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Paired Double Heavy Baryons Production in Decays of the Higgs Boson

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Abstract: Rare decays of the Higgs boson into a pair of diquarks $cc(bb)$ and $\bar{c}\bar{c}$ $(\bar{b}\bar{b})$ are studied within the perturbative Standard Model and relativistic quark model. The relativistic corrections determined by the relative motion of quarks are calculated using the production amplitude and the diquark wave functions. Numerical values of the decay widths of the Higgs boson into a baryon pair ccq (bbq) and $\bar{c}\bar{c}\bar{q}$ $(\bar{b}\bar{b}\bar{q})$ are obtained.

Keywords: Higgs boson decay; relativistic quark model

1. Introduction

The discovery of the Higgs boson [1,2] gave a new impulse to research into the Higgs sector. The study of the decays of the Higgs boson, connected with its transformation into a pair of heavy quarks or $W^{\pm}$, Z-bosons, is important for determining the corresponding coupling constants $g(HZZ)$, $g(HWW)$, $g(H\tau\tau)$, $g(H\mu\mu)$, $g(Hcc)$, and $g(Hbb)$ [3]. It can be said that the period has come for a more accurate study of both the parameters of the Higgs boson itself and the Higgs sector as a whole [4]. Experimental studies of various decay channels are constantly expanding. If initially the emphasis was on the main modes of decay into two photons, a pair of Z- and W-bosons, and a pair of leptons, now works have begun to appear in which rare decays are also sought, including, for example, processes with pair production of J/Ψ mesons. Such rare decays with the formation of bound states of heavy quarks are of interest, on the one hand, because they can be used to obtain additional information about the mechanisms of production of heavy mesons. On the other hand, interest in such decays is also due to the search for manifestations of the New Physics, which lies beyond the limits of the Standard Model. The accuracy of such experiments is still low. So far, only an upper bound on the branching ratios has been obtained [5]:

$$B(H \to J/\Psi, J/\Psi) < 3.8 \times 10^{-4}, \quad B(H \to \Upsilon(1S), \Upsilon(1S)) < 1.7 \times 10^{-3}. \tag{1}$$

Theoretical calculations of the decay of the Higgs boson through various channels began long before its discovery. Initial estimates for the width of the Higgs boson decay with the production of a pair of heavy mesons were made in the nonrelativistic approximation for some decay mechanisms in the works in [6,7]. New theoretical studies of the processes of pair production of quarkonia were carried out in [8–14], taking into account relativistic corrections and one-loop corrections. At the same time, an analysis of various decay mechanisms was performed. In recent papers [15,16], the production of doubly heavy single baryons in the decay of the Higgs boson has also been studied within the framework of the quark–gluon decay mechanism.

This work continues our previous studies [12,13] of rare exclusive decays of the Higgs boson. We consider decay processes in which a pair of diquarks (cc), $(\bar{c}\bar{c})$ is formed. Then, each of these diquarks turns into a doubly heavy baryon. Thus, in the decay of the H boson, a baryon-antibaryon pair is produced. Our calculation of the decay width is carried out

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within a nonrelativistic limit and with an account of relativistic effects of various nature. To calculate the decay width, we use the relativistic quark model (RQM), on the basis of which calculations of relativistic effects were previously carried out for various processes in [12,17]. Decays of this type are rare, and estimating the decay width is important, because it can serve as a guide for finding them in Higgs boson factories. The purpose of this work is to study the processes of pair production of heavy diquarks in the decay of the Higgs boson, since such processes have not been studied previously. Let us pay attention from the very beginning to two important points in our calculations [12,13]:

- We investigate different production mechanisms (quark–gluon and Z-boson) of the pair diquark (cc), (bb) production;
- Relativistic corrections that are determined by the relative motion of the bound quarks in the pair production amplitude and in the bound state wave functions are taken into account.

A new numerical estimation is obtained for the Higgs boson decay width into the pair baryon–antibaryon. Rare exclusive processes such as pair baryon production in H decay are important for testing the Standard Model. The production and decay of heavy quark bound states for various physical processes is an important physical task that has been studied for a long time [18]. On the basis of a relativistic quark model, we can consistently take into account relativistic effects in the processes of the production of bound states of quarks [12,17].

2. General Formalism

In the production of diquark states of the same quarks (cc) and $(\bar{c}\bar{c})$, there are two basic production mechanisms: the quark−gluon production mechanism, and Z-boson production mechanism. In the case of the quark−gluon production mechanism, the Higgs boson transforms into a heavy quark−antiquark pair. Then, after a heavy quark or antiquark emits a gluon, it can create another quark–antiquark pair. After this, heavy quarks and antiquarks can form diquark bound states with a certain probability.The Z-bosonic decay mechanism is characterized by the transformation of the original Higgs boson into a pair of Z-bosons, followed by the decay of each into a quark–antiquark pair and the formation of diquark states. These two mechanisms are shown in Figures 1 and 2. The general factor determining the order of the first contribution can be represented as $\alpha_s M_{QQ}^2/M_H^4$, where M_H is the mass of the Higgs boson and M_{QQ} is the mass of a heavy diquark. It is convenient to extract such a factor when analyzing the order of the amplitude, bearing in mind the structure of the interaction vertices and propagators. The order of the second Z-boson contribution is determined by the following factor $\alpha M_Z M_W/M_H^4$, where M_Z and M_W are the masses of the Z and W bosons. This means that both contributions must be taken into account to obtain the total decay width.

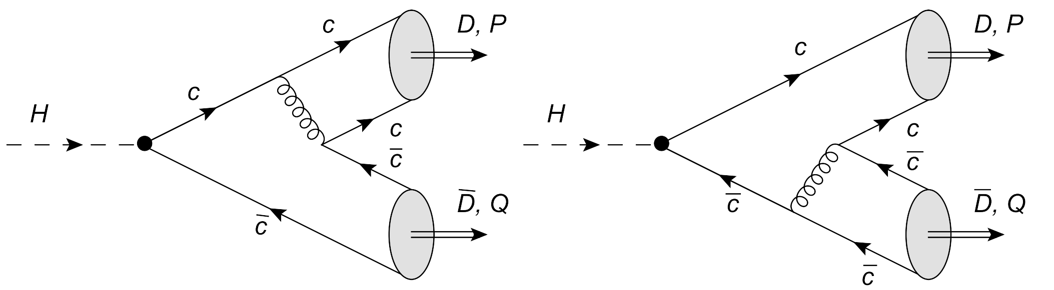

Figure 1. The pair diquark production in the quark–gluon mechanism . The Higgs boson is indicated by the dashed line, and the gluon by a wavy line.

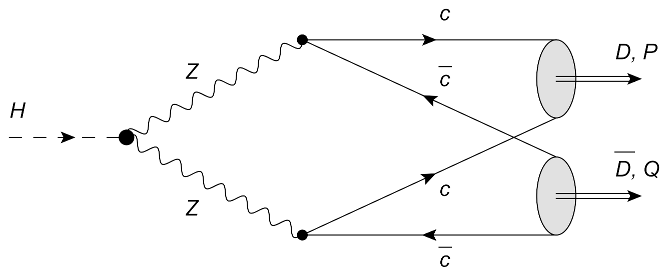

Figure 2. The pair diquark production in the Z-boson mechanism . The Higgs boson is indicated by the dashed line, and the Z-boson by a wavy line.

We further consider the diquarks (cc), (bb), which are two-particle bound states of heavy quarks c, b in an antisymmetric color state with zero angular momentum, positive parity, and spin 1 (axial vector). Between two quarks there are attractive forces in the antisymmetric color state, which lead to the formation of a bound state. The energy levels of such states can be calculated in the quark model in the same way as for quark–antiquark states. The diquark constructed from two heavy quarks (b and c) may be considered as a nucleus of double heavy baryons.

We begin with the quark–gluon production mechanism of pared diquarks shown in Figure 1. Here, it is appropriate to note that even in the case of c-quarks, relativistic effects play such an important role that all estimates of the observed quantities in the nonrelativistic approximation are ultimately invalid. Therefore, when constructing the interaction amplitudes, it is necessary to take into account all possible sources of corrections connected with the relative momenta of the quarks. On the basis of the quasipotential method, the decay amplitude is a convolution of a perturbative production amplitude of two c–quark and $\bar{c}$–antiquark pairs and quasipotential wave functions of axial vector diquarks [9,10,12,13,19]:

$$\mathcal{M}^{ij}(P,Q) = -i\delta^{ij}(\sqrt{2}G_F)^{\frac{1}{2}}\frac{\pi}{3}M_{QQ}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times$$
$$\times Tr\Big\{\Psi^{\mathcal{AV}}(p,P)\Gamma_1^{\nu}(p,q,P,Q)\Psi^{\mathcal{AV}}(q,Q)\gamma_\nu + \Psi^{\mathcal{AV}}(q,Q)\Gamma_2^{\nu}(p,q,P,Q)\Psi^{\mathcal{AV}}(p,P)\gamma_\nu\Big\}, \quad (2)$$

where M_{QQ} is the diquark mass; δ_{ij} is the color factor of the decay amplitude; p_1, p_2 are the four-momenta of the first and second c-quark in the pair forming the diquark $(\mathcal{QQ})$; and q_1, q_2 are the four-momenta of anti-quarks $\bar{c}$ in the anti-diquark $(\bar{\mathcal{Q}}\bar{\mathcal{Q}})$. For the purpose of further transformations, it is convenient to express these four-momenta in terms of the total and relative momenta, as follows:

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (pP) = 0; \qquad q_{1,2} = \frac{1}{2}Q \pm q, \quad (qQ) = 0, \quad (3)$$

A superscript $\mathcal{AV}$ for wave functions indicates the axial vector diquark $(\mathcal{QQ})$. The vertex functions $\Gamma_{1,2}$ are written out clearly below in leading order over α_s and shown in Figure 1. Below, we have presented the vertex functions $\Gamma_{1,2}$ in leading order (see also Figure 1). The generated heavy quarks c, b and antiquarks $\bar{c}$, $\bar{b}$ are not in an intermediate state on the mass surface: $p_{1,2}^2 \neq m^2$, so $p_1^2 - m^2 = p_2^2 - m^2$. This means that there is a symmetrical exit of particles from the mass surface [12].

The integration in Formula (2) is performed over the relative three-momenta of quarks $\mathbf{p}$ and antiquarks $\mathbf{q}$. The account of relative quark momenta p and q in all functions in (2) is necessary to increase the accuracy of the calculation. Relative momenta $p = L_P(0,\mathbf{p})$ and $q = L_Q(0,\mathbf{q})$ are obtained after the Lorentz transformation of the four-vectors $(0,\mathbf{p})$ and $(0,\mathbf{q})$ to the reference frames moving with the four-momenta P and Q [12,17].

After a series of transformations that take into account the law of transformation of the two-particle wave function during the transition from a rest frame to a moving one with four-momenta P and Q, we can present relativistic wave functions of diquarks in the following form [12,13]:

$$\Psi^{\mathcal{AV}}(p,P) = \frac{\Psi_0(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat{p}}{2m}\right]\times\hat{\varepsilon}(P,S_z)$$
$$(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat{p}}{2m}\right], \quad (4)$$

$$\Psi^{\mathcal{AV}}(q,Q) = \frac{\Psi_0(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{\epsilon(q)+m}{2m}\right]}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat{q}}{2m}\right]\times\hat{\varepsilon}(Q,S_z)$$
$$(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat{q}}{2m}\right], \quad (5)$$

where the hat symbol means a contraction of the four-vector with the Dirac gamma matrices; $v_1 = P/M_{QQ}$, $v_2 = Q/M_{QQ}$; $\epsilon(p) = \sqrt{m^2+\mathbf{p}^2}$, m is $c(b)$-quark mass, and $\varepsilon^{\lambda}(P,S_z)$ is the polarization vector of the vector diquark [12,17].

The relativistic wave functions in Equations (4) and (5) are the product of diquark wave functions in the rest frame $\Psi_0(\mathbf{p})$ and spin projection operators that are accurate at all orders in $|\mathbf{p}|/m$ [12,13]. Spin projection operators (4)–(5) can be considered form factors for the transition of quarks from a free to a bound state. An expression of the spin projector in different forms was first derived in [20], where spin projectors are written in terms of heavy quark momenta $p_{1,2}$ lying on the mass shell.The transformation law of the bound state wave function from the rest frame to the moving one with four momenta P was discussed in the Bethe–Salpeter approach in [21] and in quasipotential method in [22]. We use the quasipotential approach and write the transformation law of the bound state wave function as follows: [12,13,22]:

$$\Psi_P^{\rho\omega}(\mathbf{p}) = D_1^{1/2,\,\rho\alpha}(R^W_{L_P})D_2^{1/2,\,\omega\beta}(R^W_{L_P})\Psi_0^{\alpha\beta}(\mathbf{p}),$$
$$\bar{\Psi}_P^{\lambda\sigma}(\mathbf{p}) = \bar{\Psi}_0^{\varepsilon\tau}(\mathbf{p})D_1^{+\,1/2,\,\varepsilon\lambda}(R^W_{L_P})D_2^{+\,1/2,\,\tau\sigma}(R^W_{L_P}), \quad (6)$$

where R^W is the Wigner rotation, L_P is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ is defined by [10,17]

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}D^{1/2}_{1,2}(R^W_{L_P}) = S^{-1}(\mathbf{p}_{1,2})S(\mathbf{P})S(\mathbf{p}), \quad (7)$$

where the explicit form for the Lorentz transformation matrix of the four-spinor is

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p)+m}{2m}}\left(1+\frac{(\boldsymbol{\alpha}\mathbf{p})}{\epsilon(p)+m}\right). \quad (8)$$

For further transformation of the initial expression of the amplitude, the following relations are applied [10,17]:

$$S_{\alpha\beta}(\Lambda)u^{\lambda}_{\beta}(p) = \sum_{\sigma=\pm1/2} u^{\sigma}_{\alpha}(\Lambda p)D^{1/2}_{\sigma\lambda}(R^W_{\Lambda p}),$$
$$\bar{u}^{\lambda}_{\beta}(p)S^{-1}_{\beta\alpha}(\Lambda) = \sum_{\sigma=\pm1/2} D^{+\,1/2}_{\lambda\sigma}(R^W_{\Lambda p})\bar{u}^{\sigma}_{\alpha}(\Lambda p). \quad (9)$$

Also using the transformation property of the Dirac bispinors to the rest frame

$$\bar{u}_1(\mathbf{p}) = \bar{u}_1(0)\frac{(\hat{p}'_1+m_1)}{\sqrt{2\epsilon_1(\mathbf{p})(\epsilon_1(\mathbf{p})+m_1)}}, \quad p'_1 = (\epsilon_1,\mathbf{p}),$$

$$v_2(-\mathbf{p}) = \frac{(\hat{p}_2' - m_2)}{\sqrt{2\epsilon_2(\mathbf{p})(\epsilon_2(\mathbf{p}) + m_2)}} v_2(0), \;\; p_2' = (\epsilon_2, -\mathbf{p}), \tag{10}$$

we can introduce a projection operator onto a diquark state with spin 1 [23,24]:

$$u_i(0)u_j(0) = \left[\frac{(1+\gamma_0)}{2\sqrt{2}}\hat{\varepsilon}C\right]_{ij}, \;\; v_i(0)v_j(0) = \left[\frac{(1-\gamma_0)}{2\sqrt{2}}\hat{\varepsilon}C\right]_{ij}, \tag{11}$$

where C is the charge conjugation matrix.

The spin 1 diquark wave function is symmetric. Two identical quarks (cc) or (bb) are in the ground S-wave state. The color part of the diquark wave function in the production amplitudes (4) and (5) is taken as $\frac{1}{\sqrt{2}}\varepsilon_{ijk}$ (color indexes $i, j, k = 1, 2, 3$). The only quantum number that can be realized for colored antitriplet diquarks of the same flavors in the S-state is the axial vector. Therefore, the color part of the amplitude is obtained in the form

$$\frac{1}{\sqrt{2}}\varepsilon_{rki}\frac{1}{\sqrt{2}}\varepsilon_{slj}T^a_{rs}T^a_{kl} = -\frac{2}{3}\delta_{ij}, \tag{12}$$

where $T^a (a = 1, 2, ..., 8)$ are the generators of the color group SU(3).

Working with the three-dimensional quasipotential method, we write the double diquark production amplitude in the Higgs boson decay as a convolution of the production vertex function projected onto the positive energy states by means of the Dirac bispinors (free quark wave functions) and bound state quasipotential wave functions describing diquarks in the reference frames moving with four momenta, P and Q [12,17]. Further transformations include the known transformation law of the bound state wave functions to the rest frame (6). After the introduction of projection operators (11) into a production amplitude of two heavy quarks and two heavy antiquarks and a series of transformations, the amplitude of the Higgs boson decay to the pair of diquarks in the case of quark–gluon mechanism in the leading order in strong coupling constant α_s can be presented in the form (see additional transformations and intermediate expressions of the interaction amplitudes in [12,17]):

$$\mathcal{M}^{ij} = \frac{2\pi}{3}\delta^{ij}M_{QQ}\alpha_s\Gamma_Q\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\mathrm{Tr}\{\mathcal{T}_{12}(p,P,q,Q) + \mathcal{T}_{34}(p,P,q,Q)\}, \tag{13}$$

$$\mathcal{T}_{12}(p,P,q,Q) = \Psi^{\mathcal{AV}}(p,P)\left[\frac{\hat{p}_1 - \hat{r} + m}{(r-p_1)^2 - m^2}\gamma_\mu + \gamma_\mu\frac{\hat{r} - \hat{q}_1 + m}{(r-q_1)^2 - m^2}\right]D^{\mu\nu}(k_2)\Psi^{\mathcal{AV}}(q,Q)\gamma_\nu, \tag{14}$$

$$\mathcal{T}_{34}(p,P,q,Q) = \Psi^{\mathcal{AV}}(q,Q)\left[\frac{\hat{p}_2 - \hat{r} + m}{(r-p_2)^2 - m^2}\gamma_\mu + \gamma_\mu\frac{\hat{r} - \hat{q}_2 + m}{(r-q_2)^2 - m^2}\right]D^{\mu\nu}(k_1)\Psi^{\mathcal{AV}}(p,P)\gamma_\nu, \tag{15}$$

where $\alpha_s = \alpha_s\left(\frac{M_H^2}{4\Lambda^2}\right)$, $\Gamma_Q = m(\sqrt{2}G_F)^{\frac{1}{2}}$. The gluon four-momenta are $k_1 = p_1 + q_1$, $k_2 = p_2 + q_2$. Four-momentum of Higgs boson squared $r^2 = M_H^2 = (P+Q)^2 = 2M_{Q\bar{Q}}^2 + 2PQ$. The general structure of the amplitude in the case of another Z-bosonic mechanism is presented in the Appendix A.

The accounting for relativistic effects mentioned above means that the terms in the amplitude of the production of a diquark pair, which contain the relative momenta p, q, are consistently taken into account. But in some of these factors, such as gluon propagators and quark propagators, the relative momenta can simply be omitted as compared to the mass of the Higgs boson M_H without a significant loss in calculation accuracy. Simplifying the denominators of the quark and gluon propagators in this way, we obtain

$$\frac{1}{(p_1+q_1)^2} \approx \frac{1}{(p_2+q_2)^2} = \frac{4}{M_H^2}, \tag{16}$$

$$\frac{1}{(r-q_1)^2-m_1^2}=\frac{1}{(-r-p_1)^2-m_1^2}=\frac{1}{(r-p_2)^2-m_1^2}=\frac{1}{(-r-q_2)^2-m_1^2}=\frac{2}{M_H^2}. \quad (17)$$

In (16) and (17), we omit corrections of the form $|\mathbf{p}|/M_H$, $|\mathbf{q}|/M_H$. However, we take into account in the amplitudes (13), (14) the second-order correction for small ratios $|\mathbf{p}|/m$, $|\mathbf{q}|/m$ relative to the nonrelativistic result when $p=q=0$. The interaction amplitude (13) contains a common trace from the product of Dirac factors, which is calculated using the package FORM [25]. Then, we obtain the relativistic decay amplitude of the paired diquark production as follows:

$$\mathcal{M}_{\mathcal{AVAV}}{}^{ij,(1)}=\frac{128\pi}{3M_H^4}\delta^{ij}(\sqrt{2}G_F)^{\frac{1}{2}}mM_{QQ}\alpha_s\varepsilon_1^\lambda\varepsilon_2^\sigma F_{1,\mathcal{AVAV}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{AV}}(0)|^2, \quad (18)$$

where the polarization vectors of axial vector diquarks are ε_1^λ, ε_2^σ. The superscript in amplitude designation and subscript in tensor function $F_{1,\mathcal{AVAV}}^{\lambda\sigma}$ designation denote the contribution of the quark–gluon mechanism in Figure 1 (see also [12]). The tensor function $F_{1,\mathcal{AVAV}}^{\lambda\sigma}$ in (18) has the following general structure:

$$F_{1,\mathcal{AVAV}}^{\alpha\beta}=g_1^{(1)}v_1^\alpha v_2^\beta+g_2^{(1)}g^{\alpha\beta}, \quad (19)$$

where the functions $g_1^{(1)}$, $g_2^{(1)}$ are obtained explicitly in the form:

$$g_1^{(1)}=-1+\frac{1}{9}\omega_1^2,\quad g_2^{(1)}=-\frac{1}{2}+\frac{1}{2}r_3^2+\frac{1}{18}\omega_1^2-\frac{1}{18}\omega_1^2r_3^2-r_1+\frac{2}{3}r_1\omega_1+\frac{1}{3}r_1\omega_1^2, \quad (20)$$

where the parameters $r_1=\frac{m}{M_{QQ}}$, $r_3=\frac{M_H}{M_{QQ}}$, ω_1 and $\tilde{\Psi}_{\mathcal{AV}}(0)$ are relativistic parameters, which are discussed below.

Another contribution to the Higgs boson decay is determined by the Z-boson mechanism presented in Figure 2. It contains other coupling constants and factors and must be considered to obtain the total amplitude of the decay process. The general expression for such a decay amplitude has the form:

$$\mathcal{M}_{\mathcal{AVAV}}{}^{ij,(2)}=\frac{16\pi\alpha}{(\frac{M_H^2}{4}-M_Z^2)^2}\delta^{ij}(\sqrt{2}G_F)^{\frac{1}{2}}\frac{M_ZM_W}{\cos\theta_W\sin^2 2\theta}\varepsilon_1^\lambda\varepsilon_2^\sigma F_{2,\mathcal{AVAV}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{AV}}(0)|^2, \quad (21)$$

where the tensor $F_{2,\mathcal{AVAV}}^{\lambda\sigma}$ has the same structure as in (19) with its own functions like $g_1^{(2)}$, $g_2^{(2)}$:

$$g_1^{(2)}=-\frac{1}{8}r_2^2-\frac{1}{12}\omega_1r_2^2-\frac{1}{72}\omega_1^2r_2^2+\frac{1}{3}a_1\omega_1r_2^2-\frac{1}{3}a_1^2\omega_1r_2^2+\frac{1}{64}r_3^2r_2^4+\frac{1}{96}\omega_1r_3^2r_2^4+ \frac{1}{576}\omega_1^2r_3^2r_2^4-\frac{1}{24}\omega_1a_1r_3^2r_2^4+\frac{1}{24}\omega_1a_1^2r_3^2r_2^4, \quad (22)$$

$$g_2^{(2)}=-\frac{1}{4}-\frac{1}{6}\omega_1-\frac{1}{36}\omega_1^2+\frac{1}{2}a_1+\frac{1}{3}a_1\omega_1+\frac{1}{18}a_1\omega_1^2-\frac{1}{2}a_1^2-\frac{1}{3}a_1^2\omega_1-\frac{1}{18}a_1^2\omega_1^2+ \frac{1}{16}r_3^2r_2^2+\frac{1}{24}\omega_1r_3^2r_2^2+\frac{1}{144}\omega_1^2r_3^2r_2^2-\frac{1}{6}a_1\omega_1r_3^2r_2^2+\frac{1}{6}a_1\omega_1^2r_3^2r_2^2-\frac{1}{128}r_3^4r_2^4- \frac{1}{192}\omega_1r_3^4r_2^4-\frac{1}{1152}\omega_1^2r_3^4r_2^4+\frac{1}{48}a_1\omega_1r_3^4r_2^4-\frac{1}{48}a_1^2\omega_1r_3^4r_2^4, \quad (23)$$

where the parameter $a_1=2|e_Q|\sin^2\theta_W$, $r_2=M_{QQ}/M_Z$.

The decay width of the Higgs boson into a pair of vector diquark states is determined by the following expressions (see also [9,10,12,13]):

$$\Gamma_{\mathcal{AVAV}} = \frac{2^{14}\sqrt{2}\pi\alpha_s^2 m^2 G_F|\tilde{\Psi}_{\mathcal{AV}}(0)|^4\sqrt{\frac{r_3^2}{4}-1}}{9M_H^5 r_3^5}\sum_{\lambda,\sigma}|\varepsilon_1^{\lambda}\varepsilon_2^{\sigma}F^{\lambda\sigma}_{\mathcal{AVAV}}|^2, \tag{24}$$

$$F^{\lambda\sigma}_{\mathcal{AVAV}} = \left[g_1^{(1)} + \frac{3}{8}\frac{\alpha}{\alpha_s(1/4-r_3^2r_2^2)^2}\frac{M_W M_Z}{mM_{QQ}\cos\theta_W\sin^2 2\theta_W}g_1^{(2)}\right]v_1^{\sigma}v_2^{\lambda} + \left[g_2^{(1)} + \frac{3}{8}\frac{\alpha}{\alpha_s(1/4-r_3^2r_2^2)^2}\frac{M_W M_Z}{mM_{QQ}\cos\theta_W\sin^2 2\theta_W}g_2^{(2)}\right]g^{\lambda\sigma}. \tag{25}$$

We present in square brackets in (25) two terms corresponding to contributions of the quark–gluon mechanism in Figure 1 and the Z-boson mechanism. In (24), we identified a common factor that is included in the amplitude of the quark–gluon decay mechanism (compare with [12,13]).

The decay rate (24) contains various parameters, including those that determine the relativistic corrections. For a group of used parameters, such as quark masses, the diquark masses are determined within the framework of quark models as a result of calculating the different observed quantities for heavy mesons and baryons [10,12]. The parameters of the quark models are determined using the condition of the best agreement with the experimental data [3]. For the calculation of other relativistic parameters, we can also use the quark model and carry out momentum integration with the wave functions of bound states [10,12,13]. The closedness of the approach based on the relativistic quark model lies in the fact that, within its framework, the numerical values of all used parameters can be found.

To determine the values of relativistic parameters with diquark wave functions $\Psi_{AVD_{cc}}(\mathbf{p})$, $\Psi_{AVD_{bb}}(\mathbf{p})$, we suppose that the interaction of quarks in a diquark is described by the Breit Hamiltonian in quantum chromodynamics in the center-of-mass reference frame [26–31]:

$$H = H_0 + \Delta U_1 + \Delta U_2,\quad H_0 = \sqrt{\mathbf{p}^2+m_1^2} + \sqrt{\mathbf{p}^2+m_2^2} - \frac{2\tilde{\alpha}_s}{3r} + \frac{1}{2}(Ar+B), \tag{26}$$

$$\Delta U_1(r) = -\frac{\alpha_s^2}{6\pi r}[2\beta_0\ln(\mu r) + a_1 + 2\gamma_E\beta_0],\quad a_1 = \frac{31}{3} - \frac{10}{9}n_f,\quad \beta_0 = 11 - \frac{2}{3}n_f, \tag{27}$$

$$\Delta U_2(r) = -\frac{\alpha_s}{3m_1m_2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{\pi\alpha_s}{3}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\delta(\mathbf{r}) + \frac{2\alpha_s}{3r^3}\left(\frac{1}{2m_1^2}+\frac{1}{m_1m_2}\right)(\mathbf{S}_1\mathbf{L}) + \\ + \frac{2\alpha_s}{3r^3}\left(\frac{1}{2m_2^2}+\frac{1}{m_1m_2}\right)(\mathbf{S}_2\mathbf{L}) + \frac{16\pi\alpha_s}{9m_1m_2}(\mathbf{S}_1\mathbf{S}_2)\delta(\mathbf{r}) + \frac{2\alpha_s}{m_1m_2r^3}\left[\frac{(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})}{r^2} - \frac{1}{3}(\mathbf{S}_1\mathbf{S}_2)\right] - \\ - \frac{\alpha_s^2(m_1+m_2)}{2m_1m_2r^2}\left[1 - \frac{4m_1m_2}{9(m_1+m_2)^2}\right], \tag{28}$$

where $\mathbf{L} = [\mathbf{r}\times\mathbf{p}]$, $\mathbf{S}_1$, $\mathbf{S}_2$ are spins of heavy quarks, n_f is the number of flavors, and $\gamma_E \approx 0.577216$ is the Euler constant. We also add to the potential (26) the spin-confining potentials obtained in [27,32]:

$$\Delta V^{hfs}_{conf}(r) = f_V\frac{A}{8r}\left\{\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16}{3m_1m_2}(\mathbf{S}_1\mathbf{S}_2) + \frac{4}{3m_1m_2}[3(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - (\mathbf{S}_1\mathbf{S}_2)]\right\}, \tag{29}$$

where we take the parameter $f_V = 0.9$. The dependence of the QCD coupling constant $\tilde{\alpha}_s(\mu^2)$ on the renormalization point μ^2 in the pure Coulomb term in (26) is determined using the three-loop result [33]:

$$\tilde{\alpha}_s(\mu^2) = \frac{4\pi}{\beta_0 L} - \frac{16\pi b_1 \ln L}{(\beta_0 L)^2} + \frac{64\pi}{(\beta_0 L)^3}\left[b_1^2(\ln^2 L - \ln L - 1) + b_2\right], \quad L = \ln(\mu^2/\Lambda^2), \quad (30)$$

whereas for the other terms of the Hamiltonians (27) and (28), we take the leading order approximation. The coefficients b_i are written explicitly in [33]. The parameters of the linear confinement potential A and B have the usual values of quark models [31].

In our case, the bound system contains two identical quarks (fermions). The diquark wave function must be antisymmetric when they are interchanged. In this case, the coordinate part and the spin part of the wave function are symmetric, and the antisymmetry is determined by the color part of the wave function. The coordinate part of the wave function is determined from the solution of the quasipotential equation with the Hamiltonian (26). We take the Breit Hamiltonian (26) and construct an effective potential model as in [10,12,13,30], making a rationalization of the kinetic energy operator. Relativistic parameters in the decay amplitude (see Table 1) are obtained after finding the numerical solution of the Schrödinger equation [34]. There are no free diquarks to study the effective interaction between two heavy quarks within them. Therefore, to check our model of bound states of heavy quarks, we performed calculations of various observable quantities, such as the masses of charmonium, bottommonium, and B_c mesons. The obtained results are in good agreement with experimental data and the calculations of other quark models.

Table 1. Numerical values of relativistic parameters (21)–(23) in the relativistic quark model.

$n^{2S+1}L_J$	Diquark	M_{QQ}, GeV	$\tilde{R}(0)$, GeV$^{3/2}$	ω_1	ω_2
1^3S_1	AVD_{cc}	3.233	0.386	0.0422	0.0032
1^3S_1	AVD_{bb}	9.845	0.720	0.0180	0.0010

The decay amplitudes (19) and (22) are expressed in terms of the functions $F^{\lambda\sigma}_{i,\mathcal{AV}\mathcal{AV}}$, which are presented in the form of an expansion in $|\mathbf{p}|/m$, $|\mathbf{q}|/m$ up to terms of the second order. As a result of algebraic transformations, we express relativistic corrections in terms of relativistic parameters $\tilde{\psi}_{\mathcal{AV}}(0) = \tilde{R}(0)/\sqrt{4\pi}$, ω_1. The parameter ω_2 is not included in the decay width, since corrections of the order $O(\mathbf{q}^4)$, $O(\mathbf{p}^4)$ connected with it are omitted. Since we are studying the production of S-wave bound states, for them the values of the relativistic parameters ω_n in our model are determined using the following integral expressions [12,13]:

$$I_n = \int_0^\infty p^2 R(p) \frac{(\epsilon(p)+m)}{2\epsilon(p)} \left(\frac{\epsilon(p)-m}{\epsilon(p)+m}\right)^n dp, \quad (31)$$

$$\tilde{R}(0) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty \frac{(\epsilon(p)+m)}{2\epsilon(p)} p^2 R(p) dp, \;\; \omega_1 = \frac{I_1}{I_0}, \;\; \omega_2 = \frac{I_2}{I_0}, \quad (32)$$

where $R(p)$ is the radial diquark wave function in momentum representation.

The results of numerical calculations of the decay widths corresponding to two mechanisms in Figures 1 and 2 are presented in Tables 2 and 3.

Table 2. The Higgs boson decay widths within the nonrelativistic limit and accounting for relativistic corrections.

Final State $(QQ)(\bar{Q}\bar{Q})$	**Nonrelativistic Decay Width** $\Gamma^{nr}_{\mathcal{AV}\mathcal{AV}}$ **in GeV**	**Relativistic Decay Width** $\Gamma^{rel}_{\mathcal{AV}\mathcal{AV}}$ **in GeV**
AVD_{cc} + $AVD_{\bar{c}\bar{c}}$ $1^3S_1 + 1^3S_1$	6.15×10^{-14}	2.09×10^{-14}
AVD_{bb} + $AVD_{\bar{b}\bar{b}}$ $1^3S_1 + 1^3S_1$	3.46×10^{-14}	0.89×10^{-14}

Table 3. The contributions of the Z-boson and quark–gluon processes to the Higgs boson decay widths in GeV.

Contribution of two mechanisms to (24) taking into account relativistic effects		
Contribution	$H \to AVD_{cc} + AVD_{\bar{c}\bar{c}}$	$H \to AVD_{bb} + AVD_{\bar{b}\bar{b}}$
Figure 1	0.64×10^{-17}	0.50×10^{-16}
Figure 2	2.09×10^{-14}	0.88×10^{-14}
Total contribution	2.09×10^{-14}	0.89×10^{-14}

3. Numerical Results and Conclusions

The ongoing intensive research in the field of the Higgs sector aims to study various reactions and various interactions, and refine the numerous parameters that determine these interactions. Rare decays of the Higgs boson occupy an important place in this area of research. In this work, we have attempted to give an initial estimate for the decay widths of the Higgs boson, to produce a pair of doubly heavy baryons in the final state. The processes of pair production of doubly heavy mesons and baryons belong to the rare class, but the study of such reactions is of obvious interest, since the effects of small and large distances of interacting particles begin to manifest themselves in them.

We distinguish two stages in the process of baryon production. In the first stage, a diquark with spin 1 (anti-diquark) is formed, which will be the nucleus of the future baryon (antibaryon). In the second stage, a diquark (anti-diquark) picks up a light quark (antiquark) with a probability close to unity, and baryons of spin 1/2 and 3/2 are formed. Thus, the resulting formula for the decay width (24) is simultaneously a formula for the probability of baryon–anti-baryon pair production. The obtained numerical results are presented in detail in various Tables 1–3. In our calculations, we take into account the relativistic effects determined by the relative momenta of quarks, both in the decay amplitude itself and in the wave functions of bound states, including their law of transformation from a rest frame to a moving reference frame. In the rest frame, relativistic effects are taken into account by generalizing the Breit potential. For definiteness, in this work, we take the second-order relativistic corrections. Using the formalism of the relativistic quark model, we can also consider higher-order relativistic corrections on the basis of the obtained general expressions. In our approach, all relativistic parameters arising (31) and (32) are determined using convergent integrals in the relativistic quark model (see Table 1) [12,13]. The relativistic corrections calculated in our approach allow an essential change in the numerical values of the decay rate in nonrelativistic approximation. It is necessary to emphasize that, at the nonrelativistic limit, we set $p = 0$, $q = 0$ and neglect relativistic corrections in the quark interaction Hamiltonian. The basic parameter that greatly changes the nonrelativistic results is the modified radial wave function at zero $\tilde{R}(0)$, which enters the fourth power of the decay width.

An experimental study of such rare decays is only possible at Higgs boson factories, in which a significant number of H bosons will be produced. Exploring two decay mechanisms in our work, we have identified the main one, which is connected with the initial production of a pair of Z bosons (see Figure 2). Comparing the results of our calculations of pair production of baryons with the results of the pair production of charmonium and bottomonium from our previous work, we can see that the decrease in the decay width is slightly less than two orders of magnitude. This decrease is primarily explained by a significant decrease in the diquark wave function at zero.

We present separately, in Table 3, numerical values of the contributions from different decay mechanisms with an accuracy of two significant figures after the decimal point before the factor $10^{-17} \div 10^{-14}$. In each considered decay mechanism, there are radiative corrections $O(\alpha_s)$ that were not taken into account. These represent a major source of theoretical uncertainty, which we estimated to be about 30% in α_s [9,10,13,14]. Our estimates also show that other theoretical errors connected with errors in determining the parameters of the quark model and next-order relativistic effects amount to approximately 10%. The resulting branchings of the decay of the Higgs boson into the pairs (ccq), $(\bar{c}\bar{c}\bar{q})$ and (bbq), $(\bar{b}\bar{b}\bar{q})$ are equal to, respectively, 0.65×10^{-11} and 0.28×10^{-11}.

The numerical values we determined for the widths of the exclusive decay of the Higgs boson with the formation of a pair of diquarks are significantly less than its total decay width. To observe such rare decay processes with the formation of a pair of baryons at the LHC, it is necessary to significantly increase its luminosity. The prospects of studying rare exclusive decays of the Higgs boson, generating (ccq) and $(\bar{c}\bar{c}\bar{q})$, are connected with future Higgs boson factories. Several Higgs boson factory projects have been actively discussed in recent years, such as the ILC (International Linear Collider), the CLIC (Compact Linear Collider) [35], and the FCC (Future Circular Collider) [36]. To study rare events in particle colliders, it is necessary to observe a large number of such events. The probability of an event is determined by the cross.section of the corresponding reaction and the luminosity of the collider. Currently, discussions are ongoing about various parameters of the future colliders ILC, CLIC, and FCC [10]. In the FCC proton–proton collider, the main process of Higgs boson production will be the process of gluon–gluon fusion. As calculations have shown, the cross-section for such a process will reach a value of 802 pb at an energy of 100 TeV [36]. This reaction will produce approximately 2×10^{10} Higgs bosons per year (see Table 20 from [10,36]). Taking into account the obtained decay branchings, we can expect only 0.1 events per year in which a baryon–antibaryon pair will be produced.

In recent years, we have carried out a number of studies of rare, exclusive decays of the Higgs bosons that produced a pair of mesons or baryons [9,10,12,13]. If we compare the results of this work with the results for paired charmonium production [12,13], we can note the different structure of the amplitudes, which make the main contribution to the decay width. In the case of paired charmonium production, there is a mass factor that leads to a significant increase in the decay width (by approximately two orders of magnitude) compared to paired diquark production. Paired production of diquarks is characterized by contributions from only crossed amplitudes in Figures 1 and 2. In this sense, the processes of pair production of diquarks are similar to the processes of pair production of B_c mesons [9,10]. It is no coincidence that the numerical values of the decay widths of the H boson into a pair of diquarks and B_c mesons are close in magnitude [9,10].

We investigated two pair production mechanisms: Z-bosonic and quark–gluon. Our calculations show that, in the end, the Z-boson mechanism makes the main contribution to the Higgs boson decay width. This result is fairly consistent with our previous calculations of the pair production of mesons (J/Ψ, Υ) [12,13], where the Z-boson mechanism was also decisive. Recent work [16] has studied the production of single doubly heavy diquarks in the decay of the Higgs boson and shown that the quark–gluon mechanism for such processes makes the main contribution to the decay width. In this regard, at least two circumstances should be noted. First, the transition from the amplitudes of the production of a doubly heavy diquark and a free quark–antiquark pair (amplitudes of type 1 in [16]) to the amplitudes of the production of a pair of doubly heavy diquarks (amplitudes of type 2 in this work) leads to the appearance, in the case of the quark–gluon mechanism, of a large mass of the order of the Higgs boson mass in the denominator (gluon propagator), which significantly reduces the contribution of the amplitudes of the quark–gluon pair production mechanism.For the Z-boson mechanism, the transition from amplitudes of type 1 to amplitudes of type 2 is not connected with the appearance of the Higgs boson mass in the amplitude denominator. Second, the renormalization scale changes from the values $2m_c$, $2m_b$ to the value $m_H/2$, as a result of which, the value of the strong interaction constant, which is included in the decay width squared in the case of the quark–gluon mechanism, decreases.

To estimate the width of the Higgs boson decay into a pair of baryons, we assume that the axial vector diquark (cc) can fragment either into a baryon Ξ_{cc} with spin $J = 1/2$ containing a light quark u , d, or into a baryon Ξ^*_{cc} with spin $J = 3/2$. The width of the H decay into a baryon–anti-baryon pair $(B\bar{B})$ is then equal to

$$\Gamma_{B\bar{B}} = \int_0^1 dz_1 \int_0^1 dz_2 \frac{d\Gamma}{dz_1 dz_2}(H \to D\bar{D}) \cdot D_{D\to B}(z_1) \cdot D_{\bar{D}\to\bar{B}}(z_2), \tag{33}$$

where z_i is the part of the baryon momentum that is carried away by the diquark. Since the baryon has almost the same momentum as the diquark, the diquark fragmentation function $D_{D\to B}(z)$ can be represented as [37]

$$D_{D\to B}(z) = P_{D\to B} \cdot \delta(1-z), \tag{34}$$

where $P_{D\to B}$ is the total probability of fragmentation of a diquark into a baryon. This probability can be taken as equal to unity for the fragmentation of a diquark into a baryon (ccq): $\int_0^1 D_{D\to B}(z)dz = 1$. Then, the obtained decay width (24) can be used to estimate baryon–antibaryon pair production.

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Appendix A. The Amplitude of Pair Axial Vector Diquark Production via the Z-Boson Mechanism

The general form of the amplitude of pared diquark production (Figure 2) is the following:

$$\begin{aligned}\mathcal{M}^{ij} = &\frac{\delta^{ij}\pi\alpha(\sqrt{2}G_F)^{1/2}M_Z M_W}{\cos\theta_W \sin^2 2\theta_W}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{\Psi_0(\mathbf{p})}{\frac{\epsilon(p)}{m}\frac{(\epsilon(p)+m)}{2m}}\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{\Psi_0(\mathbf{q})}{\frac{\epsilon(q)}{m}\frac{(\epsilon(q)+m)}{2m}}D_{\mu\alpha}(P)D_{\nu\alpha}(Q)\times\\ &Tr\Big\{[\frac{\hat{v}_1-1}{2}+\frac{\hat{v}_1 p^2}{2m(\epsilon(p)+m)}-\frac{\hat{p}}{2m}]\hat{\varepsilon}_1(\hat{v}_1+1)[\frac{\hat{v}_1+1}{2}+\frac{\hat{v}_1 p^2}{2m(\epsilon(p)+m)}+\frac{\hat{p}}{2m}]\gamma_\mu[\frac{1-\gamma_5}{2}-a_1]\Big\}\times\\ &Tr\Big\{[\frac{\hat{v}_2-1}{2}+\frac{\hat{v}_2 q^2}{2m(\epsilon(q)+m)}-\frac{\hat{q}}{2m}]\hat{\varepsilon}_2(\hat{v}_2+1)[\frac{\hat{v}_2+1}{2}+\frac{\hat{v}_2 q^2}{2m(\epsilon(q)+m)}+\frac{\hat{q}}{2m}]\gamma_\nu[\frac{1-\gamma_5}{2}-a_1]\Big\},\end{aligned} \tag{A1}$$

where $\varepsilon_{1,2}$ are the polarization four-vectors describing the axial vector diquark states. $D_{\mu\alpha}(k)$ is the Z-boson propagator. After all transformations described in Section 2, the amplitude (A1) takes the form (21).

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