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AN OPEN-SOURCE PYTHON TOOL FOR THE MAXWELL EIGENVALUE PROBLEM AND MULTIPACTING ANALYSIS IN AXISYMMETRIC ELLIPTICAL CAVITY STRUCTURES

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Abstract

Multipacting is a phenomenon arising from the emission and subsequent multiplication of charged particles in accelerating radiofrequency (RF) cavities, which can limit the achievable RF power. Predicting field levels at which multipacting occurs is crucial for optimising cavity geometries. This paper presents an open-source Python code (*PyMultipact*) for analysing multipacting in 2D axisymmetric cavity structures. The code leverages the NGSolve framework to solve the Maxwell eigenvalue problem (MEVP) for the electromagnetic (EM) fields in axisymmetric RF structures. The relativistic Lorentz force equation governing the motion of charged particles is then integrated using the calculated fields within the domain to describe the motion of charged particles. Benchmarking against existing multipacting analysis tools is performed to validate the code's accuracy.

INTRODUCTION

Multipacting is a phenomenon arising from the emission and subsequent multiplication of charged particles in accelerating radiofrequency (RF) cavities, which can limit the achievable RF power. Accurate prediction of the EM field levels at which multipacting occurs is crucial for designing and optimising RF cavities. Several programs, free [1] and commercial [2], exist to predict and analyse multipacting in RF structures. A status report of the different multipacting codes developed to predict multipacting in superconducting (SRF) cavities as of 2001 was given in [3]. Currently, the codes reported in [3] are either inaccessible or without proper documentation; therefore, this situation motivated this work.

This paper presents an open-source Python code for multipacting analysis of 2D axisymmetric RF structures. The code leverages the NGSolve framework [4] to solve the MEVP to obtain the cavity's resonant modes' EM fields. The relativistic Lorentz force equation governing the motion of charged particles is then integrated using the fields within the cavity. Benchmarking against existing multipacting analysis tools is performed to validate the code's accuracy. Figure 1 shows a block diagram of the general workflow for multipacting codes. The code is based on the concepts outlined in [5]. Its open-source nature fosters further development and customisation for specific applications.

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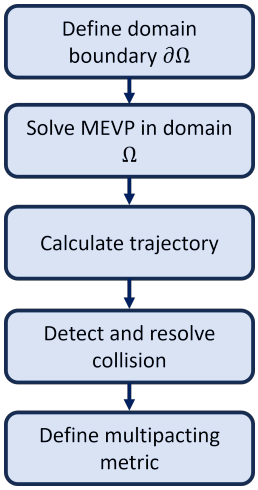

Figure 1: Block diagram of the general workflow of multipacting codes.

MAXWELL EIGENVALUE PROBLEM

For the solution domain Ω, the MEVP is written mathematically as

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) - \lambda \mathbf{E} = 0 \quad & \mathbf{E} \in \Omega, \\ \nabla \cdot \mathbf{E} = 0 \quad & \mathbf{E} \in \Omega, \\ \mathbf{n} \times \mathbf{E} = 0 \quad & \mathbf{E} \in \partial\Omega_1, \\ \mathbf{n} \times (^{-1} \times \mathbf{E}) = 0 \quad & \mathbf{E} \in \partial\Omega_2, \end{aligned}$$

where $\mathbf{n}$ is the surface normal, $\mathbf{E}$ is the electric field, $\lambda = (\omega/c)^2$ represents eigenvalues, ω is the angular frequency, c is the speed of propagation of electromagnetic waves in vacuum, $\partial\Omega := \partial\Omega_1 \cup \partial\Omega_2$ is the boundary of Ω. A similar equation could be written for the $\mathbf{H}$ field. The assumption intrinsic in the equation is that the domain Ω is a vacuum.

In variational form, following the Galerkin method, the weak form of the problem for axisymmetric domains reads as

$$\left[\int_\Omega r \operatorname{curl} \mathbf{u} \cdot \operatorname{curl} \mathbf{v} \, \mathrm{d}V\right] \mathbf{e} = \lambda \left[\int_\Omega r \mathbf{u} \cdot \mathbf{v} \, \mathrm{d}V\right] \mathbf{e}, \quad (1)$$

where r is the radial coordinate, $\mathbf{u}$ represents the basis functions, $\mathbf{v}$ is the test function, same as the basis function, and $\mathbf{e}$ is the vector of coefficients to be calculated. The natural functional space for the curl − curl problem is the $H(\mathrm{curl}, \Omega)$ functional space. Therefore, $\mathbf{u}$ and $\mathbf{v}$ here represent the edge or Nédélec basis functions.

In matrix form,

$$\mathbf{K}\mathbf{e} = \lambda \mathbf{M}\mathbf{e} \quad (2)$$

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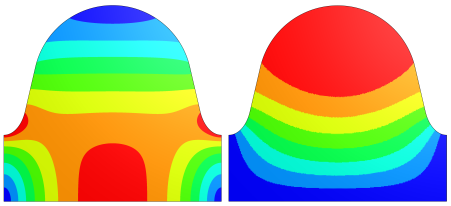

Figure 2: Electric (left) and magnetic (right) field of the TM010 mode of the TESLA cavity's mid-cell geometry.

which is the generalised eigenvalue problem. Figure 2 shows the electric and magnetic fields of the fundamental (TM010) mode of the TESLA cavity's mid-cell geometry [6]. The mode frequency is 1300.02 MHz.

TRAJECTORY CALCULATION

The motion of charged particles in electromagnetic fields is described by the Lorentz force equation. For relativistic particles ($\beta = 1$ or near relativistic $\beta \approx 1$), the relativistic Lorentz force Eq. ([5]) is given as

$$\begin{aligned} \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= -\frac{q}{m}\left(1-\left(\frac{\|\mathbf{v}\|}{c}\right)^2\right)^{1/2}\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}-\frac{1}{c^2}(\mathbf{v}\cdot\mathbf{E})\mathbf{v}\right), \\ \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} &= \mathbf{v}, \end{aligned} \tag{3}$$

where q and m are the charge and mass of the charged particle (in this case, electron), respectively. Consider a structure with domain Ω with boundary $\partial\Omega$. We define a set

$$X := \partial\Omega \cup \Psi, \tag{4}$$

which is the set of particles on the cavity wall where $\Psi = \{\psi : \psi \in [0, 2\pi]\}$ is the set of the field phase and $\partial\Omega = \{\mathbf{x}_i : \mathbf{x}_i \in \mathbb{R}^2\}$ is the set of points on the boundary. Each particle in the set X has coordinates $(\mathbf{x}_i, \psi_i)$ in the phase space. The simulation starts by releasing several particles from the structure's surface. The codes outlined in [3] utilise a range of multi-step and multi-stage methods to solve the initial value problem. The code presented in this paper implements the classical multi-step Runge-Kutta method.

COLLISION DETECTION AND RESOLUTION

For a fixed electromagnetic field level, the initialised particles in X are released with an initial velocity $v_0\mathbf{n}$[1] and advanced in the phase space within the structure until they collide with the boundary of the structure. Let

$$\begin{aligned} L_k = \{(x,y) \mid\ & x = x_k + t(x_{k+1} - x_k), \\ & y = y_k + t(y_{k+1} - y_k),\ 0 \le t \le 1\} \end{aligned} \tag{5}$$

be the line segment connecting any two points $\mathbf{x}_k, \mathbf{x}_{k+1} \in \mathbb{R}^2$ representing the position of a particle at time step k and $k+1$, respectively. Let $\partial\Omega := \bigcup \partial_i\Omega$, where $\partial_i\Omega$ represents discrete line segments in $\partial\Omega$. A collision occurs *iff* $|L \cap \partial_i\Omega| > 0$ and odd. If the electric field at the moment of collision with the surface points away from the wall, i.e. $\mathbf{n}\cdot\mathbf{E} > 0$, a particle is released with the same initial velocity $v_0\mathbf{n}$ and advanced until it hits the wall again. The process is repeated for the duration of the simulation. On the other hand, the particle is lost if $\mathbf{n}\cdot\mathbf{E} < 0$, see Fig. 3.

[1] $\mathbf{n}$ here refers to the unit normal of $\partial\Omega$ pointing inside Ω.

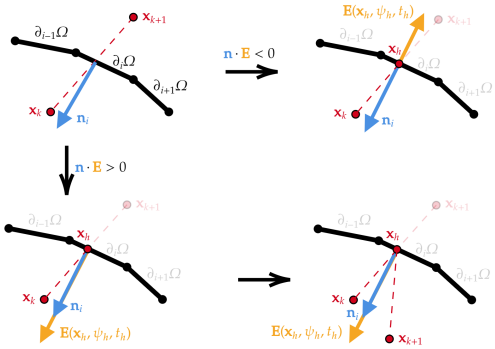

Figure 3: Illustration of collision handling.

MULTIPACTING METRIC

Several metrics exist for analysing the multipacting phenomenon, as discussed in [3]. The counter function (CF) and enhanced counter function (ECF) implemented by the Helsinki group [5] are implemented in this code. The CF is the ratio of the number of particles released to the number of particles left after n impacts. It serves to identify surface sites and field magnitudes that support multipacting. After each hit of a particle, the number of secondary particles emitted is calculated from the surface's secondary emission yield (SEY) function δ to incorporate the SEY properties. The SEY function could be based on model functions or experimental data [7, 8]. This function depends on the energy and angle of impact of the particle. The number of secondary electrons for a single particle after n impacts is defined in [5] as

$$N_n := \prod_{k=1}^{n} \delta(\mathbf{x}_k, E_k), \tag{6}$$

where E_k is the kinetic energy of the particle in electron volts at the kth impact calculated as

$$E_k = \frac{1}{q_0}(\gamma_k - 1)m_0c^2, \tag{7}$$

where $\gamma_k := \gamma(\mathbf{v}_k)$ is the Lorentz factor and q_0 is the elementary charge. The ECF is defined for P initial particles as

$$e_n := \sum_{j=1}^{P} N_n^j, \tag{8}$$

RESULTS

The mid-cell of the TESLA cavity geometry was analysed for multipacting as a test case and compared with the results

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from using the *MultiPac* code. The simulation for both codes considered two surface points, 72 initial phases, 179 peak electric field values ranging from 1 to 90 MV/m and an initial velocity of $v_0 = 2$ eV as inputs. A similar analysis using the *Multipac* code is documented in [9].

Figures 4, 5, and 6 compare the CF, final impact energy *Ef*, and ECF of the new code *PyMultipact* with results from *Multipac*. The figures show good qualitative agreement between the results of the newly developed code and those obtained using *Multipac*. The quantitative discrepancies in the results could be a result of using different initial particle emission sites and different time steps. When writing this paper, the time step used in *MultiPac* could not be determined and therefore could not be compared. The initial particle emission sites are plotted as black and red dots in Figs. 7 and 8. Other factors that could explain the discrepancies are the field interpolation scheme and collision treatment implemented in *MultiPac*.

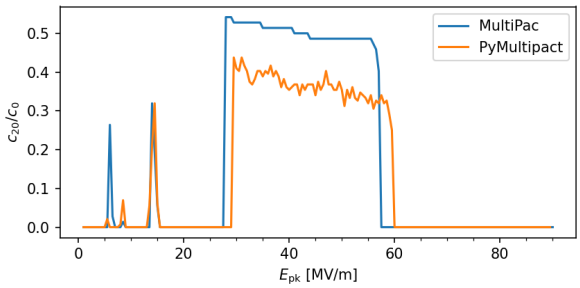

Figure 4: Counter function comparison.

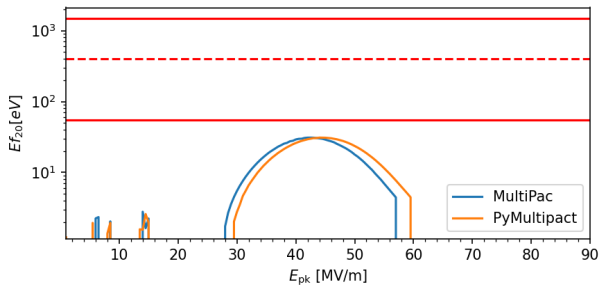

Figure 5: Final impact energy comparison.

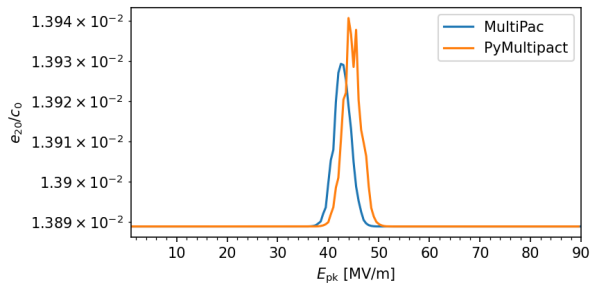

Figure 6: Enhanced counter function comparison.

Figures 7 and 8 plot the particle trajectories obtained from both codes for peak surface electric field $E_{pk} \approx 14.0$ MV/m and E_{pk}=42.5 MV/m, respectively. This also shows a good correspondence between the results from both codes as they predict similar multipacting sites.

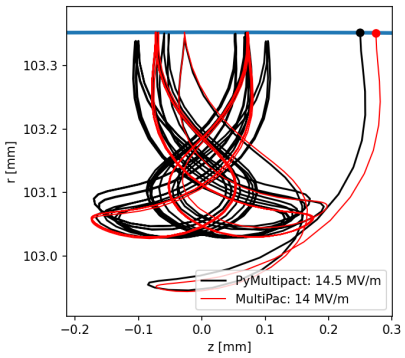

Figure 7: Trajectory of a particle for peak surface electric field $E_{pk} \approx 14.0$ MV/m.

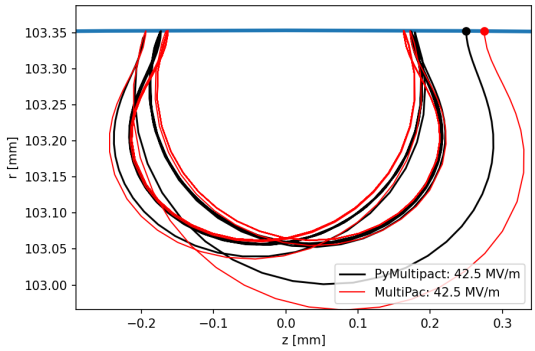

Figure 8: Trajectory of a particle for peak surface electric field E_{pk}=42.5 MV/m.

CONCLUSION AND OUTLOOK

A 2D multipacting analysis code for axisymmetric elliptical cavity structures has been developed, featuring an internal eigenmode solver based on the NGSolve framework. A comparison was conducted with an existing code using the mid-cell of the TESLA cavity geometry as a case study. The application of this code extends, in principle, to similar axisymmetric RF structures. Further test cases are planned to demonstrate this. Further developments include the incorporation of additional multipacting analysis metrics. The code and its documentation are under development and can be found at [10].

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