Multi-field open inflation and instanton

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Abstract

In the open inflation scenario, phase transition from false vacuum inflation at a local minimum to slow-roll inflation is mediated by the quantum tunneling. In this work, we present a concrete multi-field open inflation model, explicitly showing how the bubble is nucleated by tunneling of scalar fields, and how slow-roll inflation in the nucleated bubble follows. This model is the first viable open inflation model with a simple potential. Multi-field tunneling is described by multi-field instanton method, which is an natural extension to the Coleman-De Luccia(CDL) instanton method. Evolution of the universe in the bubble obeys equations of motion for the open Friedmann-Lemaître-Robertson-Walker(FLRW) universe.

1 Introduction

Recently, quantum tunneling of a scalar field in inflationary era is keenly studied in the context of string landscape, which predicts many local minima in the potential for scalar fields, where scalar fields are stable classically but unstable due to quantum effects [1–3]. Quantum tunneling of a scalar field with gravity was first studied by Coleman and De Luccia[4, 5], and they found that the tunneling proceed via bubble nucleation, inside of which can be regarded as open FLRW universe because of the O(4)-symmetry of the instanton. In the case slow-roll inflation after the tunneling lasts for moderate number of e-foldings, say, about 50 or 60, the universe becomes almost flat at the end of the inflation but small cuvatrture remains, and $\Omega_{K,0} \sim 10^{-2} - 10^{-3}$ might be observed in future observation. In fact, Freivogel et al. suggested that this kind of situation is preferred in string landscape[6]. The inflation scenario in which the universe bears small negative curvature because of the tunneling before slow-roll inflation is often called open inflation[7, 8]. Since open infaltion can be seen as the outcome of string landscape, studies of open inflation might give us some indication about string landscape.

It is known that there has been a trouble in cnstructing a single-field open inflation model, that only a model with very artificial potential is possible[9]. This is because the condition for existence of a CDL instanton require the potential barrier to be steep, however, the condition for slow-roll inflation require flat potential tilt, and very artificial potential is needed in order to match these two oppsitely oriented condition at the same time.

However, let us consider a model with two scalar fields. One of which called "tunneling field" plays a major role in quantum tunneling, and the other called "inflaton" plays a major role in slow-roll inflation. In this case, it is expceted that open infaltion is realized with simple potential by choosing the potenial barrier for the tunneling field steep while the potential for the inflaton after the tunneling flat. Systems with many scalar fields in inflationary era are also motivated by string landscape.

In this study, we propose a concrete multi-field model with simple potential, and make sure that open inflation scenario is indeed realized by explictly studying the evolution, which includes quantum tunneling and slow-roll inflation after the tunneling. This multi-field open inflation model is the first open inflation model with simple potenial.

2 Scenario and Model

The brief scenario of our multi-field open inlflation model is as follows. In the begining, two scalar fields are trapped at a false vacuum for a long time. Then, quantum tunneling happens where the tunneling

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field play a main role. On the other side of the potential barrier, the inflaton starts to roll slowly due to the coupling between the two scalar fields, and causes slow-roll inflation. The slow-inflation continues until the inflaton reach the true vacuum where reheating occurs in the usual manner. The schematic picture of this scenario is drawn in the Fig. 1.

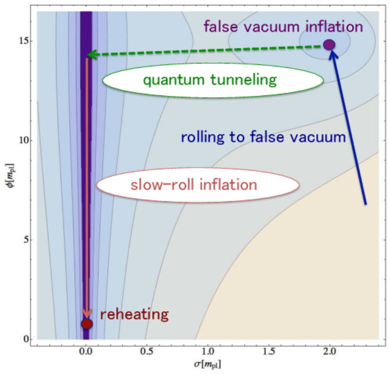

Figure 1: The schematic picture of our multi-field open inflation scenario. The color corresponds to the potential: lighter region is higher and darker region is lower. The vertical axis is inflaton's field value, and the horizontal axis is tunneling field's field value.

To realize this multi-field open inflation scenario, we propose a concrete model with a simple potential,

$$V(\sigma,\phi) = \alpha\sigma^2\left\{(\sigma-\sigma_0)^2 + M_V^2\right\} + \left(\frac{1}{2}m_\phi^2\phi^2 + \frac{\beta}{2}\sigma^2(\phi-\phi_0)^2\right). \tag{2.1}$$

Here, σ is the tunneling field and ϕ is the inflaton, first part of the potential is important in tunneling, and second part makes it possible for ϕ to slow-roll only after the tunneling by the coupling to σ. We take the parameters as $\alpha = 0.1$, $\sigma_0 = 2m_{pl}$, $M_V = 0.2m_{pl}$, $\beta = 0.1$, $m_\phi = 10^{-6}m_{pl}$, $\phi_0 = 15m_{pl}$. The false vacuum is located near (σ_0, ϕ_0), and the true vacuum is located at $(0, 0)$.

3 Bubble Nucleation and Multi-filed Instanton

In this section, we study the quantum tunneling of scalar fields via bubble nucleation. Multi-field tunneling with gravity is expected to be described with multi-field instanton method, which is a natural extension to the CDL instanton method[10]. In the multi-field instanton method, we solve equations of motion for an instanton,

$$\frac{\bar{a}''}{\bar{a}} + \frac{1}{3m_{pl}^2}\left(\bar{\sigma}'^2 + \bar{\phi}'^2 + V(\bar{\sigma},\bar{\phi})\right) = 0\,,$$
$$\bar{\sigma}'' + 3\frac{\bar{a}'}{\bar{a}}\bar{\sigma}' - V_{\bar{\sigma}}(\bar{\sigma},\bar{\phi}) = 0\,, \qquad \bar{\phi}'' + 3\frac{\bar{a}'}{\bar{a}}\bar{\phi}' - V_{\bar{\phi}}(\bar{\sigma},\bar{\phi}) = 0\,, \tag{3.1}$$

with boundary condition,

$$\bar{a}(0) = \bar{a}(\tau_{\rm end}) = 0\,,\quad \bar{a}'(0) = -\bar{a}'(\tau_{\rm end}) = 1\,,\quad \bar{\sigma}'(0) = \bar{\sigma}'(\tau_{\rm end}) = 0\,,\quad \bar{\phi}'(0) = \bar{\phi}'(\tau_{\rm end}) = 0\,. \tag{3.2}$$

Here, τ is imaginary time and $'$ is derivative with respect to τ, $\bar{a}$ is an instanton for the scale factor of the universe in Euclidean spacetime, and $\bar{\phi}$ and $\bar{\sigma}$ are instantons for ϕ and σ, respectively. It is known from the CDL argument[5] that the action for the instanton gives the tunneling rate and that the inside of the bubble is written as an open FLRW universe which statrts with scalar fields whose values are given by the instanton's value at $\tau = 0$.

We nemerically find the solution which satisfies these equations with the potential given by Eq. (2.1), and the result is ploted in Fig. 2.

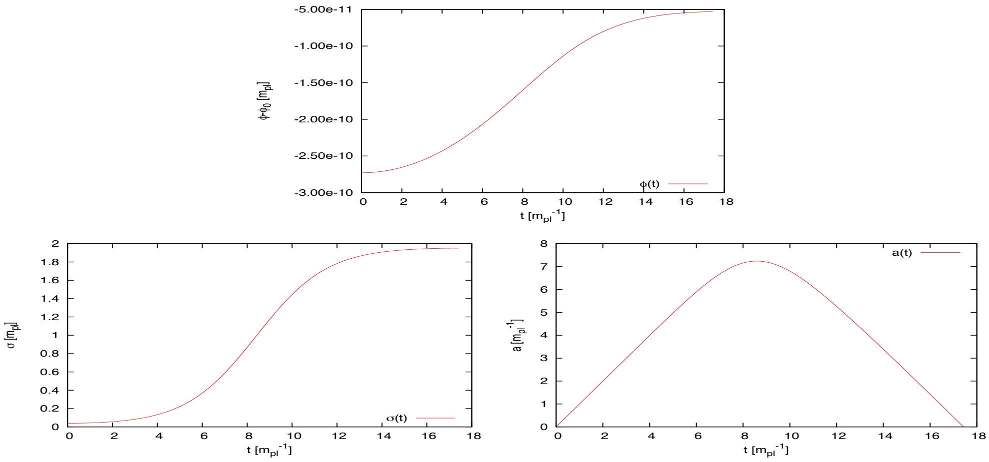

Figure 2: Multi-field instanton for the potenial give by Eq. (2.1)[10].

4 Slow-roll Inflation Inside the Bubble

In this section, we study the evolution of the univese inside the bubble. The evolution equation are those for a open FLRW universe,

$$\frac{\ddot{a}}{a}+\frac{1}{3m_{pl}^2}\left(\dot{\sigma}^2+\dot{\phi}^2-V(\sigma,\phi)\right)=0\,,$$

$$\ddot{\sigma}+3\frac{\dot{a}}{a}\dot{\sigma}+V_\sigma(\sigma,\phi)=0\,,\qquad \ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}+V_\phi(\sigma,\phi)=0\,, \tag{4.1}$$

where $\dot{}$ denotes a derivative with respect to t and a is the scale factor of the open FLRW universe. The initial conditions at $t=0$ are given from the multi-field instanton method as $a(0)=0$, $\sigma(0)=\bar{\sigma}(0)$, $\phi(0)=\bar{\phi}(0)$, as we mentioned before.

We calculated these equations numerically with the potential given by Eq. (2.1), and the result is plotted in Fig. 3. Contribution to the Hubble square H^2, or the energy density of the univese, from each component are plotted in Fig. 4, and it is apparent that the inflaton indeed causes slow-roll inflation.

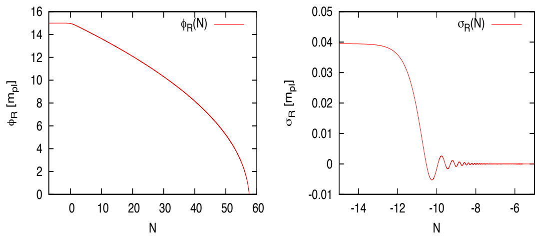

Figure 3: The evolution of the scalr fields in the nucleated bubble[10]. left:evolution of the inflaton. right:evolution of the tunneling field. The inflaton slowly rolls down the potential for about 60 numbers of e-foldings, from the nucleation point, $\phi\sim 15m_{pl}$, to the bottom of the potential, $\phi=0$. The tunneling field soon reaches its potential minimum and it doesn't play a role in the evolution of the universe since then.

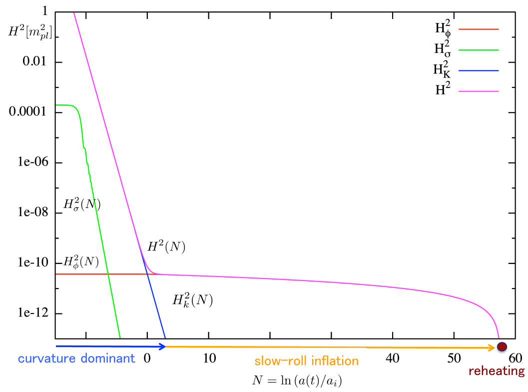

Figure 4: The contribution to Hubble square H^2 from each component, H_ϕ^2, H_σ^2, H_k^2 which are from infalton, tunneling field, and curvature of the universe, respcetively. Curvature dominance after the bubble nucleation soon comes to an end, and then slow-roll inflation by the inflaton follows.

5 Conclusion

We proposed a multi-field open inflation in Sec. 2 which is expected to realize multi-field open inflation scenario. The tunneling via bubble nucleation and the slow-roll inflation inside the bubble is explicitly calculated in Sec. 3 and Sec. 4, respectively. In Sec. 4, we can see that slow-roll inflation indeed occurs in the bubble, and now we are sure that open inflation scenario is indeed realized in our model of multi-field open inflation model. In this model, we assume two scalar fields, one is a tunneling field the other is an inflaton, which plays a major role in tunneling and slow-roll inflation, respectively. Since the potenial for these two scalar fields has simple form, it can be said that we succeed in constructing a open inflation model with simple potential for the first time.

Now, we have a concrete model for the multi-field open inflation and know how the evloution of the universe proceeds in this model. Thus, it is possible to calculate some observable quantity in our model like the powerspecturm for the curvature perturbation, which is already known for a single field model[11, 12].

It is also interesting to study the bispecturm for the curvature perturbation in open inflation model, which is now we are working for, since the bispectrum is now getting available thanks to the development of the technology [13, 14]. A trace of open inflation might be found in the bispectrum, and as we mentioned in introduction, it might be a trace of string landscape.

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