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# Current Topics In Weak Interaction Physics

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## INTRODUCTION

Weak Interaction physics has been amply reviewed on a number of recent occasions. Instead of attempting yet another general survey, let me simply declare that the standard empirical and theoretical lore is well known to you, at least in its major outlines. This will permit me to focus on a small subset of topics that appear to be of special interest just now. The standard lore, of course, may not be right, and in any case, it is certainly not complete. What we know about the weak interactions, after all, is still based on the very limited phenomenology of a few low energy decay processes and the early stages of high energy neutrino physics.

On the conventional picture, one supposes that all leading weak processes are adequately described to lowest order in an effectively local self interaction of a charged V, A current composed of leptonic and hadronic parts. This familiar current x current scheme is only in part based on clean experimental evidence. It incorporates those interactions which rather directly provide for the known semi-leptonic processes, as well as for the purely leptonic process of $\mu$ meson decay. But treated in a straightforward and perhaps naive way, it also leads to sharp predictions concerning $(\nu_e e)(\nu_e e)^\dagger$ and $(\nu_\mu \mu)(\nu_\mu \mu)^\dagger$ interactions; namely, that these should have the same structure and strength as the mixed leptonic interaction $(\nu_e e)(\nu_\mu \mu)^\dagger$ responsible for muon decay.

These so-called "diagonal" couplings would for example manifest themselves in the form of $\nu_e$-e or $\nu_\mu$-$\mu$ elastic scattering; equivalently, in the high energy neutrino process $\nu_\mu + (Z) \rightarrow \nu_\mu + \mu^+ + \mu^- + (Z)$. Reines and Gurr[1] have recently reported negative results in a search for the reaction $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$ induced by anti-neutrinos from the Savannah River reactor. The upper limit on cross section is expressed by $\sigma_{exp} < 4\ \sigma_{theory}$. With respect to the upcoming round of neutrino experiments at NAL, there is is a fair hope that the reaction $\nu_\mu + (Z) \rightarrow \nu_\mu + \mu^+ + \mu^- + (Z)$ could be detected at the cross section level implied by the standard theory. This matter is clearly a very critical issue for our present picture of the weak interactions.

On the current-current scheme, the same hadronic currents which figure in semi-leptonic processes are supposed to fully account for non-leptonic processes, through the self coupling of these hadronic currents. Owing to the complexities of strong interaction effects, however, there is no reliable way to test this hypothesis. It is useful here to recall the contrast with semi-leptonic reactions. For the latter processes, the relevant piece of the Hamiltonian is

$$\mathcal{H}_{semi\,leptonic} = \frac{G}{\sqrt{2}} J_\mu \ell_\mu^{\dagger} + h.c. \qquad (1)$$

where

$$\ell_\mu = i \sum_{\ell=e,\mu} \bar{\psi}_{\nu_\ell} \gamma_\mu (1+\gamma_5) \psi_\ell$$

is the lepton current and $J_\mu$ is the hadron current, formed in some unspecified way (at this stage) out of fundamental hadron fields. To lowest order in the weak coupling constant G, the amplitude for a semi-leptonic process such as $\nu_\ell + \alpha \to \beta + \ell$, where $\alpha$ and $\beta$ are hadron systems, is

$$i \frac{G}{\sqrt{2}} \langle \beta | J_\mu | \alpha \rangle \bar{u}_\ell \gamma_\mu (1+\gamma_5) u_\nu . \qquad (2)$$

The leptonic part appears as a trivial, known factor, all the complexities of the strong interactions residing in the hadronic matrix element $\langle \beta | J_\mu | \alpha \rangle$. Even if we can say nothing a priori about this matrix element, the factorized form of the overall amplitude imposes a restriction on the structure of the cross section; and this structure is subject to experimental test. On the other hand, for a weak non-leptonic process $\alpha \to \beta$, the amplitude

$$\langle \beta | \mathcal{H}_{\text{non leptonic}} | \alpha \rangle$$

is measured "whole". There is no way from experiment to look inside $\mathcal{H}$nonleptonic to test whether it really has the structure

$$\mathcal{H}_{nonleptonic} = \frac{G}{\sqrt{2}} J_\mu J_\mu^\dagger$$

implied by the current-current model. Indeed, one can only say negative things. Without the introduction of neutral currents or other speculative inputs, it is difficult to account for the empirical success of the $\Delta I = 1/2$ rule for non-leptonic reactions.

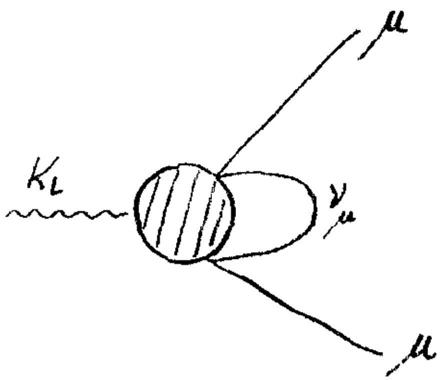
But even apart from this, the limitation must be kept in mind that the current-current picture is less a theory than a phenomenological framework. Because the weak interactions are supposed to be so very weak (at least at present energies), one is instructed to compute only to first order in the weak coupling constant G. Where a process is already allowed in this order, higher order corrections are presumed to be negligible. Yet if one takes the Hamiltonian seriously and proceeds to compute the correction terms according to standard rules of perturbation theory, the integrals typically diverge! Similarly, processes which cannot arise in lowest order are supposed to be hopelessly weak. So far as we know they really are, yet naive computations don't always confirm this - again the integrals diverge; e.g., consider the second order process $\nu_e + \mu \rightarrow \nu_e + \mu$. These remarks hold whether or not the weak interactions are mediated by heavy vector bosons. The simple current-current picture (with or without the intermediate vector bosons) is of course not ruled out on these grounds - the fault may lie with our pertubative procedures. But at the present stage we are effectively left with a first order phenomenology. And precisely for this reason enormous interest attaches to the experimental search for evidences of higher order effects, e.g., non locality effects in high energy neutrino reactions.

At the more conventional level, weak interaction theory and phenomenology are most richly developed for semi-leptonic processes. We deal here with weak hadronic currents and their matrix elements - a branch of strong interaction physics as probed by the weak interactions. These currents, along with their sister electromagnetic current, have in recent years acquired a considerable status as interesting theoretical entities in their own right. In part on the basis of experimental evidence and in part from rather free invention, theorists have indulged in all sorts of speculations about their properties: think of the catch phrases CVC, PCAC, current algebra, light cone structure, etc. The standard phenomenology reduces largely to a study of currents. At the same time one is on the look out for developments that don't fit into the standard framework.

With this brief introduction out of the way, let me now turn to the details of a few recent developments.

## $K_L \to \mu^+ + \mu^-$ DECAY

Among conceivable but not yet detected weak interaction processes, the reaction $K_L \to \mu^+ + \mu^-$ has certain theoretical features which are of considerable interest. This is the kind of process that could arise in first order from the coupling of neutral lepton and hadron currents, something which is not required on present phenomenology but which could well have a natural place there. It could also arise as a second order effect in conventional weak interactions, as illustrated in the diagram




One expects such second order effects to be very small, but as already emphasized, there is no way to be quantitative about this. There is yet another mechanism for the reaction however, one whose operation would seem to be in no doubt even as to general order of magnitude: $K_L \to$ two real or virtual photons $\to \mu^+ + \mu^-$. The second step of the sequence is presumably describable by standard spinor electrodynamics, and as for the first step, $K_L \to 2\gamma$ decay is both expected in order $Ge^2$, and seen. On this mechanism an obvious estimate would suggest that[2]

$$\frac{\Gamma(K_L \to 2\mu)}{\Gamma(K_L \to 2\gamma)} \approx \left(\frac{1}{137}\right)^2 ,$$

hence

$$B.R. = \frac{\Gamma(K_L \to 2\mu)}{\Gamma(K_L \to all)} \approx 2 \times 10^{-8}.$$

This is a very tiny branching ratio, one which seems to leave ample room for other, more exotic mechanisms to make their contributions, if any. But Messr's Clark, Elliof, Field, Frisch, Johnson, Keith, and Wentzel[3] have earnestly looked for this process and have failed to detect it. At the 90% confidence level they report an upper limit corresponding to

$$B.R._{exp} \lesssim 1.8 \times 10^{-9}. \tag{3}$$

Either the estimate for the conventional mechanism is wrong, or there are additional exotic contributions which happen to add destructively- or worse! Namely, let us start with the reasonable assumption that

CP violating effects can be ignored, so that it is $K_L$ decay into the CP-odd $^1S_o$ state of $\mu^+$, $\mu^-$ that is in question. Clearly the decay amplitude can be no smaller, in modulus, than its imaginary, or absorptive part. But the absorptive part can be expressed, via unitarity, as a sum of contributions from physical intermediate states; and to lowest relevant order in the fine structure constant only the $2\gamma$, $3\pi$, and $2\pi\gamma$ states need be considered. So to compute the absorptive amplitude and thereby set a lower bound on $K_L \to 2\mu$ decay, one only needs the amplitudes for $K_L \to$ intermediate state and intermediate state $\to 2\mu$. But the amplitude for $K_L \to 2\pi\gamma$ decay is known to be sufficiently small (the process has not been seen; B. R. $\lesssim 4 \times 10^{-4}$) and this state can be reliably ignored on the present scale of interest. We are thus left only with the $2\gamma$ and $3\pi$ states (spin-parity $0^-$). Let us for the moment systematically ignore the $3\pi$ state. Then the needed $K_L \to 2\gamma$ amplitude, whose modulus is known from the measured rate for this reaction, is purely real; and of course the $2\gamma \to 2\mu$ amplitude is supposed to be reliably given by standard spinor electrodynamics. A trivial unitarity calculation now leads to the lower bound for $K_L \to 2\mu$ decay[4]

$$\mathrm{B.R.} \gtrsim 6 \times 10^{-9}, \qquad (4)$$

a value three times bigger than the experimental upper bound! Let us now reinstate the $3\pi$ intermediate state. This has two kinds of effect.

For one thing, reinstatement of the $3\pi$ state puts the contributions of the $2\gamma$ state into doubt, since the $K_L \to 2\gamma$ amplitude itself now acquires a problematic absorptive part via the unitarity sequence $K_L \to 3\pi \to 2\gamma$. Although we can appeal to experiment for information on the $K_L \to 3\pi$ amplitude, or rather, its modulus, no such experimental information is available for $3\pi \to 2\gamma$; and apart from order of magnitude estimates, this process is not subject to reliable theoretical control. Moreover, the $3\pi$ state makes a direct contribution to the absorptive $K_L \to 2\mu$ amplitude and again we have no reliable information on $3\pi \to 2\mu$.

Despite all of these qualifications on the "naive" unitarity bound, Eq. (4), estimates made by Martin, de Rafael, and Smith[5] suggest that the $3\pi$ contributions cannot easily be made big enough to account for the observed discrepancy. Whether or not one accepts this, it is clear that something very interesting is going on here.

We have so far ignored the possibility that CP violation may be playing a role in this situation. On this question two different lines have recently been pursued. According to one of these,[6] it is supposed that one can continue to ignore the very tiny ($\sim 10^{-3}$) CP impurity in the state $K_L$, but allowance is made for substantial CP violation in the electromagnetic processes $K_L \to 2\gamma$ and $K_L \to 2\mu$, i.e., one contemplates $K_L$ decay into the $0^+$ states of $2\gamma$ and of $2\mu$ ($^3P_0$ state). Of course the experimental bound on $K_L \to 2\mu$ decay refers to the incoherent sum of transitions to $^1S_0$ and $^3P_0$. As for the <u>observed</u> decay $K_L \to 2\gamma$, we do not yet know directly what fraction goes to the CP violating $0^+$ state of $2\gamma$. Now for the CP violating channels there is an important simplification; namely, the absorptive amplitudes receive no contributions from the $3\pi$ state. Thus the absorptive amplitude for $K_L \rightarrow 2\mu$ $(^3P_o)$ receives a contribution only from the $2\gamma$ $(0^+)$ state (quantitatively, the $2\pi\gamma$ contribution can again be ignored). Moreover, the CP violating $K_L \rightarrow 2\gamma$ amplitude is now real, again because the $3\pi$ state cannot contribute to the absorptive part. So one can now set a reliable lower bound on CP violating $K_L \rightarrow 2\mu$ decay in terms of the rate for CP violating $K_L \rightarrow 2\gamma$ decay. Turning this around one learns from the experimental limit on $K_L \rightarrow 2\mu$ decay (both CP conserving and CP violating) that at most $\sim 37\%$ of the observed $K_L \rightarrow 2\gamma$ rate refers to the CP violating channel.

The above line of reasoning does nothing to resolve the problem of the missing $K_L \rightarrow 2\mu$ reactions. It merely exploits the situation to extract some information on CP violation in $K_L \rightarrow 2\gamma$ decay. The burden of the missing $K_L \rightarrow 2\mu$ reactions is still carried by the $3\pi$ contributions in the CP conserving channels, contributions which are surprisingly big.

Another approach with respect to CP violation has been taken by Christ and Lee.[7] They suppose that the $3\pi$ contributions are in fact not big enough to resolve the problem and can indeed be ignored; but instead, they allow for the possibility that the tiny CP impurity in the $K_L$ state produces effects which are magnified by an unusually large rate for $K_S \rightarrow 2\mu$, equally unexpected. The analysis goes as follows. First recall the definitions

$$K_{\substack{S\\L}} = \left[2(1+|\epsilon|^2)\right]^{-1/2}\left[(1+\epsilon)K^0 \pm (1-\epsilon)\bar{K}^0\right] \tag{5}$$

where the tiny complex parameter $\epsilon$ is a measure of CP impurity in the $K_S$, $K_L$ states. With the phase convention CPT $|\mu^+\mu^-$, CP $= \pm 1\rangle =$ $\pm\ |\mu^+\mu^-$, CP $= \pm 1\rangle$, define

$$\text{amp}\ (K^0 \to \mu^+\mu^-;\ CP=\pm 1) = b_\pm + i\,a_\pm \tag{6}$$

where $a_\pm$ is the absorptive part, $b_\pm$ the dispersive part. From CPT invariance it follows that

$$\text{amp}\ (\bar{K}^0 \to \mu^+\mu^-;\ CP=\pm 1) = \pm\left(b_\pm^* + i\,a_\pm^*\right) \tag{7}$$

Now define the $K_L$ and $K_S$ amplitudes

$$\begin{aligned} L_\pm &= \text{amp}\ (K_L \to \mu^+\mu^-;\ CP=\pm 1) \\ S_\pm &= \text{amp}\ (K_S \to \mu^+\mu^-;\ CP=\pm 1) \end{aligned} \tag{8}$$

and let $L_\pm^{abs}$ be the absorptive part of $L_\pm$.

Finally, introduce the 2 component complex vectors

$$A = \begin{pmatrix} L_+^{abs} \\ L_-^{abs} \end{pmatrix}, \quad B = \begin{pmatrix} Re\, L_+ \\ -i\, Im\, L_- \end{pmatrix}, \quad C = Re\,\epsilon \begin{pmatrix} -Re\, S_+ + i\, Im\, S_+ \\ Re\, S_- - i\, Im\, S_- \end{pmatrix}. \tag{9}$$

To first order in the parameter $\epsilon$ one finds

$$A - B = C, \tag{10}$$

hence by the triangle inequality

$$|B| > \big|\, |C| - |A| \,\big| \tag{11}$$

Here

$$|A| \equiv L^{abs} = \left\{ |L_+^{abs}|^2 + |L_-^{abs}|^2 \right\}^{1/2} \tag{12}$$

$$|B| = \left\{ (Re\, L_+)^2 + (Im\, L_-)^2 \right\}^{1/2} < \left\{ \Gamma(K_L \to 2\mu) \right\}^{1/2} \tag{13}$$

$$|C| = Re\,\epsilon \left\{ \Gamma(K_S \to 2\mu) \right\}^{1/2}. \tag{14}$$

In accordance with the present philosophy, we now suppose that the $3\pi$ (and $2\pi\gamma$) intermediate states can be systematically ignored in the computation of absorptive parts. Then the simple unitarity contributions from the $2\gamma$ states give

$$\lambda v \leq L^{abs} \{\Gamma(K_L \to 2\gamma)\}^{1/2} \leq \lambda$$

$$\lambda^2 = \frac{1}{2}\frac{\alpha^2}{v}\left(\frac{m_\mu}{m_K}\right)^2 \left[\ln\left(\frac{1+v}{1-v}\right)\right]^2 \approx 1.2 \times 10^{-5}, \qquad (15)$$

where v is the muon velocity in the K rest frame. From Eq. (11)-(15) one finds the inequalities

$$\lambda v \{\Gamma(K_L \to 2\gamma)\}^{1/2} - \{\Gamma(K_L \to 2\mu)\}^{1/2} \leq \mathrm{Re}\,\epsilon \{\Gamma(K_S \to 2\mu)\}^{1/2}$$

$$\leq \lambda \{\Gamma(K_L \to 2\gamma)\}^{1/2} + \{\Gamma(K_L \to 2\mu)\}^{1/2}. \qquad (16)$$

Putting in the experimental bound on $\Gamma(K_L \to 2\mu)$, and using Re $\epsilon \sim 1.4 \times 10^{-3}$, one finds

$$1 \times 10^{-5} \gtrsim \frac{\Gamma(K_S \to 2\mu)}{\Gamma(K_S \to all)} \gtrsim 5 \times 10^{-7}. \qquad (17)$$

It is the right hand inequality that is so surprising here. One would "normally" expect the indicated branching ratio to be of order $10^{-10}$ - $10^{-11}$. The Christ-Lee analysis of course doesn't explain <u>why</u> the $K_S \to 2\mu$ rate should be so big; it simply requires such a result to explain why $K_L \to 2\mu$ is so small. The present experimental upper bound on the $K_S \to 2\gamma$ branching ratio is $7.3 \times 10^{-6}$. An order of magnitude improvement is needed.

## SECOND CLASS CURRENTS

An obvious and important question for semi-leptonic processes has to do with the properties of the weak hadronic current with respect to strong interaction symmetries like strangeness and isospin (and, to the extent that it is a strong interaction symmetry, $SU_3$). For example, from the very occurrence in nature of $n \to p + e + \nu$ decay one learns that $J_\mu$ has an isovector, strangeness conserving piece; from $\Lambda \to p + e + \nu$ decay, that it has an $I = 1/2$, $\Delta S = \Delta Q = 1$ piece. There is no firm evidence to date for the presence of other conceivable terms, e.g., $\Delta S = -\Delta Q$ or $|\Delta S| = 2$, etc. However, you will realize that the scope is severely limited insofar as evidence is sought for only amongst decay processes: what conceivable decay reaction could test, e.g., for a $\Delta S = 4$ term? The observational possibilities are of course widened for high energy neutrino experiments, and one may well hope for new revelations in the upcoming round of experiments.

In any case, for the $\Delta S = 0$ terms (the well established isovector part, but also for conceivable terms with $I > 1$) there is one further classification scheme to be considered, namely, classification with respect to G parity. Let us denote by $V_\mu$ and $A_\mu$ the $\Delta S = 0$ vector and axial vector currents. We may imagine a priori, that each decomposes into pieces which are respectively even and odd under G parity.

Following Weinberg,[8] let us designate as "first class" the even G parity part of $V_\mu$ and the odd G parity part of $A_\mu$. The converse pieces are called "second class." The existence of first class vector and axial vector pieces is established by the occurrence, respectively, of $\pi^+ \to \pi^0 + e^\pm + \nu$ and $\pi^\pm \to \mu^+ + \nu$ decays. No equally simple tests are available, unfortunately, for second class currents. To get at these, given the practical realities, one must turn to processes connecting hadron states of indefinite G parity, e.g., nuclear $\beta$ decay, where first and second class currents can both simultaneously contribute. The problem is to find whether second class effects can be unambiguously identified. This has proved to be a difficult task. Nevertheless it is an important one, precisely because there is presently no natural place for second class currents in the standard theoretical lore - CVC, Cabibbo model, current algebra, etc. To discover those effects that would signal the existence of second class currents, let's see what things are implied in the absence of such currents. For definiteness, let us suppose that the $\Delta S = 0$ currents are purely isovector. Also, it will simplify the discussion if we ignore the possibility of time reversal violation; insofar as we restrict ourselves to observational effects which are even under time reversal there is no loss of generality in this. We now consider two general kinds of situations.

1. First consider a $\Delta S = 0$ semi-leptonic process between hadron states belonging to a common isotopic multiplet, e.g., $n \to p + e + \nu$.

For definiteness, suppose that $\Delta Q = +1$, i.e., that the transition is $I_z \to I_z + 1$. One encounters here the matrix element

$$\langle I_z+1, \alpha' \,|\, V_\mu + A_\mu \,|\, I_z, \alpha \rangle,$$

where $\alpha$ and $\alpha'$ specify spins and momenta. Let us now focus on, say, the vector current, in particular on its spatial parts $V_i$. By hypothesis the vector current has even G parity, so with $G = CU$, $U = e^{i\pi I_2}$, we have

$$(CU)\, V_i\, (CU)^{-1} = V_i \,.$$

On the other hand, CPT invariance implies that

$$(CPT)\, V_i\, (CPT)^{-1} = V_i^\dagger,$$

and of course, since $V_i$ is a vector, we have

$$P V_i P^{-1} = -V_i$$

Thus

$$V_i = -U^{-1} T^{-1} V_i^\dagger T U \tag{18}$$

It therefore follows that

$$\langle I_z+1, \alpha' | V_i | I_z, \alpha \rangle = -\langle -I_z, \alpha_\tau | V_i | -I_z-1, \alpha'_\tau \rangle$$

$$= -\langle I_z+1, \alpha_\tau | V_i | I_z, \alpha'_\tau \rangle, \qquad (19)$$

where the subscript $\tau$ means: reverse all spins and momenta. This equation implies a restriction on the structure of the matrix element, following from the presumed first class character of the currents. Any departure would signal the effect of second class currents. Incidentally, notice that we have not had to assume T invariance for this result. Similar reasoning applied to the axial vector current gives

$$\langle I_z+1, \alpha' | A_i | I_z, \alpha \rangle = -\langle I_z+1, \alpha_\tau | A_i | I_z, \alpha'_\tau \rangle . \qquad (19')$$

As an example, consider neutron $\beta$ decay, where the most general structure of the matrix elements, apart for G parity considerations, is

$$\langle p | V_\mu | n \rangle = i \bar{u}(p) \left\{ g_V \gamma_\mu + \frac{G_V}{2m} \sigma_{\mu\nu} q_\nu + i \frac{h_V}{2m} q_\mu \right\} u(n),$$

$$\langle p | A_\mu | n \rangle = i \bar{u}(p) \left\{ g_A \gamma_\mu \gamma_5 + i \frac{G_A}{2m} q_\mu \gamma_5 + \frac{h_A}{2m} \sigma_{\mu\nu} q_\nu \gamma_5 \right\} u(n), \qquad (20)$$

where $q = p_n - p_p$ and where the form factors depend on $q^2$. The form factors must be relatively real if T invariance is valid. Independent of this, one observes from Eqs. (19) that the form factors $h_v$ and $h_A$ are forbidden for first class currents; i.e., they can arise only from second class currents, whereas the remaining terms arise only from first class currents. So one must look for the distinctive spectrum and spin correlation effects induced by the form factor $h_v$ and $h_A$. Unfortunately the coefficients $h_v$ and $h_A$ multiply terms that are small for the small momentum transfers that occur in β decay, so that even if these coefficients were substantial, the observed effects would be small. For high energy neutrino processes like $\nu_\mu + n \rightarrow p + \mu^-$ this difficulty does not arise. On the other hand the necessity that spin correlations be observed in order to disentangle $h_v$ and $h_A$ from the other form factors imposes severe experimental problems once again. The above examples refer to a $J = 1/2 \rightarrow J = 1/2$ transition within an isomultiplet. The structure for an arbitrary $J \rightarrow J$ transition within a multiplet can easily be worked out,[9] but the general formulas will not be given here. The general features are unchanged.

2. The second kind of situation we want to consider involves the comparison of a mirror pair of semi-leptonic processes: the $\Delta Q = +1$ process connecting hadron states $\alpha$ and $\beta$ (which now belong to different multiplets), and the $\Delta Q = -1$ process connecting $\tilde{\alpha}$ and $\tilde{\beta}$, where $|\tilde{\alpha}> = U|\alpha>$, $|\tilde{\beta}> = U|\beta>$, e.g., $|\tilde{p}> = |n>$, $|\tilde{\Sigma}^+> = |\Sigma^->$, etc.

From Eq. (18), taken together with the assumption of time reversal invariance, we have

$$U V_\mu U^{-1} = V_\mu^{+}$$

and similar reasoning for the axial vector current gives also

$$U A_\mu U^{-1} = A_\mu^{+}$$

The matrix elements for the two mirror processes are therefore related by

$$\langle \tilde{\beta} | V_\mu^{+} + A_\mu^{+} | \tilde{\alpha} \rangle = \langle \beta | V_\mu + A_\mu | \alpha \rangle \tag{21}$$

i.e., the amplitudes are identical. In fact, insofar as we look only at effects which are even under time reversal, the final consequences are independent of the validity of T invariance.

One immediate consequence, in the context of nuclear β decay, is that the decay rates for a mirror pair of processes should be equal in the absence of second class currents. Of course, the whole argument presumes that electromagnetic violations of isospin can be ignored. In fact some part of the electromagnetic effects are allowed for in the comparison not of rates but of ft values (this corrects for phase volume differences and for final state Coulomb interactions). But additional isospin impurity effects are more problematic; i.e., a small (order $\alpha Z$) difference in Gamow-Teller matrix elements for a pair of mirror processes might originate from electromagnetic rather than second class current effects.

I've belabored this question of second class currents in part to remind that after all these years they are still badly tested for; but in part because some tantalizing positive indications have appeared in the past year. Wilkinson and Alburger[10] have surveyed the available data on mirror pairs of $\beta$ decays and report a systematic pattern of discrepancies, as big as 20%, between the ft values. The discrepancies [ $(ft)^+ > (ft)^-$ ] seem roughly to scale with the sum of positron and electron end point energies, $W_o^+ + W_o^-$, as would be expected if these discrepancies arise from a second class form factor $h_A$ in the fundamental process of <u>nucleon</u> $\beta$ decay. On this interpretation $h_A$ would have to be substantial, several times as big as the first class coefficient $g_A$. However these same authors[11] have also recently studied the (I: $1 \rightarrow 0$, J: $2^+ \rightarrow 2^+$) mirror decays $Li^8 \rightarrow Be^8 + \beta^- + \bar{\nu}$ and $B^8 \rightarrow Be^8 + \beta^+ + \nu$. They exploit the large width of $Be^8$ (unstable against decay into a pair of alpha particles) to study the ft dependence on the "variable" end point energies. They find no such dependence, setting an upper limit on $h_A$ some three times smaller than indicated above (so still a rather large upper limit). This suggests that the observed ft discrepancies arise from isospin violating nuclear overlap effects. We must leave it to the nuclear theorists whether this can be understood quantitatively. Better still would be direct experimental search for second class form factors in $\beta$ decay between members of a common isomultiplet.

Cleaner tests for second class currents may some day become practical for high energy neutrino processes. The idea here would be to compare two $\Delta S = 0$ mirror processes.

$$\nu_\mu + \alpha \to \mu^- + X$$
$$\bar{\nu}_\mu + \tilde{\alpha} \to \mu^+ + \tilde{X} .$$

The simplest version arises for the case where the hadron target is its own mirror, $\alpha = \tilde{\alpha}$ (deuteron, carbon, etc.) and where one sums over all $\Delta S = 0$ channels X and $\tilde{X}$. In the absence of second class currents the familiar structure functions $W_i^{(\nu)}$ and $W_i^{(\bar{\nu})}$ should be identical.

$$W_i^{(\nu)} = W_i^{(\bar{\nu})} . \qquad (\alpha = \tilde{\alpha})$$

DEEP INELASTIC LEPTON - HADRON SCATTERING

For my last topic let me take up briefly some further aspects of these structure functions for deep inelastic lepton-hadron scattering. This is a very popular subject nowadays and an enormous literature has grown up around it - increasingly, a literature about the light cone structure of current commutators. The theoretical approaches are too varied for anything like a comprehensive review, and anyhow I have time only to bring out a very few points. As usual, let us declare that the kinematics and some of the elementary theoretical (and experimental) lore are well known. We are considering processes of the sort $e + p \to e + X$, $\nu + p \to \mu^- + X$, $\bar{\nu} + p \to \mu^+ + X$, where one sums over all hadron channels for given momentum transfer q between the leptons. Let the hadron target momentum be denoted by p. In computing the cross section one encounters

$$W_{\nu\mu} = (2\pi)^3 \sum_X \langle p| J_\nu^+ |X\rangle\langle X| J_\mu |p\rangle \, \delta(p_X - p - q)$$

$$= \frac{1}{2\pi}\int dx\, e^{-iq\cdot x} \langle p| \left[ J_\nu^+(x), J_\mu(0) \right] |p\rangle$$

$$= W_1 \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{W_2}{m^2}\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)$$

$$+ \frac{1}{2}\frac{W_3}{m^2}\epsilon_{\nu\mu\alpha\beta}\, q_\alpha p_\beta + \cdots \qquad (23)$$

where the omitted terms are proportional to $q_\mu$ or $q_\nu$ and where the structure functions $W_i$ depend on $q^2$ and on $\nu = -q\cdot p/m$. For neutrino processes $J_\mu$ is the charge raising semi-leptonic current, for anti-neutrino processes the charge lowering semi-leptonic current, and for electron scattering $J_\mu = J_\mu^+$ is the electromagnetic current (in this case $W_3 = 0$ ). In all cases, as is seen from the second equality of Eq. (23), one is studying the Fourier transform with respect to q of a current commutator. Let us define

$$F_1 = W_1 , \quad F_2 = \frac{\nu}{m} W_2 , \quad F_3 = \frac{\nu}{m} W_3$$

and regard the $F_i$ as functions of $q^2$ and $\omega = q^2/2m\nu$ $(0 \le \omega \le 1)$. It is Bjorken's[12] idea (supported by the SLAC-MIT experiments on deep inelastic electron scattering ) that the $F_i$ have finite limits, at least some of them nontrivial, as $q^2 \to \infty$, $\omega$ fixed.

Let's first observe that in this so-called scaling limit one is probing the structure of the current commutator near the light cone, $x^2 \approx 0$. Namely, suppose that $\vec{q}$ points along the 3-axis so that, in Eq. (23),

$$\exp(-iq\cdot x) = \exp \frac{i}{2} \left\{ (q_0 - q_3)(x_0 + x_3) + (q_0 + q_3)(x_0 - x_3) \right\} .$$

We are in the rest frame of the hadron target, where $q_o = \nu$, and we are considering $q_o = \nu \to \infty$, $\omega$ fixed. In this limit

$$q_3 \to q_0 + m\omega$$

The integral of Eq. (23) is damped by rapid oscillations of the exponential factor unless

$$|x_0 + x_3| \lesssim \frac{1}{m\omega}, \qquad |x_0 - x_3| \lesssim \frac{1}{2q_0}$$

But the commutator vanishes unless $\chi^2 = (\vec{\chi})^2 - \chi_o^2 \leq 0$. Therefore the integral is governed by the leading singularities of the commutator on the light cone, $\chi^2 \approx 0$; and the existence of scaling suggests that the light cone structure may be simple. At any rate, this is the hope, and it has engaged a lot of attention recently.

As already mentioned, the light cone has been approached from a wide variety of view points, most notably, on the basis of the parton model— which does not explicitly use the language of light cones. I won't review the parton model here but instead, following Gell-Mann,[13] let us see how some of the results of that model emerge from a more abstract picture based on the light cone structure of current commutators suggested by the free-quark model. In this latter model the currents are formed in the familiar way out of quark fields, and we may define

$$J_\mu^a(r;x) = i\,\bar{\psi}(x)\,\frac{\lambda^a}{2}\,\gamma_\mu\,(1+r\gamma_5)\,\psi(x), \qquad r=\pm 1, \qquad (24)$$

where $\lambda^a$ are the $SU_3$ matrices and the subscript a is an $SU_3$ index. In the absence of strong interactions the quark fields satisfy the free anti-commutation relations

$$\{\psi(x),\ \psi(y)\} = 0$$

$$\{\psi(x),\ \bar{\psi}(y)\} = -i\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - M\right) D(x-y)$$

where

$$D(x) \xrightarrow[x^2\approx 0]{} -\frac{1}{2\pi}\,\epsilon(x_0)\,\delta(x^2). \qquad (25)$$

It is an easy matter to work out the current commutators $[\,J_\nu^a(r,x),\ J_\mu^b(r',y)]$ for arbitrary x , y; and, in particular, for the limit $(x-y)^2 \approx 0$ which is of special interest to us here. One finds that the commutator vanishes for $r \neq r'$, and

$$\left[ J_\nu^a(r,x), J_\mu^b(r,y) \right] \underset{(x-y)^2 \approx 0}{\longrightarrow} i f_{abc} \left\{ J_\nu^c(r,S;x,y)\delta_{\mu\alpha} + J_\mu^c(r,S;x,y)\delta_{\nu\alpha} \right.$$

$$- \delta_{\mu\nu} J_\alpha^c(r,S;x,y) - r\,\epsilon_{\nu\alpha\mu\rho} J_\rho^c(r,A;x,y) \Big\} \frac{\partial}{\partial x_\alpha} D(x-y)$$

$$+ d_{abc} \Big\{ J_\nu^c(r,A;x,y)\delta_{\mu\alpha} + J_\mu^c(r,A;x,y)\delta_{\nu\alpha} - \delta_{\mu\nu} J_\alpha^c(r,A;x,y)$$

$$\left. - r\,\epsilon_{\nu\alpha\mu\rho} J_\rho^c(r,S;x,y) \right\} \frac{\partial}{\partial x_\alpha} D(x-y) , \qquad (26)$$

where

$$J_\mu^a(r, {}^S_A ; x,y) = i\,\bar{\psi}(x)\frac{\lambda^a}{2}\gamma_\mu(1+r\gamma_5)\psi(y) \pm (x \leftrightarrow y) . \qquad (27)$$

These are <u>bilocal</u> operators, where the label "S" refers to a bilocal hermitean operator symmetric under $x \leftrightarrow y$; "A" is anti-symmetric and anti-hermitean. The idea now is to adopt the singularity, $SU_3$ and tensor <u>structure</u> of Eq. (26) as a conjecture for the light cone properties of currents in the real, interacting world, dropping the further specifics of Eq. (27). This structure is already, in itself, informative and incorporates the results of parton models based on the identification partons with quarks.

To see how this comes about, let us write

$$\langle p | J_\mu^a(r, {}^S_A ; x, 0) | p \rangle = \frac{p_\mu}{m} \int d\omega\, e^{-i\omega p\cdot x} G^a(r, {}^S_A ; \omega) + \cdots \qquad (28)$$

where the omitted term is proportional to $x_\mu$ and will play no role in the following argument. Here $G_S$ is even, $G_A$ is odd in $\omega$.

In computing the structure functions, we encounter, e.g.,

$$\int dx\, e^{-iq\cdot x} \langle p | J_\mu^a(r, {}^S_A ; x, 0) | p \rangle \frac{\partial}{\partial x_\nu} D(x)$$

$$= \frac{p_\mu}{m} \int d\omega\, G^a(r, {}^S_A ; \omega) \int dx\, e^{-i(q+\omega p)\cdot x} \frac{\partial}{\partial x_\nu} D(x)$$

$$= \frac{i p_\mu}{m} \int d\omega\, G^a(r, {}^S_A ; \omega) \int dx\, e^{-i(q+\omega p)\cdot x} (q+\omega p)_\nu D(x) . \qquad (29)$$

Using the fact that

$$\int dx\, e^{-iQ\cdot x} D(x) = -2\pi i\, \epsilon(Q_0)\, \delta(Q^2) \qquad (30)$$

we find from Equs. (23), (26), (29), (30)

$$
\begin{aligned}
2\omega \, F_1^{(a,b)}(r,\omega) &= F_2^{(a,b)}(r,\omega) \\
\frac{1}{\omega} F_2^{(a,b)}(r,\omega) &= i f_{abc} G^c(r,S;\omega) + d_{abc} G^c(r,A;\omega) \\
\frac{1}{r} F_3^{(a,b)}(r,\omega) &= i f_{abc} G^c(r,A;\omega) + d_{abc} G^c(r,S;\omega) \\
\omega &= q^2/2my \, .
\end{aligned}
\tag{31}
$$

From Eqs. (31) one recovers the familiar parton formulas, e.g., for $\Delta S = 0$ transitions,

$$F_3^{(\nu p)} - F_3^{(\nu n)} = 12 \left( F_1^{(\gamma p)} - F_1^{(\gamma n)} \right)$$

$$F_1^{(\gamma p)} + F_1^{(\gamma n)} > \frac{5}{18} \left( F_1^{(\nu p)} + F_1^{(\nu n)} \right)$$

The important question is whether the structure of Eq. (26) survives when one switches on strong interactions, as mediated say by isosinglet vector or scalar gluons. The answer seems to be, formally, that it does,[14] where "formal" means, as determined by canonical equal time commutation (anti-commutation) relations for the gluon and quark fields. The bilocal operators now have a more complicated form than indicated by Eq. (27), but the structure of Eq. (26) survives.

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