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CHIRAL SYMMETRY BREAKING AND THE QUARK MODEL: UNIFICATION OF BARYON AND MESON CONSTRAINTS*

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I. INTRODUCTION

A decade after the introduction of the notion of quarks [1], the quark model continues to provide remarkable patterns and insights into almost every aspect of elementary particle physics; it also serves as the foundation of such symmetries as SU_3 (perhaps now SU_4) and chiral symmetry. Aside from the fundamental commutation relations, however, the initial development of the theory of chiral symmetry breaking [2,3] was independent of quark model considerations. In particular, the attractive $(3,\bar{3})$ chiral breaking scheme proposed by Gell-Mann, Oakes, and Renner [3] (GMOR) predicts reasonably small chiral breaking effects - in fact smaller than present phenomenology suggests in some cases [4,5]. These predictions are based upon simple SU_3 assumptions of certain chiral breaking meson matrix elements which appear to have no direct relation to the quark model itself.

In order to refine the $(3,\bar{3})$ chiral breaking model, which certainly gives a better qualitative account of the data than do other $SU_3 \times SU_3$ breaking representations, we recently suggested [6] returning to the quark picture and incorporating the scaling (quark-parton) structure of the deep inelastic scattering of baryons in a manner closely related to that suggested by Jaffe and Llewellyn-Smith [7]. A consistent pattern emerges for many baryon processes such as for chiral breaking effects in pion photoproduction off nucleons [6]. In this paper we shall expand in detail upon the chiral breaking baryon matrix elements in our scheme and extend the approach to include chiral breaking meson matrix elements and electromagnetic (isospin breaking) effects as well.

The distinction between our approach and that of GMOR is our use of the light cone transformation properties [7,8] for the chiral breaking "bad" quark operators [9]. The rationale for this procedure is the assumption of the "goodness" of the light plane (or infinite momentum frame) SU_3 and $SU_{6,W}$ charges [10], under which the hadron states are presumed to transform irreducibly. Alternative justification for our chiral breaking scheme comes from the behavior of "fixed poles" in Compton-like processes [7,11-13]. Such a unified picture of chiral symmetry breaking points [6] to a non-strange quark mass $\hat{m}$ of size $\hat{m} \sim m_\pi$ (derivable from baryon matrix elements) and a quark mass ratio of $m_s/\hat{m} \sim 5$-6 (extracted from both baryon and meson matrix elements). These values in turn imply that the partially conserved axial-current hypothesis (PCAC) for pions and kaons is on a somewhat different footing. Pion PCAC is found always to hold provided it is treated in a dispersion-theoretic sense [9] (neutral PCAC) rather than in an operator sense (strong PCAC) since the chiral breaking parameter c turns out to be approximately -0.8 to -0.9 rather than near the chiral $SU_2 \times SU_2$ limit of $-\sqrt{2}$ as in the GMOR case.

We begin this analysis by first reviewing in Sec. II the general $(3,\bar{3})$ theory of chiral symmetry breaking, the quark model, the light cone and Melosh transformations, scaling, fixed poles, and quark probability distribution integrals. In Sec. III we investigate all chiral symmetry breaking baryon matrix elements, including those in baryon mass formulae, baryon σ terms, threshold pion photoproduction, and Goldberger-Treiman discrepancies. Meson matrix elements are analyzed in a similar manner in Sec. IV, including a survey of PCAC constraints, $K_{\ell 3}$ and $\pi^0 \to 2\gamma$ decay; a description of the ninth pseudoscalar meson state is also given. In Sec. V, an isospin violating term is introduced in the quark mass matrix and its effect is explored in electromagnetic mass differences and $\eta_{3\pi}$ decay for our chiral breaking scheme. Throughout this paper we contrast our results with those obtained in the GMOR chiral breaking approach.

II. THEORETICAL FOUNDATIONS

A. Chiral Symmetry Breaking

The theory of chiral symmetry as embodied in the chiral symmetric Hamiltonian density H_0, where

$$[Q_i, H_0] = 0 \qquad [Q_i^5, H_0] = 0 \tag{2.1}$$

($i = 1, \ldots, 8$), has stood up to a decade of analysis of its underlying charge algebra coupled with its implied approximate axial current conservation (pion PCAC). In recent years, therefore, it has become of interest to link together chiral symmetry <u>breaking</u> with the more conventional SU_3 (Gell-Mann-Okubo) <u>breaking</u> via the (semi-strong) Hamiltonian density

$$H = H_0 + H' , \tag{2.2}$$

where H' does not commute with the vector and axial-vector charges

$$Q_i = \int d^3x\, V_i^o(\vec{x}, 0) , \quad Q_i^5 = \int d^3x\, A_i^o(\vec{x}, 0) . \tag{2.3}$$

The hope is to find H' such that it is simultaneously "small" compared to H_0 in the SU_3 sense and in the $SU_2 \times SU_2$ sense. To this end, it was proposed [2,3] that H' transform according to the $(3,\bar{3}) + (\bar{3},3)$ representation of $SU_3 \times SU_3$ (henceforth denoted as $(3,\bar{3})$)

$$H' = \epsilon_0 u_0 + \epsilon_8 u_8 \sim u_0 + c u_8 , \tag{2.4}$$

where the scalar densities u_i are related to the pseudoscalar densities v_i via the $(3,\bar{3})$ commutation relations

$$[Q_i, u_j] = i f_{ijk} u_k \qquad [Q_i, v_j] = i f_{ijk} v_k \tag{2.5a}$$

$$[Q_i^5, u_j] = -i d_{ijk} v_k \qquad [Q_i^5, v_j] = i d_{ijk} u_k . \tag{2.5b}$$

This $(3,\bar{3})$ representation is most compelling for four reasons:

i) The quark model can be incorporated in this scheme because the quark mass matrix term $\bar{q}\mathcal{M}q$ transforms according to the $(3,\bar{3})$ representation. We shall discuss this possibility in greater detail shortly.

ii) The pion mass and divergence of the axial-vector current become related in a simple manner (with $f_\pi \approx 93$ MeV),

$$\partial\cdot A_\pi = m_\pi^2 f_\pi \phi_\pi = -(\epsilon_0/\sqrt{3})(\sqrt{2}+c)v_\pi \ , \tag{2.6}$$

so that $c = -\sqrt{2}$ corresponds to the chiral $SU_2 \times SU_2$ limit $m_\pi = 0$.

iii) The chiral-broken value of c becomes linked to the chiral-broken meson mass ratio $m_\pi^2/m_K^2 \simeq 1/13$ by comparing (2.6) and its kaon analog for vacuum to pseudoscalar matrix elements (taking $f_K = f_\pi \neq 0$):

$$\frac{m_\pi^2}{m_K^2} = \left(\frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c}\right)\frac{\langle 0|v_\pi|\pi\rangle}{\langle 0|v_K|K\rangle} \ . \tag{2.7}$$

iv) The pion matrix element of the $\pi\pi\sigma$ term, defined as

$$\sigma^{\pi\pi} \equiv [Q_\pi^5, i\partial\cdot A_\pi] = (\epsilon_0/3)(\sqrt{2}+c)(\sqrt{2}u_0+u_8) \tag{2.8}$$

now appears to be experimentally [14] consistent with Weinberg's low energy $\pi\pi$ analysis (following from (2.8) coupled with pion PCAC), corresponding to the $(3,\bar{3})$ value [15]

$$\langle\pi_i|\sigma^{\pi\pi}|\pi_j\rangle = \sigma_{\pi\pi}^{\pi\pi}\delta_{ij} = m_\pi^2\delta_{ij} \tag{2.9}$$

and independent of the value of c.

In order to maximize the predictive power of the theory, it is necessary to make a further assumption as to the specific SU_3 transformation properties of the pseudoscalar matrix elements of the densities u_i and v_i, so far unspecified in (2.5) - (2.9). Needless to say, GMOR made the simplest ansatz that the matrix elements of the u_i and v_i have the simple SU_3 structure (which we shall refer to as the "GMOR SU_3 assumption")

$$<P_i|u_j|P_k> \sim d_{ijk} \qquad <0|v_j|P_k> \sim \delta_{jk} \tag{2.10}$$

for $i,j = 1,\ldots,8$ (pion PCAC implies that $j = 0$ can be included in (2.10) as well). This assumption follows from the SU_3 transformation law $Q_i|P_j> = if_{ijk}|P_k>$ and (2.5a) applied to $<P_i|[Q_j,u_k]|P_m>$; i.e., if the hadronic states are assumed to transform irreducibly under the static (spacelike) charges defined by (2.3) - a proposition most likely untrue [10] as we shall discuss in Sec. IIB. Given this GMOR SU_3 assumption (2.10), one concludes that

$$<0|v_\pi|\pi>/<0|v_K|K>\big|_{GMOR} = 1 , \tag{2.11}$$

and consequently (2.7) then demands that

$$c_{GMOR} = -\sqrt{2}\left(\frac{m_K^2-m_\pi^2}{m_K^2+\frac{1}{2}m_\pi^2}\right) \approx -1.25 . \tag{2.12}$$

Since c is then near the chiral $SU_2 \times SU_2$ limit $-\sqrt{2}$, pion PCAC becomes almost exact in an operational sense, and a chiral perturbation theory in the strong interaction parameters ϵ_0 and ϵ_8 becomes feasible [16,17].

Unfortunately, however, all $\pi\pi\,\sigma$ terms are also then forced to be small, being proportional to $\sqrt{2}+c$ in (2.8), and this is not always borne out by experiment. Of particular significance is the present phenomenological value of the $\pi N\,\sigma$ term [18],

$$\sigma_{NN}^{\pi\pi} = <N|\sigma^{\pi\pi}|N> \approx 65 \pm 5 \text{ MeV} . \tag{2.13}$$

In the GMOR scheme, $\epsilon_8(u_8)_N/2m_N$ transforms like $(\lambda_8)_N$ and must therefore correspond to the nucleon-SU_3 baryon-mass difference of -210 MeV. Further, the ratio $(u_0/u_8)_N$ ought to be near unity [4] (or smaller), for otherwise

$SU_3 \times SU_3$ breaking is not of the order of SU_3 breaking for the baryons and higher order terms such as H'^2 or H'^3 would then have to be included in the strong interaction expansion of $\langle N|H|N\rangle$. Moreover, the phenomenological value of the isoscalar direct channel KNσ term, $\sigma(I_s=0) \approx 0$, reconfirms $(u_0/u_8)_N \approx 1$ in the GMOR scheme [5,19] (we will return to this point later). Consequently the GMOR SU_3 assumption leads to the prediction [4]

$$\sigma_{NN}^{\pi\pi}(\text{GMOR}) = (\epsilon_0/3)(\sqrt{2}+c)(\sqrt{2}u_0+u_8)_N\,|_{SU_3} \approx 20 \text{ MeV}\ , \tag{2.14}$$

substantially smaller than (2.13). We take this fact as a reasonable justification for questioning the GMOR SU_3 assumption, but still prefer to work within a $(3,\bar{3})$ formulation for the reasons (i) to (iv) already stated.

B. Quark Model, Light Cone, and Melosh Transformations

The simplest description of quark dynamics is given by the free Lagrangian density [20]

$$\mathscr{L} = \bar{q}(i\partial\!\!\!/ - \mathscr{M})q \tag{2.15}$$

where the quark field q is the SU_3 triplet of up, down, and strange fields and $\mathscr{M}$ is the quark mass matrix, diag. (m_u, m_d, m_s). Alternatively, one might suppose the quarks interact via vector gluons described by the quark Lagrangian

$$\mathscr{L} = \bar{q}(i\partial\!\!\!/ - gB\!\!\!/ - \mathscr{M})q\ . \tag{2.16}$$

In either case the chiral decomposition $H = H_0 + H'$ obeys (2.1) with a chiral breaking part given by

$$H' = \bar{q}\mathscr{M}q = \hat{m}(\bar{u}u + \bar{d}d) + m_s\bar{s}s \tag{2.17}$$

where $m_u = m_d = \hat{m}$ in the SU_2 conserving limit.

In this quark language, the currents are $V_i^\mu = \frac{1}{2}\bar{q}\gamma^\mu\lambda_i q$ and $A_i^\mu = \frac{1}{2}i\bar{q}\gamma^\mu\gamma_5\lambda_i q$, and the scalar and pseudoscalar densities are

$$u_i = \bar{q}\lambda_i q \qquad\qquad v_i = \bar{q}\lambda_i\gamma_5 q\ . \tag{2.18}$$

They satisfy the equal-time commutation relations (2.5) with, for example,

$$u_0 = \sqrt{\tfrac{2}{3}}\,(\bar{u}u + \bar{d}d + \bar{s}s) \tag{2.19a}$$

$$u_8 = \sqrt{\tfrac{1}{3}}\,(\bar{u}u + \bar{d}d - 2\bar{s}s)\,. \tag{2.19b}$$

It is then possible to write (2.17) in the form $\epsilon_0 u_0 + \epsilon_8 u_8$ provided

$$\epsilon_0 = \sqrt{\tfrac{1}{6}}\,(m_s + 2\hat{m}) \tag{2.20a}$$

$$\epsilon_8 = -\sqrt{\tfrac{1}{3}}\,(m_s - \hat{m})\,. \tag{2.20b}$$

Evidently, it is possible to express the chiral breaking parameter $c = \epsilon_8/\epsilon_0$ in terms of the quark mass ratio

$$c = -\sqrt{2}\left(\frac{X-1}{X+2}\right) \tag{2.21}$$

where

$$X \equiv m_s/\hat{m}. \tag{2.22}$$

While the $SU_3 \times SU_3$ chiral limit corresponds to $m_s = \hat{m} = 0$ and the SU_3 limit to $m_s = \hat{m}$ or $X = 1$, the chiral $SU_2 \times SU_2$ limit means $m_s >> \hat{m}$ and $X \to \infty$. For the GMOR value of $c \approx -1.25$, (2.21) implies $X_{GMOR} \approx 25$.

It is also natural to discuss the light plane representation of the quark fields [8,9,21]. For $x^+ = \frac{1}{\sqrt{2}}(x^0+x^3) = 0$, the quark field $q_+ = \frac{1}{2}\gamma_+\gamma_- q$ is determined by the light plane generators but the field $q_- = \frac{1}{2}\gamma_-\gamma_+ q$ is dynamically related to q_+ via the field equations corresponding to the Lagrangian densities (2.15) or (2.16). Choosing a representation for the γ matrices which diagonalizes the two quark fields $q_\pm$,

$$q_+ = \begin{pmatrix}\phi\\0\end{pmatrix} \qquad q_- = \begin{pmatrix}0\\ \chi\end{pmatrix} \tag{2.23}$$

where $q = q_+ + q_-$, the two component dynamical constraint equation for the quark-gluon Lagrangian (2.16) is [9,21]

$$\chi(x) \sim \vec{\sigma}_\perp \cdot (\vec{\nabla}_\perp \nabla_-^{-1} + ig\nabla_-^{-1}\vec{B}_\perp)\phi(x) + \mathcal{M}\nabla_-^{-1}\phi(x) , \tag{2.24}$$

where the $B_\perp$ term in (2.24) is absent for the free quark Lagrangian (2.15). It is then possible to express various chiral symmetry quark field operators on the light plane as, for example, the "good" operators

$$\bar{q}\gamma^+ q \sim \phi^\dagger \phi \tag{2.25a}$$

$$\bar{q}\gamma^+\gamma_5 q \sim \phi^\dagger \sigma_3 \phi \tag{2.25b}$$

and the "bad" operators

$$\bar{q}q \sim \phi^\dagger \chi - \chi^\dagger \phi \tag{2.26a}$$

$$\bar{q}\gamma_5 q \sim \phi^\dagger \sigma_3 \chi + \chi^\dagger \sigma_3 \phi \tag{2.26b}$$

$$\bar{q}\sigma_\perp q \sim \phi^\dagger \sigma_\perp \chi + \chi^\dagger \sigma_\perp \phi . \tag{2.26c}$$

The light plane charges Q^L and $Q^{5,L}$ can be expressed in terms of the good quark operators (2.25),

$$Q_i^L = \int d^4x\, \delta(x^+) V_i^+(x) \sim \int dx^- d^2x^\perp\, \phi^\dagger(x)\lambda_i \phi(x) \tag{2.27a}$$

$$Q_i^{5,L} = \int d^4x\, \delta(x^+) A_i^+(x) \sim \int dx^-\, d^2x^\perp \phi^\dagger(x)\lambda_i\sigma_3\phi(x) , \tag{2.27b}$$

but the scalar and pseudoscalar densities, appearing in chiral symmetry breaking theories, must be expressed in terms of the bad quark operators of (2.26), such as

$$u_i \sim \phi^\dagger \lambda_i \vec{\sigma}_\perp \cdot (\vec{\nabla}_\perp \nabla_-^{-1} + ig\nabla_-^{-1}\vec{B}_\perp)\phi + \phi^\dagger \lambda_i \mathcal{M}\nabla_-^{-1}\phi + \text{h.c.} \tag{2.28a}$$

$$v_i \sim \phi^\dagger \lambda_i \sigma_3 \vec{\sigma}_\perp \cdot (\vec{\nabla}_\perp \nabla_-^{-1} + ig\nabla_-^{-1}\vec{B}_\perp)\phi + \phi^\dagger \lambda_i \mathcal{M}\sigma_3 \nabla_-^{-1}\phi + \text{h.c.} \tag{2.28b}$$

As to the type of commutation relations for the bad operators u_i and v_i, there are two cases:

(i) For commutation relations involving the static (spacelike) charges Q_i and Q_i^5, the usual equal-time commutators (2.5) hold.

(ii) For commutation relations involving the light plane charges Q_i^L and $Q_i^{5,L}$, one must employ the fundamental light plane commutators obtained from (2.27)

$$[Q_i^L, \phi] = -\tfrac{1}{2}\lambda_i\phi \qquad [Q_i^{5,L}, \phi] = -\tfrac{1}{2}\lambda_i\sigma_3\phi \ , \tag{2.29}$$

along with the light plane expansions (2.28).

Thus, commutators of the type $[Q_i^L, u_j]$ reveal that the first two terms of (2.28a) and (2.28b) transform as a normal SU_3 octet plus singlet. Because of the additional quark mass matrix $\mathcal{M}$, however, the third terms of (2.28a) and (2.28b) transform as octet plus singlet only when weighted by suitable quark mass factors u_i/m_q, where the $i = 0,8$ components are mixed. Commutators of the type $[Q_i^{5,L}, u_j]$ indicate that the first two terms of (2.28a) and (2.28b) have the usual $(3,\bar{3})$ d-type commutation relations analogous to (2.5b) since $\{\sigma_\perp, \sigma_3\} = 0$; the third terms of (2.28a) and (2.28b), however, when weighted as u_i/m_q, are found to obey (1,8) f-type commutation relations since $[1, \sigma_3] = 0$.

While the quark density transformation properties under the light plane charges Q_i^L, $Q_i^{5,L}$ are easily obtained, as above, knowledge of the transformation properties of the hadron expectation values of these densities requires further input. It is well known that, even in free quark theories, $Q_i = Q_i^L$ only for theories of unbroken SU_3 symmetry [22] and that they are not equal [22,23] when $\hat{m} \neq m_s$. However, at least in free quark theories, even with $\hat{m} \neq m_s$ the hadron states continue to transform irreducibly under the light plane charges [23], i.e.,

$$Q_i^L |P_j\rangle = if_{ijk} |P_k\rangle \tag{2.30}$$

Indeed, in the free quark case, the full SU_6 hadron multiplet continues to transform irreducibly under the SU_6 generated by the light plane charges Q_i^L, $Q_i^{5,L}$, and the other 18 similar generators, when $\hat{m} \neq m_s$. In the real world, however, the analytic structure of current matrix elements makes it impossible [23] to identify exactly the Q_i^L with the operators (let us call them W_i, W_i^5, etc.) which in this real world classify the hadron states ($Q_i^L = W_i$ only for $i = 1$, 2, 3, and 8). However, this identification is clearly worst for the generators other than Q_i^L which relate different SU_3 multiplets within a given SU_6 multiplet (equivalently, SU_6 breaking is stronger than SU_3 breaking). Thus we shall continue to assume that light plane quark distributions as measured in deep inelastic scattering, and their integrals, for hadrons in a given SU_3 multiplet transform irreducibly (as represented in (2.30)) under the Q_i^L. In particular, for consistency of our approach, this symmetry of the distribution integrals under the Q_i^L should be distinctly better than SU_3 symmetry for hadron masses; to repeat, this has been shown to be the case in free quark models with $\hat{m} \neq m_s$.

The above distinctions become important for matrix elements of the bad operators u_i and v_i. Thus the GMOR SU_3 assumption (2.10) is invalidated if (2.30) is true and the third terms in (2.28a,b) dominate the matrix element; on the other hand, the GMOR SU_3 would be approximately valid if the first two terms in (2.28a,b) dominate the matrix element. We shall see it is in fact likely that the third terms in (2.28a,b) dominate most matrix elements of the u_i or v_i. Indeed, with relatively reasonable assumptions, this can be proven, as we show in a following section. In this case the SU_3 structure of the u_i's is effectively altered so that (u_1, u_2, u_3), $[\frac{1}{2}(1+X)]^{-1}(u_4, u_5, u_6, u_7)$, $3^{-\frac{1}{2}}(\sqrt{2}\, u_0 + u_8)$, and $3^{-\frac{1}{2}}X^{-1}(u_0 - \sqrt{2}\, u_8)$, and not the u_i themselves, transform as the SU_3 octet plus singlet. A similar observation holds for the v_i [24].

The distinction between Q_i^5 and $Q_i^{5,L}$ is also of significance. The dynamical PCAC assumption for pions naturally leads to commutators involving Q_π^5 as in (2.5b) and not $Q_\pi^{5,L}$ which annihilates the vacuum [22]. Since SU_3 is broken ($m_s \neq \hat{m}$), $Q_i \neq Q_i^L$, and hence (2.30) indicates that if pion PCAC is always good, kaon (and eta) PCAC may be, at times, bad. We shall return to this point again in Sec. IVB.

Finally, since the light plane states do not have simple angular momentum properties [10,25], it has proved useful to search for a unitary transformation which relates the W-spin generators of the constituent quark states, $SU(6)_{W,\text{strong}}$, to the $SU(6)_W$ light plane charges associated with the quark currents. For noninteracting quark states, Melosh obtained the unitary transformation [25]

$$V_{\text{free}} = \exp\left\{\frac{i}{2}\int d^4x\, \delta(x^+)\phi^\dagger(x)\tan^{-1}(\vec{\gamma}_\perp \cdot \vec{\nabla}_\perp/\kappa)\phi(x)\right\} \qquad (2.31)$$

where κ represents a quark-hadron mass scale. This unitary transformation has been successfully applied to (chiral symmetric) hadron decays [26] and to other hadronic transitions [27]. It has also been suggested as a tool to analyze the chiral symmetry breaking properties of H' [28].

C. Scaling, Fixed Poles, and Quark Probability Distribution Integrals

As will be discussed in the next section, the baryon matrix elements of the bad operators u_i are related to regulated integrals of the structure functions found in electron and neutrino deep inelastic scattering. Alternatively, the matrix elements of u_i can be related to the $\alpha = 0$ fixed poles found in forward current-baryon scattering. We therefore review the interrelationships of these amplitudes and fixed poles in the present section.

The conventionally defined structure amplitude for deep inelastic scattering off a spin-averaged nucleon target is [29]

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4y\, e^{iq\cdot y} \langle p|[J_\mu(y), J_\nu^\dagger(0)]|p\rangle = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu)$$

$$+ \frac{1}{m_N^2}\left(p_\mu - \frac{\nu}{q^2} q_\mu\right)\left(p_\nu - \frac{\nu}{q^2} q_\nu\right) W_2(q^2, \nu)$$

$$- i\, \frac{\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta}{2m_N^2} W_3(q^2, \nu) + \frac{q_\mu q_\nu}{m_N^2} W_4(q^2, \nu) + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2m_N^2} W_5(q^2, \nu) \quad (2.32)$$

where $\nu = q\cdot p$. For electron scattering $J_\mu = V_\mu^{EM} = \bar{q}\gamma_\mu \frac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8)q$, and parity and current conservation demand that $W_3^{eN} = W_4^{eN} = W_5^{eN} = 0$. For neutrino/antineutrino scattering, $J_\mu = (V-A)^{\nu/\bar{\nu}} = \bar{q}\gamma_\mu(1-i\gamma_5)\frac{1}{2}(\lambda_1 \mp i\lambda_2)q$; in what follows only the isotopic even combination $W_i^+ = W_i^{\nu p} + W_i^{\bar{\nu}p}$ will be important. An analysis using leading light cone singularities or the Bjorken-Johnson-Low limit [30] yields the scaling properties

$$\lim W_1(q^2, \nu) = F_1(x)$$

$$\lim \frac{\nu}{m_N^2} W_{2,3}(q^2, \nu) = F_{2,3}(x)$$

$$\lim \frac{\nu^2}{m_N^4} W_{4,5}(q^2, \nu) = F_{4,5}(x) \quad (2.33)$$

in the scaling limit $-q^2$, $\nu \to \infty$ with $x = -q^2/2\nu$ fixed. Experimentally, this seems to be verified in the case of the well-measured functions W_1^{ep} and W_2^{ep}.

The deep inelastic structure functions W_i are, of course, the absorptive part of the structure functions for the forward scattering of currents off spin-averaged nucleon targets. For the latter, it is advantageous to employ a set of KSF (kinematic singularity free) covariants which are slightly different from the covariants employed in (2.32):

$$
\begin{aligned}
T_{\mu\nu} &= i \int d^4y\, e^{iq\cdot y} \langle p \,|\, T^* \left(J_\mu(y) J_\nu^\dagger(0) \right) |\, p \rangle \\
&= \left(-q^2 g_{\mu\nu} + q_\mu q_\nu \right) A_1(q^2, \nu) \\
&\quad + \left(q^2 p_\mu p_\nu - \nu (p_\mu q_\nu + p_\nu q_\mu) + \nu^2 g_{\mu\nu} \right) A_2(q^2, \nu) \\
&\quad - i \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta A_3(q^2, \nu) + g_{\mu\nu} A_4(q^2, \nu) \\
&\quad + (p_\mu q_\nu + p_\nu q_\mu - \nu g_{\mu\nu}) A_5(q^2, \nu) + p_\mu p_\nu A_6(\nu)
\end{aligned} \tag{2.34}
$$

where, to avoid kinematic singularities in A_2, one includes a sixth invariant amplitude, A_6, which is independent of q^2. For convenience, the isotopic spin notation for the A is taken to be the same as for the W. In addition to the Regge singularities known from purely hadronic amplitudes, the light cone structure of two-current amplitudes gives rise to fixed poles (i.e., Regge intercept α not a function of momentum transfer) in $T_{\mu\nu}$; in particular, one can find $\alpha = 0$ fixed poles in the crossing symmetric amplitudes. It is commonly assumed and can, in fact, be proved [12, 13, 31] that the residues of fixed poles in kinematic singularity free amplitudes are polynomials in q^2 so that they are real and do not appear in the absorptive parts of the amplitudes (the deep inelastic structure functions W). Thus, from the known Regge behavior of the amplitudes, one can isolate the part of A_i which corresponds to the $\alpha = 0$ fixed pole for large ν [13],

$$
\begin{aligned}
A_1^{fp} &= C_1(q^2)\, \nu^0 \\
A_2^{fp} &= C_2(q^2)\, \nu^{-2} \\
A_3^{fp} &= C_3(q^2)\, \nu^{-1} \\
A_4^{fp} &= C_4(q^2)\, \nu^0 \\
A_5^{fp} &= C_5(q^2)\, \nu^{-1} \\
A_6^{fp} &= C_6\, \nu^{-2} \quad ,
\end{aligned} \tag{2.35}
$$

where $C_i(q^2)$ are polynomials in q^2. If one further assumes that the A_i scale as their imaginary parts W_i, then one finds that νA_1, $\nu^2 A_2$, νA_3, νA_4, $\nu^2 A_5$, and $\nu^2 A_6$ scale. When this scaling behavior is applied to (2.35), one concludes that all the C_i are constants [31] and that $C_1 = C_4 = C_5 = 0$. Writing dispersion relations for A_i, one then relates the fixed pole residues to integrals over the deep inelastic scaling functions. Thus [7, 13, 32]

$$C_1 = C_4 = C_5 = 0 \tag{2.36a}$$

$$C_2 = 2 \int_0^\infty \frac{dx}{x^2} \tilde{F}_2(x) \tag{2.36b}$$

$$C_6 = 4 M_N^2 \int_0^\infty dx\, \tilde{F}_5(x) \tag{2.36c}$$

where the tilde indicates that all Regge behavior with $\alpha > 0$ has been subtracted; this Regge subtraction contributes to the integral in the interval $1 < x < \infty$. The result for C_2 in (2.36b) can also be derived by use of the DGS representation along with the scaling structure of νW_2 [32]. Thus the $\alpha = 0$ fixed pole residues can be found either through a knowledge of the current-nucleon scattering amplitudes or via the deep inelastic scaling functions for all x; the latter must be known extremely well, however, in order that the Regge subtracted integral yield a reliable result.

The fixed pole residue C_2^{ep} at $q^2 = 0$ has been estimated from photoproduction data [33]:

$$\begin{aligned} C^{\gamma p} &\equiv \frac{1}{2} C_2^{ep}(0) \\ &= 1 + \frac{1}{2\pi^2 \alpha} \int_0^\infty \tilde{\sigma}_{\gamma p}(\nu)\, d\nu \approx 1 \end{aligned} \tag{2.37}$$

where $\tilde{\sigma}_{\gamma p}$ is the Regge subtracted photoproduction cross section.

The value for the neutron fixed pole [34] is more poorly determined being consistent with 0 (as often guessed at from the $\nu \to 0$ Thomson limit which yields the result $C^{\gamma p} \approx 1$ in the case of the proton) but also consistent with the value required by our later analysis ($C^{\gamma n} \sim 2/3$).

The $\alpha = 0$ fixed poles in A_6 are unknown since $F_5(x)$ is difficult to determine experimentally and since high energy axial current-nucleon scattering is unknown. Theoretically, one can show that $F_5(x) = 0$ in parton and scalar gluon models; this is not the case, however, in vector gluon models (with spin 1/2 quarks) where F_5 is proportional to the quark-gluon coupling constant [13]. Nonetheless we shall present a number of arguments which imply that the fixed pole, C_6, for strangeness nonchanging currents, proportional to a subtracted integral over F_5, (2,36c), is zero. In particular, we show below that, assuming the validity of pion PCAC, (2.6), $C_6 \neq 0$ would lead to fixed poles in hadronic amplitudes in contradiction to bilinear unitarity. Later we shall argue for the phenomenological necessity of $C_6 = 0$.

Beginning with the amplitude expansion, (2.34), we see that the amplitude for double axial divergence-scattering is given by

$$q_\mu T_{\mu\nu} q_\nu = q^2 \cdot A_4 + q^2 \cdot \nu A_5 + \nu^2 A_6 \tag{2.38}$$

From the results of (2.36) the fixed pole behavior of the double divergence scattering is then $\sim C_6 \nu^0$ (independent of ν and q^2). Using PCAC (2.6), double divergence scattering is proportional to the amplitude for πN scattering and hence if $C_6 \neq 0$, this purely hadronic amplitude will have a fixed pole, not allowed by bilinear unitarity. An essentially equivalent argument considers the single divergence-current scattering amplitude and finds an $\alpha = 0$ fixed pole in it for $C_6 \neq 0$. By the arguments of Cheng and Tung [35] this also implies the presence of $\alpha = 0$ fixed poles in purely hadronic amplitudes of the type $Vp \to \pi p$. Thus, subject to the limitations of PCAC, we conclude that $C_6 = 0$. If the corrections to πPCAC are smaller than order m_π^2/m_N^2 or if the dispersive corrections to PCAC cannot generate cancelling fixed pole behavior, then neglect of C_6 is justified. As we shall see, the quark masses must not be too small (i.e., they cannot be of GMOR size) if we are to trust our approximation of $C_6 = 0$ in chiral breaking applications [36].

Several other arguments for a small value of C_6 are possible. It was shown in ref. 13 that C_6 is proportional to the quark-gluon coupling constant g (multiplied by a quark mass). According to most analyses, in the context of asymptotic freedom, ψ-decay, etc., g is a relatively small number implying an unexpectedly small value for C_6. The fact that $g \to 0$ yields $C_6 = 0$ is no surprise, since for free quarks, i.e., in the weak binding approximation, $F_5 \equiv 0$ identically and hence $C_6 = 0$. Further, we remind the reader again that this discussion has been for $\Delta S = 0$ currents. The $\Delta S = 1$ case will be discussed in the next section.

Finally, we will be discussing various probability distribution integrals of quarks in baryons. With the help of the formal definition in the quark-parton model, one can write

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}\left(u(x)+\bar{u}(x)\right) + \frac{1}{9}\left(d(x)+\bar{d}(x)\right) + \frac{1}{9}\left(s(x)+\bar{s}(x)\right) \qquad (2.39)$$

where $u(x)$ ($\bar{u}(x)$) is the probability of finding an up-quark (up-antiquark) in the proton between x and x+dx, etc. The valence values are well known in terms of the integrals $\int(u-\bar{u})\,dx = 2$, $\int(d-\bar{d})\,dx = 1$, and $\int(s-\bar{s})\,dx = 0$. The probability sums such as $u(x)+\bar{u}(x)$, however, can be strongly affected by the presence of quark-antiquark pairs in the baryons. The quark distributions of interest in the following sections are $\tilde{f}_i$ where, for instance,

$$\tilde{f}_u = \int_0^{\infty} \frac{dx}{x} \left(u(x) + \bar{u}(x) \right)^{R.s.} \tag{2.40}$$

where R.s. indicates that Regge behavior with $\alpha > 0$ has been subtracted. This subtraction includes the Pomeron, but even though the effect of quark-antiquark pairs is thereby suppressed, the result that $\tilde{f}_u$, $\tilde{f}_d$, and $\tilde{f}_s$ turn out to be close to the valence values of 2, 1, and 0 is surprisingly simple. This will be discussed further in later sections; the valence values are consistent with the fixed pole estimate [7, 11, 12] of (2.36b) and (2.37):

$$\begin{aligned} C^{\gamma p} &= \int_0^{\infty} \frac{dx}{x^2} \tilde{F}_2^{ep}(x) = \frac{4}{9} \tilde{f}_u + \frac{1}{9} \tilde{f}_d + \frac{1}{9} \tilde{f}_s \\ &\approx 1 \quad . \end{aligned} \tag{2.41}$$

While the fixed pole scale $C^{\gamma n} = \frac{1}{2} C_2^{en}$ is not well enough determined to further constrain the values of $\tilde{f}$, it is the right order of magnitude so that valence-type values for the $\tilde{f}$'s are not ruled out [12, 34].

III. CHIRAL SYMMETRY BREAKING BARYON MATRIX ELEMENTS

A. Relation Between Light Plane Behavior, Fixed Pole Assumptions and the Infinite Momentum Frame

The connection between the matrix elements of the bad operators $u_{\underset{-}{i}}$ and the fixed poles of the deep inelastic structure functions can most easily be seen in terms of the nucleon matrix elements of the sigma term written in quark language as

$$2\,m_N\,\sigma_{NN}^{\pi\pi} = \hat{m}\,(\bar{u}u + \bar{d}d)_{NN} \quad . \tag{3.1}$$

Following the procedures outlined by Jaffe and Llewellyn-Smith [7] one first takes the double divergence of $W_{\mu\nu}$ in (2.32) for neutrino (or axial-vector) scattering and uses the divergence condition $\partial \cdot A_i = -\hat{m}\,\bar{q}\,\gamma_5\,\lambda_i\,q$ for i = 1, 2, 3, to isolate the most singular term on the light cone,

$$4\,x\,F_4(x) - 2\,F_5(x) = \left(\hat{m}^2/m_N^2\right) F_2(x)/x^2 \quad . \tag{3.2}$$

Next one computes the single divergence of $W_{\mu\nu}$ to obtain the Regge subtracted sum rule

$$\sigma_{NN}^{\pi\pi} = \frac{1}{2} m_N \int_0^\infty dx \left[4x\,\tilde{F}_4^+(x) - \tilde{F}_5^+(x)\right] \quad , \tag{3.3}$$

which, when combined with (2.36b) and (3.2) can be rewritten as [7]

$$\begin{aligned}\sigma_{NN}^{\pi\pi} &= \frac{\hat{m}^2}{2m_N} \int_0^\infty \frac{dx}{x^2}\,\tilde{F}_2^+(x) + \frac{1}{2}\,m_N \int_0^\infty dx\,\tilde{F}_5^+(x) \\ &= \left(\hat{m}^2/4m_N\right) C_2^+ + \left(1/8\;m_N\right) C_6^+ \quad .\end{aligned} \tag{3.4}$$

Thus one sees that the πN sigma term can be expressed completely in terms of the two $\alpha = 0$ fixed pole residues. Inverting (3.1) and similar expressions it is readily verified that the nucleon matrix elements of the bad operators u_i can be expressed in terms of $m_q C_2$ and C_6/m_q.

The relation between the light cone expression (2.28a) for u_i and the fixed pole description is now apparent: the quark mass matrix third term in (2.28a) corresponds to $m_q C_2$, while the first two terms in (2.28a) correspond to the C_6/m_q (this latter term is actually <u>independent</u> of quark mass because C_6 is proportional to m_q [11]). Put another way, setting $C_2 = 0$ corresponds to the GMOR SU_3 assumption; (2.37), however, argues that C_2 is not zero, although $(\hat{m}^2/m_N)\, C_2$ would be suppressed if $\hat{m} \ll m_N$. The fact that $\sigma^{\pi\pi}_{NN}$ is indeed large [18] indicates by (2.14) that the GMOR SU_3 assumption is suspect and that the C_2 term dominates (3.4). Finally it may be argued on the basis of the discussion in the previous section that it is perhaps a good approximation to neglect the non-current conserving fixed pole residue C_6 especially as our quark masses turn out to be large. Hence we take as our fundamental assumption for π-like axial currents

$$C_6^+ = \int_0^\infty \tilde{F}_5(x)\, dx = 0 \tag{3.5}$$

so that in the scaling language of quark probability distribution integrals we obtain $\langle p \,|\, \bar{u}u \,|\, p \rangle = 2\hat{m}\tilde{f}_u$ and $\langle p \,|\, \bar{d}d \,|\, p \rangle = 2\hat{m}\tilde{f}_d$ from (3.1), (3.4), and (3.5) [6, 11, 12].

As mentioned previously, this analysis applies only to $\Delta S = 0$ currents; $\Delta S = 1$ currents may be treated similarly to (3.1) - (3.5) with certain crucial differences:

(i) We consider only the axial parts of the $\Delta S = 1$ currents (since the vector parts also have non-zero divergence for $\hat{m} \neq m_s$) so that the R.H.S. of (3.2) becomes proportional to $(\hat{m} + m_s)^2$ and all F's refer to axial currents only.

(ii) It is not plausible that F_5 and, hence, $C_6 (\Delta S = 1) = 0$. Indeed naive parton model traces show that F_5 depends on the difference between initial and final quark masses. More precisely

$$F_5 \begin{pmatrix} u \to s \\ \bar{s} \to \bar{u} \end{pmatrix} = \frac{\hat{m}^2 - m_s^2}{2 m_N^2 x} \begin{pmatrix} + f_u(x) \\ - f_{\bar{s}}(x) \end{pmatrix}$$

(iii) Consequently the combination of (3.4) and (3.1) appropriate to kaons yields

$$(\hat{m} + m_s) \langle p | \begin{pmatrix} \bar{u}u \\ \bar{s}s \end{pmatrix} | p \rangle = \left[(m_s + \hat{m})^2 \begin{pmatrix} + \\ - \end{pmatrix} \begin{pmatrix} \hat{m}^2 - m_s^2 \end{pmatrix} \right] \begin{pmatrix} \tilde{f}_u \\ \tilde{f}_s \end{pmatrix}$$

so that we regain $\langle p | \bar{u}u | p \rangle = 2 \hat{m} \tilde{f}_u$ and obtain the new result $\langle p | \bar{s}s | p \rangle = 2 m_s \tilde{f}_s$, i.e., the corresponding quark mass factor always appears multiplying the quark field distribution integral.

Non-zero values for F_5 do, of course, imply (following the argument in Section II) J = 0 fixed pole behavior in $\Delta S = 1$ axial-divergence scattering. Strict application of kaon PCAC to $\Delta S = 1$ axial current divergences would then imply a J = 0 fixed pole in the K-nucleon scattering amplitude. Thus if we trust the naive parton model (weak-binding) results, we must presume a breakdown of kaon PCAC for calculating J = 0 fixed pole residues. This is not implausible as corrections to kaon PCAC are, naively, as large as the value of F_5 (and hence C_6) calculated, for $\Delta S = 1$, above. The opposite choice, i.e., trusting kaon PCAC for the J = 0 fixed pole behavior, would lead to serious inconsistencies in the type of phenomenology which follows.

In summary, for nucleon matrix elements of the $\bar{q}q$ fields, we have

$$\langle p|\bar{u}u|p\rangle = 2m_u\tilde{f}_u \qquad \langle p|\bar{d}d|p\rangle = 2m_d\tilde{f}_d \qquad \langle p|\bar{s}s|p\rangle = 2m_s\tilde{f}_s$$

$$\langle n|\bar{u}u|n\rangle = 2m_u\tilde{f}_d \qquad \langle n|\bar{d}d|n\rangle = 2m_d\tilde{f}_u \qquad \langle n|\bar{s}s|n\rangle = 2m_s\tilde{f}_s \quad . \tag{3.6}$$

Equations (3.6) are, of course, fully covariant [29]. However, an especially transparent alternate derivation employs the infinite momentum frame where $\bar{q}\gamma_\mu q = \bar{q}q\,p_\mu/m$ in the forward direction leads immediately to (3.6) if Z diagram contributions are negligible [11, 12]. Thus, there are four interpretations of (3.6):

(i) Dominance of the quark mass matrix term in the light-cone decomposition of the $\langle N|u_i|N\rangle$.

(ii) Suppression of the $p_\mu p_\nu$ $\alpha = 0$ fixed pole for $\Delta S = 0$ axial-vector nucleon scattering, and, hence, of $\alpha = 0$ fixed poles in hadronic πN scattering.

(iii) Suppression of Z diagrams in the infinite momentum frame for $\langle N|\bar{q}\lambda_i q|N\rangle$.

(iv) Small quark gluon coupling constant, i.e., the weak binding approximation.

Since the arguments leading to (3.6) can be repeated for hyperon targets, one can immediately obtain the hyperon SU_3 analogs [12] of (3.6):

$$\langle\Sigma^+|\bar{u}u|\Sigma^+\rangle = 2m_u\tilde{f}_u,\quad \langle\Sigma^+|\bar{d}d|\Sigma^+\rangle = 2m_d\tilde{f}_s,\quad \langle\Sigma^+|\bar{s}s|\Sigma^+\rangle = 2m_s\tilde{f}_d$$

$$\langle\Sigma^-|\bar{u}u|\Sigma^-\rangle = 2m_u\tilde{f}_s,\quad \langle\Sigma^-|\bar{d}d|\Sigma^-\rangle = 2m_d\tilde{f}_u,\quad \langle\Sigma^-|\bar{s}s|\Sigma^-\rangle = 2m_s\tilde{f}_d$$

$$\langle\Sigma^0|\bar{u}u|\Sigma^0\rangle/m_u = \langle\Sigma^0|\bar{d}d|\Sigma^0\rangle/m_d = \tilde{f}_u+\tilde{f}_s,\quad \langle\Sigma^0|\bar{s}s|\Sigma^0\rangle = 2m_s\tilde{f}_d \ .$$

$$\langle\Lambda^0|\bar{u}u|\Lambda^0\rangle/m_u = \langle\Lambda^0|\bar{d}d|\Lambda^0\rangle/m_d = \frac{1}{3}\left(\tilde{f}_u + 4\tilde{f}_d + \tilde{f}_s\right)$$

$$\langle\Lambda^0|\bar{s}s|\Lambda^0\rangle/m_s = \frac{2}{3}\left(2\tilde{f}_u - \tilde{f}_d + 2\tilde{f}_s\right) \tag{3.7}$$

$$\langle\Xi^0|\bar{u}u|\Xi^0\rangle = 2m_u\tilde{f}_d,\quad \langle\Xi^0|\bar{d}d|\Xi^0\rangle = 2m_d\tilde{f}_s,\quad \langle\Xi^0|\bar{s}s|\Xi^0\rangle = 2m_s\tilde{f}_u$$

$$\langle\Xi^-|\bar{u}u|\Xi^-\rangle = 2m_u\tilde{f}_s,\quad \langle\Xi^-|\bar{d}d|\Xi^-\rangle = 2m_d\tilde{f}_d,\quad \langle\Xi^-|\bar{s}s|\Xi^-\rangle = 2m_s\tilde{f}_u$$

We re-emphasize that in (3.6) and (3.7), the quark masses are those which appear in the fully perturbed Lagrangian (and also in 2.28a). The results (3.7) are derived using the fact that the $\tilde{f}_i$ for other baryons may be determined by SU_3 symmetry from those we have defined for the proton, $\tilde{f}_u$, $\tilde{f}_d$ and $\tilde{f}_s$; that is, quark light cone distributions transform according to SU_3 as implied by the simple state transformation properties under the light plane charges (2.30). The matrix elements $\langle B|\bar{q}q|B\rangle$, themselves are not directly related by SU_3 but, rather, display the $\bar{q}q\ m_q$ of the third term in (2.28a).

It is perhaps also important to stress that in the present approach, as embodied in (3.6) and (3.7), there is no inconsistency between the $SU_2\times SU_2$ limit and $C_6 = 0$ of the type discussed in ref. 13. The fundamental assumption behind the inconsistency proof was that the u_8 part of H' generated all SU_3 splittings. Here u_0, because of the unequal quark masses appearing in the strange and non-strange $\langle\bar{q}q\rangle_B$'s, also generates part of the SU_3 mass splittings.

B. Baryon Mass Formulae

Taking the octet baryon matrix elements of the strong interaction Hamiltonian leads to the possible forms

$$\langle B \mid H \mid B \rangle = 2 m_B^2,\ 4 \bar{m}_B m_B \quad . \tag{3.8}$$

Substituting $H_0 + H'$ for H in (3.8), leads to the Gell-Mann-Okubo octet-breaking form

$$m_\Sigma^n + 3 m_\Lambda^n = 2 m_N^n + 2 m_\Xi^n \tag{3.9}$$

for linear masses (n = 1) or quadratic masses (n = 2). To decide between these alternatives in our quark model scheme, we parallel Caser and Testa's argument for the pseudoscalar mesons [37]. The equation of motion for $\partial \cdot V_{K^-}$ implies, via (2.4) and (2.5), the relation

$$\partial \cdot V_{K^-} = -i \frac{\sqrt{3}}{2} \epsilon_8 u_{K^-} . \tag{3.10}$$

Even though the u's are "bad" operators, u_{K^-} can transform only as λ_{K^-}. Also, the Ademollo-Gatto theorem applied to the vector current between baryon spinors leads to $\langle B_f \mid V_j^\mu \mid B_i \rangle = i f_{fji} \gamma^\mu + \ldots$. Combining these two statements with (3.10) then gives the linear mass formula [38] in (3.9).

While both the linear and quadratic forms of (3.9) are reasonably well satisfied phenomenologically, the average masses [39] $m_N = 939$ MeV, $m_\Sigma = 1193$ MeV and $m_\Xi = 1318$ MeV predict $m_\Lambda = 1107$ MeV (1128 MeV) from the linear (quadratic) version of (3.9). In the quark model scheme where H' is pure octet, such corrections to m_Λ can be interpreted as H'^2 terms arising from the perturbative expansion of (3.8) which include parts transforming as $\underline{27}$. When scaled to the octet part, this $\underline{27}$ part is 0 (3%) in either the linear or quadratic formulation [40] (but with opposite signs). Following (3.10), we shall assume a linear mass dependence in (3.8) and account for the slight 27 contamination. While such refinements have a marginal effect on the semi-strong mass difference constraints presented in this section, they will have a bearing on the determination of the ninth pseudoscalar mass discussed in Section IVE.

In our quark model scheme, (2.17), (3.6), (3.7) and the linear form of (3.1) can be combined to eliminate the 27 part in the octet d/f ratio,

$$\begin{aligned}(d/f)_{ss} &= \frac{\tilde{f}_u - 2\tilde{f}_d + \tilde{f}_s}{\tilde{f}_u - \tilde{f}_s} \\ &= -\frac{3}{5}\left(\frac{3\,m_\Sigma - m_N - m_\Lambda - m_\Xi}{m_\Xi - m_N}\right) \approx -\frac{1}{3} \quad .\end{aligned} \tag{3.11}$$

The quadratic mass formula gives $(d/f)_{ss} \approx -0.28$. Then the singlet and octet masses

$$\begin{aligned}\bar{m}_B &= \frac{1}{8}\left(2\,m_\Xi + 2\,m_N + m_\Lambda + 3\,m_\Sigma\right) \approx 1151 \text{ MeV} \\ m_B^8 &= \frac{1}{10}\left(8\,m_\Xi - 2\,m_N + 3\,m_\Lambda - 9\,m_\Sigma\right) \approx 128 \text{ MeV}\end{aligned} \tag{3.12}$$

can be combined to form the only other (octet) constraint on the quark parameters,

$$\left(m_s^2 - \hat{m}^2\right)\left(\tilde{f}_u - \tilde{f}_d\right) = 2\,\bar{m}_B\, m_B^8 \approx 0.29 \text{ GeV}^2 \ . \tag{3.13}$$

If we now assume that the strange quarks are negligible in the proton so that $\tilde{f}_s = 0$ (later justified), then the proton Compton fixed pole scale (2.37) together with (3.11) determine $\tilde{f}_u$ and $\tilde{f}_d$ [6, 12]:

$$\tilde{f}_u \approx 2 \qquad \tilde{f}_d \approx \frac{4}{3} \qquad \tilde{f}_s = 0 \quad , \tag{3.14}$$

quite close indeed to the valence values 2, 1, and 0. Given the absence of an accurate determination of the neutron fixed pole scale, it is possible to slightly alter $\tilde{f}_u$ as well as $\tilde{f}_d$ in (3.14) from the valence values and still obey (2.37). However, the relevant quantity in (3.13), $\tilde{f}_u - \tilde{f}_d$, then changes very little. In particular, given (3.14), (3.13) then implies

$$m_s^2 - \hat{m}^2 \approx 0.44 \text{ GeV}^2 \approx 23\, m_\pi^2 \quad , \tag{3.15}$$

which is an important constraint on the size of the quark masses [6].

A similar analysis also holds for the decuplet masses. Since no structure functions are accessible for decuplets, one can only apply the valence values in this case. This may not be too bad an approximation, however, because the deviation of (3.14) from the valence values is a measure of the $\Sigma - \Lambda$ mass difference. For decuplets, however, the equal splitting rule is consistent with (but does not imply) valence values for the distribution integrals. With this assumption, the octet mass combination analogous to (3.13) is ($\bar{m}_D \approx 1380$ MeV)

$$m_s^2 - \hat{m}^2 = \frac{2}{5}\, \bar{m}_D\, (m_\Omega + m_{\Xi^*} - 2\, m_\Delta) \approx 0.42 \text{ GeV}^2 \, , \tag{3.16}$$

very close indeed to the octet baryon value (3.15).

If the baryon mass formulae were linear rather than quadratic at the quark level, i.e., if C_6 were non-vanishing and in fact dominated $\langle B | \bar{q} q | B \rangle$ (rather than (3.6)) as in the GMOR scheme, then the octet mass differences would give $m_s - \hat{m} \sim 190$ MeV whereas the decuplet mass differences would imply $m_s - \hat{m} \sim 140$ MeV. Furthermore, a linear combination of the quadratic mass structure and linear mass structure would not lead to a simpler consistent solution than (3.14) - (3.16). One might then argue that this consistency is further indication that fixed pole (or Z diagram) corrections to (3.6) are in fact small and that (3.5) is approximately correct.

To complete the picture, the octet-decuplet mass difference of $\Delta m_{DB} \approx 1380 - 1150 \approx 230$, which is an SU_6 effect, presumably due to a quark spin-spin interaction in H_0 satisfying (2.1), may be explained in terms of the scaling functions which also (but independently) help specify chiral symmetry breaking effects as in (3.6) and (3.7). For example, it has recently been argued [41] that this SU_6 mass difference, when formulated as the difference between the Δ and N Regge trajectory spacings, is linked to the SU_6 broken value of $F_2^{en}/F_2^{ep} \sim \frac{1}{4}$ (rather than the SU_6 value of $\frac{2}{3}$) near $x \sim 1$.

C. Baryon σ Terms

As was stressed earlier, the large experimental value of the πN σ term [18] gave the first clue that the GMOR SU_3 assumption for the bad operators u_i and v_i may not be correct. This value is obtained from on-shell πN data ($q^2 = q'^2 = m_\pi^2$), but extrapolated below threshold, via singly and doubly-subtracted dispersion relations, to [4, 42] $\nu = 0$, $t = 2m_\pi^2$, where the axial-vector-nucleon corrections to the σ term are $O(m_\pi^4)$ in the forward, isospin even πN amplitude,

$$\bar{F}^{(+)}\left(\nu = 0,\ t = 2m_\pi^2\right) \doteq \sigma_{NN}^{\pi\pi}/f_\pi^2 + O\left(m_\pi^4\right) . \tag{3.17}$$

Using the latest and most accurate data, fixed t and independently interior dispersion relations, many analyses [18] have obtained values for (3.17) averaging $(1.05 \pm 0.07)\ m_\pi^{-1}$, corresponding to (2.13) for $f_\pi \approx 93$ MeV. Before 65 MeV is to be accepted as the true value of $\sigma_{NN}^{\pi\pi}$, the subtle sign change in (3.17) must be appreciated. In the limit of both pion momenta being soft, the exact low energy theorem for $\bar{F}^{(+)}$ is

$$\bar{F}^{(+)}\left(\nu = 0,\ t = 0;\ q^2 = q'^2 = 0\right) = -\sigma_{NN}^{\pi\pi}/f_\pi^2 \tag{3.18}$$

The presence of the Adler zero [43] at

$$\overline{F}^{(+)}\left(\nu = 0,\ t = m_\pi^2;\ q^2 = 0,\ q'^2 = m_\pi^2\right) = 0 \tag{3.19}$$

is what causes the sign change between (3.17) and (3.18). This zero is close to being satisfied at the on-shell point [44] $\nu = 0$, $t = m_\pi^2$. If c were near $-\sqrt{2}$, then pion PCAC could be invoked in an operational sense and an expansion in q^2 and q'^2 satisfying (3.19) would convert (3.18) to (3.17) provided correction terms $O(H'^2)$ were small [4, 45]. Since, however, $\sigma_{NN}^{\pi\pi}$ is roughly 3 times the GMOR value, it is not clear that pion PCAC is valid in such an operational sense and this in turn would cast doubt on the validity of (3.17) and therefore on the meaning of the large value of $\overline{F}^{(+)}\left(0,\ 2m_\pi^2\right)$.

There are in fact two arguments which reaffirm the correctness of (3.17) and the validity of $\sigma_{NN}^{\pi\pi} \approx 65$ MeV. Firstly, even if c is not near $-\sqrt{2}$, pion PCAC can always be tested in a dispersion-theoretic sense (neutral PCAC). In the case of πN scattering, the Adler zero is manifested to leading order in the invariants q^2, q'^2, and t via the analytic expansion.

$$\begin{aligned}\overline{F}^{(+)}(\nu, t; q^2, q'^2) = \frac{\sigma^{\pi\pi}}{f_\pi^2}&\left[(1-\beta)\left(\frac{q^2+q'^2}{m_\pi^2} - 1\right) + \beta\left(\frac{t}{m_\pi^2} - 1\right)\right] \\ &+ a\nu^2 + b\, q'\cdot q + O\left(\frac{q'^2 q^2}{m_N^4},\ \text{etc.}\right)\end{aligned} \tag{3.20}$$

Phenomenologically it appears that [46] $\beta \approx 0.4$ and does not vanish as assumed in the formal chiral expansion with $\sigma_{NN}^{\pi\pi} \sim \sqrt{2} + c$ manifestly small. Nevertheless (3.20) still implies (3.17) - (3.19), with on-shell PCAC corrections to (3.19) measured to be $-.16\ m_\pi^{-1}$ or about 10% per pion [44, 47].

A second argument in favor of (3.17) is an independent, but much less accurate, estimate of $\sigma_{NN}^{\pi\pi}$ by the Fubini-Furlan extrapolation [48] of the soft result (3.18) along the parabola $\nu \sim q^2$ up to the physical threshold $\nu = m_\pi$, t=0. In this case the sign of $\sigma_{NN}^{\pi\pi}$ does not change and one finds ($\nu = (s - u)/4\, m_N$)

$$f_\pi^2\, F^{(+)}\, (m_\pi,\ 0) = -\,\sigma_{NN}^{\pi\pi} + R^{(+)} \tag{3.21a}$$

$$f_\pi^2\, F^{(-)}\, (m_\pi,\ 0) = \frac{1}{2}\, m_\pi + R^{(-)} \quad , \tag{3.21b}$$

where $R^{(\pm)}$ are s-wave rescattering integrals, presumably dominated by the resonances N' (1535), N'' (1700), and Δ(1650), leading to $R^{(+)} \sim 4R^{(-)}$. The experimental s-wave scattering lengths applied to (3.21) then lead to the estimate [5]

$$\sigma_{NN}^{\pi\pi} = 66 \pm 18 \text{ MeV} . \tag{3.22}$$

While this estimate is certainly less reliable than the on-shell value of 65 ± 5 MeV, it does verify the sign change in (3.17). Moreover the scattering length $a_1 + 2a_3$ contributes with opposite sign to (3.17) and (3.21a), and its magnitude ($\sim -\,0.02$ m_π^{-1}) therefore provides a sensitive test as to any difference between the values (2.13) and (3.22). Consequently one may conclude that (2.13) is correct.

In quark language, one can combine (3.1) with (3.6) to obtain the Jaffe-Llewellyn-Smith form [6, 7, 12],

$$\sigma_{NN}^{\pi\pi} = \frac{\hat{m}^2}{m_N} \left(\tilde{f}_u + \tilde{f}_d\right) . \tag{3.23}$$

Since $\sigma_{NN}^{\pi\pi}$ then depends quadratically upon $\hat{m}^2$ and upon an insensitive combination of the distribution integrals $\tilde{f}_u$ and $\tilde{f}_d$, it is reasonable to combine (3.23) with (2.13) and (3.14) to estimate the size of the non-strange quark mass,

$$\hat{m} \approx m_\pi \approx 140 \text{ MeV} . \tag{3.24}$$

If $\sigma_{NN}^{\pi\pi}$ should be substantially reduced in the future to, say, 45 MeV (at the expense of pion PCAC and large corrections to (3.19), the value of $\hat{m}$ would change only slightly to 120 MeV. Given (3.24), one can apply the quark mass scale set by the baryon mass splittings in this scheme, (3.15), to estimate the size of the strange quark mass,

$$m_s \approx 4.9\, m_\pi \approx 680 \text{ MeV}, \tag{3.25}$$

or [6]

$$X = m_s/\hat{m} \approx 5 \ . \tag{3.26}$$

Furthermore, with the additional assumption that $\tilde{f}_s$ strictly vanishes as in (3.14) (to be independently verified shortly), (3.8) and (3.23) can be combined to extract the chiral octet baryon mass

$$m_0 = m_N \left(1 - \sigma_{NN}^{\pi\pi}\Big/2\,\overline{m}_B\right) \approx 910 \text{ MeV} \tag{3.27}$$

and the perturbative condition $\langle B \mid H' \mid B\rangle \ll \langle B \mid H_0 \mid B\rangle$ is seen to hold,

$$m_N - m_0 \approx 30 \text{ MeV} \ll m_0 \ , \tag{3.28}$$

which preserves the internal consistency of this scheme.

These results can be converted into the usual chiral symmetry breaking language. The ratio $X \sim 5$ applied to (2.21) immediately yields [6]

$$c = -\sqrt{2}\left(\frac{X-1}{X+2}\right) \approx -0.8 \ . \tag{3.29}$$

While this value of c is not as close to $-\sqrt{2}$ as is the GMOR value, it is midway between this $SU_2 \times SU_2$ chiral limit and the SU_3 limit $c = 0$. This argues in favor of a dispersive "neutral" PCAC rather than "strong" PCAC ($c \sim -\sqrt{2}$) or "weak" PCAC ($c \sim 0$). Evidently this is still adequate to allow pion PCAC to be a useful tool to probe chiral symmetry and chiral symmetry breaking in low energy πN and $\pi\pi$ scattering and in many other strong, electromagnetic and weak processes involving pions. Another conclusion in the chiral symmetry breaking language is that for a vanishing $\tilde{f}_s$, (2.19) and (3.6) imply [6, 49]

$$\left(u_0/u_8\right)_N = \sqrt{2}\left\{\frac{\hat{m}\left(\tilde{f}_u + \tilde{f}_d\right) + m_s \tilde{f}_s}{\hat{m}\left(\tilde{f}_u + \tilde{f}_d\right) - 2m_s \tilde{f}_s}\right\} \approx \sqrt{2} \tag{3.30}$$

as might be expected from the Zweig rule. However, $(u_8)_N$ does <u>not</u> transform like λ_8 in our scheme and $\epsilon_8(u_8)_N/2m_N$ is <u>not</u> -210 MeV as would be the case if the GMOR SU_3 assumption were valid. Instead, (2.19), (3.6), and (3.26) indicate that [50]

$$\frac{\langle N \mid \epsilon_8 \quad u_8 \mid N\rangle}{2m_N} = \left(\frac{2}{X+1}\right)\left(m_N - \bar{m}_B\right)$$

$$\approx \frac{1}{3}(-210 \text{ MeV}) \quad . \tag{3.31}$$

Combining (3.29) - (3.31) with the form (2.14) again reproduces $\sigma^{\pi\pi}_{NN} \sim 65$ MeV as expected, and one can see how the quark mass ratio X in (3.29) and (3.31) conspires to keep $\sigma^{\pi\pi}_{NN}$ large while the ratio $\left(u_0/u_8\right)_N$ remains near unity as is necessary so that $SU_2 \times SU_2$ breaking is of order SU_3 breaking [4] .

The phenomenological values of the $I_s = 0, 1$ KN σ terms tend to reinforce this picture of chiral symmetry breaking. They are usually defined as

$$\sigma^{KK}_{NN}(I_s = 0) = \epsilon_0\left(\frac{\sqrt{2} - \frac{1}{2}c}{3}\right)\left(\sqrt{2}\, u_0 - \frac{3}{2}\sqrt{3}\, u_3 - \frac{1}{2} u_8\right)_{pp} \tag{3.32a}$$

$$\sigma^{KK}_{NN}(I_s = 1) = \epsilon_0\left(\frac{\sqrt{2} - \frac{1}{2}c}{3}\right)\left(\sqrt{2}\, u_0 + \frac{\sqrt{3}}{2} u_3 - \frac{1}{2} u_8\right)_{pp} \tag{3.32b}$$

and with (2.19), $u_3 = \bar{u}u - \bar{d}d$ and (3.6) they can be expressed in quark language as [6]

$$\sigma_{NN}^{KK}(I_s = 0) = \left(\frac{m_s + \hat{m}}{2m_N}\right) \left[\hat{m}\left(-\tilde{f}_u + 2\tilde{f}_d\right) + m_s \tilde{f}_s\right] \tag{3.33a}$$

$$\sigma_{NN}^{KK}(I_s = 1) = \left(\frac{m_s + \hat{m}}{2m_N}\right) \left(\hat{m}\tilde{f}_u + m_s \tilde{f}_s\right) . \tag{3.33b}$$

These quantities can only be extracted from experiment via the Fubini-Furlan extrapolation to threshold [48], analogous to (3.21). One finds

$$f_K^2 F_{I_{s=0}}(m_K, 0) = -\sigma_{NN}^{KK}(I_s = 0) + R_0 \tag{3.34a}$$

$$f_K^2 F_{I_{s=1}}(m_K, 0) = -\sigma_{NN}^{KK}(I_s = 1) - m_K + R_1 , \tag{3.34b}$$

where again $R_{0,1}$ are s-wave rescattering integrals, presumably large in order to measure the not insignificant kaon PCAC corrections, now $O(m_K^2/m_N^2)$ in our dispersive, neutral version of PCAC. It turns out, however, that due to the absence of exotic KN resonances, R_0 is in fact very small. Both R_0 and R_1 are controlled by the u channel $\bar{K}N$ spin $\frac{1}{2}^-$ Λ' (1405) and Σ'' (1750) resonances, but the isospin crossing matrices suppress R_0 and enhance R_1. Further, since experimentally the s-wave scattering lengths obey $a_{I_s=1}^{KN} \gg a_{I_s=0}^{KN} \approx 0$, it is clear from (3.34a) that $\sigma_{NN}^{KK}(I_s = 0) \approx 0$ [19]. Taking u_i to transform like λ_i in (3.32a) then leads again to the conclusion that [4] $(u_0/u_8)_N^{GMOR} \sim 1$ in the GMOR scheme, and consequently to the low estimate $\sigma_{NN}^{\pi\pi}(\text{GMOR}) \approx 20$ MeV as in (2.14). In the quark language, however, the combination $(-\tilde{f}_u + 2\tilde{f}_d)$ in (3.33a) is near the valence value of zero and so (3.33a) is a true measure of $\tilde{f}_s$ which is then of order $\left(2m_N/m_s^2\right)\sigma_{NN}^{KK}(I_s = 0) \approx 0$. Thus $\tilde{f}_s$ most certainly is very nearly zero as one might expect a measure of finding strange quarks in the proton to be [6].

A more quantitative but also more model-dependent statement can be extracted from (3.34a) and (3.34b) if one accepts the dynamical estimates of [51] $R_0 \approx 29$ MeV and $R_1 \approx 288$ MeV (again consistent with the model independent estimate $R_0 \ll R_1$) which then lead to $\sigma_{NN}^{KK}(I_s = 0) \sim 0 \pm 30$ MeV and $\sigma_{NN}^{KK}(I_s = 1) \sim 180$ MeV. These estimates pin down $\tilde{f}_s$ to be very near zero, and (3.33b) also predicts that, given (3.23) and $\sigma_{NN}^{\pi\pi} \approx 65$ MeV,

$$X \sim 8 \quad , \tag{3.35}$$

not inconsistent with the more accurate estimate (3.26).

The on-shell (Cheng-Dashen) method has also been applied to the KN system [45]. Unfortunately a formula analogous to (3.17) for the crossing-symmetric KN amplitude evaluated at the point $\nu = 0$, $t = 2m_K^2$ can have $O(m_K^4)$ corrections as large as 30% and cusp corrections arising from the 2π cut. Furthermore, the resulting estimates of $\sigma_{NN}^{KK}(I_s = 1)$ vary from 600 MeV [52] down to 100 - 200 MeV [53], depending upon how one treats the $\bar{K}N$ scattering lengths, phase shifts, and poles (Λ, Σ, Y_1^*)—all absent in the Fubini-Furlan off-shell KN analysis described above. While the latter estimate implies $X \sim 4 - 8$, consistent with our previous determinations, the former value of 600 MeV leads to the GMOR value of $X \sim 25$. Clearly, then, no distinction between the two chiral breaking schemes can be made on the basis of the on-shell KN analysis at the present time.

D. Threshold Photoproduction

Chiral breaking corrections to the standard low energy theorems of pion photoproduction off nucleons have been obtained by use of equal-time commutators of the axial-vector charge and its time derivative with the electromagnetic current evaluated in the Breit frame [54]. This leads to a Fubini-Furlan type of expression for the isoscalar photon electric s-wave multipole $E_{0+}^{(0)}$ at threshold,

$$m_v^2 f_\pi \left[\frac{E_{0+}^{(0)}}{m_\pi} + \frac{g_A}{4 m_N f_\pi} - R_v^{(0)} \right] =$$

$$= -\lim_{\vec{p} \to 0} \frac{\int d^3x \langle p(\vec{p}) \,|\, \left[i \partial \cdot A_3(\vec{x}), \vec{V}_s(0) \right] \cdot \hat{p} \,|\, p(-\vec{p}) \rangle}{\langle p(\vec{p}) \,|\, \vec{\sigma} \cdot \hat{p} \,|\, p(-\vec{p}) \rangle} \quad , \qquad (3.36)$$

where the g_A term in (3.36) is the nucleon pole contribution in the soft limit [55] $R_v^{(0)}$ is the isoscalar rescattering (and vector dominance) correction obtained from πN scattering, and $\vec{V}_s$ corresponds to the three-vector, isoscalar part of the electromagnetic current, $V_{em} = V_v + V_s = V_3 + 3^{-1/2} V_8$. The chiral breaking equal-time commutator in (3.36) can be expressed in quark language as

$$\left[i \partial \cdot A_3(\vec{x}), \vec{V}_s(0) \right] = -\frac{1}{3} \hat{m} (\bar{u} \vec{\sigma} u - \bar{d} \vec{\sigma} d) \delta^3(\vec{x}) \quad . \qquad (3.37)$$

The nucleon matrix elements of (3.37) can be analyzed in terms of tensor currents and SU_6 symmetry [54]. In our quark model-infinite momentum frame approach of Section IIIA, we prefer to parallel the derivation of (3.6) and evaluate the nucleon matrix elements of (3.37) by keeping the leading spin-flip term in the light cone expansion of $\bar{q} \sigma_\perp q$ in (2.26c) combined with (2.24) or alternatively by evaluating $\langle \bar{N} | \vec{\sigma} | N \rangle$ and $\bar{q} \vec{\sigma} q$ in the infinite momentum frame while neglecting possible Z diagrams [6]. In either case, the result is the same, e.g.,

$$\langle p' \,|\, \bar{q}_i \vec{\sigma} q_i \,|\, p \rangle = \left(m_i / m_N \right) \tilde{g}_i \, \bar{u}(p') \vec{\sigma} u(p) \quad , \qquad (3.38)$$

where m_i is the quark mass of type i in the proton (i = u, d, s) and $\tilde{g}_i$ is the i^{th} quark probability distribution integral, similar to $\tilde{f}_i$ except that it "counts" the difference between + helicity and - helicity quarks (relative to the proton's helicity), whereas the $\tilde{f}_i$ counts their sum. These distribution integrals $\tilde{g}_i$ also appear in the nucleon matrix elements of $\bar{q}\gamma_5 q$, e.g.,

$$\langle p' | \bar{q}_i \gamma_5 q_i | p \rangle = \tilde{g}_i \bar{u}(p') \gamma_5 u(p) \tag{3.39}$$

where it is important to note that no quark mass factor occurs in (3.39) as they do in (3.6) and (3.38) because the leading term on the light cone in (2.28b) must transform like $\sigma_\perp$ (spin-flip) and this arises in combination with the first two terms of (2.28b) via $\sigma_3 \sigma_\perp \nabla_\perp$ rather than in combination with the quark mass term $\sigma_3 \mathcal{M}$. Alternatively, (3.39) can be deduced in the infinite momentum frame from the two-component spinor reduction of $\bar{q}\gamma_5 q$ to $i\vec{\sigma} \cdot (\vec{p}' - \vec{p})$, where no quark mass term appears. Finally, then, one may derive a sum rule for the $\tilde{g}_i$ in terms of the axial-vector ratio $g_A = 1.25$ by use of the quark model relation $\partial \cdot A_3 = -\hat{m} v_3$

$$\langle p' | \partial \cdot A_3 | p \rangle = -\hat{m} \langle p' | v_3 | p \rangle = -m_N g_A \bar{u}(p') \gamma_5 \tau_3 u(p) \tag{3.40}$$

The resulting sum rules are obtained by combining (3.38) - (3.40):

$$g_A = (\hat{m}/m_N)(\tilde{g}_u - \tilde{g}_d) \tag{3.41a}$$

$$\langle p' | \left[i\partial \cdot A_3(\vec{x}), \vec{V}_s(0) \right] | p \rangle = -\frac{1}{3} \hat{m} g_A \langle p' | \vec{\sigma} | p \rangle \quad . \tag{3.41b}$$

It is then possible to combine the threshold theorem (3.36) with the quark model sum rule (3.41b), and, with the estimate [54] $R_V^{(0)} \approx 0.12\, E_{0+}^{(0)}/m_\pi$ and $m_V \approx 850$ MeV, we have

$$.88 \frac{E_{0+}^{(0)}}{m_\pi} = -\frac{g_A}{4 m_N f_\pi} + \hat{m} \frac{g_A}{3 m_V^2 f_\pi} \quad . \tag{3.42}$$

This result can also be obtained from the tensor current-SU_6 analysis of ref. 54. Thus (3.42) can be used to extract an independent estimate of $\hat{m}$ provided $E_{0+}^{(0)} < 0$ is known to sufficient accuracy (there being a partial cancellation with the $g_A/4m_N f_\pi > 0$ term). With the use of the isotopic decompositions

$$E_{0+}^{(0)} = \frac{1}{2}\left(E_{0+}^{(\pi^o p)} - E_{0+}^{(\pi^o n)}\right) \tag{3.43a}$$

$$= \frac{1}{2\sqrt{2}}\left(E_{0+}^{(\pi^- p)} + E_{0+}^{(\pi^+ n)}\right), \tag{3.43b}$$

$E_{0+}^{(0)}$ can be extracted from the threshold extrapolations of the <u>sum</u> of two <u>small</u> numbers in (3.43a) (since $E_{0+}^{(\pi^o p)}$ amd $E_{0+}^{(\pi^o n)}$ are of opposite sign) or the <u>difference</u> of two <u>large</u> numbers in (3.43b) (since $E_{0+}^{(\pi^- p)}$ and $E_{0+}^{(\pi^+ n)}$ are also of opposite sign). In the former case, the dispersion-theoretic extrapolation of von Gehlen [56] yields

$$E_{0+}^{(0)} \approx -0.062\ m_\pi^{-3} \quad . \tag{3.44}$$

Two recent energy-independent analyses of low energy pion photoproduction data [57] work with isotopic 1/2 and 3/2 combinations of the multipole amplitudes near threshold and both obtain a value for $E_{0+}^{(0)}$ (i.e., by use of (3.43a) and (3.43b)) in almost perfect agreement with (3.44). It is therefore reasonable to apply (3.44) to (3.42) to find [6]

$$\hat{m} \sim 130\ \text{MeV}, \tag{3.45}$$

which is roughly the <u>same</u> estimate as found from $\sigma_{NN}^{\pi\pi}$, (3.24). This estimate (3.45) is also valid in the GMOR approach!

One could, in principle, apply the same technique to the isovector threshold multipole amplitude $E_{0+}^{(+)}$ to again constrain the non-strange quark mass [54]. We hesitate to do so, however, because $E_{0+}^{(+)}$ is even smaller than $E_{0+}^{(0)}$ and not nearly so well-determined [57]. Furthermore the $I = 3/2\ \Delta$ isobar can contaminate the $E_{0+}^{(+)}$ chiral breaking equation, whereas isospin conservation prevents it from contributing at all to (3.42). For these reasons, we concentrate only upon the $E_{0+}^{(0)}$ chiral breaking constraint, and given the apparent consistency between the independent phenomenological values of $\hat{m}$, (3.24) and (3.45), we conclude that our alternative to the GMOR scheme as proposed in Section IIIA is on a reasonably firm footing.

E. Goldberger-Treiman Corrections

In the chiral $SU_2 \times SU_2$ limit, the Goldberger-Treiman identity is

$$m_N g_A = f_\pi g_{\pi NN}(0) \quad , \tag{3.46}$$

where $g_{\pi NN}(0)$ corresponds to the zero-mass π^0 coupling constant with protons. For physical pions, therefore, the deviation from (3.46), as given by the discrepancy

$$\Delta_{\pi NN} = 1 - (m_N g_A / f_\pi g_{\pi NN}) \quad , \tag{3.47}$$

is a measure of chiral $SU_2 \times SU_2$ breaking [16, 58]. Experimentally, using $m_N = 938.9$ MeV, $g_A = 1.25$, $f_\pi = 93$ MeV, and $g_{\pi NN} = 13.40$, this discrepancy, including present measured and estimated errors, is [5]

$$\Delta_{\pi NN} = 0.058 \pm 0.013 \quad . \tag{3.48}$$

Chiral breaking matrix elements so far considered were not contaminated by pseudoscalar meson poles. In the case of Goldberger-Treiman discrepancies, however, since $\partial \cdot A_\pi = -\hat{m} v_\pi$ contains a pion pole, it must be removed before chiral breaking properties can be investigated. Calling $\langle N | \overline{v}_\pi | N \rangle$ the non-pole coefficient of the Dirac structure $\overline{u}_{p'} \gamma_5 u_p$ in $\langle N | v_\pi | N \rangle$, the πNN discrepancy can be obtained with the aid of (3.40) as

$$\Delta_{\pi NN} = \frac{\hat{m}}{f_\pi g_{\pi NN}} \langle N | \overline{v}_\pi | N \rangle \tag{3.49}$$

which manifests the chiral $SU_2 \times SU_3$ limit for $\hat{m} \to 0$. Likewise, the kaon Goldberger-Treiman discrepancies can be written in quark language as

$$\Delta_{KN\Lambda} = \frac{\frac{1}{2}\left(m_s + \hat{m}\right)}{f_K\, g_{NK\Lambda}} \langle N \,|\, \bar{v}_K \,|\, \Lambda \rangle \tag{3.50a}$$

$$\Delta_{KN\Sigma} = \frac{\frac{1}{2}\left(m_s + \hat{m}\right)}{f_K\, g_{NK\Sigma}} \langle N \,|\, \bar{v}_K \,|\, \Sigma \rangle \tag{3.50b}$$

where $g_{NK\Lambda} = g_{pK^+\Lambda}$ and $g_{NK\Sigma} = g_{pK^+\Sigma}$.

As the non-pole matrix elements $\langle B_f \,|\, \bar{v}_j \,|\, B_i \rangle$ are not measurable, it is necessary to relate them via SU_3 symmetry. To this end, we note that in our quark model scheme the spin-flip matrix elements $\langle B_f \,|\, v_j \,|\, B_i \rangle$, as represented by (3.39), do not contain quark mass factors as do the non-flip matrix elements $\langle B_f \,|\, u_j \,|\, B_i \rangle$ of (3.6) and (3.7). As such, $\langle B_f \,|\, v_j \,|\, B_i \rangle$ are effectively "good" matrix elements on the light cone and therefore should obey the GMOR SU_3 assumption once the pole terms are removed, i.e.,

$$\langle B_f \,|\, \bar{v}_j \,|\, B_i \rangle = \langle B \,\|\, \bar{v} \,\|\, B \rangle \left(d_v\, d_{fji} + f_v\, i f_{fji}\right) , \tag{3.51}$$

where $d_v + f_v = 1$. Combining (3.49) - (3.51), it is possible to express the quark mass ratio in the SU_3 breaking form [5]

$$X + 1 \approx \left(\frac{2}{\Delta_{\pi NN}}\right)\left[\frac{f_K}{f_\pi} A - \frac{m_N g_A}{f_\pi g_{\pi NN}} \left(\frac{m_\Sigma + m_\Lambda + 2 m_N}{4 m_N}\right) B\right] \tag{3.52}$$

where

$$A = \left(-\sqrt{3}\, g_{KN\Lambda} + g_{KN\Sigma}\right) / 2 g_{\pi NN} \tag{3.53a}$$

$$B = \left(-\sqrt{3}\, g_A^{KN\Lambda} + g_A^{KN\Sigma}\right) / 2 g_A \tag{3.53b}$$

both become unity in the SU_3 limit.

Following the discussion of ref. 5, we note that, experimentally, A and B now appear to be unity with roughly a 10% error since the combinations in (3.53) are very insensitive to SU_3 breaking. Moreover a reasonable estimate of f_K/f_π is 1.22 with perhaps a 2% error. With these values and (3.48), (3.52) implies

$$X \approx 6 \tag{3.54}$$

with roughly a 20% error. This is roughly the same as (3.26) and the cruder estimate (3.35), $X \sim 8$. Since we have also found $\hat{m}$ in two independent ways, (3.24) and (3.45), we are encouraged by the overall consistency of this scheme. By way of contrast here, the baryon Goldberger-Treiman discrepancies also imply the value $X \sim 6$ in the GMOR scheme due to the SU_3 approximation (3.51), while the meson formula (2.7) and (2.11) gives the very different value $X_{GMOR} \sim 25$.

F. Bag Model and Heavy Quark Models

One of the key motivations in describing the quark confinement in hadrons via a surface [59] "bag" potential is that, for massless quarks, the static SU_6 value of g_A is shifted from 5/3 down to 1.08, not far from the experimental value of 1.25. Recently, Golowich [60] has shown that for a non-strange quark mass of size

$$\hat{m} \approx 122 \text{ MeV}, \tag{3.55}$$

a modified bag model calculation gives $g_A = 1.25$. Even more recently, however, it was pointed out [61] that the experimental values of the πN σ term and the proton charge radius prefer a smaller value of $\hat{m}$, approximately

$$\hat{m} \approx 44 \text{ MeV} , \tag{3.56}$$

giving $g_A = 1.14$ and a quark mass ratio of

$$X \sim 7 \tag{3.57}$$

These results are strikingly similar to our conclusions, and while we do not understand the discrepancy between (3.55) and (3.56), we offer the following observations:

(i) The bag calculation of g_A involves $\hat{m}$ in an implicit dynamical manner; g_A is not proportional to an overall $\hat{m}$ factor. This is expected since $\bar{q}\gamma^+\gamma_5 q$ is a "good" operator on the light cone.

(ii) While our chiral breaking calculation of $\sigma_{NN}^{\pi\pi}$, (3.23), is proportional to $\hat{m}^2$, the bag model calculation explicitly displays only one factor of $\hat{m}$, the other $\hat{m}$ factor perhaps being implicit in the bag wave functions. "Wee" quark dynamics not presently incorporated in bag models, however, alter the $\sigma_{NN}^{\pi\pi}$ bag estimate so as to support the higher value (3.55).

(iii) While (3.55) and (3.56) differ by a factor of three, both estimates are much greater than the presently preferred [62] GMOR value of $m \sim 5$ MeV.

In either the chiral symmetry breaking picture or the bag model, the masses $\hat{m}$ and m_s may be taken to zero, in which limit the $SU_3 \times SU_3$ or bag model modified SU_6 limits are respectively recovered. Alternative pictures [63] have been suggested recently, based upon the non-relativistic Fermi-Breit type of reduction of a vector gluon potential. In this form of reduction the $\hat{m} \to 0$ limit cannot be taken because of $1/\hat{m}$ factors. Phenomenologically these pictures explain both the SU_6 breaking decuplet mass differences and the $\Sigma - \Lambda$ mass difference, provided one takes

$$\hat{m} = 336 \text{ MeV} \qquad \text{and} \qquad X = \frac{m_s}{\hat{m}} = 1.6 \tag{3.58}$$

(Note that these are approximately the values obtained in the weak binding limit,

$\hat{m} = \frac{1}{3} m_N \approx 313$ MeV and $m_s = \frac{1}{3} m_{\Omega^-}$.) In our approach we accommodate naturally the $\Sigma - \Lambda$ mass difference but have no definite prediction for the octet-decuplet splitting. Certainly the non-relativistic reduction is appropriate in the charmed quark sector but its application to the highly relativistic normal baryons may be misleading, particularly in light of the difficulties with the chiral limit.

Yet other approaches to chiral symmetry breaking, within the context of σ models [64], have been suggested, which yield large quark masses and a value of X similar to that in (3.58).

IV. CHIRAL SYMMETRY BREAKING MESON MATRIX ELEMENTS

A. Constraints on $\langle 0 | v_P | P \rangle$

The fundamental pseudoscalar meson relation (2.7) can be expressed in quark language as (with $f_K = f_\pi$)

$$\frac{m_K^2}{m_\pi^2} = \frac{1}{2}(1 + X) \frac{\langle 0 | v_K | K \rangle}{\langle 0 | v_\pi | \pi \rangle} \tag{4.1}$$

If we relax the GMOR SU_3 assumption (2.11), then (4.1) does not constrain X to the GMOR value of 25. Instead, as in the case of baryon matrix elements, one must investigate v_i to determine its SU_3 structure.

Assuming again that the hadronic (pseudoscalar) states transform as irreducible representations of the SU_3 group generated by Q_i^L, as in (2.30), means that $\langle 0 | v_j | P_k \rangle$ is not required to have the Wigner-Eckart structure δ_{jk} as in (2.10) unless the first (spin-flip) term in the light cone expansion (2.28b) controls $\langle 0 | v_P | P \rangle$. In this case, there is no argument based on fixed poles in hadronic amplitudes or Z-graphs to decide the issue; however, the non-spin flip structure of $\langle 0 | v_P | P \rangle$ is similar to the spin averaged or non-flip behavior of $\langle B | u_i | B \rangle$ occurring in mass formulae and σ-terms. Therefore, in parallel with our baryon analysis, we will assume that the quark mass (non-flip) term in (2.28b) dominates $\langle 0 | v_P | P \rangle$. In this case

$$\langle 0 | v_\pi | \pi \rangle \sim 2\hat{m} \qquad \langle 0 | v_K | K \rangle \sim m_s + \hat{m} \ . \tag{4.2}$$

Caser and Testa [65] give the infinite momentum frame interpretation of (4.2), noting that such a structure corresponds to the "direct" quark-anti-quark infinite momentum saturation $\langle 0 | v_P | \bar{q} q \rangle \langle q \bar{q} | P \rangle$, while "exchange" saturation correlates with the GMOR SU_3 structure (2.11). In our language, if we compute $\langle 0 | v_P | P \rangle$ in the infinite momentum frame by representing $|P\rangle$ as a superposition of on-shell quark and antiquark spinors, sharing equally the $|P\rangle$ momentum, then (4.2) results. This procedure is equivalent to a weak binding approximation which was one condition under which our earlier choice $C_6^+ = 0$ was valid in the baryon analysis.

Given (4.2), the ratio

$$\langle 0 | v_K | K \rangle / \langle 0 | v_\pi | \pi \rangle = \frac{1}{2}(1 + X) \tag{4.3}$$

can be combined with (4.1) and then leads to the meson-determined quark mass ratio [21]

$$X = 2 m_K / m_\pi - 1 \approx 6 \tag{4.4}$$

or alternatively $c \approx -0.9$ from (2.21). We believe it is significant that (4.4) is consistent with $X \approx 5 - 8$, as obtained from the baryon mass formulae, fixed poles, σ-terms and Goldberger-Treiman discrepancies.

An alternative derivation of (4.4) follows from $SU_3 \times SU_3$ considerations on the light cone. Sazdjian and Stern [21] note that the quark mass term in $\partial \cdot A_i$ (i.e., in v_i of (2.28b)) transforms as $(1, 8)_L$ on the light cone. Octet light cone states transforming like $(1, 8)_L$ can also be constructed by operating the vector and axial-vector currents V^+ and A^+ on the vacuum. The simplest state constructed in this way is the $j = 0$ "octet-axial" state dominated by the pion for $Y = 0$ and kaon for $Y = 1$ in the low mass region, and one is therefore led back to (4.2) - (4.4). Although these low lying $SU_3 \times SU_3$ light cone states are the simple hadron states, which by themselves are divorced from any light cone considerations, the above procedure was made more convincing by a parallel treatment of the $j = 1$ light cone vector and axial-vector light cone states [21] giving results similar to the usual saturation of the Weinberg first and second sum rules.

Another argument, by Fuchs [24], begins with the light plane expression for v_i, (2.28b), and then transforms it to the constituent quark basis by applying the free Melosh transformation (2.31). To leading order in quark orbital angular momentum, one is again led to the structure (4.2). Finally, in the present case, an argument for (4.2), to be presented later, may be made on the basis of spontaneous symmetry breaking of the Nambu-Jona-Lasinio type. For the moment, we proceed to show that the phenomenology of meson chiral symmetry strongly supports (4.2).

B. The PCAC Approximation

One of the central inputs in any chiral breaking theory is the PCAC approximation. For baryon matrix elements such as σ_{NN}^{KK} or the Goldberger-Treiman corrections, chiral breaking and PCAC effects are independently accounted for. For pseudoscalar states, however, the two effects can become subtly intertwined. In the GMOR scheme, the SU_3 assumption (2.10) coupled with the $(3, \bar{3})$ commutation relation (2.5b) implies that the PCAC operation for all $q_i \to 0$ in $\langle P_i | u_j | P_k \rangle$ (this we term SU_3 PCAC) is exact for $i = 1, \ldots, 8$ (provided $f_\pi = f_K = f_{\eta_8}$).

In the chiral breaking scheme described in Section IIIA, as noted earlier, the fact that u_π and u_K (or v_K and v_π) are not in the same SU_3 multiplet (whereas $u_\pi/\hat{m}$ and $u_K/(m_s + \hat{m})$ are in the same multiplet) means that "SU_3 PCAC" will not be exact and in fact may be a bad approximation in some cases. On the other hand, SU_3 PCAC ought to be a reasonable approximation in other cases.

In particular, the SU_3 PCAC limit of soft π, K, and η_8 applied to the diagonal pseudoscalar matrix elements of the chiral breaking Hamiltonian density H' yields [3] as $q_i \to 0$,

$$\langle P_i | H' | P_i \rangle \approx -if_i^{-1} \langle 0 | [Q_i^5, H'] | P_i \rangle$$

$$\approx f_i^{-1} \langle 0 | \partial \circ A_i | P_i \rangle \approx m_i^2 \ , \tag{4.5}$$

independent of the chiral breaking transformation properties of H'. Furthermore, (4.5) is a powerful constraint on the theory, but one that is reasonable in the light of the Goldstone property [66] of H_0,

$$\lim_{q_i \to 0} \langle P_i | H_0 | P_i \rangle = 0 \ . \tag{4.6}$$

More specifically, GMOR noted that in the free meson model

$$\langle P_i | H_0 | P_j \rangle = \delta_{ij} (q_{0i} q_{0j} + \vec{q}_i \circ \vec{q}_j) \tag{4.7}$$

vanishes according to (4.6). However it is clear that (4.7) is a rapidly varying (SU_3 singlet) object which is $m_i^2 \delta_{ij}$ on-mass shell in the rest frame. When the latter value is combined with the total Hamiltonian constraint [29]

$$\langle P_i | H | P_i \rangle_{rest} = 2m_i^2 \tag{4.8}$$

along with $H \doteq H_0 + H'$, one deduces that (4.5) holds in the rest frame and should therefore be approximately valid in the soft limit because H' is not presumed to be rapidly varying. Thus one assumes

$$\langle \pi | H' | \pi \rangle \approx m_\pi^2 \qquad \langle K | H' | K \rangle \approx m_K^2 \tag{4.9}$$

are always valid; we shall return to the implications of (4.9) in the next section. One should not infer from (4.9), however, that the off-diagonal matrix elements of $\langle P_i | u_j | P_k \rangle$ need obey the SU_3 PCAC property, for if they did, the GMOR SU_3 assumption (2.10) would be inescapable. We shall return to this point later.

Finally, one must investigate the SU_3 PCAC property as it applies to $\langle 0 | v_P | P \rangle$. Assuming it to be valid leads to the ratio (with $f_K = f_\pi$)

$$\frac{<0|v_K|K>}{<0|v_\pi|\pi>} = \frac{\sqrt{2} - \frac{1}{2}b}{\sqrt{2} + b} \tag{4.10}$$

where

$$b \equiv \frac{<0|u_8|0>}{<0|u_0|0>} \quad . \tag{4.11}$$

In the GMOR scheme, b measures the SU_3 breaking of the vacuum and must be small if (4.10) is to remain near unity as required by the GMOR SU_3 assumption. On the other hand, in our approach u_8 does not transform like λ_8 for single particle baryon states and one might suspect the same holds true for vacuum matrix elements. We shall investigate this possibility in Section IVD. To the extent that this is true, b need <u>not</u> vanish and the ratio in (4.10) need not be unity while preserving the PCAC approximation applied to $<0|v_P|P>$. In other words, the Coleman theorem [67], stating that "the symmetry of the vacuum is the symmetry of the world," could still apply to our scheme of chiral symmetry breaking but with a non-vanishing value of $<0|u_8|0>$ because u_8 could contain a part transforming like λ_0.

C. Constraints on $<P|u_i|P>$

Following the pattern of $<B|u_i|B>$ (non-flip) and $<0|v_P|P>$, it is reasonable to assume that the dominant term in the (non-flip) matrix elements $<P|u_i|P>$ correspond to the quark mass term in the light cone representation (2.28a). Thus in our chiral breaking approach, the analogs of (3.6) and (3.7) for pseudoscalar meson states are (for $m_u = m_d \equiv \hat{m}$)

$$\begin{aligned} &<\pi|\bar{u}u|\pi> = <\pi|\bar{d}d|\pi> = 2\hat{m}\tilde{h}, \ <\pi|\bar{s}s|\pi> = 2m_s\tilde{h}_s, \ <K^\pm|\bar{u}u|K^\pm> = 2\hat{m}\tilde{h}, \\ &<K^\pm|\bar{d}d|K^\pm> = 2\hat{m}\tilde{h}_s, \ <K^\pm|\bar{s}s|K^\pm> = 2m_s\tilde{h}, \ <K_{6,7}|\bar{u}u|K_{6,7}> = 2\hat{m}\tilde{h}_s, \\ &<K_{6,7}|\bar{d}d|K_{6,7}> = 2\hat{m}\tilde{h}, \ <K_{6,7}|\bar{s}s|K_{6,7}> = 2m_s\tilde{h}, \ <\eta_8|\bar{u}u|\eta_8> = \\ &= <\eta_8|\bar{d}d|\eta_8> = \frac{2}{3}\hat{m}(\tilde{h} + 2\tilde{h}_s), \ <\eta_8|\bar{s}s|\eta_8> = \frac{2}{3}m_s(4\tilde{h} - \tilde{h}_s). \end{aligned} \tag{4.12}$$

Given the PCAC conditions (4.9), the equations (4.12) can be applied to H' $= \hat{m}(\bar{u}u+\bar{d}d) + m_s\bar{s}s$ to obtain

$$m_\pi^2 \approx 4\hat{m}^2\tilde{h} + 2m_s^2\tilde{h}_s \tag{4.13a}$$

$$m_K^2 \approx 2(\hat{m}^2+m_s^2)\tilde{h} + 2\hat{m}^2\tilde{h}_s\ , \tag{4.13b}$$

whereas $\sigma^{\pi\pi} = \hat{m}(\bar{u}u+\bar{d}d)$ leads to

$$\sigma_{\pi\pi}^{\pi\pi} = 4\hat{m}^2\tilde{h}\ . \tag{4.14}$$

Since π PCAC implies $\sigma_{\pi\pi}^{\pi\pi} = m_\pi^2$ which appears to be in reasonable agreement with experiment, a comparison of (4.13a) with (4.14) reveals $m_s^2\tilde{h}_s$ as small, and $\hat{m}_s \gg \hat{m}$ then indicates that

$$\tilde{h}_s \approx 0\ . \tag{4.15}$$

Combining (4.15) with (4.13) leads to the PCAC prediction of the quark mass ratio,

$$X_{PCAC} = \sqrt{2m_K^2/m_\pi^2 - 1} \approx 5\ . \tag{4.16}$$

Once again we note the apparent consistency between the value of X as derived from the mesons and from the baryons. The slight difference between the meson values (4.4) and (4.16) is a measure of K PCAC corrections to (4.13b). In our approach it is not an accident that

$$X_{PCAC}^2 = X_{GMOR} \tag{4.17}$$

because of the inherent quadratic quark mass dependence of $H' = \hat{m}^2(\bar{u}u+\bar{d}d)/\hat{m} + m_s^2\bar{s}s/m_s$ with $\bar{q}q/m_q$ having simple SU_3 transformation properties.

We note, however, that SU_3 PCAC cannot be applied to the off-diagonal matrix elements $\langle\pi|u_K|K\rangle$ or $\langle\eta_8|u_K|K\rangle$ in our scheme. Further, assuming π PCAC is good (i.e., $\tilde{h}_s \approx 0$), implies that K PCAC will be bad for $\langle K|u_{3,8}|K\rangle$.

While these matrix elements do not affect (4.9) or (4.10), we do not fully understand why SU_3 PCAC should fail in our approach for just the above four matrix elements.

Finally, although the quark mass scale is difficult to set for the mesons, if we assume it is the same as found for the baryons, i.e., $\hat{m} \sim m_\pi$, then (4.13a) and (4.15) indicate that $\tilde{h} \sim \frac{1}{4}$. While this is not near the valence value $\tilde{h} = 1$, the structure of the quark-antiquark mesons and the implied Regge subtractions in $\tilde{h}$ (as in the $\tilde{f}$'s) does not make such a connection compelling in our scheme. In other words, comparisons between baryon and meson chiral breaking matrix elements such as via quark counting are suspect in our approach.

D. Constraints on $\langle 0|u_i|0\rangle$

Paralleling the analysis for $\langle P|u_i|P\rangle$, we note that since the vacuum has no spin-flip component, the quark mass term ought to dominate the light cone representation (2.28a) for vacuum matrix elements. In this case $\langle 0|u_8|0\rangle \sim m_s - \hat{m}$ which does not vanish for $m_s \neq \hat{m}$ and (4.11) then becomes [68]

$$b = -\sqrt{2}\left(\frac{X-1}{X+2}\right) = c\ . \qquad (4.18)$$

Since $c \sim -0.8$ to -0.9, it is clear from (4.18) that the SU_3 symmetry of the vacuum is not measured by b being small in our approach. Instead, (4.18) is a simple realization that u_8 contains a large λ_0 as well as a λ_8 piece (in the quark mass matrix term).

Given (4.18), one can see that in fact the SU_3 PCAC property can be applied to $\langle 0|v_P|P\rangle$ because (4.10) is then

$$\frac{\langle 0|v_K|K\rangle}{\langle 0|v_\pi|\pi\rangle} = \frac{\sqrt{2} - \frac{1}{2}c}{\sqrt{2} + c} \sim 3 \qquad (4.19)$$

which is identical to (4.3). Alternatively, assuming SU_3 PCAC to be valid in this case again leads to (4.4) and $X \approx 6$. In short, there is a pattern of internal consistency for meson matrix elements in our chiral breaking approach which is similar to the consistency within the GMOR scheme. The value of the vacuum expectation ratio, b, must be as given in (4.18) if $<0|v_K|K> \propto m_s + \hat{m}$, etc., (4.2), and if PCAC is valid for both the expectation values $<0|v_K|K>$ and $<0|v_\pi|\pi>$.

Theoretically the vacuum expectation value $<0|\bar{q}q|0>$ is trivially proportional to $2m_q$ where m_q is the quark mass in the spinor multiplying the creation and annihilation operators in the field q. Our model requires that such spinors always be solutions of the Dirac equation with the full quark mass, whether they appear here or in expectation values of the mass Hamiltonian H'. (This is equivalent to using the full mass matrix $\mathcal{M}$ in writing 2.28 for $\bar{q}\lambda_i q$ and $\bar{q}\gamma_5\lambda_i q$. The vacuum is, by definition, then a vacuum with respect to the massive quark field operators; and the expectation value of the number operator for massive quarks in a hadron must transform simply under SU_3. This type of approach appears to be consistent with models in which the entire quark mass is generated spontaneously or self-consistently. Indeed the original Nambu-Jona-Lasino [69] model for spontaneous mass generation begins with a Lagrangian with a 4-fermion interaction but without a mass term. Solutions exist for which the ground state of the theory is not a vacuum with respect to the original massless quark field annihilation operator; rather one defines a new vacuum $|0_m>$ and a massive quark field q_m such that

$$< 0_m|\bar{q}_m q_m|0_m> \propto 2m \tag{4.20}$$

Perturbation theory is performed in the $|0_m>$ basis and the perturbing Lagrangian, for consistency, must not generate additional quark mass. The spontaneous generation must not maintain SU_3 symmetry if $m_s \neq \hat{m}$ and if all mass arises in this fashion.

This is similar to what happens in a σ model where the quark mass is generated by a $\sigma_q \bar{q}q$ interaction term. To obtain unequal quark masses $\langle\sigma_q\rangle_0$ must be different for strange and non-strange quarks. To the extent that σ represents a Hartree-Fock approximate form for the quark operator $\langle\bar{q}q\rangle_0$ itself, i.e.,

$$\langle\sigma_q\rangle_0 \sim \langle\bar{q}q\rangle_0 \tag{4.21}$$

we again obtain (4.18). Again quark field operators "know" the full quark mass.

E. The Ninth Pseudoscalar State

The quark model naturally extends the octet of pseudoscalars to a nonet, and the possibility then arises of mixing the SU_3 octet state η_8 with the singlet state η_1. A number of ambiguities and difficulties arise, however, in implementing this effect in chiral symmetry breaking theories.

The first ambiguity lies in the nature of the octet meson mass formula. The structure of this relation is seen to be quadratic in the masses once one assumes the SU_3 PCAC relation (4.5),

$$m_\pi^2 + 3m_{\eta_8}^2 - 4m_K^2 = 0 . \tag{4.22}$$

This quadratic relation can also be derived by using the Ademollo-Gatto theorem on the matrix elements $\langle\pi^0|V^\mu_{4-i5}|K^+\rangle$ and $\langle\eta_8|V^\mu_{4-i5}|K^+\rangle$ by use of the usual SU_3 transformation properties. The Heisenberg equations of motion imply $\partial V_{4-i5} \sim u_{4-i5}$ (note that since only one component of u appears, one does not have to worry about its "bad" SU_3 transformation properties [37]). While (4.22) implies $m^2_{\eta_8} \approx 17\, m_\pi^2$ whereas $m_\eta^2 \approx 16\, m_\pi^2$, it is possible that this octet relation can be altered by a $\underline{27}$ piece, presumably of $O(H'^2)$. Such an effect can be seen in the baryon mass formulae, where it is roughly 3% of the baryon mass (see Section IIIB). This could shift $m^2_{\eta_8}$ by about m_π^2 from the value of $17\, m_\pi^2$

predicted by (4.22).

The second ambiguity concerns the identity of the isoscalar pseudoscalar meson which mixes with the physical η. The conventional choice is the $\eta'(958)$, with the mixing angle defined by

$$|\eta\rangle = \cos\theta \, |\eta_8\rangle - \sin\theta \, |\eta_1\rangle$$
$$|\eta'\rangle = \sin\theta \, |\eta_8\rangle + \cos\theta \, |\eta_1\rangle \tag{4.23}$$

with $m^2_{\eta_1} + m^2_{\eta_8} = m^2_\eta + m^2_{\eta'}$. The pure octet mass formula (4.22) predicts $m^2_{\eta_8} \approx 17m^2_\pi$ or $\theta \approx -11^\circ$, while a $\underline{27}$ contaminated mass formula could give $m^2_{\eta_8} \approx 18m^2_\pi$ or $\theta \approx -14^\circ$. The various meson decays and high energy charge exchange cross sections are reasonably consistent with quark model singlet to octet ratios and an η-η' mixing angle somewhere between -10° and -20°, with $\theta \sim -10^\circ$ now favored because of the reduction of the experimental width of the η [72]. It has been suggested, however, that it is the E(1420) rather than the $\eta'(958)$ which mixes with the η; in this case, $\theta \sim -6^\circ$. Finally, three particle mixing (η, η', gluons?) may occur rather than the simple two particle mixing scheme of (4.23).

The main difficulty with the ninth pseudoscalar meson arises because of the existence of a ninth axial-vector current in quark models [73]. At this point, it is simplest to use a basis constructed of non-strange and strange quarks:

$$|\eta_{NS}\rangle = \frac{1}{\sqrt{3}}(\sqrt{2}|\eta_1\rangle + |\eta_8\rangle) = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle)$$
$$|\eta_S\rangle = \frac{1}{\sqrt{3}}(|\eta_1\rangle - \sqrt{2}|\eta_8\rangle) = |\bar{s}s\rangle \tag{4.24}$$

In this basis the axial current divergences reveal their simplest form:

$$\partial A_i = \begin{cases} -\hat{m} v_i & i = 1,2,3,NS \\ -\frac{1}{2}(\hat{m} + m_s)v_i & i = 4,5,6,7 \\ -m_s v_i & i = S \end{cases} \tag{4.25}$$

Combining (4.25) and (2.6) with $f_\pi = f_8 = f_0$ (assuming U_3 symmetry) together with the analogs of (4.2), one can readily derive in our scheme

$$\frac{m^2_{\eta_{NS}}}{m^2_\pi} = \frac{<0|\partial A_{NS}|\eta_{NS}>}{<0|\partial A_\pi|\pi>} = \frac{<0|v_{NS}|\eta_{NS}>}{<0|v_\pi|\pi>} = 1 \tag{4.26}$$

$$\frac{m^2_{\eta_S}}{m^2_\pi} = \frac{<0|\partial A_S|\eta_S>}{<0|\partial A_\pi|\pi>} = X\frac{<0|v_S|\eta_S>}{<0|v_\pi|\pi>} = X^2 . \tag{4.27}$$

These results are <u>identical</u> to the GMOR values with $X^2 \approx 25$ in (4.27) replaced by $X \approx 25$ in the latter analysis. The physical ninth pseudoscalar particle ($\eta_9 = \eta'$ or E) then has a mass, given $m^2_9 + m^2_\eta = m^2_{\eta_S} + m^2_{\eta_{NS}}$ or $m_9 \sim 3m_\pi$ in this U_3 limit (but independent of any SU_3 PCAC assumptions). Clearly no such light isoscalar pseudoscalar meson exists [74,75].

One way out of this difficulty is to assume that the ninth axial charge does not commute with H_0: $[Q^5_0, H_0] = ig$ and thus $\partial A_0 = g - \sqrt{\frac{2}{3}}\,(\epsilon_0 v_0 + \epsilon_8 v_8)$. <u>If</u> g is not a total divergence, this then indicates that η_1 is not a Goldstone boson, and the analog of (4.6) reads in this case

$$\lim_{q\to 0} <\eta_1|H_0|\eta_1> = m^2_0 \tag{4.28}$$

One can then proceed as in (4.5-9) or as in (4.26,27) to find [76]

$$m^2_{\eta_{NS}} = m^2_\pi + \frac{2}{3}m^2_0 \tag{4.29}$$

$$m^2_{\eta_S} = X^2 m^2_\pi + \frac{1}{3}m^2_0 . \tag{4.30}$$

The constraints of (4.22) and (4.29,30) coupled with $m^2_9 + m^2_\eta = m^2_{\eta_{NS}} + m^2_{\eta_S} = m^2_{\eta_8} + m^2_{\eta_1}$ and the orthogonality of the physical states $<\eta_9|H|\eta> \sim <\eta_9|\eta> = 0$ then is sufficient to determine the system. In particular we find

$$m_9^2 = m_{\eta_8}^2 + \frac{\left[(2X^2-1)m_\pi^2 - m_{\eta_8}^2\right]^2}{8(m_{\eta_8}^2 - m_\eta^2)} . \tag{4.31}$$

This formula, however, is <u>exceedingly</u> sensitive to the value of m_{η_8} (which is only approximately known through the mass formula (4.22)) and even more sensitive to the value of X. For instance, for $X = 5$, $m_{\eta_8}^2 = 17\, m_\pi^2$, one finds $m_9 =$ 1685 MeV, which might indicate [37] that the ninth pseudoscalar meson is the E(1420); on the other hand, the values $X = 5$, $m_{\eta_8}^2 = 20\, m_\pi^2$ or $X = 4.1$, $m_{\eta_8}^2 =$ $17\, m_\pi^2$ give $m_9 \approx m_{\eta'}$. Furthermore, the derivation of (4.31) involves approximations, and so the expression is of little practical value. In gauge models, an Adler-type vector gluon anomaly gives rise to a term g in ∂A_0. Unfortunately this term is itself a total divergence, and a new axial-vector current can be defined which is divergenceless in the chiral limit. Hence m_0 in (4.28) is zero and the solution to the ninth pseudoscalar meson problem outlined above is invalid. Kogut and Susskind [77] have suggested that the particle associated with the ninth axial vector particle is actually a dipole consisting of a positive and negative metric boson; the singularities due to this dipole cancel out of any physical matrix element, and hence there is no physical ninth pseudoscalar meson.

In a theory with a Goldstone symmetry breaking interaction for η, such as is implied by (4.29,30), Caser and Testa [37] have explored the possibility of three-particle mixing, where the third particle is a multigluon external state. They find that the glue state does not mix and conclude in effect that (4.31) holds; thus they claim the ninth pseudoscalar meson is the E(1420) but, as has been noted, this conclusion is unwarranted. Fuchs [78], on the other hand, has considered a similar scheme but judged (4.30) to be of questionable validity; rejecting this equation, he found the η' to have a large component of glue and the other glue plus quark state to have a high mass.

In any event, one may conclude from the above discussion that the ninth pseudoscalar meson state does not determine the quark mass ratio or scale in a model independent way. In fact, in the conventional η - η' mixing scheme, both the GMOR and our quark model breaking theory lead to the same picture of the ninth meson state.

F. Meson σ Terms

Given the quark model definition $\sigma^{\pi\pi} = \hat{m}(\bar{u}u + \bar{d}d)$, the meson matrix elements in our chiral breaking scheme can be computed with the aid of (4.12) and their U_3 singlet analogs [76]:

$$\sigma^{\pi\pi}_{KK}/\sigma^{\pi\pi}_{\pi\pi} = \tfrac{1}{2}[1 + (\tilde{h}_s/\tilde{h})] \tag{4.32a}$$

$$\sigma^{\pi\pi}_{\eta_8\eta_8}/\sigma^{\pi\pi}_{\pi\pi} = \frac{1}{3}[1 + 2(\tilde{h}_s/\tilde{h})] \tag{4.32b}$$

$$\sigma^{\pi\pi}_{\eta_1\eta_1}/\sigma^{\pi\pi}_{\pi\pi} = \frac{2}{3}[1 + \tfrac{1}{2}(\tilde{h}_s/\tilde{h})] \tag{4.32c}$$

$$\sigma^{\pi\pi}_{\eta_1\eta_8}/\sigma^{\pi\pi}_{\pi\pi} = \frac{\sqrt{2}}{3}[1 - (\tilde{h}_s/\tilde{h})] \,. \tag{4.32d}$$

Recalling that the SU_3 PCAC statement (4.9) leads to (4.15) as well as the reasonable estimate (4.16), the pion PCAC value $\sigma^{\pi\pi}_{\pi\pi} = m^2_\pi$ and $\tilde{h}_s \approx 0$ convert (4.32) to

$$\begin{array}{ll} \sigma^{\pi\pi}_{KK} \approx \tfrac{1}{2}m^2_\pi & \sigma^{\pi\pi}_{\eta_8\eta_8} \approx \frac{1}{3}m^2_\pi \\ \sigma^{\pi\pi}_{\eta_1\eta_1} \approx \frac{2}{3}m^2_\pi & \sigma^{\pi\pi}_{\eta_1\eta_8} \approx \frac{\sqrt{2}}{3}m^2_\pi \,, \end{array} \tag{4.33}$$

which are precisely the values in the GMOR scheme. Here again a distinction between the two $(3,\bar{3})$ chiral breaking models in terms of a quark mass ratio or quark mass scale cannot be made from the meson σ terms.

Phenomenologically, therefore, estimates consistent with (4.33) serve only to reconfirm the underlying $(3,\bar{3})$ structure of H'. The $K\pi$ low energy scattering lengths are not yet known with sufficient accuracy to extract $\sigma^{\pi\pi}_{KK}$. While a recent estimate of $\sigma^{\pi\pi}_{KK}$ from the $\Delta I = 3/2$ Dalitz plot $K_{3\pi}$ slopes gave [79] $3/2\ m^2_\pi$ to $2m^2_\pi$, it was based on an assumption concerning the momentum variation of $<\pi|H^{3/2}_W|K>$, which perhaps is incorrect. If the $\eta' \to \eta\pi\pi$ decay amplitude were pure σ-term [5,80], the value implied by (4.33),

$$\sigma^{\pi\pi}_{\eta'\eta} = -\cos\theta\sin\theta(\sigma^{\pi\pi}_{11} - \sigma^{\pi\pi}_{88}) + (\cos^2\theta - \sin^2\theta)\sigma^{\pi\pi}_{18}$$

$$\approx 0.5\ m^2_\pi\ , \tag{4.34}$$

would be roughly one-sixth experiment, as implied by the conventional $\eta - \eta'$ mixing picture with $\theta \approx -10^\circ$. Since the $\eta' \to \eta\pi\pi$ rate goes as [5] $\Gamma_{theory}/\Gamma_{experiment} \sim \sin^4\theta$, it might appear that $\theta \sim -20^\circ$. Alternatively, it has been suggested [81] that the nearby $\delta(970)$ resonance enhances the $\eta' \to \eta\pi\pi$ rate and effectively masks $\sigma^{\pi\pi}_{\eta'\eta}$. Finally, the σ term $\sigma^{\pi\pi}_{\eta\eta}$ given by (4.33) as

$$\sigma^{\pi\pi}_{\eta\eta} = \cos^2\theta\ \sigma^{\pi\pi}_{88} - 2\sin\theta\cos\theta\ \sigma^{\pi\pi}_{18} + \sin^2\theta\ \sigma^{\pi\pi}_{11} \approx 0.5\ m^2_\pi \tag{4.35}$$

can only be extracted from the $\eta \to 3\pi$ electromagnetic decay amplitude and we postpone discussion of it until Section V.

G. Meson-Meson Amplitudes

Working with the assumption that the isotensor σ term in $\pi\pi$ scattering is zero, which is an automatic consequence in the quark model, Weinberg [15] was able to obtain (2.9) as well as the low energy analytic expansion of the $\pi\pi$ amplitude

$$f^2_\pi <\pi_a\pi_b|T|\pi_c\pi_d> = \delta_{ab}\delta_{cd}(s-m^2_\pi) + \delta_{ac}\delta_{bd}(t-m^2_\pi) + \delta_{ad}\delta_{bc}(u-m^2_\pi) \tag{4.36}$$

where the Mandelstam invariants s,t,u are defined in the usual way and satisfy

$s+t+u = q_a^2 + q_b^2 + q_c^2 + q_d^2$. One might hope to extend (4.36) to the SU_3 partners of the pion. Osborn [81], working in the GMOR chiral breaking scheme, in effect assumed the validity of (4.33) and SU_3 PCAC in its strong GMOR sense and obtained, for example, for $\pi_{a,c} \to \pi$ and $P_{b,d} \to K$ or η_8

$$<\pi' P | T | \pi P> = \frac{\sigma_{PP}^{\pi\pi}}{f_\pi^2}\left[(1 - \beta_P)\left(\frac{q'^2_\pi + q_\pi^2}{m_\pi^2} - 1\right) + \beta_P\left(\frac{t}{m_\pi^2} - 1\right)\right] \tag{4.37}$$

with $\beta_K = \frac{1}{2}$ and $\beta_{\eta_8} = 0$. Pagels and co-workers were able to extend this result to a complete $SU_3 \times SU_3$ generalization of (4.36) by the replacement of $\delta_{ab}\delta_{cd}$ with $\frac{2}{3}\delta_{ab}\delta_{cd} + d_{abe}d_{ecd}$ where e is summed from 1 to 8, using it extensively in their chiral perturbation theory [17,82].

While results like (4.37) are similar to the baryon analog (3.20), manifesting the Adler zero and the sign change at the on-shell point $t = 2m_\pi^2$, the specific values of β_K and β_{η_8} very definitely depend upon the strongest version of kaon and eta PCAC, respectively. In Section IVB we have stressed that such a strong version of SU_3 PCAC is valid only in the GMOR scheme, while in our quark model scheme, kaon PCAC does have large corrections in general and applied to (4.37) in particular, although it appears to have small corrections in (4.9). On the other hand, η_8 PCAC appears to be well approximated in $<\eta_8|u_i|\eta_8>$, so perhaps $\beta_{\eta_8} = 0$ is a reasonable estimate in our quark model chiral breaking scheme. There also exists, however, the problem of extrapolating the Adler zero for one soft η_8 at $t = m_8^2 \approx 18\ m_\pi^2$ down within the analytic circle of convergence determined by the 2π cut in (4.37) to be $t = 4m_\pi^2$.

H. Other Meson Decays

The only meson process which appears to have a bearing on the distinction between the GMOR and our quark model scheme of chiral symmetry breaking is

$K_{\ell 3}$ decay, involving the chiral breaking object

$$<0|[Q_K, \partial\cdot A_\pi]|K> \sim f_K m_K^2 \hat{m}/(m_s+\hat{m}) = (X+1)^{-1} f_K m_K^2 \, . \tag{4.38}$$

Using (non-covariant) charge commutators, Hakim, Legonini, and Paver obtain the consistency relation [83]

$$(X-1)\hat{m}^2 = \frac{2m_K^2}{X+1} - \frac{m_\pi^2}{(f_K/f_\pi f_+(0))} \tag{4.39}$$

which, in covariant language, presumably unifies the current algebra Ward identities obtained from V_K^μ and $\partial\cdot V_K$. This relation can also be obtained from light cone considerations [84] if the LHS is weighted by a scaling integral estimated to be near unity. The interesting aspect of (4.39) is that it admits the two solutions $X \sim 25$, $\hat{m} \sim .01\, m_\pi$ (GMOR) and $X \sim 5$ and $\hat{m} \sim m_\pi$ which is near the values suggested by our analysis. With further model-dependent assumptions concerning the kappa meson, it is possible, however, to conclude that the GMOR scheme is consistent with a divergence form factor slope $\lambda_0 > 0$, while our version would imply $\lambda_0 < 0$. Experiment has yet to decide conclusively between these two possibilities [72].

Chiral breaking pion mass corrections could, in principle, be important in $\pi^o \rightarrow 2\gamma$ decay because the pion PCAC scale is presumably set by the Sutherland zero [85] and not by the Adler anomalous correction [86] to the Ward identity. In fact, a subtraction constant in the (neutral) PCAC dispersion relation, generated by the Bjorken limit, involves the chiral breaking equal-time commutator,

$$e\int d^3x e^{i\vec{q}\cdot\vec{x}} < \gamma_{k'}|[\partial\cdot A(\vec{x},0), \vec{V}_{em}]|0> \equiv \Sigma\, \vec{\epsilon}\,' \times \vec{k}' \, , \tag{4.40}$$

leading to the $\pi^o \rightarrow 2\gamma$ amplitude [87]

$$F_\pi \approx -\frac{2\alpha S}{\pi f_\pi} + \frac{\Sigma}{f_\pi m_V^2} \, , \tag{4.41}$$

where S is the average non-strange quark charge of 1/6 (without color) or 1/2 (with three colors). Unfortunately the color ambiguity weakens the predictive power of (4.41) to constrain $\hat{m}$, but a quark model analysis of (4.41) is nonetheless of interest. Since the commutator in Σ is the same as occurs for photoproduction, use of (3.37) tells us that the quark bilinear of interest is $\bar{q}\,\sigma_\perp q$. Then the light cone relation (2.26c) coupled with (2.24) reveals that the non-flip $L_z = 0$ leading term in $<\gamma\,|\chi^+\sigma_\perp\,\phi\,|0>$ comes from the $\vec{\sigma}_\perp \cdot \vec{\nabla}_\perp$ term in (2.24) and is <u>not</u> proportional to an additional quark mass factor as is the nucleon matrix element (3.38). Thus $<\gamma\,|\chi^+\sigma_\perp\,\phi\,|0>$ transforms simply under SU_3, and can be analyzed using <u>model-dependent</u> PCTC (partially conserved tensor current) methods combined with vector dominance. The result is roughly [87, 88] $\Sigma \sim -e^2\hat{m}m_V/g_V^2(0)$ where $g_V^2(0)/4\pi \sim 2$.

Unfortunately, meson processes do not appear to set a quark mass scale in a simple manner. If, however, one applies the quark mass $\hat{m} \sim m_\pi$ (as indicated by baryon processes in our scheme) to Σ in (4.41), then the chiral breaking term is as large as the Adler term (with $S = 1/6$) and of the <u>same sign</u>. In this case a color enhancement of the first term in (4.41) ($S = 1/2$) would lead to too large a value for F_π [88].

V. ELECTROMAGNETIC CHIRAL SYMMETRY BREAKING EFFECTS

A. The SU_2 Breaking Quark Hamiltonian

The possible existence of a u_3 term in the hadronic Hamiltonian, which is roughly the same strength as the second-order electromagnetic Hamiltonian, has been recognized for a long time. In quark language, if the SU_2 breaking effect of $m_u \neq m_d$ is allowed in the quark mass matrix (2.17), then (2.4) becomes

$$H' = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s = \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3 \tag{5.1}$$

where (2.19) - (2.22) remain valid with $\hat{m} = \frac{1}{2}(m_u+m_d)$ and

$$\begin{aligned} u_3 &= \bar{u}u - \bar{d}d \\ \epsilon_3 &= \tfrac{1}{2}(m_u - m_d) \, . \end{aligned} \tag{5.2}$$

Since on the light cone the bad operators u_0, u_3, and u_8 can have mixed transformation properties of λ_0, λ_3, and λ_8, it is useful to reexpress $H_0 + H'$ in its most general SU_3 form

$$H = H_{0,SU_3} + H_8 + \chi H_3 \tag{5.3}$$

where H_{0,SU_3} conserves I^2, I_3, and Y, H_8 is the Gell-Mann-Okubo part transforming like λ_8, and H_3 is the isospin-violating part transforming like λ_3. The number χ measures the strength of H_3 relative to H_8. In the GMOR scheme, where u_i transforms like λ_i, χ_{GMOR} is just ϵ_3/ϵ_8, which in the quark model is

$$\chi_{GMOR} = \sqrt{3}\,\frac{m_u - m_d}{m_u + m_d - 2m_s} \, . \tag{5.4}$$

On the other hand, in our quark model approach with $\bar{q}q/m_q$ transforming simply under SU_3,

$$\chi = \sqrt{3}\,\frac{m_u^2 - m_d^2}{m_u^2 + m_d^2 - 2m_s^2} \, . \tag{5.5}$$

The general SU_3 form (5.3) makes the definite prediction that the scale of baryon and meson matrix elements of H_8 and H_3 is given by a single value of χ once the Wigner-Eckart theorem is applied. Further, within the octet baryons themselves, H_8 and H_3 belonging to the same octet implies that the d/f ratio with d+f = 1 in

$$\langle B_f | H_j | B_i \rangle = \langle B \| H \| B \rangle (d d_{fji} + f i f_{fji}) \tag{5.6}$$

is the same for H_3 as for the semistrong H_8 baryon mass differences, d/f ≈ -1/3 given by (3.11). Since H_3 is isospin breaking, it is clear that the H_3 scale and d/f can be probed by the baryon and meson electromagnetic mass differences. In order to perform this analysis we must first separate off contributions which arise from other than explicit quark mass differences. These are the finite second-order current-current contributions from one photon loop. Thus we write [89]

$$H_{em} = H_{JJ} + \chi H_3 \tag{5.7}$$

where H_{JJ} corresponds to the finite part of the second-order em photon loop,

$$H_{JJ} = -\frac{ie^2}{2}\int d^4x\, D^{\mu\nu}(x) T^*(V_\mu^{em}(x) V_\nu^{em}(0))_{\text{finite}} \, . \tag{5.8}$$

We will discuss this separation further in the next section.

To extract χ, we first examine the pseudoscalar meson electromagnetic mass differences using the quadratic form (4.22) and the SU_3 structure of (5.6) with f = 0 for the mesons,

$$m_{\pi^+}^2 - m_{\pi^o}^2 = (H_{JJ})_{\pi^+} - (H_{JJ})_{\pi^o} \tag{5.9a}$$

$$m^2_{K^+} - m^2_{K^o} = (H_{JJ})_{K^+} - (H_{JJ})_{K^o} + \chi \langle P \| H \| P \rangle \qquad (5.9b)$$

where (5.9a) has been approximately verified in pole saturation [90] and hard pion current algebra models [91], and $\langle P \| H \| P \rangle$ is determined from the semi-strong mass splitting to be $-\frac{2}{\sqrt{3}}(m_K^2 - m_\pi^2)$. If we accept the $SU_3 \times SU_3$ Dashen theorem [92] to eliminate the H_{JJ} pieces in (5.9),

$$(H_{JJ})_{K^+} - (H_{JJ})_{K^o} = (H_{JJ})_{\pi^+} - (H_{JJ})_{\pi^o} , \qquad (5.10)$$

then χ can be obtained from (5.9) and (5.10) as

$$\chi = -\frac{\sqrt{3}}{2} \frac{\left(m^2_{K^+} - m^2_{K^o}\right) - \left(m^2_{\pi^+} - m^2_{\pi^o}\right)}{m_K^2 - m_\pi^2} \approx 0.020. \qquad (5.11)$$

Since estimates of the JJ pieces from single pole saturation [90] of (5.8) or chiral breaking corrections [93] are consistent with the Dashen theorem (5.10) to within 20% and since this theorem can also be derived using pion PCAC on $\langle \pi | H_{JJ} | \eta \rangle$, we accept (5.10) and the value of χ given by (5.11).

For the baryon octet, one can use the values derived in (3.11) and (3.12) to predict the contribution of χH_3 to the electromagnetic mass differences. The value of $(\chi H_3)_B$ can then be subtracted from the experimental electromagnetic mass differences to obtain a prediction for the baryon matrix elements of H_{JJ}. For $\chi \approx 0.018$, one obtains a value for $(H_{JJ})_B$ which is reasonably close to estimates [94] made by octet and decuplet saturation of matrix elements of H_{JJ}; this result holds for either the linear or the quadratic form of the baryon mass formula. It should be noted that this fit favors $\langle p | H_{JJ} | p \rangle - \langle n | H_{JJ} | n \rangle \approx$ (1.3 MeV)$2m_N$. Vector meson and decuplet baryon electromagnetic mass differences, though hard to extract from data, tend to further reconfirm the existence of H_3 with [95] $\chi \approx 0.02$.

In quark language, therefore, the quark mass matrix (5.1) appears to correctly map out the physical Hamiltonian (5.3). The phenomenological value $\chi \approx 0.018 - 0.020$ gives, from (5.4),

$$(m_u - m_d)/\hat{m}\,|_{GMOR} \approx -\tfrac{1}{2} \tag{5.12}$$

for the GMOR SU_2-breaking quark mass difference and from (5.5),

$$(m_u - m_d)/\hat{m} \approx -\tfrac{1}{4} \tag{5.13}$$

in our scheme. While (5.13) is a larger SU_2-breaking effect than one might expect, (5.12) is twice this size. One might assume that m_s sets the scale for this SU_2-breaking, in which case (5.12) and (5.13) would be of the order of a few percent. In absolute terms (5.13) coupled with $X \approx 5$ and our baryon estimate of the quark mass scale, $\hat{m} \approx m_\pi$, leads to the set of chiral-breaking quark masses

$$m_u \sim 125 \text{ MeV} \qquad m_d \sim 155 \text{ MeV} \qquad m_s \sim 680 \text{ MeV} \tag{5.14}$$

whereas in the GMOR scheme, (5.12), $X \approx 25$ and the estimate [62] $\hat{m} \sim 6$ MeV then implies $m_u \sim 4.5$ MeV, $m_d \sim 7.5$ MeV, and $m_s \sim 150$ MeV.

B. Dynamical Origin of the SU_2-Breaking Tadpole

We discuss further, now, the renormalization of the second order electromagnetic mass shifts, as represented in (5.8). At the quark level it is clear [11,12,96] that there is a contribution to the em hadron mass shift, $\delta m^2 = 2E\delta E$, coming purely from the em mass shifts of the quarks themselves, $\delta m_q^2 = 2E_q \delta E_q$, which infinite momentum language ($E_q = xE$) allows one to express rigorously as

$$\delta m^2_{singular} = \sum_q \delta m_q^2 \tilde{f}_q \tag{5.15}$$

where $\tilde{f}_q$ are the distribution integrals defined earlier. This contribution, if evaluated using second-order QED expressions for the quark mass shifts, becomes

$$\delta m^2_{\text{singular}} = \frac{3\alpha}{2\pi} \sum_q \lambda_q^2 m_q^2 \tilde{f}_q \, \ell n \, \Lambda^2/m_q^2 \tag{5.16}$$

i.e., *it is divergent in the ultraviolet cutoff*, Λ. The complete expression for the second-order δm^2 is given in terms of the Cottingham formula

$$\delta m^2 = \frac{-i\alpha}{8\pi^3} \int \frac{d^4 q}{q^2 - i\epsilon} \; T^\mu_\mu (p,q) \tag{5.17}$$

which may be evaluated in terms of the structure functions in the contraction of (2.34)(after performing the usual steps of Wick rotating, writing dispersion relations for A_1 and A_2 with appropriate Regge subtractions, and using standard fixed pole information, i.e., no $\alpha=0$ fixed pole in A_1). By employing various sum rules it can be shown [12] that the infinite part of δm^2 (in the ultraviolet cutoff Λ) is precisely given by (5.16). Thus it is possible to carry out a renormalization program at the quark level such that

$$\delta m^2 = \delta m^2_{\text{finite}} + \delta m^2_{\text{singular}} \tag{5.18}$$

where $\delta m^2_{\text{singular}}$ is as given in (5.15) with δm_q^2, after renormalization, being the physical quark mass shifts. The contribution $\delta m^2_{\text{singular}}$ is thus naturally associated with the tadpole term in (5.1); the form of this contribution is precisely like that of the semistrong baryon and meson mass formula (3.6)-(3.8) and (4.13), implying that H_3 of (5.3) has the same d/f ratio as appropriate to H_8. H_{JJ} is then clearly to be identified with the explicitly finite bound state contributions denoted by $\delta m^2_{\text{finite}}$. Various approximate evaluations of H_{JJ} have been suggested. For instance saturation of H_{JJ} by the lowest hadron states (i.e., the "elastic" contribution), combined with the quark masses (5.14) and distribution integrals $\tilde{f}_q$ given earlier, provides a reasonable description of the octet baryon electromagnetic mass splittings. More modern approximations to H_{JJ} suggest themselves as well; for instance, it might be reasonable to suppose that the dominant contributions to H_{JJ} arise from the Coulomb interactions between the various possible quark pairs in the bound state [97].

We are still left, however, in any approach, with the fundamental problem of why $m_u < m_d$. No truly natural explanation exists at the moment.

C. The Decay $\eta_{3\pi}$

If, as in the previous section, we assume that H_3 belongs to the $(3,\bar{3})$ representation of $SU_3 \times SU_3$, the amplitude for the G parity violating decay $\eta_{3\pi}$ ought to be uniquely determined.

In the past there was a problem associated with the Sutherland theorem [98] and the possible breakdown of pion PCAC. In the soft pion limit, the decay amplitude T for $\eta_{3\pi}$ is

$$\lim_{p_a \to 0} T = -\lim_{p_a \to 0} \langle \pi_a \pi_b \pi_c | H_{em} | \eta \rangle = \frac{i}{f\pi} \langle \pi_b \pi_c | [Q_a^5, H_{em}] | \eta \rangle . \tag{5.19}$$

Sutherland has shown that (5.19) vanishes (or is very small) for H_{em} given by H_{JJ} alone. The presence of the tadpole term, H_3, however, allows one to make a more reasonable estimate of the $\eta_{3\pi}$ decay rate and Dalitz plot slope, provided one takes into account pole terms which vary rapidly off the pion mass shell [99] (such as the pion pole $\eta \xrightarrow{em} \pi \to 3\pi$). This type of analysis can be done in the tree graph approximation to a nonlinear Lagrangian [74,100]. An equivalent dispersion theoretic approach has been proposed which allows a more flexible treatment of the meson-meson scattering vertices which appear in the rapidly varying pole terms: one writes [95,101]

$$T = \bar{T} + T_P \tag{5.20}$$

where the pole terms vary rapidly with the pion momenta but the background amplitude $\bar{T}$ does not. Thus one can take the soft limit of (5.20) to find

$$\bar{T} = \lim_{p_a \to 0} (T - T_P) \tag{5.21}$$

In U_3 broken $(3,\bar{3})$ models (i.e., $m_0 \neq 0$ in (4.28)), both T and T_P are equal to $f_\pi^{-2}\langle\pi|\epsilon_3 u_3|\eta\rangle$ in the soft limit; hence $\bar{T} = 0$ so that the pole model of $\eta \to 3\pi$ is exact [95,101]:

$$T = T_P \tag{5.22}$$

Due to the isotopic structure of T_P, all of the soft pion limits are satisfied; hence there is no breakdown of pion PCAC.

More recently another puzzle has arisen in connection with the ninth axial-vector current (and its associated Goldstone boson) which is implicit in the $U_3 \times U_3$ symmetry of quark-gluon models. This puzzle can be seen most simply as follows: First we note that η and π are SU_3 states and that we are only working to first order in SU_2 breaking; hence $H_{em} = \frac{1}{2}(m_u - m_d)u_3 + H_{JJ}$. This is true in either our scheme or the GMOR scheme. Second, we note that the soft pion limit in (5.19) yields

$$\lim_{p_a \to 0} T = \frac{1}{2f_\pi}(m_u - m_d)\delta_{a3}\langle\pi_b\pi_c|v_{NS}|\eta\rangle \tag{5.23}$$

where we use the basis of pure strange/nonstrange quarks of (4.24). Due to (4.25), however, (5.23) is proportional to $\langle\pi_b\pi_c|\partial A_{NS}|\eta\rangle$ in the quark model. By momentum conservation, therefore, the total divergence nature of the operator indicates that <u>all</u> soft pion limits of T are zero [73,102]. Two points must be made:

(i) The fact that all soft pion limits of T are zero does <u>not</u> imply that the on-shell value of T is zero; one must take into account rapidly varying poles.

(ii) This $\eta_{3\pi}$ puzzle should not be treated until the simpler and more basic problem of the pseudoscalar mass spectrum is solved.

Turning first to the problem of the mass spectrum, we recall from Section IVE that one way of accounting for the pseudoscalar masses is to introduce a term into the divergence of the ninth axial current: $\partial A_0 = g - \sqrt{\frac{2}{3}}(\epsilon_0 v_0 + \epsilon_8 v_8)$. This implies that (5.23) should be written

$$\lim_{p_a \to 0} T = - \frac{m_u - m_d}{2 f_\pi \hat{m}} \delta_{a3} \langle \pi_b \pi_c | \partial A_{NS} - \sqrt{\frac{2}{3}} g | \eta \rangle \tag{5.24}$$

If g is not a total divergence, the right-hand side of (5.24) is no longer zero and the difficulty vanishes; one can account for the pseudoscalar masses (Sec. IVE) and previous analyses of the $\eta_{3\pi}$ decay remain valid [95,101]. In vector-gluon models, however, g is a total divergence and one is left with both problems; if one assumes that the Goldstone boson associated with the ninth current is actually the positive/negative metric dipole mentioned in Sec. IVE, then the mass problem is solved and, as Weinberg has shown in the nonlinear Lagrangian framework [73], the ninth axial-vector current and its (dipole) Goldstone boson decouple from the $\eta_{3\pi}$ analysis. In operational terms, (5.24) is then no longer zero.

Although it is a pseudo problem to try to analyze the $\eta_{3\pi}$ amplitude without first resolving the pseudoscalar mass spectrum problem, it is nevertheless of interest to see how the U_3 structure of the quark model leaves the pole model result (5.22) unchanged in the current algebra - PCAC (rapidly varying pole) method of analysis. To take the simplest case, consider the decay $\eta_{NS} \to 3\pi$ with $g = 0$ so that $m_{\eta_{NS}} = m_\pi$. In general the background term $\bar{T}$ in (5.21) vanishes in the $(3,\bar{3})$ model; in the U_3 symmetric case, $\lim_{p_a \to 0} T$ in (5.21) also vanishes because of the total divergence nature of (5.23). Hence $\lim_{p \to 0} T_P$ also vanishes, and one can easily verify that the π and η_{NS} poles do cancel when $p \to 0$ if $m_{\eta_{NS}} = m_\pi$ and $\beta_{NS} = 0$ (see Sec. IVE). It is also clear that the pole denominators vanish in this limit, however, and that formal arguments concerning U_1 symmetry should be treated with caution. While each term in (5.21) is then zero, the on-shell amplitude is given by (5.22) and is not zero; the value obtained, of course, makes no sense until the pseudoscalar mass spectrum is accounted for in a satisfactory manner. It is also of interest to see how the vanishing of $N = \langle \pi_b \pi_c | v_{NS} | \eta_{NS} \rangle$ in (5.23) for U_3 quark models can be explained in rapidly varying pole language. Noting that N has an η_{NS} rapidly varying pole term as well as a constant background term, one writes [95,101] $N = \bar{N} + N_P$. These two terms ($\bar{N}$ and N_P) are both proportional to $\langle 0 | v_{NS} | \eta_{NS} \rangle$ and exactly cancel when $m_{\eta_{NS}} = m_\pi$, thus recovering $N = 0$.

The upshot of this discussion is that in any $(3,\bar{3})$ chiral breaking scheme (i.e., in our approach or that of GMOR), the structure of H_{em} leads to a vanishing of $\bar{T}$ in (5.21) - either due to the exact cancellation between (5.19) and the soft limit of the pole amplitude T_P in the case of a U_3 broken mass spectrum, or, alternatively, due to the separate vanishing of the two terms on the right-hand side of (5.21) in U_3 symmetric theories. For theories with a correct mass spectrum, the resulting on-shell pole term (5.22) is dominated by the pion pole if $\beta_\eta = 0$, both in the slope and in the rate. This implies a Dalitz plot slope structure $s - \frac{4}{3} m_\pi^2$, consistent with the data [72] and also with the nonlinear Lagrangian solution [100]. The η_{+-0} decay rate is then $\sim$ 70 eV for $\eta = \eta_8$ ($\theta_{\eta'\eta} = 0$) and $\sim$ 120 eV for $m_{\eta_8}^2 \approx 18 m_\pi^2$ ($\theta_{\eta'\eta} \approx -14^\circ$). While both of these values fall short of the new measured rate [72] of 204 ± 22 eV, the $(3,\bar{3})$ quark model cannot be discounted because of the $\eta_{3\pi}$ decay. Neither, for that matter, can a distinction be made between the two $(3,\bar{3})$ chiral breaking schemes on the basis of $\eta_{3\pi}$ decay.

VI. CONCLUSION

We have seen that a description of chiral symmetry and SU_3 breaking based on the Hamiltonian

$$H' = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \tag{6.1}$$

(q and m_q refer to quark field and mass of any one type) when formulated in the most general fashion, does not conform to the original GMOR assumptions. Indeed, we have argued in a variety of ways that hadron expectation values of the fundamental densities

$$u_i = \bar{\psi}\lambda_i\psi \tag{6.2}$$

do not transform under SU_3 like λ_i. Rather the quantities $\left\langle\frac{\bar{q}q}{m_q}\right\rangle_{\text{hadron}}$ are the simple SU_3 objects. Similar statements apply to expectation values of the type $\langle 0|v_P|P\rangle$, P being any pseudoscalar meson, and $\langle 0|\bar{\psi}\lambda_i\psi|0\rangle$. This alteration of the original GMOR scheme has profound implications for the phenomenology of chiral symmetry breaking.

In particular, we have shown that the πN σ term, baryon mass differences, and proton Compton amplitude fixed pole value combine to determine strange and nonstrange quark mass values

$$\frac{m_u + m_d}{2} = \hat{m} \sim 140 \text{ MeV} \qquad m_s \sim 680 \text{ MeV} \tag{6.3}$$

The value of the parameter

$$c = -\sqrt{2}\left(\frac{X-1}{X+2}\right) \quad , \tag{6.4}$$

where $X = m_s/\hat{m} \sim 5$ is no longer near $-\sqrt{2}$; rather $c \approx -.8$. Within the framework of baryon phenomenology we also consider sum rules for the axial coupling g_A and for low energy π photoproduction off nucleons that allow an independent determination of $\hat{m}$, confirming the result (6.3). Various additional independent determinations of X, using Goldberger-Treiman discrepancies and KN σ terms, are shown to yield again

$$X = m_s/\hat{m} \sim 5\text{-}6 \tag{6.5}$$

This type of consistent phenomenology for the baryons is impossible if one assumes that expectation values of the u_i (6.2) transform like λ_i under SU_3.

An investigation of meson PCAC and chiral symmetry breaking phenomenology begins with the relation indicated by the GMOR analysis

$$\frac{m_K^2}{m_\pi^2} = \frac{1}{2}(1+X)\frac{<0|v_K|K>}{<0|v_\pi|\pi>} \quad . \tag{6.6}$$

Combined with

$$\frac{<0|v_\pi|\pi>}{<0|v_K|K>} = \frac{2\hat{m}}{m_s+\hat{m}} \tag{6.7}$$

as implied by the above discussed properties of such expectation values, we obtain once again the value (6.5) for X. The GMOR assumption that (6.7) has the value 1 yields X = 25, which is well known to be inconsistent with the πN σ term when also evaluated in their framework. A second determination of X, using directly the PCAC results $<\pi|H'|\pi> = m_\pi^2$ and $<K|H'|K> = m_K^2$ combined with our formalism for $<P|\bar{q}q|P>$ expectation values, yields again $X \sim 5$. That is, for most meson matrix elements PCAC remains a good approximation since

$$\frac{<P|\bar{q}q|P>}{m_q} \qquad \frac{<0|v_P|P>}{<m_q>_P} \qquad \frac{<0|\bar{q}q|0>}{m_q}\ , \tag{6.8}$$

which are related by PCAC, all transform simply under SU_3 in our approach.

Unfortunately, it is impossible to determine the absolute mass scale for $\hat{m}$ and m_s using mesons alone since there is no determination of the quark distribution integral scales such as is provided in the two baryon cases by the Compton fixed pole and by g_A, respectively.

A variety of additional processes, especially meson decays, were investigated and shown to be consistent with our approach but provide no additional constraints at the present time. A future model dependent test of our formalism (as opposed to the GMOR method) using $K\ell_3$ decay may be possible. Electromagnetic mass splittings and related subjects were also considered and shown to be consistent with a SU_2 mass difference

$$\frac{m_u - m_d}{\hat{m}} \approx -\frac{1}{4} . \tag{6.9}$$

In summary, we have developed a completely consistent approach to chiral symmetry breaking, for both mesons and baryons, which provides a large variety of independent determinations of the quark masses. These turn out to be

$$\hat{m} \sim 140 \text{ MeV} \qquad \text{and} \qquad m_s \sim 680 \text{ MeV},$$

much larger than previous values but of the same size as those found in the bag model and other recent approaches.

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$Q_i = Q_i^L$. However, for $\epsilon_0 \neq 0$, $\epsilon_8 \neq 0$ (broken SU_3 symmetry), $dQ_i/dt \neq 0$ and $dQ_i^L/dx^+ \neq 0$ imply $Q_i \neq Q_i^L$.

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