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Ş. Mişicu* and M. Rizea

Speeding of α Decay in Strong Laser Fields

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Abstract: An extremely high-intensity laser interaction with a nucleus that is undergoing spontaneous α-decay is investigated in the framework of the time-dependent one-body Schrödinger equation solved by a Crank-Nicolson scheme associated with transparent boundary conditions. The wave-packet dynamics are determined for various laser intensities and frequencies for continuous waves and for sequences of few-cycle pulses. We show that pulse sequences containing an odd number of half-cycles determine an enhancement of the tunneling probability and therefore a drastic decrease of alpha half-lives compared to the field-free case and the continuous wave case.

Keywords: Alpha decay, laser-nucleus interaction

PACS: 25.20.-x,23.60.+e

1 Introduction

The advent of chirped pulse amplification techniques opens up the possibility of producing electromagnetic field intensities as high as 10^{26} W/cm^2 [1]. Under such conditions, a new era is beginning, where the laser control of various nuclear processes could become an invaluable tool for fundamental and applied research at the sub-atomic level [2, 3].

The influence of laser radiation on nuclei is, however, confronted by the problem that on, one hand many low-energy nuclear excitations involve energies in the range 10^{-3}-10 MeV, whereas laser photon energies, such as those produced with the Ti:sapphire laser (λ=800 nm), do not currently exceed 1 eV. On the other hand state-of-the-art facilities that employ X-ray free-electron lasers (XFEL) are already able to produce pulses with wavelengths as low as 0.634 Å and durations of 10^{-14} s [4].

Due to the steady progress towards laser sources with higher intensities and frequencies, in recent years, there has been an increasing focus on direct laser-nucleus reactions (see [5] and references therein). These reactions can be resonant or non-resonant. The latter can occur in particle (β, proton) or heavy-ion (α, cluster) radioactivity. Very recently, we discussed how the dynamics of α-decay in a spherical nucleus is modified by a linearly polarized ultra-intense laser field, using a quantum time-dependent formalism [6]. In the present paper we summarize the main results reported earlier, and discuss new, related aspects such as the electron-positron pair production that may accompany the decay process.

2 α-decay in a periodic field

Our approach to α-decay of a nucleus subjected to a short pulse of a linearly polarized, ultra-intense laser consists of a non-perturbative formalism. In this we solve the time-dependent Schrödinger equation describing the relative alpha cluster-daughter nucleus motion under the effect of a continuous wave (cw) or a sequence of very short pulses of duration no longer than 1 attosecond (as). Our primary goal is to determine how the interaction of radiation with a decaying nucleus could accelerate the tunneling dynamics.

A well-known phenomenon at the atomic level, analogous to the one studied in this article, is the ionization of an electron moving in a static Coulomb potential and interacting with an intense laser field [7, 8].

We commence our considerations by stating the Schrödinger equation for two charged nuclei, located in the laboratory frame at positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, and subject to a time-dependent external field:

$$i\hbar\frac{\partial\psi(\boldsymbol{r}_1,\boldsymbol{r}_2,t)}{\partial t} = H(t)\psi(\boldsymbol{r}_1,r_2,t) \qquad (1)$$

For an α cluster of mass m_1 and charge $Z_1 = 2$ and a daughter nucleus of mass m_2 and charge $Z_2 = Z - 2$ coupled to an external electromagnetic field, described by the transverse vector potential $\boldsymbol{A}$, the Hamiltonian in the Coulomb

***Corresponding Author: Ş. Mişicu:** National Institute for Nuclear Physics-HH, Bucharest-Magurele, P.O.Box MG6, Romania, E-mail: misicu@theor1.theory.nipe.ro
M. Rizea: National Institute for Nuclear Physics-HH, Bucharest-Magurele, P.O.Box MG6, Romania

gauge reads as [9] :

$$H(t) = \sum_{i=1}^{2} \frac{1}{2m_i} \left[\boldsymbol{p}_i - eZ_i\boldsymbol{A}(\boldsymbol{r}_{1,2}, t)\right]^2 + V(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \quad (2)$$

where $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the momenta of the two nuclei. In order to separate the center-of-mass motion (C.M.), the center-of-mass variables $(\boldsymbol{R}, \boldsymbol{P})$ and the relative variables $(\boldsymbol{r}, \boldsymbol{p})$ are introduced :

$$\boldsymbol{R} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_1 + m_2}, \qquad \boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2 \quad (3)$$

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad \boldsymbol{p} = \frac{m_2\boldsymbol{p}_1 + m_1\boldsymbol{p}_2}{m_1 + m_2} \quad (4)$$

The two charges are assumed to form a quasi-bound state whose dimensions, $R_N \sim r_0(A_1^{1/3} + A_2^{1/3})$, are small compared to the radiation wavelength $\lambda = 2\pi c/\omega$. Proceeding in full analogy to the atomic case, we adopt the long-wavelength approximation [10], i.e. we assume that $2\pi R_N \ll \lambda$. Then to order 0 in R_N/λ, $\boldsymbol{A}(\boldsymbol{r}_1)$ and $\boldsymbol{A}(\boldsymbol{r}_2)$ are replaced by $\boldsymbol{A}(0)$ since the vector potential varies over distances of the order of λ [9]. Consequently the center-of-mass part of the Hamiltonian decouples from the relative component, the latter taking the form

$$H_{\text{rel}} = \frac{1}{2\mu} \left[\boldsymbol{p} - eZ_{\text{eff}}\boldsymbol{A}(t)\right]^2 + V(r) \quad (5)$$

where

$$Z_{\text{eff}} = \frac{Z_1A_2 - Z_2A_1}{A_1 + A_2}, \quad \mu = \frac{m_1m_2}{m_1 + m_2}$$

.

The dynamics of the C.M. are described by Volkov states [11]; in what follows we neglect this effect on the α-daughter relative quantum motion.

When the wave-function of the relative motion is denoted by ϕ, the corresponding time-dependent Schrödinger equation (TDSE) reads

$$i\hbar\frac{\partial\phi(\boldsymbol{r}, t)}{\partial t} = H_{\text{rel}}\phi(\boldsymbol{r}, t) \quad (6)$$

Since our intent is to grasp the salient features of the α-decay process in a ultra-high intense laser field we introduce a transverse electric field

$$\boldsymbol{E}(t) = -\frac{\partial\boldsymbol{A}}{\partial t} \quad (7)$$

that we assume to be represented by a modulated linearly-polarized and monochromatic plane wavefunction (single-mode field) of frequency ω

$$E(t) = \mathcal{E}_0\mathcal{F}(t)\sin\omega t \quad (8)$$

where $\mathcal{E}_0$ is the electric field strength, $\mathcal{F}(t)$ is the pulse shape function (envelope) and $\omega = 2\pi/T$ is the frequency of the radiation pulse. In this study we use an envelope function of the type

$$\mathcal{F}(t) = \sum_{p=0}^{N}(-)^p\theta(t - \tau_p) \quad (9)$$

such that the laser pulses act in the time intervals $[\tau_0 = 0, \tau_1]$, $[\tau_2, \tau_3]$, etc. We recover the cw case, when $\tau_1 \longrightarrow t_{\text{pulse}}$, where t_{pulse} is the total laser pulse duration. In the general case we consider short pulses with constant amplitude, of length equal to an integer number of half-cycles, and separated by intervals of similar length; i.e. if at the time τ_{2i} the field is again turned on and then at τ_{2i+1} is again turned off, then the duration of the $(i + 1)$th short pulse is

$$\tau_{2i+1} - \tau_{2i} = n_{i+1}\frac{T}{2} \quad (10)$$

where n_{i+1} is the number of half-cycles. Along with the assumption made above that the electric field is linearly polarized we assume also that the direction $\boldsymbol{E}(t)$ is parallel to the relative-coordinate vector $\boldsymbol{r}$, i.e. we select a fixed orientation (x-axis) of the decaying system and discard from further consideration all possible non-axial configurations with respect to the field. We postpone for future study the two-dimensional approach to this problem. In the present paper, we adopt a one-dimensional geometry. In this approximation the TDSE (6) describing the tunneling process is recast as follows:

$$i\hbar\frac{\partial\phi}{\partial t} = \frac{1}{2\mu}\left(\frac{\hbar}{i}\frac{\partial}{\partial x} - eZ_{\text{eff}}A(t)\right)^2\phi + V(x)\phi \quad (11)$$

The above equation describes the α-particle one-dimensional quantum motion, initially found in a metastable state, inside a static potential $V(x)$. During the tunnelling process, this motion is perturbed by a periodic time-dependent interaction with an external field. The static α-daughter $V(x)$ potential comprises a long-range Coulomb potential, and the nuclear potential in the Woods-Saxon (WS) form

$$V_{\text{nucl}} = -V_0\left[1 + \exp\left(\frac{|x| - R_n}{a}\right)\right]^{-1} \quad (12)$$

where V_0 is the depth, $R_n = r_0 \times A_2^{1/3}$ and a the potential diffuseness.

Note that instead of equation (11) we could have used the length-gauge [9], where coupling of the nuclear system to the laser would be transferred from the kinetic part to a time-dependent potential that would be linear in the coordinate x. It was shown in [12] that such a dipole interaction may lead to numerical instabilities at the computational boundaries. This is the price paid for it being a numerical scheme containing a first-order derivative with respect to x, in addition to the second-order derivative used in the field-free case. Despite this disadvantageous feature, Eq. (11) is preferable for practical calculations.

From the various schemes to integrate TDSE proposed in the literature, we chose a method of the Crank-Nicolson (CN) type, which includes the first derivative with respect to t of $H(x, t)$, the one-dimensional variant of the relative-motion Hamiltonian (5). Accordingly we consider the formal solution of the above equation:

$$\phi(x, t + \Delta t) = e^{-\frac{i\Delta t}{\hbar} H(x,t)}\phi(x, t) \tag{13}$$

where Δt is the time step. Performing a Taylor series expansion of H up to the second-order in t and using the approximation $e^z \approx \frac{1+z/2}{1-z/2}$ (Padé approximant of order [1, 1]) we arrive at the following propagation scheme:

$$\left(1 + \frac{i\Delta t}{2\hbar}H + \frac{i\Delta t^2}{4\hbar}\dot{H}\right)\phi^{n+1} = \left(1 - \frac{i\Delta t}{2\hbar}H - \frac{i\Delta t^2}{4\hbar}\dot{H}\right)\phi^{n} \tag{14}$$

Here ϕ^n is the solution at the moment t, ϕ^{n+1} is the solution at the next moment $t+\Delta t$ and $\dot{H} = \frac{\partial H}{\partial t}$. The error in time is proportional to Δt^3. In practice, the derivatives with respect to x appearing in H and $\dot{H}$ are approximated by finite differences on spatial mesh points and the solution at each time step is obtained by solving a linear system.

Among the advantages of the CN method are the necessity of defining the initial wave function only at the starting value of time, and that it must be unconditionally stable, unitary, and conserve the norm [13]. Also, if the derivatives with respect to x are approximated by the usual 3-point formulas, at each time step a tridiagonal linear system should be solved, which can be done quickly and accurately up to the machine's precision.

In the numerical calculations we use a spatial mesh size Δx of 1/8 fm and time step Δt, depending on the practical conditions imposed by the initial wave function or the laser field parameters. The value $\Delta t = 1/8$ in units of 10^{-22}s was satisfactory for most calculations. For testing purposes we used smaller spatial and temporal steps, obtaining the same behavior and very close results. The maximum time limit we achieved was $\sim 10^{-15}$s which is roughly 4 orders of magnitude higher than the limit attained previously by us [14].

Reflections of the propagated wave-function at the grid frontiers can cause errors in the calculation of physical quantites. To avoid these errors, we use a special boundary condition algorithm, namely the *Transparent Boundary Conditions* (TBC) as suggested in [15]. Since the treatment of the two boundaries is identical we are focusing only on the right boundary (corresponding to the grid point x_M). The idea is to assume near the boundary the following form of the solution: $\phi = A\exp(ik_x \cdot x)$, where A and k_x are complex constants. When the Crank-Nicolson scheme is applied, linear relations between ϕ^{n+1}_{M+1}, ϕ^{n}_{M+1}, the values beyond the numerical boundary, and respectively ϕ^{n+1}_{M}, ϕ^{n}_{M} result. They are subsequently used in the finite difference formulas for the derivatives at x_M.

To solve the TDSE an initial wavefunction has to be selected at $t = 0$. Like in our previous paper on bremmstrahlung in α-decay [14] we use a recipe [16] that provides this initial wavefunction, $\phi(x, 0)$, as a bound state with energy E_α - the eigenvalue of the stationary Schrödinger equation in the modified static potential. Note that the potential $V(x)$ has a constant value $V(\pm x_{\text{mod}}) > E_\alpha$ for a distance $|x| > |x_{\text{mod}}|$ beyond the top of the barrier.

Any solution of the TDSE (11) satisfies the continuity equation for the probability density

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}J = 0 \tag{15}$$

where ρ is the probability density

$$\rho(x, t) = |\phi(x, t)|^2 \tag{16}$$

and J is the one-dimensional probability current density or flux

$$\begin{aligned} J(x, t) &= -\frac{i\hbar}{2\mu}\left[\phi^*(x, t)\frac{\partial}{\partial x}\phi(x, t) - \phi(x, t)\frac{\partial}{\partial x}\phi^*(x, t)\right] \\ &\quad -\frac{eZ_{\text{eff}}}{\mu}A(t)\rho(x, t) \end{aligned} \tag{17}$$

as can be checked by direct calculation.

The time-dependent tunneling probability measuring the escape likelihood of the α-particle from the nuclear+Coulomb potential well is defined by

$$P_{\text{tun}}(t) = \left(\int_{x_b}^{\infty} + \int_{-\infty}^{-x_b}\right)\rho(x, t)dx = 1 - \int_{-x_b}^{x_b}\rho(x, t)dx \tag{18}$$

where $\pm x_b$ are the static barrier positions. If we take the derivative with respect to t of the above equation, and use the continuity equation, we obtain a relation between the tunneling rate and the flux across the nuclear surface.

$$\dot{P}_{\text{tun}}(t) = J(x_b, t) - J(-x_b, t) \tag{19}$$

On the other hand we introduce the total norm inside the numerical grid, i.e. we integrate the probability density between the left, x_{left}, and right, x_{right}, boundaries:

$$P_{\text{int}}(t) = \int_{x_{\text{left}}}^{x_{\text{right}}}\rho(x, t)dx \tag{20}$$

we obtain the flux across the numerical grid

$$\dot{P}_{\rm int}(t) = J(x_{\rm left}, t) - J(x_{\rm right}, t) \quad (21)$$

With the TBC assumption, the flux leaving the right boundary $x_{\rm right}$ is given by :

$$J(x_{\rm right}, t) = \frac{1}{\mu}\left[{\rm Real}\,(\hbar k_x) - eZ_{\rm eff}A(t)\right] |\phi(x_{\rm right}, t)|^2 \quad (22)$$

Formula (22) shows that an outgoing or ingoing flux develops across the boundary depending on the sign of ${\rm Real}\,(\hbar k_x) - eZ_{\rm eff}A(t)$. Due to the action of the time-dependent field, the real part of k_x is no longer constrained to be always positive, as happens for α-decay in the absence of an external perturbation [14]. Note that in this latter process the overall change in energy from the right boundary is always negative, and thus the wave function flows out of the grid region. In the present case, back-flow of the wave-function inside the numerical domain is expected to occur due to the reversal of the electric field polarity. In practice we can compute the outgoing flux and add it to the norm if a part of the wave function goes out of the domain. We therefore have full control on the norm used in the calculation of tunneling probability and decay rate. Employing the TBC procedure, one can obtain values for the physical quantities with a much smaller extension of the spatial grid than is necessary without the TBC procedure.

We defined in Eq.(18) the probability that the α particle is found beyond the Coulomb barrier $|x_b|$ which by reference separates the zone inside the barrier from the external one. Then the α-decay rate for large times (remember that for early times the decay is not exponential !) is provided by the formula

$$\lambda(t) = -\frac{\dot{P}_{\rm int}(t)}{P_{\rm int}(t)} \quad (23)$$

The relation between the electric field strength E_0 and the laser intensity $I = \frac{1}{2}c\epsilon_0\mathcal{E}_0^2$ [3] can be recast for practical purposes as $\mathcal{E}_0$[V/cm]= $27.44\left\{I[{\rm W/cm^2}]\right\}^{1/2}$. This shows that at the maximum intensity foreseen at ELI [2], i.e. $I_0 \approx 10^{25}{\rm W/cm^2}$, the electric field is $\mathcal{E}_0 \approx 8.64\times10^{13}$V/cm or, in nuclear units, $e\mathcal{E}_0$= 8.64×10^{-6}MeV/fm. For such a value the time-dependent part of the potential affects the barrier only negligibly. For this reason, in this paper, we explore laser intensities up to $I_0 \approx 10^{33}{\rm W/cm^2}$, thus $\mathcal{E}_0 \approx 10^{18}$V/cm or $e\mathcal{E}_0 \approx 10^{-2}$MeV/fm. We find convenient to express the frequency in nuclear units as $\hbar\omega$ = $1240[{\rm MeV\cdot fm}]/\lambda[{\rm fm}]$. Thus for a wavelength of λ = 1 Å, $\hbar\omega$=12.4 keV and $T = 2\pi/\omega = 3.33\cdot10^{-20} = 0.0333$ as.

We should also mention that the laser intensities and frequencies used in this paper are far to meet the relativistic onset conditions. The square of the dimensionless parameter $\eta = v/c = Z_{\rm eff}e\mathcal{E}_0/(\mu\omega c)$, represents the ratio of the maximum quiver velocity v of the decaying system in the laser field to the velocity of light. This parameter indicates the role of relativistic effects, provided η^2 ≥1; for the above choice of electric strength, this condition is not fulfilled since $\eta \approx 1.7\cdot10^{-3}$.

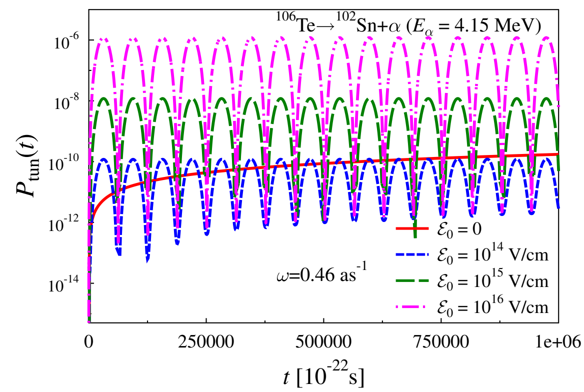

Figure 1: Tunneling probabilities in the logarithmic scale for three different field $\mathcal{E}_0$ amplitudes and the same frequency ω of the cw are compared to the tunneling probability without laser field.

3 Numerical results

We specify the parameters of the nuclear potential according to the α-nucleus optical potential compilation from [17]. For ^{106}Te, the parameters of the WS potential are : V_0=-137.7 MeV, a=0.76 fm and r_0=1.235 fm, and are consistent with the energy E_α=4.15 MeV if one solves the stationary Schrödinger equation with the potential modified at $|x_{\rm mod}|$=25 fm. The spatial border is specified by $x_{\rm right,left}$ = ±192 fm.

We first consider the interaction of a continuous sinusoidal laser wave with a dinuclear system. The comparison of the tunneling probabilities for the field-free case and for the case when a continuous wave laser of fixed frequency, ω=0.46 as ($\hbar\omega$ ≈0.3 KeV, or λ ≈4.13 nm), is impinging on the decaying system at various intensities is provided in Figure 1. As expected, the oscillation amplitude of $P_{\rm tun}(t)$ increases with the laser intensity, a feature that from a classical electromagnetism standpoint is related to the oscillation of the α nucleus in the periodically fluctuating electric field.

It can also be inferred that, at the lowest applied field intensity $\mathcal{E}_0 = 10^{14}$V/cm ($e\mathcal{E}_0$ = 10 eV/fm), which is very

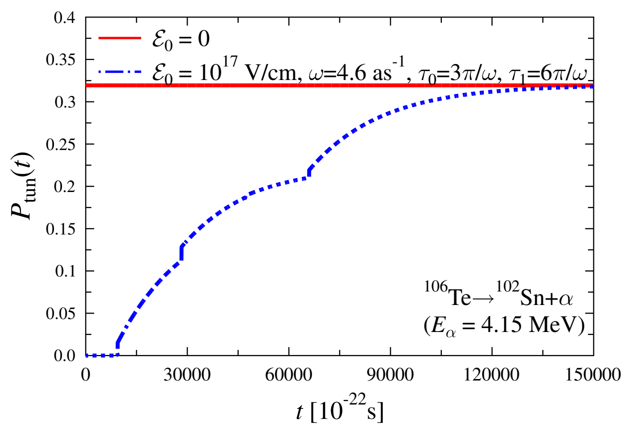

Figure 2: Tunneling probabilities of the α particle from ^{106}Te for the field-free case and for the 4 pulses sequences of duration $3\pi/\omega$ interrupted by three breaks of equal duration with characteristics $\mathcal{E}_0$=10^{17} V/cm and ω=15.2 as^{-1}. Note that the field-free $P_{tun}(t)$ is multiplied by a factor of 850.

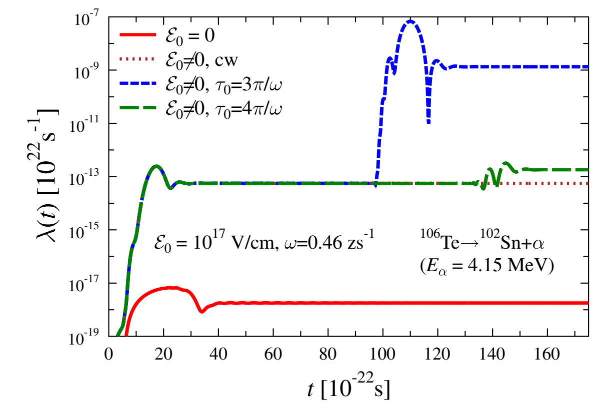

Figure 3: α decay rates of ^{106}Te in 4 instances : field-free case, continuous wave, short pulse of 3 half-cycles duration, short pulse of 4 half-cycle duration. The characteristics of the laser field are $\mathcal{E}_0$=10^{17} V/cm and ω=0.46 zeptoseconds(zs)$^{-1}$.

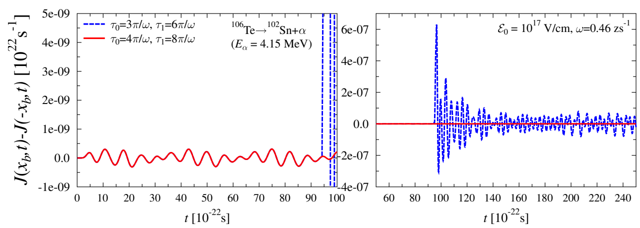

Figure 4: Total flux across the nuclear domain in the case of a short pulse of 3 half-cycles duration (full curve), and a second one of 4 half-cycles duration (short dashes). The values of the radiation field and frequency are $\mathcal{E}_0$=10^{17} V/cm and ω=0.46 zs^{-1}. Note that the pulse is repeated after a time equal to the corresponding pulse duration.

small compared to the barrier ($e\mathcal{E}_0 \ll V_{barrier}$), the maximum of P_{tun} is bypassed by the field-free tunneling probability after $\sim 10^{-16}$s. Similar to the case of atom ionization by lasers [7], we conclude a quasi-stabilization of the decaying system induced by the laser field.

The tunneling probabilities for a 4-pulse laser signal of length $3\pi/\omega$ and for the field-free case are compared in Figure 2. Note that the amplification of $P_{tun}(t)$ when an odd number of half-cycles is applied is ~850 after ~15 as.

In Figure 3 we show how the decay rates, defined by eq.(23), increase in a step-like manner when the electric field is turned off. This behavior is caused by the box-like shape of the laser signal envelope. For an envelope of Gaussian or $\sin^2$ type a smooth increase of $\lambda(t)$ is expected. When comparing the case of a pulse with an odd number of half-cycles to one with an even number of half-cycles, we conclude that the jump of $\lambda(t)$ in the latter case is much less pronounced compared to the case when the laser is turned off. From inspection of Figure 3, we conclude that $T_{1/2}$ decreases by approximately 9 orders of magnitude! Thus, the α-decay of ^{106}Te is speeded up to times of the order of femtoseconds, compared to the microsecond timescales in the field-free case.

In Figure 4 we explored the behavior of the total flux leaving from, or returning inside, the nuclear surface (as reference we take the Coulomb barrier position), with the intent to explain the jump in the decay rates. After turning off the laser field, due to an imbalance between the left border flux and the right border flux, the quantum flux for a pulse with an odd number of half-cycles oscillates with a larger amplitude than a pulse with an even number of half-cycles.

An issue that can play a role in the problem investigated in our study concerns electron-positron pair creation by a strong laser field in the presence of a heavy nucleus (see [5] and references therein). The probability of e^--e^+ pair production on nuclei is controlled by two dimensionless parameters: $\xi = e\mathcal{E}_0/m_e c\omega$, which expresses the energy of interaction of an electron of rest mass m_e with the laser field, and $\zeta = \mathcal{E}_0/E_{crit}$, the ratio between the strength of the laser's electric field and the critical vacuum (Schwinger) field $E_{crit} = m_e^2c^3/e\hbar$=1.3·$10^{16}$V/cm [18]. Note that the process of e^-- e^+ pair production shares common features with both the tunnelling ionization process and the α-decay reaction studied in this work. These last two processes can be described as the decay of the vacuum of a neutral gas of atoms and of a compound nucleus under the influence of an external electric field. Production of e^-- e^+ pairs in the vicinity of a heavy nucleus of charge Z was recently set under scrutiny in ref. [19]. Optimal conditions for the e^{-}- e^{+} pair production are expected in the strong-field regime, i.e. for $\xi \gg 1$. In the case of subcritical fields ($\zeta \ll 1$), as is the case for optical or infrared lasers, the above quoted article provides the following quasi-classical formula for the pair creation rate:

$$\dot{\mathcal{W}} = \frac{(Z\alpha)^2}{2\sqrt{\pi}} m_e \left(\frac{\zeta}{2\sqrt{3}}\right)^{2.5} \exp\left(-\frac{2\sqrt{3}}{\zeta}\right) \quad (24)$$

For $e\mathcal{E}_0$=10 eV/fm ($I_0 = 1.33 \cdot 10^{25}$W/cm^2) and λ=1240 nm, we have $\xi \approx 38.6$, $\zeta \approx 0.077$ and $\dot{\mathcal{W}} \ll 1$. Instead, for supercritical fields the recommended formula for this quantity is

$$\dot{\mathcal{W}} = \frac{13(Z\alpha)^2}{6\sqrt{3}\pi} m_e \zeta \left[\ln\left(\frac{\zeta}{2\sqrt{3}}\right) - 2.0644\right] \quad (25)$$

For the case $e\mathcal{E}_0$=10 keV/fm ($I_0 = 1.33 \cdot 10^{31}$W/cm^2), $\xi \gg 1$, $\zeta \gg 1$ and $\dot{\mathcal{W}} \gg 1$. We estimated that in this case around $N_{e^-,e^+} \sim 350$ pairs are likely to be created in the nuclear Coulomb + ultra-high laser field environment for a very short pulse duration, $t_{\rm pulse}$= 1 as. Since for a linearly polarized laser the pairs are predicted to move colinearly to the field, we expect a major alteration of the dinuclear system dynamics. However in an earlier reference [20], it was concluded that pair creation by a linearly polarized pulse in the vicinity of a nucleus is negligibly small for all intensities, no matter how large they are, due to the occurrence of a suppressing factor $\exp(-m_e c^2/\hbar\omega)$ in $\mathcal{W}$. Clearly this issue deserves to be tackled in more depth in the near future. A first improvement would be to go beyond the point-like nucleus approximation as happens in the case of static strong fields [18].

Another effect induced by overcritical laser fields in nuclei is related to the change in the proton density that is conjectured to occur at intensities ($I \geq 10^{30}$W/cm^2) due to the ac Stark shifts of proton states [21]. However as we can conclude from Figure 1 of that reference, the changes in the proton root-mean-square radii are too small at the highest intensity considered in this paper, i.e. 10^{33}W/cm^2, and consequently we expect that the proton distributions of the two nuclei are not distorted enough to influence the tunnelling dynamics.

To conclude, in this paper we proposed a numerical algorithm based on the CN method to solve the TDSE, which allowed us to examine the detailed dynamics of the α-decay process under the influence of an ultra-intense monochromatic laser field. Our primary goal was to establish the characteristics of the laser pulse that cause a major modification of the tunneling probabilities, decay rates and thus of half-lives. The most important result of our study was that short pulses containing an odd number of half-cycles affect this type of nuclear radioactivity more than those with even numbers of half-cycles.Repeated application of such pulses leads to faster decay of an α-radioactive nucleus. We proved that, at sub-attosecond timescales, it was possible to increase the decay rates by several orders of magnitude. To substantiate this effect, we used ultra-intense laser fields in our theoretical study; production of such fields will require the next generations of laser facilities. Laser control of nuclear decay processes is a new facet of the emerging field of direct laser-nucleus reactions. Better knowledge of the mechanism governing this type of phenomenon could also find other applications, including the use of high-power lasers in disposal of radioactive waste. All in all, this contribution draws attention to the possibility of controlling spontaneous cluster radioactive decays with super-strong electromagnetic fields.

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