Eur. Phys. J. C (2024) 84:823
https://doi.org/10.1140/epjc/s10052-024-13197-4

THE EUROPEAN PHYSICAL JOURNAL C

Comment

Critical remarks on Finslerian modified gravity

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Received: 3 June 2024 / Accepted: 5 August 2024

Abstract We disagree with the analysis presented in the paper entitled "Finslerian extension of an anisotropic strange star in the domain of modified gravity" (by Sourav Roy Chowdhury et al., published in Eur Phys J C 84:472, 2024). They applied the Finsler space-time to develop the Einstein field equations in the extension of modified geometry. According to the Finsler metric structure, they have considered $\overline{Ric} > 1$ to develop a stable physical system and have also shown that the Finslerian background is a strong candidate for describing the compact system.

There is a recent interest in new physics beyond the Standard Model related to Finsler's theories of curved space-time and quantum gravity and possible applications in modern cosmology. Such theories constructed on tangent bundles of space-time manifolds are positive with local Lorentz violations, which may be related to new directions in particle physics and dark energy and dark matter models in cosmology. Researchers in particle physics and cosmology must become more familiar with some important methods and perspectives and possible applications of Finsler geometry in standard and non-standard theories of physics.

In the theory of general relativity (GR), the Einstein field equations (EFE) relate the geometry of space-time to the distribution of matter in it. Misuse of concepts in each part affects the opposite side, challenging physical and geometric interpretations. We want to explain some methodological issues. First, it is important to know that the concept and definition of Ricci and flag curvature in Finsler geometry should be properly examined.

The problem with choosing $\overline{Ric}$ values

To define the stellar structure, the authors assumed the Finsler structure as the following [1],

$$\mathfrak{F}^2 = -\mathrm{e}^{\lambda(\mathrm{r})}\mathrm{y}^\mathrm{t}\mathrm{y}^\mathrm{t} + \mathrm{e}^{\nu(\mathrm{r})}\mathrm{y}^\mathrm{r}\mathrm{y}^\mathrm{r} + \mathrm{r}^2\overline{\mathfrak{F}}^2, \tag{1}$$

where $\overline{\mathfrak{F}}$ is the "Finslerian sphere" that is equivalent to the spherical symmetry of Riemannian space and is defined as follows,

$$\overline{\mathfrak{F}} = \frac{\sqrt{(1-\epsilon^2\sin^2\theta)y^\theta y^\theta + \sin^2\theta y^\phi y^\phi}}{1-\epsilon^2\sin^2\theta} - \frac{\epsilon\sin^2\theta y^\phi}{1-\epsilon^2\sin^2\theta}, \tag{2}$$

with $0 \le \epsilon < 1$. It is obvious that the metric (2) returns to the Riemannian sphere while $\epsilon = 0$ (see more Ref. [2] pp. 4-6).

In 2002, Shen [3] investigated two-dimensional Finsler metrics of constant curvature, and Randers–Finsler space [4] with constant flag curvature was classified by Bao et al. [5]. Therefore $\overline{\mathfrak{F}}$ is the spherical metric with constant flag curvature $K = 1$.

On the other hand, in Ref. [6], (p. 216) is mentioned "*Every Finsler metric with constant flag curvature K must be Einstein with constant Ricci scalar $(n-1)K$.*" it means that

$$Ric = (n-1)K, \tag{3}$$

where n is the dimension of space. According to the explanations, in 2D space $Ric = K$.

In Ref. [1], the authors considered different values for $\overline{Ric}$, which is geometrically impossible. $\overline{Ric}$ can only accept one value $\overline{Ric} = 1$ that in this case, Einstein field equations are

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Published online: 17 August 2024

similar to Riemannian case [7], and the results precisely are the same as Ref. [8]. Furthermore, they mentioned:

According to the present study, it is clear that with the increase in the coupling parameter and the Finsler parameter, the radii of the system decrease, and the central density increases significantly, which indicates that the Finslerian background is a strong candidate for describing the compact system.

GR is known as the geometric theory of gravitation. It is clear that $\overline{Ric}$ is a geometric quantity, and the geometrical conditions can impose restrictions on it that do not allow us in this particular case $\overline{Ric} > 1$. Since the Finsler parameter ($\overline{Ric} = 1$) is constant, only the increase in the coupling parameter causes the system's radii to decrease and the central density to increase. So, the Finsler structure cannot describe the compactness of the system. The Finslerian background affects the motion of particles because the geodesic equations depend on the Finsler parameter (ϵ) [2].

Funding No funding was received for this research.

Data Availability Statement My manuscript has no associated data. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.].

Code Availability Statement My manuscript has no associated code/software. [Author's comment: Code/Software sharing not applicable to this article as no Code/Software was generated or analysed during the current study.].

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