PHILOSOPHICAL TRANSACTIONS A

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Review

Cite this article: Townsend PK. 2022 Aether, dark energy and string compactifications. *Phil. Trans. R. Soc. A* **380**: 20210185.
https://doi.org/10.1098/rsta.2021.0185

Received: 16 August 2021
Accepted: 9 September 2021

One contribution of 12 to a theme issue 'The future of mathematical cosmology, Volume 2'.

Subject Areas:
cosmology, electromagnetism, high-energy physics, relativity, string theory

Keywords:
aether, branes, dark energy

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THE ROYAL SOCIETY PUBLISHING

Aether, dark energy and string compactifications

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The nineteenth-century aether died with special relativity but was resurrected by general relativity in the form of dark energy; a tensile material with tension equal to its energy density. Such a material is provided by the D-branes of string-theory; these can support the fields of supersymmetric particle-physics, although their energy density is cancelled by orientifold singularities upon compactification. Dark energy can still arise from supersymmetry-breaking anti-D-branes but it is probably time-dependent. Recent results on time-dependent compactifications to an FLRW universe with late-time accelerated expansion are reviewed.

This article is part of the theme issue 'The future of mathematical cosmology, Volume 2'.

1. Introduction

When Einstein introduced his 1905 theory of special relativity (SR), as it was soon to be called, he was motivated by the need to reconcile observations implied by Maxwell's electrodynamics for inertial observers in constant relative motion, in particular, an invariable *in vacuo* speed of light c. The hypothesis that light waves are disturbances of a space-filling 'aether' was, as he put it, unnecessary [1]. It appears that Einstein was not motivated by the failure of Michelson and Morley to detect the motion of the Earth through the hypothetical aether, but SR explained their null results and the aether hypothesis was eventually abandoned. However, SR does not refute the aether hypothesis; it just restricts the aether to be some material that is Lorentz invariant, which implies that its stress-energy tensor is proportional to the Minkowski metric.

Put differently, the aether (if it exists) must be a shear-free tensile material with a constant tension T that is equal to its energy density $\mathcal{E}$. No such material is known but one may still ask whether it is theoretically possible.

Ironically, the answer to this question was implicitly provided by Einstein himself when he modified his 1915 gravitational field equations of general relativity (GR) to incorporate his cosmological constant λ, with dimensions of inverse length squared [2]. In modern notation, this modified equation is

$$G_{\mu\nu} + \lambda g_{\mu\nu} = \kappa^2 \Theta_{\mu\nu} \quad \text{and} \quad \kappa^2 = \frac{8\pi G_N}{c^4}, \tag{1.1}$$

where G is the Einstein tensor for the space–time metric g, and Θ is the matter stress-energy tensor (and G_N is Newton's constant). However, there is an alternative interpretation of this equation, due to Lemaître [3]: we can rewrite it as the unmodified Einstein field equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}, \tag{1.2}$$

but with the modified matter stress-energy tensor

$$T_{\mu\nu} = -\Lambda g_{\mu\nu} + \Theta_{\mu\nu} \quad (\Lambda = \kappa^{-2}\lambda). \tag{1.3}$$

This shows that the introduction by Einstein of a positive cosmological constant is equivalent to an assumption that the Universe is filled with an ideal fluid of negative pressure equal to its constant energy density: $\mathcal{E} = \Lambda$. This 'dark energy' can be equivalently described as a tensile material of tension Λ that is locally Lorentz invariant (since $g = \eta$ in local inertial frames). Dark energy is therefore a kind of aether, but is it the kind that could serve as a medium for the propagation of electromagnetic waves?

The short answer is 'yes but no but maybe'. The preliminary 'yes' arises from consideration of the dynamics of a D3-brane of IIB superstring theory, but this has to be integrated into a bigger picture that involves all the fundamental forces, including gravity. This leads to consideration of Calabi–Yau compactifications of Type IIB superstring theory with D-branes and orientifold planes, which is a string-theory construction of supersymmetric versions of particle physics models coupled to supergravity; a brief sketch will suffice here. At this point, the answer to our question is 'no' because unbroken supersymmetry does not allow for dark energy; it has been unavoidably cancelled by the orientifold planes. The introduction of anti-D-branes is one way to both break supersymmetry and introduce dark energy, so the 'no' becomes a 'maybe', but here we leave the branes and proceed more phenomenologically to a review of the difficulties of incorporating dark energy in string-theory compactifications, and some of the ideas about how it might be achieved by allowing the compact space to be time-dependent.

The first part of this article (the relation of branes to the aether and the connection to dark energy) was developed by the author for talks (unpublished) at conferences in 2005 and 2015 marking the centenaries of Einstein's works on, respectively, SR and GR. The last part (time-dependent compactifications to accelerating universes) is based mainly on articles from 2003 to 2019 written in collaboration with Mattias Wohlfarth, Julian Sonner and Jorge Russo.

2. Strings, branes and aether

To develop some intuition, let us imagine a one-dimensional world filled with a material of constant tension T, and let us suppose that it is embedded in a three-dimensional Euclidean space. What we then have is a string of tension T; a guitar string would be an example. According to SR, a string of mass density μ will have energy density $\mathcal{E} = \mu c^2$, while $T \sim 10^{-11}\mathcal{E}$ for a typical string on a guitar. The tension can be increased by a tuning that stretches the string, which will also slightly decrease its mass density, so stretching will increase the ratio $T/\mathcal{E}$ but not by much before the string breaks. So let us further imagine that this guitar-string world is unbreakable, and can be stretched as much as we wish; can we then make the ratio $T/\mathcal{E}$ arbitrarily large? The answer is no because SR imposes a fundamental upper limit on this ratio. Small amplitude disturbances of the string (perhaps created by the plucking of a guitarist) will travel along it at a speed

$$v = \sqrt{\frac{T}{\mu}} = c\sqrt{\frac{T}{\mathcal{E}}}, \tag{2.1}$$

but SR requires $v \leq c$, and hence

$$T \leq \mathcal{E}. \tag{2.2}$$

This inequality suggests the following threefold classification of strings:

— $T \ll \mathcal{E}$. These are non-relativistic strings. Our typical guitar string is an example, as are all strings that we could make with materials available to us.

— $T \lesssim \mathcal{E}$. These could be called 'relativistic' strings. An example is the Schwarzschild black string in a five-dimensional space–time. This is just the Schwarzschild black-hole solution of Einstein's four-dimensional gravitational field equations 'lifted' to a cylindrically symmetric solution of the same equations in five dimensions; in this case $T = \frac{1}{2}\mathcal{E}$ [4].

— $T = \mathcal{E}$. These are 'ultra-relativistic' strings. An example is the Nielsen–Olesen string solution of a Higgs-type scalar field theory [5]; the 'material' that makes up the string is a line defect in the scalar field analogous to the Abrikosov vortex in a superconductor. It has a potential interpretation as a 'cosmic string', and an 'effective' (low energy) description as a Nambu-Goto string. There is also an ultra-relativistic 'magnetically' charged black string solution of five-dimensional supergravity [6]; in this context, the bound (2.2) can be interpreted as a 'BPS bound' on the energy density in terms of the tension, which acquires an interpretation as the 'magnetic' string-charge source for the Maxwell-like gauge field of five-dimensional supergravity. Finally, there is the 'fundamental' string of string theory, which is ultra-relativistic by hypothesis; in this case

$$T = \frac{\hbar c}{\ell_s^2}, \tag{2.3}$$

where ℓ_s is the 'string length' of string theory.

We are not restricted to strings; the same ideas apply to branes. For waves on a p-brane, which has p space dimensions, SR again imposes the inequality (2.2) but the dimensions of tension are now such that

$$T = \frac{\hbar c}{\ell^{(p+1)}}, \tag{2.4}$$

for some 'characteristic' length ℓ, which can be interpreted as the brane's width. The $p = 0$ case is degenerate because there are no waves in zero dimensions. Nevertheless, it is natural to suppose that $T \to mc^2$ for $p = 0$, where m is the particle's rest-mass, and $\mathcal{E} \to E$, its energy; in this case ℓ is the particle's Compton wavelength, and the inequality (2.2) is equivalent to $mc^2 \leq E$ (the standard SR formula $E = mc^2$ applies to a particle at rest).

For an ultra-relativistic p-brane, the dynamics at length scales much larger than ℓ can depend only on the p-brane's tension and the geometry of the $(1 + p)$-dimensional worldvolume that is swept out by its time evolution. Let $\{\xi^\mu; \mu = 0, 1, \ldots, p\}$ be local coordinates on this worldvolume; its local embedding in a $(1 + n)$-dimensional Minkowski space–time, with Minkowski coordinates $\{X^m; m = 0, 1, \ldots, n\}$, is then specified by functions $X^m(\xi)$. These functions are worldvolume scalar fields for the generalization to p worldspace dimensions of Dirac's membrane action [7]; in units for which $\hbar = c = 1$, the Dirac p-brane action is

$$I = -T \int d^{p+1}\xi \sqrt{-\det g}, \tag{2.5}$$

where g is the worldvolume metric induced by the Minkowski space–time metric η:

$$g_{\mu\nu}(\xi) = \partial_\mu X^m \partial_\nu X^n \eta_{mn}. \tag{2.6}$$

The Nambu–Goto string action is the $p=1$ case.

As we are interested here in fluctuations of an infinite static planar p-brane, we shall write $X^m=(X^\mu,X^I)$ for $I=1,\ldots,n-p$, and then choose X^μ to be the local worldvolume coordinates; i.e.

$$X^\mu(\xi)=\xi^\mu. \tag{2.7}$$

This is the Monge gauge choice, and in this gauge we have

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},\quad h_{\mu\nu}=\partial_\mu\mathbf{X}\cdot\partial_\nu\mathbf{X}, \tag{2.8}$$

where $\mathbf{X}$ is the Euclidean $(n-p)$-vector worldvolume field with components $X^I(\xi)$. After setting

$$\mathbf{X}=\frac{\boldsymbol{\phi}}{\sqrt{T}}, \tag{2.9}$$

we find that the Monge gauge action is

$$I_{\text{Monge}}=\int d^{p+1}\xi\left\{-T-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\boldsymbol{\phi}\cdot\partial_\nu\boldsymbol{\phi}+\mathcal{O}\left(\frac{1}{T}\right)\right\}, \tag{2.10}$$

where the $\mathcal{O}(1/T)$ terms are interaction terms that are at least quartic in derivatives of $\boldsymbol{\phi}$. We now have a relativistic field theory for scalar fields of canonical dimension propagating in a $(1+p)$-dimensional Minkowski space–time. The stress-energy tensor takes the form (for 'mostly plus' metric signature)

$$T_{\mu\nu}=-T\eta_{\mu\nu}+\Theta_{\mu\nu}(\boldsymbol{\phi})+\mathcal{O}\left(\frac{1}{T}\right), \tag{2.11}$$

where

$$\Theta_{\mu\nu}(\boldsymbol{\phi})=\partial_\mu\boldsymbol{\phi}\cdot\partial_\nu\boldsymbol{\phi}-\frac{1}{2}\eta_{\mu\nu}(\eta^{\rho\sigma}\partial_\rho\boldsymbol{\phi}\cdot\partial_\sigma\boldsymbol{\phi}), \tag{2.12}$$

which satisfies the continuity equation $\partial_\mu\Theta^{\mu\nu}=0$ as a consequence of the linearized field equation $\Box\boldsymbol{\phi}=0$. The first term in (2.11) is the stress-energy tensor of the infinite static planar brane on which the scalar fields propagate as small-amplitude fluctuations in the $n-p$ 'extra' dimensions inaccessible to denizens of the brane.

This simple example demonstrates that light-speed waves of *scalar* fields may have an interpretation as disturbances of an aether in a way that is consistent with SR. There is more to do before the same claim can be made for the vector fields of electromagnetism. An ad-hoc introduction of electromagnetic fields on the brane would explain neither why they are confined to it nor how they might be considered fluctuations of it, but supersymmetry provides the means to make this 'electromagnetism on the brane' natural because it allows scalar fields to be connected by a symmetry to electromagnetic fields. This possibility is not realized by brane solitons of Minkowski space field theories, which all have an effective description involving fields of spins $\le\frac{1}{2}$ [8,9]. A closely related fact is that soliton solutions of a maximally supersymmetric field theory preserve at most half of the supersymmetry, which implies that a 3-brane solution of a Minkowski space field theory can have at most $N=2$ four-dimensional supersymmetry (an $N=1$ example was studied in [10]). So we must look to (super)gravity.

Of most relevance here are the maximal 10-dimensional supergravity theories (IIA and IIB); these have planar static black brane solutions that are asymptotically flat in transverse directions [11,12]. A generic black p-brane of this type carries a p-form 'charge-density' of magnitude $T\le\mathcal{E}$, which is a source for a $(p+1)$-form potential among the supergravity fields ($p=1,3,5$ for the chiral IIB supergravity). The terminology 'BPS' is generally used for an ultra-relativistic black p-brane solution with $T=\mathcal{E}$, and a special feature of a BPS p-brane supergravity solution is that it preserves half the supersymmetry of the 10-dimensional Minkowski vacuum. Specifically, it breaks the 10-dimensional maximal space–time supersymmetry to a maximal worldvolume supersymmetry; this is possible because the p-form charge appears in the 10-dimensional supersymmetry algebra [13,14]. The variables relevant to the effective low-energy dynamics of the brane are the coefficients of 'zero modes' of the supergravity fields in the brane background; these depend only on the brane's worldvolume coordinates and they form a multiplet of worldvolume supersymmetry [15]. For the black 3-brane of IIB supergravity, this is the maximally supersymmetric ($N=4$) Maxwell supermultiplet [12]; its six scalar fields have the obvious interpretation as perturbations in the six transverse-space directions but these are now in the same supermultiplet as electromagnetic fields, which are therefore as relevant to the low-energy dynamics as the scalar fields. But what is this dynamics?

The 10-dimensional IIA and IIB supergravity theories are the effective theories (on length scales much greater than ℓ_s) for closed strings of the IIA and IIB superstring theories, so one might expect p-branes to be a non-perturbative feature of Type II superstring theory, and this is required by U-duality [16] (as reviewed in [17]). Remarkably, this conclusion has confirmation from string perturbation theory: Polchinski showed that the Type II superstring theories include not only closed strings but also open strings with endpoints on fixed p-planes [18], called 'D-branes' (Dp-branes if we want to specify p) because of the Dirichlet boundary conditions at the string endpoints [19]. They are branes because the massless modes of open strings attached to them can be interpreted as fluctuations of the fixed p-plane; the Dp-brane tension is [18]

$$T_{\mathrm{D}p} = \frac{\hbar c}{g_s \ell_s^{p+1}}, \tag{2.13}$$

where g_s is the dimensionless string coupling constant ($g_s \ll 1$ in string perturbation theory). Note that the tension becomes infinite in the $g_s \to 0$ limit. This is a reflection of the fact that the undisturbed Dp-brane fills a *fixed* p-plane in string perturbation theory, but that is precisely the circumstance of interest here since it provides us with a Minkowski 'vacuum' on which the open-string massless modes propagate. Moreover, the effective action for these fields (on length scales much larger that ℓ_s) can be found from superstring perturbation theory; the result is a generalization of a 1985 result of Fradkin & Tseytlin [20] who showed that the effective action for slowly varying massless fields of the open bosonic string is (a higher-dimensional version of) the Born–Infeld action for nonlinear electrodynamics [21]. The generalization fuses the Born–Infeld action and the Dirac p-brane action into the Dirac–Born–Infeld (DBI) action [22].

For the IIB D3-brane the massless open-string modes form an $N=4$ Maxwell supermultiplet, in agreement with the supergravity results. The DBI action for the bosonic fields of this supermultiplet (i.e. omitting the fermionic fields required by supersymmetry) is (again for $\hbar = c = 1$)

$$I = -T_{\mathrm{D3}} \int d^4\xi \sqrt{-\det(g + \ell_s^2 F)}, \tag{2.14}$$

where F is the antisymmetric worldvolume electromagnetic field-strength tensor. In the Monge gauge for which

$$g_{\mu\nu} = \eta_{\mu\nu} + \ell_s^2 \partial_\mu \phi \cdot \partial_\nu \phi, \quad \phi = \{\phi^1, \ldots, \phi^6\}, \tag{2.15}$$

the DBI action takes the form

$$I_{\mathrm{Monge}} = g_s^{-1} \int d^4\xi \left\{ -\ell_s^{-4} - \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \mathcal{O}(\ell_s^2) \right\}, \tag{2.16}$$

which may be compared with (2.10). Apart from the overall factor of $1/g_s$ and the omitted fermion fields, the difference is that we now have Maxwell's electrodynamics in addition to scalars. Electric charges (not included in the above action) have a higher-dimensional interpretation as the endpoints of fundamental IIB strings, and magnetic charges have a similar interpretation as endpoints of IIB D-strings (i.e. D1-branes).

Leaving aside many issues to focus on the fact that we now have an interpretation of electromagnetic waves as disturbances of an aether consistent with SR, let us consider whether the tension of this D3-brane aether can be identified with the cosmological constant; i.e. can we have $T_{\mathrm{D3}} = \Lambda$? The answer is no because in BI electrodynamics there is a maximum value of the electric field (this was its motivating feature) and the BI parameter T is the corresponding maximum energy density, but atomic physics experiments put a lower bound on any hypothetical maximum value of the electric field [23], and this is equivalent to the bound $T \gtrsim 10^{33}\,\mathrm{J\,m^{-3}}$. By contrast, cosmological observations yield $\Lambda \sim 10^{-10}\,\mathrm{J\,m^{-3}}$, a discrepancy of more than 40 orders of magnitude! This is not a problem for BI theory because we can subtract a constant from the vacuum energy density (and this is usually done) but this ad hoc subtraction is contrary to the spirit of the DBI action. Let us put this problem aside for the moment to consider another one.

(a) Gravitational aether?

We need a theory not just of electromagnetism but of all the fundamental interactions. There is no difficulty in getting other spin-1 gauge theories 'on the brane'. For example, the effective low-energy description of a parallel stack of N coincident D3-branes is a $U(N)$ gauge theory [24], but this leaves out gravity. One may wonder whether there is some other kind of brane on which a massless spin-2 field can be trapped. There are reasons to think that this is not possible; a review of the difficulties, and an interesting potential way to overcome them, may be found in [25].

A priori, it is hard to see how the dynamics implied by a presumed Einstein–Hilbert term 'on the brane' could be compatible with a brane interpretation. One difficulty is that there is no coordinate-independent *local* definition of energy in GR, which is why energy loss due to gravitational radiation can only be understood in the context of appropriate asymptotic boundary conditions, as first appreciated by Trautman [26]; coordinate transformations can move the energy around like a bump under the carpet that can be flattened locally but not globally. This would appear to rule out any interpretation of gravitational waves as fluctuations of a 'gravitational' aether.

This conclusion is supported by the 't Hooft–Susskind principle of holography that was suggested by black-hole physics [27,28]. This states that the number of quantum gravitational degrees of freedom in a given region grows as the area of the region's boundary, but the number of degrees of freedom associated with quantized fluctuations of a gravitational aether would grow with the volume (by hypothesis, essentially) and these two growth rates are generally very different; an exception is anti-de Sitter space–time because the volume of a ball in hyperbolic space grows asymptotically and (in suitable units) at the same rate as the area of its boundary (a fact that is essential for the applicability of the holographic principle in anti-de Sitter space–time [29]).

Nothing said above implies the impossibility of a gravitational aether. An 'Einstein-Aether' modification of GR has been proposed [30], for phenomenological reasons reviewed in [31], but it does not appear to arise from any larger framework such as string/M-theory that could provide a consistent quantum theory. In other words, it may be in the 'swampland', which is the currently popular term for phenomenological theories that have no 'UV completion'; the topic was recently reviewed in [32].

3. String-theory and dark energy

The standard, Kaluza–Klein, way to get a theory with gravity in four dimensions from one that is initially formulated in a higher dimension, such as superstring theory, is to look for a solution for which the 'extra' dimensions are compact. For IIB superstring theory, we must compactify six dimensions. In the remaining three space dimensions, and at distinct points in the compact space, we may place stacks of D3-branes. More generally, we may have stacks of D$(3+2k)$-branes wrapped on $2k$-cycles of the compact manifold for $k=0,1,2$. In practice, the compact space is chosen to be a Calabi–Yau manifold, which yields an effective $N=2$ four-dimensional supergravity, and the $2k$-cycles for $k>0$ are chosen to be calibrated surfaces, breaking $N=2$ supersymmetry to $N=1$. In this way, we might hope to have a physical 3-space filled with stacks of D-branes, some wrapped over cycles of the compact manifold, with each stack separated by some distance in the compact manifold. However, the total Dp-brane charge (for each p) must be zero unless there are singularities in the compact space that can act as sinks for the $(p+2)$-form electromagnetic-type fields sourced by the Dp-branes.

In string theory, there are allowed singularities of this type, orientifolds, which cancel both the D-brane charge *and* the energy density, allowing a compactification to four-dimensional

Minkowski space. In this way, one can 'engineer' quasi-realistic $N=1$ supersymmetric extensions of the standard model of particle physics, as reviewed in [33], with the particles arising as massless modes of open IIB superstrings with ends on D-branes. In the four-dimensional effective theory, valid on length scales much greater than ℓ_s, the fields for these particles are coupled to $N=1$ supergravity, but the graviton and gravitino are massless modes of closed IIB superstrings that propagate freely in the full 10-dimensional space–time as particles of (quantum) size ℓ_s. They interact weakly with the particles on the branes because ℓ_s is much greater (for $g_s \ll 1$) than the width of any of the D-branes.

The stability of these constructions depends partly on the assumption of preservation of $N=1$ four-dimensional supersymmetry, which allows the inter-brane forces to cancel, and on fluxes in the compact space that allow a stabilization of moduli (parameters of the compact space, such as its scale). We still have an interpretation of particles as quanta associated with fluctuations of branes, which collectively serve as an aether, but there is no longer any connection to the cosmological constant. This resolves the discrepancy with atomic physics experiments noted earlier, but it also leaves us without dark energy; to incorporate it we need some supersymmetry-breaking modification, which is needed anyway to arrive at the non-supersymmetric standard model of particle physics.

One possibility that maintains a connection with branes is to add anti-branes; we should then expect instabilities although an influential 2003 model incorporating anti-D3-branes was argued to have a (metastable) de Sitter vacuum [34,35]; this is the late time universe to which our Universe must approach if the cosmological constant is really constant, as it is within observational bounds [36]. The de Sitter universe may be viewed as a flat FLRW space–time with an exponential scale factor

$$ds^2 = -dt^2 + S^2\, d\mathbf{x}\cdot d\mathbf{x}, \quad S = e^{Ht},\ H = \sqrt{3c^2\lambda}. \tag{3.1}$$

This universe is expanding at a rate determined by the 'Hubble constant' $H = \dot{S}/S$, which really is constant in this case. The expansion is accelerating at a rate determined by $\ddot{S}/S$, which is the constant H^2 in this case. The accelerated expansion dilutes all but the dark energy density and smoothes out any inhomogeneities, so the de Sitter universe is a late-time 'attractor' solution of the Einstein field equations.

However, there is mounting evidence, e.g. [37–39], that all 'de Sitter compactifications' are part of the swampland. This can be seen as a conjectural generalization of an earlier no-go theorem that rules out non-singular de Sitter compactification solutions of the (higher-dimensional) Einstein field equations if the compact space (and warp factor in the case of 'warped' compactifications) are time-independent *and* the stress-energy tensor satisfies the strong-energy condition (SEC) [40]. In the context of the effective low-energy 10-dimensional or 11-dimensional supergravity theories of string/M-theory this result was rediscovered in [41], where it was also extended to cover one exceptional case for which the SEC does not hold (massive IIA). It should be appreciated that the SEC is not required by basic physical principles; its significance comes from the fact that it is satisfied by the effective low-energy 10-dimensional or 11-dimensional supergravity theories of string/M-theory. More recently, it was shown that *time-dependent* compactifications to the de Sitter universe on non-singular compact spaces can also be ruled out if the higher-dimensional stress-energy tensor satisfies both the SEC and the null energy condition (NEC) [42] (although the dominant energy condition (DEC) was stated as a premise, only the weaker NEC was actually used in the proof). The tension between string/M-theory and the astronomical observations that indicate a constant dark-energy density is therefore far from having been resolved, despite two decades of effort.

From a purely four-dimensional perspective, the effect of 'extra' inaccessible dimensions is to provide a variety of matter fields and a potential V for any scalar fields associated with moduli (of the compact space or D-brane configurations) and relative positions of anti-branes (see e.g. [35,37]). The Friedmann equations found from those of GR by requiring space–time homogeneity and isotropy then yield the possible FLRW universes obtainable by the compactification considered. In this context, the absence of any de Sitter compactification is equivalent to the statement that V has no stationary points at which $V > 0$, which means that any dark-energy density must be time-dependent. This might be compatible with observations if the time-dependence is sufficiently slow, and this possibility motivates an exploration of compactifications to FLRW universes other than de Sitter for which there is an accelerated cosmic expansion, i.e. $\dot{S} > 0$ and $\ddot{S} > 0$. It is not difficult to find simple examples of string/M-theory compactifications on *time-dependent* compact spaces that lead to flat FLRW cosmologies with a short period of accelerated expansion, i.e. 'transient acceleration' (see e.g. [43–45]) but is *late-time* acceleration possible?

The answer to this question is determined by properties of the four-dimensional scalar potential V. An instructive, and much studied, example is a single scalar field σ with the energy density

$$\mathcal{E} = \frac{1}{2}\dot{\sigma}^2 + V(\sigma), \quad V(\sigma) = \Lambda\, \mathrm{e}^{-2\alpha\sigma}, \tag{3.2}$$

for positive constant α. We may restrict attention to flat universes since this is implied by late-time accelerated expansion, and for this simple case all flat FLRW universes can be found exactly [46] (for not too large α). Typical solutions correspond to motion up and then down the potential, and accelerated cosmic expansion occurs around the stationary point of this motion, when V is approximately constant [47]. There is then a transition to a late-time 'scaling' solution with

$$S \sim t^{\eta(\alpha)} \quad \text{and} \quad \sigma \sim \alpha^{-1} \ln t. \tag{3.3}$$

The late-time expansion is accelerating only if $\eta \geq 1$, which requires $\alpha \leq 1$. Although an exponential potential is special, a late time attractor solution yielding an accelerating FLRW universe for generic positive multi-scalar potential $V(\boldsymbol{\sigma})$ will be such that [48]

$$S \sim t^{\eta(\boldsymbol{\alpha})} \quad \text{and} \quad \lim_{t\to\infty}\left[\frac{\partial(\ln V)}{\partial \boldsymbol{\sigma}(t)}\right] = -2\boldsymbol{\alpha}, \quad |\boldsymbol{\alpha}| \leq 1. \tag{3.4}$$

There is no generally accepted string/M-theory compactification to an FLRW universe with late-time accelerating expansion, but it was recently shown that this is *not* ruled out by the premises of the no-go theorems summarized above; in particular, it is permitted by the higher-dimensional SEC and DEC combined [49]. This result was found by focusing on cosmological compactifications to FLRW space–times that could serve as late-time power-law attractor solutions, and then using the higher-dimensional Einstein field equations to determine the stress-energy tensor in terms of η and another exponent ξ controlling the change of scale of the compact space. The SEC limits the values of (η, ξ) to the interior of an ellipse, and the acceleration condition $\eta \geq 1$ further limits them to a chord segment of the ellipse; the DEC (and hence the NEC) is then automatically satisfied.

In all these cases, $\mathcal{E} \sim t^{-2}$ at late times, so the later the time the lower the dark energy density, and if t_0 is a late time then the further change in $\mathcal{E}$ over a period $\Delta t \ll t_0$ will be small. However, the four-dimensional stress-energy tensor at late times was found in [49] to be an ideal fluid with a pressure equal to $w\mathcal{E}$, where the constant w satisfies

$$-\frac{1}{2} < w < -\frac{1}{3}. \tag{3.5}$$

The upper bound is needed for accelerating expansion of the four-dimensional FLRW universe. The lower bound comes from the SEC that the D-dimensional stress-energy tensor is assumed to satisfy. By contrast, the constant dark energy implicit in a late-time de Sitter universe is an ideal fluid with $w = -1$ (negative pressure equal to the energy density). Observations are consistent with $w = -1$ but *not* with $w > -\frac{1}{2}$, so we need to re-examine the assumptions leading to (3.5).

String-theory introduces higher-derivative corrections to the Einstein field equations. These could be accommodated in the analysis of [49] by rewriting the corrected field equations as the uncorrected equations with a corrected stress-energy tensor, which might violate the SEC. However, if this *is* the effect of string-theory corrections then one of the major obstacles to finding a de Sitter compactification, and hence $w = -1$, is removed. But it is precisely the difficulties that have been encountered in trying to achieve this goal that have motivated the conjecture that de Sitter compactifications belong to the swampland. For this reason, it seems unlikely that the higher-derivative corrections of string theory can help. There remains the possibility that singular compactifications allowed by string-theory (but not supergravity) will allow $w < -\frac{1}{2}$, but it is unclear how this could be achieved. The idea that dark energy can emerge from string-theory as a by-product of cosmological compactifications to FLRW universes with an accelerated cosmic expansion powered by some ideal fluid with $w \lesssim 1$ has now run into difficulties, which go beyond those mentioned here; see e.g. [50].

Despite these difficulties, there is one result that may turn out to be significant. A problem with time-dependent compactifications is that an expanding four-dimensional FLRW universe typically requires an expanding compact space, which could lead to unacceptable time-dependence of parameters of the effective four-dimensional theory. In addition, eternal expansion would imply an ultimate decompactification: the Kaluza–Klein (non-zero) modes will become directly observable at some 'decompactification time' t_{decomp}. A remarkable, and surprising, corollary of the analysis of [49] is that accelerating expansion of the FLRW universe implies *decelerating* expansion of the compact space. This allows t_{decomp} to be easily much greater than the age of our Universe, and may reduce time-dependence of four-dimensional parameters to an acceptable level.

(a) Recurrent acceleration

So far, we have focused on the *late-time* fate of the Universe, mainly because a constant dark energy density implies that this will be a de Sitter universe. This is likely to be unrealizable by any cosmological compactification of string/M-theory, so it is natural to seek alternative possibilities that could be both realizable and compatible with observations. As noted above in the context of an effective four-dimensional theory with a positive potential $V(\boldsymbol{\sigma})$ without stationary points, $\mathcal{E}(t) \leq V_{\text{max}}$ for any given flat FLRW solution, where V_{max} is a maximum value of V that is reached when $\dot{\boldsymbol{\sigma}} = \mathbf{0}$. Around this time, $\mathcal{E}(t)$ is approximately constant, and the scale factor $S(t)$ grows exponentially, as in the de Sitter universe. However, this de Sitter-like phase is transient.

For the simple one-scalar model with exponential potential discussed earlier, there is only one phase of transient acceleration if $\alpha \equiv \frac{1}{2}|V'|/V > 1$. This is likely to remain true for arbitrary one-scalar positive potentials (unless $\frac{1}{2}|V'|/V < 1$ for some values of σ) because this is needed for late-time acceleration. In any case, there is no known *one-scalar* example found by cosmological compactification (and this fact is likely related to the de Sitter swampland conjecture that requires $|V'|/V > C$ for some (unknown) constant of order 1 [39]). However, for a simple dilaton-axion model with

$$\mathcal{E} = \frac{1}{2}(\dot{\sigma}^2 + \mathrm{e}^{-2\beta\sigma}\dot{\chi}^2) + \Lambda\,\mathrm{e}^{-2\alpha\sigma}, \tag{3.6}$$

there can be multiple phases of accelerating expansion when $\alpha > 1$ (and even $\alpha \gg 1$) if the dilaton-axion coupling constant β is sufficiently large; this is the phenomenon of 'recurrent acceleration' [51]. A large dilaton-axion coupling will rapidly convert dilaton kinetic energy into axion kinetic energy, which slows the descent down the exponential potential; the energy density is now decreasing more slowly, sufficiently to drive an accelerated expansion. But then the axion kinetic energy is reconverted into dilaton kinetic energy, which speeds up the descent down the potential; the energy density is now falling rapidly and the expansion is decelerating. After a few cycles the late-time (accelerating or decelerating) scaling solution is approached and $\mathcal{E} \sim t^{-2}$. The first phase of acceleration is de-Sitter-like because, momentarily $\dot{\sigma} = \dot{\chi} = 0$, whereas subsequent phases of acceleration are presumably closer to power-law accelerated expansion.

More generally, in a universe with recurrent acceleration, the duration of a first, de-Sitter-like, phase of accelerated expansion will be prolonged by the conversion of potential energy into kinetic energy of fields on which V has no (or lesser) dependence. As pointed out in [49], there is a simple mechanical analogy to a disc rolling down a hill; potential energy is converted into a combination of translational kinetic energy (which determines the rate of descent) and rotational kinetic energy (which has no effect on the rate of descent), so the larger the moment of inertia of the disc the more its descent will be slowed. Large moment of inertia in this mechanical model corresponds to large coupling constant β in the dilaton-axion model. Potentially, our Universe could be in a phase of 'suspended de-Sitter-like' expansion, which will later turn to decelerating expansion; in this case, the late-time future will remain open, and probably unknowable.

4. Summary and outlook

Dark energy, which is about 70% of the total energy of the Universe, is possibly the simplest form of energy. Astronomical observations suggest that its density is constant, in which case it can be identified as Einstein's cosmological constant, implying an evolution towards a de Sitter universe. An equivalent interpretation is as a space-filling shear-free tensile material with tension equal to its energy density, i.e. $T=\mathcal{E}$. It has been pointed out here that such a material is a candidate for an SR-compatible aether, and that branes provide a realization of this idea since their fluctuations propagate as light-speed waves, but only 'on the brane'. In the case of D-branes of superstring theory, these include electromagnetic waves.

This idea extends to all the non-gravitational forces in the context of constructions of supersymmetric particle-physics models by compactifications of IIB superstring theory with D-branes on Calabi–Yau manifolds, since the particles are then quantized fluctuations of (stacks of) D-branes, which are well-separated in the compact space and which interact weakly with supergravity fields. However, the D-brane energy density of the four-dimensional effective theory has now been cancelled by orientifold singularities of the compact space, and there is therefore no dark energy.

The addition of anti-branes can raise the energy density, breaking supersymmetry and potentially producing a constant dark-energy density, implying a late-time de Sitter universe. However, it now seems likely that string-theory scenarios for compactification to a de Sitter universe belong to the 'swampland'; this is a conjectured extension of earlier no-go theorems based on an improved understanding of how many attempts to evade them have failed. If correct, string-theory predicts a time-dependent dark energy density, and hence some alternative late-time universe.

All FLRW universes that undergo accelerated expansion can be late-time attractor solutions of the Einstein field equations. The constant energy density that drives the de Sitter expansion is an ideal fluid with pressure to energy-density ratio $w=-1$, which is equivalent to $T=\mathcal{E}$. For $w>-1$, equivalently $T<\mathcal{E}$, we get other 'scaling' solutions with a power-law expansion that is accelerating if $w<-\frac{1}{3}$. There is no known string-theory cosmological compactification to *any* FLRW universe with late-time accelerated expansion, but one may still ask whether the premises of the de Sitter no-go theorems exclude it. They do not exclude it, but they do impose the bound $w>-\frac{1}{2}$, which is not compatible with observations.

One remaining possibility exploits the fact that *transient* de Sitter-like acceleration is generic for any compactification that leads to an effective four-dimensional theory with a positive potential V. This is easily achieved; the no-go theorems require only that V has no stationary point for $V>0$, and the de Sitter swampland conjecture imposes bounds on the magnitude of the gradient of V. Cosmological evolution involves rolling up the potential to a maximum value, and then a rolling down; this implies a moment at which the total energy density $\mathcal{E}$ is stationary and this drives a transient phase of de Sitter-like expansion. Although this may be too short, generically, its duration can be extended in multi-scalar models that exhibit recurrent acceleration because the mechanism underlying this phenomenon is one that slows the fall of $\mathcal{E}$ from its maximum value. It remains to be seen whether this slowing is sufficient.

Whatever the resolution of tension between string theory and the observed accelerated expansion of our Universe, one lesson to be learned is surely that it would be naive to suppose of any cosmological compactification of string theory that the compact space is time-independent. However, its time-dependence could easily lead to an unacceptably large time-dependence of parameters of the effective four-dimensional theory and a possible decompactification. One feature of accelerated expansion is that it can freeze the time evolution of the compact space; in the analysis leading to the lower bound on w mentioned above, this was due to the fact that an accelerating expansion of the four-dimensional universe required a decelerating expansion of the compact space.

Dark energy may be simple but it is not so simply accommodated by string/M-theory, which in most other respects is still our best collection of ideas for a unified theory of particle physics with quantum gravity. It may be time to look for new ideas!

Data accessibility. This article has no additional data.

Conflict of interest declaration. I declare I have no competing interests.

Funding. This work was partially supported by STFC consolidated grant no. ST/P000681/1.

Acknowledgements. I am grateful to Eoin Colgáin and Jorge Russo or helpful communications and conversations.

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