operators where 8 fields are redundant. It can be shown that they constitute four ghost couples of the type described above. The four 'bad ghosts' are eliminated by the four Einstein conditions.

In a similar way the dipole ghost introduced in the nonlinear spinor theory constitute a ghost couple of this type, a 'good ghost' which is an energy eigenstate and a 'bad ghost' which is not. By the condition that physical states all must be eigenstates of the energy one can eliminate the bad ghosts, and hence arrive at a physically interpretable theory, provided that the bad ghosts do not produce bound states, which is hard to check.

E. C. G. Sudarshan. I find the equality of the electron charge and the proton charge to be extremely interesting and significant: it appears to be due to the use of the same description for both kinds of particles. Could you tell me how we can be assured that the electron or μ-meson would not get transmuted into a baryon? Needless to say the conservation of baryon number is also a very significant law which should not be violated without 'due processes of law'.

H. P. Dürr. The lepton conservation, I think, is a rather interesting point in our theory, but, I feel, also quite a hazardous one, which easily may prove to lack any basis. It actually states that the superselection rules we find in nature may not all arise from group theoretical conditions (gauge invariance) but also may arise from a peculiar analytical structure. In our case, baryons and leptons are not distinguished group theoretically, they transform under the same gauge transformation, and hence have the property to cancel each other's singularities. However, the leptons are dipoles and the baryons are poles in the Green's functions. Consequently the leptons, because of their vanishing norm (good ghost) have zero interaction with all particles, except with the photons, where the norm of the states is immaterial on the basis of Ward's indentity. Hence the baryon world is only linked to the lepton world by photons, and we therefore get a separate lepton and baryon conservation, if weak interactions later on do not mess up this separation. Of course, one has to be very careful about the statement that zero norm particles are coupled to photons. If we were to use Källén's approach to q.e.d. by employing a finite mass photon in which the zero limit is taken afterwards, then zero norm particles would not couple.