Physics Letters B 803 (2020) 135296

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Improved upper limits on baryon-number violating dinucleon decays to dileptons

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ARTICLE INFO

Article history:
Received 17 October 2019
Received in revised form 6 January 2020
Accepted 11 February 2020
Available online 14 February 2020
Editor: B. Grinstein

ABSTRACT

We consider effects of $n-\bar{n}$ oscillations and resultant matter instability due to dinucleon decays. We point out that existing upper bounds on the rates for the dinucleon decays $nn \to 2\pi^0$, $nn \to \pi^+\pi^-$, and $np \to \pi^+\pi^0$ imply upper bounds on the rates for dinucleon decays to dileptons $nn \to e^+e^-$, $nn \to \mu^+\mu^-$, $nn \to \nu_\ell \bar{\nu}_\ell$, and $np \to \ell^+\nu_\ell$, where $\ell = e,\ \mu,\ \tau$. We present estimates for these upper bounds. Our bounds are substantially stronger than corresponding limits from direct searches.

1. Introduction

The violation of baryon number, B, is expected to occur in nature, because this is one of the necessary conditions for generating the observed baryon asymmetry in the universe [1]. Baryon number violation (BNV) is, indeed, predicted in many ultraviolet extensions of the Standard Model (SM), such as grand unified theories. A number of dedicated experiments have been carried out since the early 1980s to search for proton decay (and the decay of neutrons bound in nuclei). These experiments have obtained null results and have set resultant stringent upper limits for the rates of such $\Delta B = -1$ baryon-number-violating nucleon decays.

A different type of baryon number violation has also received attention, namely $n-\bar{n}$ oscillations, which have $|\Delta B| = 2$ [2–16]. It was observed early on that $n-\bar{n}$ oscillations might provide the source of baryon number violation necessary for baryogenesis [2]. The same operators that mediate $n-\bar{n}$ transitions also lead to matter instability via the dinucleon decays from nn and np initial states to respective multipion final states. Let us denote the low-energy effective Hamiltonian responsible for $n-\bar{n}$ oscillations as $\mathcal{H}_{eff}^{(n\bar{n})}$. We will assume a minimal framework in which $\mathcal{H}_{eff}^{(n\bar{n})}$ incorporates all of the physics beyond the Standard Model relevant for $n-\bar{n}$ oscillations. Rates for these dinucleon decays in matter are calculated by taking into account that in the presence of a nonzero transition amplitude $\langle \bar{n}|\mathcal{H}_{eff}^{(n\bar{n})}|n\rangle$, the physical state $|n\rangle_{phys.}$ contains a small but nonzero $|\bar{n}\rangle$ component. This leads to a nonzero amplitude for annihilation of the $|\bar{n}\rangle$ component with a neighboring neutron or proton in a nucleus.

The operators in the low-energy effective Hamiltonian for proton decay are four-fermion operators with Maxwellian mass dimension 6 and hence coefficients of mass dimension -2, whereas the operators in $\mathcal{H}_{eff}^{(n\bar{n})}$ are six-quark operators, with coefficients of dimension -5. Consequently, if one were to assume that there is a single high mass scale M_{BNV} characterizing the physics responsible for baryon number violation, proton decay would be much more important as a manifestation of baryon number violation than $n-\bar{n}$ oscillations and the corresponding dinucleon decays. However, such an assumption of a single BNV mass scale may well be overly simplistic [3]. Ref. [7] presented an explicit example of a theory in which proton decay is suppressed well beyond observable levels while $n-\bar{n}$ oscillations occur at levels comparable to existing experimental limits. In such a model, it is the $n-\bar{n}$ oscillations and the corresponding nn and np dinucleon decays to multi-meson final states that are the main manifestations of baryon number violation, rather than individual proton and bound neutron decays. Further examples of models with baryon number violation but no proton decay were given in the later work [10].

Here we point out that existing upper bounds on the rates for the hadronic dinucleon decays $nn \to 2\pi^0$, $nn \to \pi^+\pi^-$, and $np \to \pi^+\pi^0$ imply upper bounds on the rates for the dinucleon to dilepton decays $nn \to e^+e^-$, $nn \to \mu^+\mu^-$, $nn \to \nu_\ell\bar{\nu}_\ell$, and $np \to \ell^+\nu_\ell$, where $\ell = e,\ \mu,\ \tau$. We present estimates for these upper bounds. Our upper bounds are considerably stronger than direct limits on the rates for these decays.

2. $n-\bar{n}$ oscillations and dinucleon decays to hadronic final states

We recall some basic results on $n-\bar{n}$ oscillations that are needed for our analysis (for further details, see, e.g., [11]). Let us consider a general theory in which there is baryon-number violat-

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https://doi.org/10.1016/j.physletb.2020.135296
0370-2693/

ing physics beyond the Standard Model (BSM) that leads to $n-\bar{n}$ transitions and let us denote the corresponding transition amplitude as

$$\delta m = \langle \bar{n}|\mathcal{H}_{eff}^{(n\bar{n})}|n\rangle \ . \tag{1}$$

In (field-free) vacuum, one is thus led to diagonalize the matrix of the Hamiltonian in the basis $(|n\rangle, |\bar{n}\rangle)$,

$$\begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix} , \tag{2}$$

where $\lambda_n = \tau_n^{-1}$ is the decay rate of the free neutron and the equality $m_{\bar{n}} = m_n$ follows from CPT invariance. The eigenstates of this matrix are $|n_\pm\rangle = (|n\rangle \pm |\bar{n}\rangle)/\sqrt{2}$, with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$. Hence, if one starts with a pure $|n\rangle$ state at $t=0$, then there is a finite probability for it to be an $|\bar{n}\rangle$ at $t \neq 0$ given by

$$P(n(t)=\bar{n}) = |\langle \bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t} \ , \tag{3}$$

where $\tau_{n\bar{n}} = 1/|\delta m|$. The current limit on $\tau_{n\bar{n}}$ from an experiment with a neutron beam from a nuclear reactor at the Institut Laue-Langevin (ILL) in Grenoble is $\tau_{n\bar{n}} \geq 0.86 \times 10^8$ s, i.e., $|\delta m| = 1/\tau_{n\bar{n}} < 0.77 \times 10^{-29}$ MeV [6]. (This and other limits discussed here are at the 90% confidence level.)

For a neutron bound in a nucleus, the Hamiltonian matrix becomes

$$\begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix} \tag{4}$$

with $m_{n,eff} = m_n + V_n$ and $m_{\bar{n},eff.} = m_n + V_{\bar{n}}$, where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$. In the presence of the $n-\bar{n}$ mixing, the resultant physical eigenstate for the neutron state in matter has a small component of $|\bar{n}\rangle$, i.e.,

$$|n\rangle_{\rm phys.} = \cos\theta_{n\bar{n}}|n\rangle + \sin\theta_{n\bar{n}}|\bar{n}\rangle \ , \tag{5}$$

where $\tan(2\theta_{n\bar{n}}) = 2\delta m/|m_{n,eff} - m_{\bar{n},eff}|$. In contrast to the situation in field-free vacuum, where $\theta = \pi/4$ and the mixing is maximal, in matter, because the diagonal elements of the Hamiltonian matrix are different, $|\theta| << 1$. However, this is more than compensated for by the large number of nucleons in a proton decay experiment such as SuperKamiokande (SK). The nonzero $|\bar{n}\rangle$ component in $|n\rangle_{\rm phys.}$ leads to annihilation with an adjacent neutron or proton, and hence to the decays to zero-baryon, multi-meson final states consisting dominantly of several pions, $nn \to$ pions and $np \to$ pions. The rate characterizing matter instability (m.i.) due to these dinucleon decays is

$$\Gamma_{m.i.} \equiv \frac{1}{\tau_{m.i.}} \simeq \frac{2(\delta m)^2|V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2} \ . \tag{6}$$

Hence, $\tau_{m.i.} \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$. A common convention is to introduce a multiplicative factor R and write $\tau_{m.i.} = R\,\tau_{n\bar{n}}^2$ (see, e.g., the review [11]). As is evident from this relation, together with Eq. (6), the factor R reflects the different nuclear potentials felt by an n and $\bar{n}$ in a nucleus and has the value $R \sim O(10^2)$ MeV, or equivalently, $R \simeq 10^{23}$ s^{-1}, dependent on the nucleus. Lower limits on $\tau_{m.i.}$ that yield equivalent lower bounds on $\tau_{n\bar{n}}$ in the 10^8 s range have been obtained from the Kamiokande [13], Soudan [14], SNO (Sudbury Neutrino Observatory) [15], and SK [16] experiments. The best current limit on matter instability (from SK) is [16],

$$\tau_{m.i.} > 1.9 \times 10^{32} \ {\rm yr} \ , \tag{7}$$

and hence, taking into account the uncertainty in the calculation of $R \simeq 0.52 \times 10^{23}$ s^{-1} for the ^{16}O nuclei in water [9,11], the SK experiment has inferred the limit [16]

$$\tau_{n\bar{n}} > 2.7 \times 10^8 \ {\rm s}, \ \ i.e., \ \ |\delta m| < 2.4 \times 10^{-30} \ {\rm MeV}. \tag{8}$$

(From this and the value $|m_{n,eff} - m_{\bar{n},eff}| \sim 10^2$ MeV, it follows that $|\theta_{n\bar{n}}| \lesssim 10^{-31}$.)

There have also been searches for dinucleon decays to specific final states. Reflecting the dominance of the strong interactions over the electroweak interactions, these decays lead mainly to hadronic final states. From null searches for the decays $^{56}{\rm Fe} \to {}^{54}{\rm Fe} + \pi^+\pi^-$ [12], $^{16}{\rm O} \to {}^{14}{\rm O} + 2\pi^0$ [17], and $^{16}{\rm O} \to {}^{14}{\rm N} + \pi^+\pi^0$ [17], experiments have set upper bounds on the rates Γ_i, or equivalently, lower bounds on the partial lifetimes $(\tau_i/B_i) \equiv \Gamma_i^{-1}$ for these decays, where B_i denotes a branching ratio. The experiments use the notational convention of referring to these as $nn \to \pi^+\pi^-$, $nn \to 2\pi^0$, and $np \to \pi^+\pi^0$. We will follow this convention, but note that a conversion would be necessary to compute the rate for an individual pair of neighboring nucleons to undergo these decays. The limit from the Fréjus experiment [12] is

$$(\tau/B)_{nn\to\pi^+\pi^-} > 0.7 \times 10^{30} \ {\rm yr}, \tag{9}$$

and the limits from the SK experiment [17] are

$$(\tau/B)_{nn\to\pi^0\pi^0} > 4.04 \times 10^{32} \ {\rm yr} \tag{10}$$

and

$$(\tau/B)_{np\to\pi^+\pi^0} > 1.70 \times 10^{32} \ {\rm yr} \ . \tag{11}$$

We use the two more stringent bounds (10) and (11) for our analysis. The multiplicities of pion final states that would be detected in the SK detector are determined by the strong reactions, including absorption, of the pions from the initial $\bar{n}$ annihilation with a neighboring n or p as these pions propagate through the oxygen nucleus; Ref. [17] reported total and charged pion multiplicities of 3.5 and 2.2, respectively. For the purposes of our estimates, these are sufficiently close to the two-pion multiplicity of the $nn \to \pi^0\pi^0$ and $np \to \pi^+\pi^0$ decays that we do not attempt to introduce further effective multiplicity correction factors.

3. Dinucleon decays to dilepton final states

The same baryon-number-violating physics that leads to $n-\bar{n}$ oscillations and hence also the dinucleon decays $nn \to$ pions and $np \to$ pions also leads to dinucleon decays to leptonic final states, in particular, to dileptons:

$$nn \to \ell^+\ell^- \quad {\rm for} \ \ell = e, \ \mu \tag{12}$$

$$nn \to \nu_\ell\bar{\nu}_\ell \quad {\rm for} \ \nu_\ell = \nu_e, \ \nu_\mu, \ \nu_\tau \tag{13}$$

and

$$np \to \ell^+\nu_\ell \quad {\rm for} \ \ell = e, \ \mu, \ \tau \ . \tag{14}$$

As is evident, these are $\Delta B = -2$, $\Delta L = 0$ decays, where L denotes total lepton number. We will derive upper bounds on the rates for these decays by relating them to hadronic dinucleon decays and using the upper bounds on rates for the latter. We utilize a minimal theoretical framework for our analysis, namely to assume the BSM physics responsible for the $n-\bar{n}$ oscillations, but then apply only Standard-Model physics to derive these relations. With this framework, we identify and estimate the leading contributions to these dinucleon decays to dileptons. These contributions involve amplitudes each of which consists of a combination of two parts:

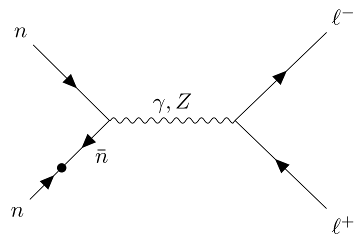

Fig. 1. Feynman diagram for $nn \to \ell^+\ell^-$ with $\ell = e,\ \mu$.

(a) The basic BNV part, involving a six-fermion operator resulting from physics operative at a mass scale $M_{BNV} >> v$, where $v = 250$ GeV is the electroweak-symmetry-breaking (EWSB) scale, and a second part involving SM physics, with a virtual timelike photon, Z, or W.

We begin with the decay $nn \to \ell^+\ell^-$. This decay can occur as follows: the $|\bar{n}\rangle$ component in a $|n\rangle_{\rm phys.}$ neutron in a nucleus leads to annihilation with a neighboring neutron to yield a virtual photon in the s channel, which then produces the final-state $\ell^+\ell^-$ pair in (12). A much smaller contribution involves a diagram with a virtual Z in the s-channel. Equivalently, one can envision this as being due to a transition in which an initial n changes to a $\bar{n}$ with transition matrix element (1), and then the $\bar{n}$ annihilates with the neighboring n to produce the virtual photon or Z, as shown in Fig. 1. Up to small corrections due to the bound state Fermi momenta of the nucleons, the center-of-mass energy is $\sqrt{s} = m_n + m_p \equiv 2m_N$ in this transition, and the ℓ^+ and ℓ^- are emitted back-to-back, each with a total energy in the lab frame equal to m_N. We denote the four-momentum of the virtual photon or Z as q and the four-momenta of the ℓ^- and ℓ^+ as p_2 and p_1, with $q = p_1 + p_2$ and $q^2 = s = (2m_N)^2$. Here and below, we neglect small effects due to Fermi momenta. The conversion reaction $e + n \to e + \bar{n}$ has been discussed in [18].

To leading order, the amplitude for $nn \to \ell^+\ell^-$ is the sum of the terms due to virtual (v) photon and Z exchange in the s-channel:

$$A_{nn\to\ell^+\ell^-} = A_{nn\to\ell^+\ell^-;\gamma} + A_{nn\to\ell^+\ell^-;Z} \ , \tag{15}$$

with

$$A_{nn\to\ell^+\ell^-;\gamma} = (\delta m)\, e^2 \,\langle 0|J^\lambda_{em}|n\bar{n}\rangle \,\frac{1}{q^2}\,[\bar{u}(p_2)\gamma_\lambda v(p_1)] \tag{16}$$

and

$$A_{nn\to\ell^+\ell^-;Z} = \sqrt{2}G_F\,(\delta m)\,\langle 0|J^\lambda_Z|n\bar{n}\rangle \Big[\bar{u}(p_2)\gamma_\lambda[(1-4\sin^2\theta_W) - \gamma_5]v(p_1)\Big], \tag{17}$$

where the δm factor represents the initial $n \to \bar{n}$ transition mediated by $\mathcal{H}^{(n\bar{n})}_{eff}$; J^λ_{em} and $J^\lambda_Z = J^\lambda_{3L} - \sin^2\theta_W\, J^\lambda_{em}$ denote the electromagnetic and neutral weak currents; and $e = \sqrt{4\pi\alpha_{em}}$, and G_F denote the electromagnetic and Fermi couplings.

We first consider the contribution from $A_{nn\to\ell^+\ell^-;\gamma}$. Since the annihilation occurs on a scale of order ~ 1 fm, a reasonable approximation is to consider the initial nn state by itself, independent of the other nucleons in the nucleus. Let us denote the wavefunction of this state as $|nn\rangle = \phi_I\,\phi_S\,\phi_L$, where I, S, and L denote the strong isospin, the spin, and the relative orbital angular momentum L of the nn pair. (To maintain standard notation, we use the same symbol, L, for orbital angular momentum and total lepton number; the context will always make clear which is meant.) This wavefunction must be antisymmetric under interchange of neutrons. The $|nn\rangle$ state has strong isospin $I = 1$, and the lowest-energy configuration has $L = 0$, so the ϕ_I and ϕ_L wavefunctions for this configuration are both symmetric under interchange of neutrons. Hence, ϕ_S is antisymmetric, corresponding to spin $S = 0$ and hence total angular momentum $J = 0$ for the nn pair. Since $\mathcal{L}^{(n\bar{n})}_{eff}$ is a Lorentz scalar, the $n - \bar{n}$ transition matrix element $\langle\bar{n}|\mathcal{L}^{(n\bar{n})}_{eff}|n\rangle$ does not change the neutron spin, so the value of S (as well as L) for the resultant $n\bar{n}$ dinucleon is the same as for the initial nn dinucleon. (This is obvious in Eq. (5).) The matrix element $\langle 0|J^\lambda_{em}|n\bar{n}\rangle$ is related by crossing symmetry to the matrix element $\langle n|J^\lambda_{em}|n\rangle$, which involves Dirac and Pauli form factors $F^{(n)}_1(q^2)$ and $F^{(n)}_2(q^2)$. For the $J = 0$ nn state, the only four-momentum on which the matrix element $\langle 0|J^\lambda_{em}|n\bar{n}\rangle$ can depend is q^λ, so $\langle 0|J^\lambda_{em}|n\bar{n}\rangle \propto q^\lambda$. But $q^\lambda\,[\bar{u}(p_2)\gamma_\lambda v(p_1)] = 0$, so that this contribution to the amplitude vanishes. Another contribution arises from an excited $|nn\rangle$ state with $L = 1$ and an antisymmetric ϕ_L, so that ϕ_S is symmetric, corresponding to $S = 1$. Then the quantum mechanical addition of L and S to yield a total angular momentum $\vec{J} = \vec{L} + \vec{S}$ can yield $J = 0, 1$, or 2. The $J = 0$ state gives zero contribution, as before, so the amplitude arises from the initial nn states with nonzero J. We denote the probability of the nn dinucleon to be in a state with $J \neq 0$ as $P_{nn,\,J\neq 0}$. Given that $J \neq 0$ so that $A_{nn\to\ell^+\ell^-;\gamma} \neq 0$, it follows that in $|A_{nn\to\ell^+\ell^-;\gamma}|^2$, the $(1/s)^2$ factor from the photon propagator is cancelled by kinematic factors of order s^2.

We next consider the contribution from $A_{nn\to\ell^+\ell^-;Z}$. The square, $|A_{nn\to\ell^+\ell^-;Z}|^2$, is negligible because of suppression by the factor $\sim (G_F s)^2 = 1.7 \times 10^{-9}$. The cross term $\text{Re}\{A_{nn\to\ell^+\ell^-;\gamma}\, A^*_{nn\to\ell^+\ell^-;Z}\}$ is also small because of the factor $\sim G_F s = 4.11 \times 10^{-5}$. Thus, although for the $J = 0$ initial nn state, the axial-vector part of J_Z has a nonzero contraction $q^\lambda[\bar{u}(p_2)\gamma_\lambda\gamma_5 v(p_1)] = 2m_\ell[\bar{u}(p_2)\gamma_5 v(p_1)]$, this contribution is suppressed both by the smallness of $2m_\ell/\sqrt{s} = m_\ell/m_N$ and by the $G_F s$ factor in the amplitude.

The two-body phase space factor for a decay of an initial state with mass $\sqrt{s}$ to final-state (fs) particles with masses m_1 and m_2 is

$$R^{(fs)}_2 = \frac{1}{8\pi}\,[\lambda(1, m_1^2/s, m_2^2/s)]^{1/2} \ , \tag{18}$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx) \ . \tag{19}$$

Hence, for the relevant case $m_1 = m_2 \equiv m$, $R_2 = (8\pi)^{-1}\sqrt{1 - 4m^2/s}$. The square root is equal to 0.9896, 1.0000, and 0.9937 for the respective decays $nn \to 2\pi^0$, $nn \to e^+e^-$, and $nn \to \mu^+\mu^-$.

We are thus led to the estimate

$$\Gamma_{nn\to\ell^+\ell^-} \sim P_{nn,\,J\neq 0}\, e^4\, \frac{R^{(\ell^+\ell^-)}_2}{R^{(2\pi^0)}_2}\,\Gamma_{nn\to 2\pi^0} \sim P_{nn,\,J\neq 0}\, e^4\, \Gamma_{nn\to 2\pi^0} \ , \tag{20}$$

where we have used the fact that $R^{(\ell^+\ell^-)}_2/R^{(2\pi^0)}_2$ is very close to unity for both $\ell = e$ and $\ell = \mu$. Utilizing the lower limit on $(\tau/B)_{nn\to 2\pi^0}$ in Eq. (10) together with the estimate (20), we thus obtain the following estimates for lower limits on the partial lifetimes for dinucleon to dilepton decays per ^{16}O nucleus:

$$(\tau/B)_{nn\to\ell^+\ell^-} \gtrsim (P_{nn,\,J\neq 0})^{-1}\,(5 \times 10^{34}\ \text{yr}) \gtrsim 5 \times 10^{34}\ \text{yr for } \ell = e,\ \mu \ , \tag{21}$$

where the final inequality follows from the fact that $P_{nn,\,J\neq 0} < 1$. Even without inserting an estimated value for the suppression factor due to $P_{nn,\,J\neq 0}$, our bound (21) is stronger than the direct limits on these two decays, which are (from the SuperKamiokande experiment) [21]:

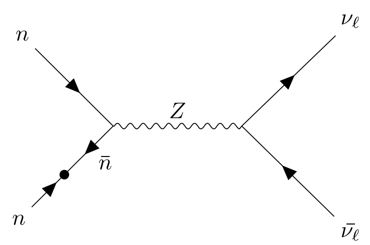

Fig. 2. Feynman diagram for $nn \to \nu_\ell \bar{\nu}_\ell$, where $\nu_\ell = \nu_e,\ \nu_\mu,\ \nu_\tau$.

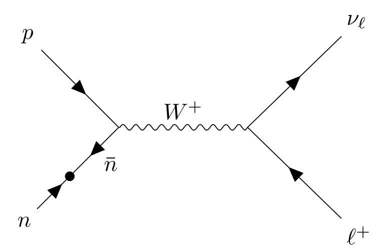

Fig. 3. Feynman diagram for $np \to \ell^+\nu_\ell$, where $\ell = e,\ \mu,\ \tau$.

$$(\tau/B)_{nn\to e^+e^-} > 4.2 \times 10^{33}\ \text{yr} \tag{22}$$

and

$$(\tau/B)_{nn\to \mu^+\mu^-} > 4.4 \times 10^{33}\ \text{yr}\ . \tag{23}$$

We next consider the decay $nn \to \nu_\ell \bar{\nu}_\ell$, where $\nu_\ell = \nu_e,\ \nu_\mu$, or ν_τ. This decay arises from a process in which the $|\bar{n}\rangle$ in $|n\rangle_{\text{phys.}}$ annihilates with a neighboring neutron to produce a virtual Z boson in the s-channel, which then yields the final-state $\nu_\ell\bar{\nu}_\ell$ pair, as shown in Fig. 2. Here and below, we shall refer to this as a tree-level process, having integrated out any loops in a BSM model to obtain the local four-fermion operators in the low-energy effective Hamiltonian $\mathcal{H}_{eff}^{(n\bar{n})}$. (More precisely, it is a tree-level process as regards SM fields.) One may again analyze the contributions of the $J=0$ and $J \neq 0$ initial nn states. For the $J=0$ initial state, by the same argument as above, the vector part of the neutral current gives a vanishing contribution, and the axial vector part gives a negligibly small contribution to the amplitude proportional to neutrino masses. Hence, the decay arises from the $J \neq 0$ initial dineutron states. We thus obtain the rough estimate

$$\Gamma_{nn\to\nu_\ell\bar{\nu}_\ell} \sim P_{nn,J\neq 0}\,(G_F s)^2\,\Gamma_{nn\to\pi^0\pi^0}\ . \tag{24}$$

Combining this with the experimental limit (10), we obtain the rough lower bound, per ^{16}O nucleus,

$$(\tau/B)_{nn\to\nu_\ell\bar{\nu}_\ell} \gtrsim P_{nn,J\neq 0}^{-1}\,(2\times 10^{41}\ \text{yr})$$
$$\gtrsim 2\times 10^{41}\ \text{yr} \quad \text{for}\ \nu_\ell = \nu_e,\ \nu_\mu,\ \nu_\tau\ . \tag{25}$$

For comparison, there is a bound from a direct search by the KamLAND experiment,[1] namely [22]

$$(\tau/B)_{nn\to\text{inv.}} > 1.4\times 10^{30}\ \text{yr} \tag{26}$$

per ^{12}C nucleus, where "inv." denotes an invisible final state, e.g., one with two neutral, weakly interacting particles which do not decay in the detector (and which could be $\nu\nu$, $\nu\bar{\nu}$, or $\bar{\nu}\bar{\nu}$, with undetermined flavors). Since the final-state (anti)neutrinos were not observed, the limit (26) applies to all of these possibilities. For the case where the final state is $\nu_\ell\bar{\nu}_\ell$, our estimated lower bound in (25) is considerably stronger than the direct experimental limit (26).

Finally, we derive a relation between the rates for $np \to \pi^+\pi^0$ and $np \to \ell^+\nu_\ell$, where $\ell^+ = e^+,\ \mu^+,\ \tau^+$. At tree level, the amplitude $np \to \ell^+\nu_\ell$ arises from the process in which the $|\bar{n}\rangle$ component in $|n\rangle_{phys.}$ annihilates with a neighboring proton to produce a virtual W^+ boson which then yields the final-state $\ell^+\nu_\ell$ pair. This is shown in Fig. 3. Denoting the four-momenta of the ν_ℓ and ℓ^+ as p_2 and p_1, we write

$$A_{np\to\ell^+\nu_\ell} = (\delta m)\frac{G_F}{\sqrt{2}}\,\langle 0|J_W^\lambda|\bar{n}p\rangle\,[\bar{u}(p_2)\gamma_\lambda(1-\gamma_5)v(p_1)]\ . \tag{27}$$

The initial np state is a mixture of $I=0$ and $I=1$ isospin states. The $I=0$ state is analogous to the deuteron, with $S=1$ and dominantly $L=0$, whence $J=1$. The $I=1$ np state has dominantly $L=0$, $S=0$, and hence $J=0$, leading to severe helicity suppression of the decays if $\ell^+ = e^+$ or $\ell^+ = \mu^+$, although this helicity suppression not so severe for $np\to\tau^+\nu_\tau$. In contrast, the decays $np \to \ell^+\nu_\ell$ from the initial np states with $J \neq 0$ are not helicity-suppressed. This is similar to the fact that there is no helicity suppression in the leptonic decays of a real W boson. It is thus expected that the dominant contribution to $np\to\ell^+\nu_\ell$ arises from the $I=0$, $J=1$ component of the initial np state. We thus estimate

$$\Gamma_{np\to\ell^+\nu_\ell} \sim (G_F s)^2\,\frac{R_2^{(\ell^+\nu_\ell)}}{R_2^{(\pi^+\pi^0)}}\,\Gamma_{np\to\pi^+\pi^0} \tag{28}$$

The phase space factor for $np \to \ell^+\nu_\ell$ decay is $R_2^{(\ell^+\nu_\ell)} = (8\pi)^{-1}[1 - m_\ell^2/(2m_N)^2]$. The expression in square brackets has the respective values 1.0000, 0.9969, and 0.1047 for $\ell = e,\ \mu,\ \tau$. In the decay $np\to\pi^+\pi^0$, $R_2^{(\pi^+\pi^0)} = (8\pi)^{-1}(0.9893)$. Combining Eq. (28) with these values for the phase space factors and the experimental limit (11), we obtain the rough lower bounds, per ^{16}O nucleus,

$$(\tau/B)_{np\to\ell^+\nu_\ell} \gtrsim 10^{41}\ \text{yrs} \quad \text{for}\ \ell = e,\ \mu \tag{29}$$

and

$$(\tau/B)_{np\to\tau^+\nu_\tau} \gtrsim 10^{42}\ \text{yr}\ . \tag{30}$$

The SK experiment has reported the limits [20]

$$(\tau/B)_{np\to e^+x} > 2.6\times 10^{32}\ \text{yr} \tag{31}$$

and

$$(\tau/B)_{np\to\mu^+x} > 2.2\times 10^{32}\ \text{yr} \tag{32}$$

per ^{16}O nucleus, where x denotes a neutrino or antineutrino (of undetermined flavor). For the cases in which $x = \nu_e$ in (31) and $x = \nu_\mu$ in (32), our bounds are much stronger than these limits from direct experimental searches. It was pointed out in [19] that data from existing searches for nucleon and dinucleon decays into multilepton final states involving e^+ and μ^+ plus (anti)neutrinos could be retroactively analyzed to set a limit on the decay $np\to\tau^+\bar{\nu}_\tau$, since the τ^+ could decay as $\tau^+\to\bar{\nu}_\tau\ell^+\nu_\ell$ with $\ell = e$ or $\ell = \mu$. Ref. [19] carried out such an analysis and obtained a lower bound $(\tau/B)_{np\to\tau^+\bar{\nu}_\tau} > 1\times 10^{30}$ yr per ^{16}O nucleus. Subsequently, from a direct search, SK obtained the limit [20]

$$(\tau/B)_{np\to\tau^+x} > 2.9\times 10^{31}\ \text{yr} \tag{33}$$

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[1] The KamLAND bound was obtained via a search for the decays of the resultant ^{10}C nucleus [22]. Although our bound applies to an ^{16}O nucleus rather than ^{12}C nucleus, one does not expect the rates to differ very much between these nuclei with almost equal numbers of nucleons. A weaker bound, $(\tau/B)_{nn\to\text{inv.}} > 1.3\times 10^{28}$ yr. per ^{16}O nucleus has been obtained by the SNO+ experiment [23].

per ^{16}O nucleus, where x is a neutrino or antineutrino (of undetermined flavor). For the case in which $x = \nu_\tau$, our bound (30) is much stronger than this direct limit. As is evident from our derivations, our limits constrain dinucleon decays that have $\Delta L = 0$. They do not constrain dinucleon decays with $\Delta L \neq 0$, such as the $\Delta L = -2$ decays $nn \to \bar{\nu}_\ell \bar{\nu}_{\ell'}$ and $np \to \tau^+ \bar{\nu}_\tau$ or the $\Delta L = +2$ decay $nn \to \nu_\ell \nu_\ell$. Using similar methods, we have derived improved upper bounds on several decay models of individual protons and bound neutrons. These are reported elsewhere [24].

Acknowledgements

This research was supported in part by the NSF Grants NSF-PHY-1620628 and NSF-PHY-1915093 (R.S.).

References

[1] A.D. Sakharov, JETP Lett. 91 (1967) 24, Pis'ma Zh. Eksp. Teor. Fiz. 5 (1967) 32.
[2] V. Kuzmin, JETP Lett. 12 (1970) 228, Pis'ma Zh. Eksp. Teor. Fiz. 12 (1970) 335.
[3] R. Mohapatra, R. Marshak, Phys. Rev. Lett. 44 (1980) 1316.
[4] S.L. Glashow, HUTP-79/A059;
L.-N. Chang, N.-P. Chang, Phys. Lett. B 92 (1980) 103;
L.-N. Chang, N.-P. Chang, Phys. Rev. Lett. 45 (1980) 1540;
T.K. Kuo, S. Love, Phys. Rev. Lett. 45 (1980) 93;
R. Cowsik, S. Nussinov, Phys. Lett. B 101 (1981) 237.
[5] S. Rao, R. Shrock, Phys. Lett. B 116 (1982) 238;
S. Rao, R. Shrock, Nucl. Phys. B 232 (1984) 143.
[6] M. Baldo-Ceolin, et al., Z. Phys. C 63 (1994) 409.
[7] S. Nussinov, R. Shrock, Phys. Rev. Lett. 88 (2002) 171601.
[8] J. Chung, et al., Soudan Collab., Phys. Rev. D 66 (2002) 032004.
[9] E. Friedman, A. Gal, Phys. Rev. D 78 (2008) 016002.
[10] J.M. Arnold, B. Fornal, M.B. Wise, Phys. Rev. D 87 (2013) 075004.
[11] D.C. Phillips, et al., Phys. Rep. 612 (2015) 1.
[12] C. Berger, et al., Fréjus Collab., Phys. Lett. B 269 (1991) 227.
[13] M. Takita, et al., Kamiokande Collab., Phys. Rev. D 34 (1986) 902.
[14] J. Chung, et al., Soudan Collab., Phys. Rev. D 66 (2002) 032004 (iron).
[15] B. Aharmim, et al., SNO Collab., Phys. Rev. D 96 (2017) 092005.
[16] K. Abe, et al., SuperKamiokande Collab., Phys. Rev. D 91 (2015) 072006.
[17] J. Gustafson, et al., SuperKamiokande Collab., Phys. Rev. D 91 (2015) 072009.
[18] S. Gardner, X. Yan, Phys. Rev. D 97 (2017) 056008.
[19] D. Bryman, Phys. Lett. B 733 (2014) 190.
[20] V. Takhistov, et al., SuperKamiokande Collab., Phys. Rev. Lett. 115 (2015) 121803.
[21] S. Sussman, et al., SuperKamiokande Collab., arXiv:1811.12430.
[22] T. Araki, et al., KamLAND Collab., Phys. Rev. Lett. 96 (2006) 101802.
[23] M. Anderson, et al., SNO+ Collab., Phys. Rev. D 99 (2019) 032008.
[24] S. Girmohanta, R. Shrock, Phys. Rev. D 100 (2019) 115025, arXiv:1910.08106.