A possible explanation of the secular increase of the astronomical unit

Takaho Miura[(a)], Hideki Arakida, [(b)] Masumi Kasai[(a)] and Syuichi Kuramata[(a)]

[(a)] *Faculty of Science and Technology,Hirosaki University, 3 bunkyo-cyo Hirosaki, Aomori, 036-8561, Japan*

[(b)] *Education and integrate science academy, Waseda University,1 waseda sinjuku Tokyo, Japan*

Abstract

We give an idea and the order-of-magnitude estimations to explain the recently reported secular increase of the Astronomical Unit (AU) by Krasinsky and Brumberg (2004). The idea proposed is analogous to the tidal acceleration in the Earth-Moon system, which is based on the conservation of the total angular momentum and we apply this scenario to the Sun-planets system. Assuming the existence of some tidal interactions that transfer the rotational angular momentum of the Sun and using reported value of the positive secular trend in the astronomical unit, $\frac{d}{dt} 15 \pm 4$(m/s),the suggested change in the period of rotation of the Sun is about 21(ms/cy) in the case that the orbits of the eight planets have the same "expansionrate."This value is sufficiently small, and at present it seems there are no observational data which exclude this possibility. Effects of the change in the Sun's moment of inertia is also investigated. It is pointed out that the change in the moment of inertia due to the radiative mass loss by the Sun may be responsible for the secular increase of AU, if the orbital "expansion"is happening only in the inner planets system. Although the existence of some tidal interactions is assumed between the Sun and planets, concrete mechanisms of the angular momentum transfer are not discussed in this paper, which remain to be done as future investigations.

1 Introduction

The Astronomical Unit (hereafter we abbreviate AU) is one of the most essential scale in astronomy which characterizes the scale of the solar system and the standard of cosmological distance ladder. AU is also the fundamental astronomical constant that associates two length unit; one (m) in International System (SI) of Units and one (AU) in Astronomical System of Units.

In the field of fundamental astronomy e.g., the planetary ephemerides, it is one of the most important subjects to evaluate AU from the observational data. However, recently Krasinsky and Brumberg reported the positive secular trend in AU as $\frac{d}{dt}\mathrm{AU} = 15 \pm 4\,(\mathrm{m/cy})$. from the analysis of radar ranging of inner planets and Martian landers and orbiters (Krasinsky and Brumberg 2004;Standish2005).

2 Tidal Acceleration in the Earth-Moon System

In this section, we briefly summarize the tidal acceleration in the Earth-Moon system.

The conservation law of the total angular momentum in the Earth-Moon system is

$$\frac{d}{dt}\left(\ell_{\mathrm{E}} + L_{\mathrm{M}}\right) = 0, \tag{1}$$

where ℓ_{E} is the rotational angular momentum of the Earth, L_{M} is the orbital angular momentum of the Moon, and for simplicity we have neglected the rotational angular momentum of the Moon, which is about 10^{-5} times smaller than L_{M}. Then, if we assume the moment of inertia, mass, and the eccentricity of the moon's orbit are constant, the conservation of the total angular momentum Eq. (1) reads

$$\frac{\dot{T}_{\mathrm{E}}}{T_{\mathrm{E}}} = \frac{1}{2}\frac{L_{\mathrm{M}}}{\ell_{\mathrm{E}}}\frac{\dot{r}}{r} \tag{2}$$

The gradual slowing of the Earth's rotation is due to the tidal force between the orbiting Moon and the Earth, or the tidal friction.

3 Application to the Sun-Planets System

In this section, we apply the same argument in the previous section to the Sun-planets system. All we need is the conservation of the total angular momentum in the solar system. We denote the mass and the orbital elements of each planet by subscript i. The length AU, denoted by a, is used to normalize r_i, then for the Earth's radius $r_3 \equiv r_{\rm E} = a$, and for the moment it is assumed that the orbits of the all planets have the same "expansion rate," i.e.,

$$\frac{\dot{r}_i}{r_i} = \frac{\dot{a}}{a} \tag{3}$$

for each i.

In the case that the Sun's moment of inertia does not change, from the analogy to Eq. (2), we obtain the change in the period of rotation of the Sun $T_\odot$ as

$$\frac{\dot{T}_\odot}{T_\odot} = \frac{1}{2}\frac{L}{\ell_\odot}\frac{\dot{a}}{a}, \tag{4}$$

where L is the sum of the orbital angular momentums of all planets

$$L = \sum_i m_i \sqrt{GM_\odot r_i (1 - e_i^2)}, \tag{5}$$

Using the value $\dot{a} \simeq 15 (\mathrm{m/cy})$ reported by Krasinsky and Brumberg, $\dot{a}/a \simeq 1.0 \times 10^{-10}$ and the right-hand-side of Eq. (4) is evaluated as

$$\frac{\dot{T}_\odot}{T_\odot} \simeq 5.7 \times \gamma_\odot^{-1} \times 10^{-10}\,(\mathrm{cy}^{-1}). \tag{6}$$

where γ is the moment of inertia factor. The moment of inertia is

$$I_\odot = \gamma_\odot M_\odot R_\odot^2. \tag{7}$$

If we use the value $\gamma_\odot = 0.0059$ and the rotational period of the Sun as $T_\odot = 25.38\,(\mathrm{days})$, the estimated value of the change in $T_\odot$ is

$$\dot{T}_\odot \simeq 21\,(\mathrm{ms/cy}). \tag{8}$$

The estimated value is sufficiently small and seems to be well within the observational limit.

4 Effect of the change in the moment of inertia

In the previous section, we have considered only the case that the moment of inertia $I_\odot$ is constant. If we generalize our result to the case that $I_\odot$ and $M_\odot$ are not constant, Eq. (4) changes to

$$-\frac{\dot{\gamma}_\odot}{\gamma_\odot} - \frac{\dot{M}_\odot}{M_\odot} - 2\frac{\dot{R}_\odot}{R_\odot} + \frac{\dot{T}_\odot}{T_\odot} = \frac{1}{2}\frac{L}{\ell_\odot}\left(\frac{\dot{M}_\odot}{M_\odot} + \frac{\dot{a}}{a}\right). \tag{9}$$

As a first approximation, we assume that the radiative mass loss occurs isotropically along radial direction and does not carry the angular momentum. The first term in the left-hand-side of Eq. (9) represents the effect of change in the internal density distribution of the Sun, and so far we do not have enough information on it in detail.

The second term in the left-hand-side of Eq. (9) represents the effect of mass loss, which can be evaluated in the following way. The Sun has luminosity at least $3.939 \times 10^{26} W$,or $4.382 \times 10^9 kg/s$,including electromagnetic radiation and contribution from neutrinos (Noerdlinger 2008). The particle mass loss rate by the solar wind is about 1.374×10^9 kg/s, according to Noerdlinger (2008). The total solar mass loss rate is then

$$-\frac{\dot{M}_\odot}{M_\odot} = 9.1 \times 10^{-12}(cy^{-1}), \tag{10}$$

which is less than a thousandth of the required value to explain the secular increase of AU (see Eq. (6)). (As pointed out by Noerdlinger (2008), Krasinsky and Brumberg (2004) unaccountably ignored the radiative mass loss $L_\odot = 3.86 \times 10^{26}$ W which is the major contribution to $\dot{M}_\odot/M_\odot$.) Therefore, we can conclude that the solar mass loss term in the left-hand-side of Eq. (9) does not make a significant contribution to the secular increase of AU, if the orbits of the eight planets have the same expansion rate.

Note that the term which is proportional to $\dot{M}_\odot/M_\odot$ also appears in the right-hand-side of Eq. (9). This term may be called as the Noerdlinger effect (Noerdlinger 2008). Noerdlinger already investigated this effect of solar mass loss, and concluded that the effect can only account for less than a tenth of the reported value by Krasinsky and Brumberg.

The third term in the left-hand-side of Eq. (9) is the contribution from the change in the solar radius. Although the very short-time and small-scale variability in the solar radius may be actually observed in the context of helioseismology, we have no detailed information on the secular change in $R_\odot$ far.

5 Case of the Sun-inner planets system

In the previous sections, we have assumed that the orbits of all the planets have the same expansion rate. However, the recent positional observations of the planets with high accuracy are mostly done within the inner planets region. Actually Krasinsky and Brumberg obtained dAU/dt by using these inner planets data, while the orbits of outer planets are given by the Russian ephemeris EPM (Ephemerides of Planets and the Moon).

Therefore, as a tentative approach, we consider the case that the expansion rates of the planetary orbits are not homogeneous but inhomogeneous in the sense that only the orbits of the inner planets expand.

In this section, we consider the case that the expansion of the planetary orbit occurs only for the inner planets.

In this case, the sum of the orbital angular momentum of all planets L is replaced by the sum of the inner planets $L_{\rm in}$:

$$L_{\rm in} \equiv \sum_{i=1}^{4} m_i \sqrt{G M_\odot r_i}. \tag{11}$$

Then Eq. (9) is now

$$-\frac{\dot{\gamma}_\odot}{\gamma_\odot} - \frac{\dot{M}_\odot}{M_\odot} - 2\frac{\dot{R}_\odot}{R_\odot} + \frac{\dot{T}_\odot}{T_\odot} = \frac{1}{2}\frac{L_{\rm in}}{\ell_\odot}\left(\frac{\dot{M}_\odot}{M_\odot} + \frac{\dot{a}}{a}\right). \tag{12}$$

Note that the sum of the angular momentum of inner planets amounts only 0.16% of the total L:

$$\frac{L_{\rm in}}{L} \simeq 1.6 \times 10^{-3}. \tag{13}$$

Therefore, under the assumption that the change in the rotational angular momentum of the Sun affects only the orbital angular momentums of the inner planets, the required values which were calculated in the previous sections to explain the secular increase of AU can now be revised to be 1.6×10^{-3} times smaller.

In particular, the right hand side of Eq. (12) is

$$\frac{1}{2}\frac{L_{\rm in}}{\ell_\odot}\frac{\dot{a}}{a} \simeq 1.5 \times 10^{-11}\,({\rm cy}^{-1}). \tag{14}$$

Interestingly, it is the same order of magnitude as (actually it is about 1.6 times larger than) $\dot{M}_\odot/M_\odot$.

Then we can conclude from Eq. (12) that the decrease of rotational angular momentum of the Sun due to the radiative mass loss has a significant contribution to the secular increase of the orbital radius of the inner planets.

6 Conclusion

In this paper, we considered the secular increase of astronomical unit recently reported by Krasinsky and Brumberg (2004), and suggested a possible explanation for this secular trend by means of the conservation law of total angular momentum. Assuming the existence of some tidal interactions that transfer the angular momentum from the Sun to the planets system, we have obtained the following results.

From the reported value $\frac{d}{dt}AU = 15 \pm 4$(m/cy),we have obtained the required value for the variation of rotational period of the Sun is about 21 (ms/cy), if we assume that eight planets in the solar system experience the same orbital expansion rate. This value is sufficiently small, and at present it seems there are no observational data which exclude this possibility.

Moreover, we have found that the effects of change in the moment of inertia of the Sun due to the radiative mass loss may be responsible for explaining the secular increase of AU. Especially, when we suppose that the orbital expansion occurs only in the inner planets region, the decrease of rotational angular momentum of the Sun has enough contribution to the secular increase of the orbital radius. Then as an answer to the question "why is AU increasing ?", we propose one possibility, namely "because the Sun is losing its angular momentum."

In present paper, we proposed the possible mechanism for explaining the secular increase of AU, nonetheless we need to verify the validity of our model by means of the some tidal dissipation models of the Sun. Moreover because the existence of dAU/dt is not confirmed robustly in terms of the independent analysis of observation by other ephemerides groups, it is important not only to perform the theoretical researches but also to re-analyze the data and to obtain more accurate value of dAU/dt adding new observations e.g., Mars Reconnaissance Orbiter, Phoenix and forthcoming MESSENGER which is cruising to the Mercury. It also seems to be meaningful to use the observations of outer planets as well, such as Cassini, Pioneer 10/11, Voyager 1/2, and New Horizons for Pluto since it is more natural situation that the variation of moment of inertia of the Sun causes the orbital changes not only of inner planets but also of outer ones. Therefore in order to reveal the origin of secular increase of AU, it is essential to investigate these sub jects in detail.

7 References

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