Heavy-ion induced double charge exchange reaction in a multi-channel approach: the research program of the NUMEN project

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Abstract. We give an updated view of the status and prospects of heavy-ion double charge exchange (HI-DCE) reaction studies performed at the Laboratori Nazionali del Sud of the Istituto Nazionale di Fisica Nucleare (INFN-LNS) in the context of the NUMEN project. The important role of HI-DCE for nuclear reaction, nuclear structure and double beta-decay investigations is outlined. A powerful way to scrutinize the nuclear response to HI-DCE is to consistently link it to the information extracted from the competing direct reactions pointing to a multi-channel description of the whole network of quasi-elastic processes. Indeed, these complementary studies are mandatory in order to minimize the systematic errors in the data analyses and build a many-facets and parameter-free representation of the systems under study.

A new research line in the field of nuclear reaction and structure has been opened by the NUMEN project [1–3], which is primarily focused on the investigations of nuclear double charge exchange (DCE) reactions. DCE reactions probe a special class of nuclear transitions $a(z,n)+A(Z,N)\rightarrow b(z\pm 2,n\mp 2)+B(Z\mp 2,N\pm 2)$, in which the initial mass partitions are conserved, but nuclear charges are redistributed in a complementary manner by two units. Such processes are of high interest for understanding nuclear isospin dynamics under controlled laboratory conditions since they provide unique access to second order isospin effects in bound nuclear systems, so far never studied systematically with nuclear probes. While nuclear isovector response is being studied extensively and successfully by single charge exchange (SCE) reactions induced by light and heavy-ion beams [4–8], DCE reactions are within the domain of heavy-ion research. Resembling the experiences gained in heavy-ion SCE reactions, special features of nuclear $|\Delta Z| = 2$ processes can be enhanced or suppressed by the selection of specific final reaction channels, e.g. favoring non–spin flip or spin–flip transitions. In this and many other respects, HI–DCE physics provides a much broader spectrum of research opportunities than the previously used pionic $(\pi^{\pm},\pi^{\mp})$ DCE reactions [9], also in consideration of the much better energy definition of the incoming beams.

The NUMEN project is mainly committed with measurements of heavy-ion induced DCE cross sections at incident energies above the Coulomb barrier [10, 11]. The reaction ejectiles are detected by modern high resolution and large acceptance magnetic spectrometers, which are the proper instruments for high precision spectroscopic studies [10–15]. Although the key aspect of the experimental strategy is the measurement of DCE cross sections, the contextual study of a multitude of related elastic and inelastic, transfer, and single charge exchange channels is also required. The access to the transitions of interest and their spectroscopy relies on the high energy resolution measurements and the accurate determination of differential cross sections at very forward angles [16]. Notoriously, in heavy–ion collisions many reaction channels are populated among which the DCE configurations account only for a tiny portion of the outgoing flux. Hence, it is a relevant experimental challenge to identify unambiguously the desired reaction products out of the debris

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originated in the projectile–target collision, requiring an effective background rejection capability by the experimental setup. The main background source consists of charged particles originated from other reaction products and beam scattering. However, as an advantage, the multi–channel features stemming out from heavy-ion collisions offer unique opportunities for the simultaneous investigation of reaction channels contributing as intermediate configurations to the DCE reactions of main interest. The multi–channel approach practiced in the NUMEN project allows to explore in an unprecedented manner the reaction mechanism and the structural details of nuclear DCE configurations. Since the various observables are measured under the same experimental conditions, systematic uncertainties are strongly reduced. Multi–channel strategies have been also adopted in other fields of heavy-ion research [17], but the systematic use of this approach for the analysis of complex sets of data is a quite recent development, belonging to the defining features of NUMEN.

On the theoretical side, an equally demanding program is necessary, aiming at a unified description of the large variety of nuclear reaction and structure phenomena. However, until recently neither nuclear reaction nor structure theory were prepared for the description of second order processes addressing the special issues of isospin dynamics. A pertinent problem in structure theory with respect to DCE configurations is clearly documented by the spread in values of nuclear matrix elements (NME) for double beta ($\beta\beta$) decay, pointing to deficiencies in our understanding of high-order isospin dynamics in many–body systems. Clearly, the problem can only be solved by applying the structure models to a variety of independent physical observables among which DCE cross sections and related spectroscopy are the ideal test range. Nuclear reaction theory has dealt with multi–step phenomena for decades but typically in a rather selective manner by considering strongly restricted subsets of interacting transfer, inelastic, and rotational channels coupled to an elastic channel. The complete description of DCE dynamics requires much broader efforts. The task starts with the derivation of optical potentials – constrained to describe elastic scattering cross sections and to be compatible for all non–elastic channels – and requires to incorporate the assumed nuclear structure effects into the reaction theoretical formalism and to determine the appropriate residual ion–ion interactions. Hence, as on the experimental side, also reaction and structure theory must pursue a multi–channel methodology.

In the context of the theory project embedded in NUMEN, the nuclear structure theory activities indeed span a broad spectrum of approaches: nuclear ground states and single particle spectra are obtained by Hartree–Fock-Bogoliubov (HFB) mean–field theory; the quasi-particle random phase approximation (QRPA) is used for spectroscopic purposes high up beyond the particle emission threshold; single particle and nucleon pair spectroscopic amplitudes are derived by advanced shell–model and interacting boson model methods.

The reaction theoretical methods are based on coupled channels studies of the measured quasi-elastic cross sections. In particular, elastic and inelastic scattering are studied to constrain, for each projectile–target system, the optical potentials and the mean field description [18–21]. Single nucleon and pair transfer processes are described by large enough coupling schemes accounting for non–orthogonality and finite range effects to probe the goodness of the adopted mean field [22–28]. The SCE, incorporating both two–step nucleon transfer as well as one–step meson exchange contributions, give important constraint on the nucleon-nucleon T–matrix interaction [29–31].

Once the nucleon transfer components are adequately constrained, the crucial question whether the leading reaction mechanism of DCE reactions is the exchange of nucleons by mean-filed dynamics can be addressed. As shown in [32], the mutual exchange of pairs of neutron and protons between the colliding ions gives a negligible contribution to DCE cross sections at the incident energies considered here. Thus, in leading order transfer DCE (TDCE) can be neglected, implying that the observed DCE cross sections must result from a different kind of reaction mechanism. As discussed in [33–35], DCE reactions proceed also by collisional nucleon–nucleon (NN) second–order processes, appropriately described by meson exchange interactions, which generalize those entering in SCE one-step reactions. On the reaction side, this is described by a newly developed second order (two–step) distorted wave theory [33, 34] which gives direct access to the desired spectral distributions, the underlying nuclear transition form factors and the related DCE–NME.

The direct component of DCE reactions proceed in general by two distinct collisional interaction scenarios. In the double single charge exchange (DSCE) mechanism the isovector NN T-matrix acts twice in a two–step sequence of SCE interactions, where each interaction is sustained by the same ingredients as known from single charge exchange reactions. In [8, 36] particular attention was paid to obtain a DSCE formalism compatible with the previously developed theory of SCE reactions. A completely different mechanism is underlying the competing mesonic or Majorana DCE (MDCE) process [37]. In the Majorana scenario the second order character of the DCE process is taken care of by virtual $\pi^{\pm}$ exchange between the interacting ions, from correlated pairs of nucleons, thus being a genuine two–nucleon process. Differently from the DSCE mechanism, MDCE dynamics is determined by the isovector pion–nucleon T–matrix. A subclass of processes corresponds in fact to virtual off–shell $(\pi^{\pm}, \pi^{\mp})$ reactions, inducing mutual DCE processes in projectile and target. An incoming charged pion transfers its charge to a nucleon and leaves the vertex as a neutral pion which interacts with a second nucleon in the same nucleus, initiating a π^0+N $\rightarrow$ $\pi^{\pm}$+N' pion–nucleon charge exchange reaction. The charged pion is transmitted to the second nucleus where it initiates a complementary DCE transition. Hence, in each nucleus MDCE is a two–nucleon process of second order in the pion–nucleon isovector T–matrix where the two isovector vertices are correlated by π^0–exchange. As a result, the nucleus is left in either a configuration with two protons added and two neutrons removed (p^2n^{-2}) or viceversa (n^2p^{-2}), which subsequently evolves into the eigenstates of the $\Delta Z = \pm 2$ daughter nucleus. The complementary process occurs in the reaction partner, thus conserving the total charge of the projectile–target system. Obviously, each of the MDCE–pions may be replaced by other mesons, although only the pionic MDCE processes has been considered so far.

Aside from the strong relevance and importance for nuclear physics, DCE reactions are unique and natural probes for physical quantities entering into $\beta\beta$ decay processes. The $\beta\beta$–oriented research is extending the successful use of hadronic SCE reaction for single beta–decay studies to the hitherto widely unexplored region of $\Delta Z = \pm 2$ transitions. As a major issue, DCE reactions are perfectly well suited to probe the nuclear structure aspects in practically all stable nuclei under scrutiny for $\beta\beta$ studies. In fact, new projects have been proposed recently in Italy [1, 38] and Japan [39], respectively, with the aim to investigate the nuclear DCE response especially of the isotopes of interest for neutrinoless double beta decay ($0\nu\beta\beta$) in present and future dedicated experiments.

So far, since the first observation in 1987 [40], $\beta^{\mp}\beta^{\mp}$ decays have been observed in about a dozen medium– to heavy–mass nuclides [41], but always in connection with the emission of two anti-neutrinos or neutrinos. This two-neutrino double beta decay ($2\nu\beta\beta$) is in full agreement with lepton number conservation and all known physics laws. As discussed in detail in [34] the DSCE reaction amplitude can be cast into a form matching in structure the NME of $2\nu\beta\beta$ decay, thus establishing an important connection between seemingly well distinct areas.

Among the $\beta\beta$ processes, $0\nu\beta\beta$ is of special interest by addressing a class of nuclear processes generating spontaneously new matter. Two electrons (or positrons) are emitted by the decay of a nucleus into an isobar daughter nucleus differing by two units of charges, a process which is strictly forbidden by lepton number conservation. These theoretically postulated phenomena are eagerly searched for by large scale underground experiments as signals for *beyond the standard model* (BSM) physics. If confirmed experimentally, $0\nu\beta\beta$ decay would unveil that lepton number conservation is violated, a result with profound consequences in many aspect of the known physics, also including a possible explanation of the puzzling matter-antimatter asymmetry observed in the Universe [42, 43].

How can nuclear physics – and especially heavy-ion physics – contribute to the clarification of this exciting and demanding open problem? For an answer to that question, a closer look to the physics ingredients of the $0\nu\beta\beta$ decay rates is helpful. Accounting for light (L) and heavy (H) Majorana neutrino exchange, one finds the general relation for the $0\nu\beta\beta$ decay rate $[T^{(0\nu)}_{1/2}]^{-1}$ [44]

$$\left[T^{(0\nu)}_{1/2}\right]^{-1} = G_{0\nu} g_A^4(0) \left| M^{(0\nu)}_L \frac{\langle m_L \rangle}{m_e} + M^{(0\nu)}_H \langle \frac{M_p}{M_H} \rangle \right|^2 . \quad (1)$$

where $M^{(0\nu)}_{L,H} \simeq M^{GT}_{L,H} + (\frac{g_V}{g_A})^2 M^F_{L,H}$ includes matrix elements of Gamow-Teller (GT) and Fermi (F) transitions, mediated by axial-vector and tensor operators, and vector operators, respectively. Here g_A and g_V represent the axial–vector and vector coupling constant, respectively. All subtleties of the final state interactions of the emitted the $e^{\pm}e^{\pm}$–pair are contained in the so–called phase space factor $G_{0\nu}$ which nowadays is known to high accuracy [45]. The quantities of major interest for BSM physics are the effective Majorana mass of the light neutrinos $\langle m_L \rangle$ and the effective inverse Majorana mass of the heavy neutrino $\langle \frac{1}{M_H} \rangle$, where the former is normalized to the electron mass m_e and the latter to the proton mass M_p. However, obviously their unambiguous determination depends on the knowledge of the nuclear matrix elements

$$M^{(0\nu)}_{L,H} = \langle B | O^{(0\nu\beta\beta)}_{L,H} | A \rangle \quad (2)$$

describing the transition from the nuclear parent state A to the $\Delta Z = \pm 2$ daughter state B by the corresponding $0\nu\beta\beta$ operators $O^{(0\nu\beta\beta)}_{L,H}$, given by a superposition of vector, weak magnetic, axial-vector, and pseudoscalar terms, see e.g. Ref. [46].

In Ref. [47] a collection of $\beta\beta$–NME obtained by state–of–the-art nuclear structure calculations is given, illustrating the systematic uncertainties imposed by a spread of values by a factor of two to three, excluding uncertainties in g_A, emphasizing the sensitivity of $0\nu\beta\beta$ to details of nuclear many–body dynamics.

These results point to the urgency of independent tests of the nuclear wave functions entering into the $0\nu\beta\beta$–NME, preferentially in processes involving operators of a similar structure as encountered in $0\nu\beta\beta$ decay. A characteristic feature of $0\nu\beta\beta$ decay is that it probes a broad range of momentum components of the involved nuclear states. Indeed, the exchange of the Majorana neutrino among two nucleons generates a natural localization of about 2 fm, which is the average distance of the nucleons inside the nucleus, resulting in a momentum spread of about 100 MeV/c [42]. This aspect has a deep impact on the dynamics of the decay, making it necessary to describe intermediate virtual states up to high excitation energies and multipolarities [42, 48].

High precision experimental information from $2\nu\beta\beta$ [42], ordinary muon capture [49–51], nucleon transfer reactions [52–54], double gamma decay [55], SCE [56–58], and DCE reactions [10] are or could be used to constrain specific features of the $0\nu\beta\beta$–NME calculations [42]. Although none of these studies can provide a direct access to the $0\nu\beta\beta$–NMEs, they all give helpful information toward such an objective.

In this framework, DCE reactions, although originating from the strong interaction between projectile and target, are particularly appealing. In fact, they share the same nuclear states as the $\beta\beta$ decays and are driven by short–range second order isospin operators. In addition, they probe a broad spectrum of momenta and multipolarities in the intermediate odd-odd isobar nucleus and are sensitive to nucleon-nucleon correlations insofar the MDCE is concerned, thus resembling the key dynamical features of $0\nu\beta\beta$.

As pointed out above DCE reactions will proceed also by two direct reaction mechanisms. They are of special interest because the reaction amplitudes contain second order nuclear transition form factors of a similar structure as the $\beta\beta$–NME, albeit the DCE and the $\beta\beta$ transition operators do not match one–by–one. The extraction of the desired information from data is a demanding task even when the reaction proceeds under conditions preferable for the collisional direct mechanism by suppressing the sequential multi–nucleon transfer contributions. Of special interest is the MDCE reaction amplitude, which is determined by pion potentials resulting from the intranuclear π^0 exchange, which in MDCE theory plays exactly the same role as the neutrino potentials in $0\nu\beta\beta$, thus establishing an important connection between strong interaction DCE and weak interaction $\beta\beta$ physics.

It is worth emphasizing that the physical observables are the decay rates for $\beta\beta$ and the cross sections for DCE. Instead, neither the $\beta\beta$–NME nor the DCE transition form factors are directly accessible by experiments. Thus, in both cases additional steps of theoretical work are necessary before the nuclear many–body and the dynamical quantities of interest can be separated. In $\beta\beta$ theory, the NMEs are evaluated by state-of-art nuclear structure approaches [59, 60], ranging from proton–neutron quasiparticle random phase approximation (pnQRPA) [61, 62] and energy density functional (EDF) descriptions [63–65] to interacting shell–model methods (ISM) [66–68], sophisticated phenomenological approaches like the interacting boson model (IBM) [69], and ab–initio schemes for nuclear many–body calculations [70]. An important common feature of these techniques is to project the nuclear many–body states from the complete Hilbert space into limited model subspaces, chosen such that numerical calculation can be actually performed. The projection procedures includes the consistent redefinition of transition operators by corresponding projection techniques. The purpose is to keep, based on reasonable physical arguments, the relevant aspects of the NMEs within the model space, leaving out pieces of the wave functions (assumed to be) of minor importance. However, this condition cannot be guaranteed *a priori* and needs to be checked by comparison with appropriate experimental data. In order to remove systematic uncertainties which may be hidden as a common caveat of different models when adjusting the parameters to the same kind of data, it is necessary to have available additional completely independent sets of data, probing the same many–body system. Since DCE reaction theory does not rely on the use of a specific structure model, results of the structure approaches used for $\beta\beta$ can easily be implemented into the reaction calculations on the level of form factors and transition potentials. That is the important contribution of heavy-ion DCE physics to the possible solution of the open questions of $\beta\beta$ physics, as emphasized in [1, 10].

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