# 29 Extraction of the CP phase and the life time difference from penguin free tree level $B_s$ decays

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**Abstract** In this talk I present alternative methods for the extraction of the CP phase $2\beta_s$ and lifetime difference $\Delta\Gamma_s$ using penguin-free tree level two body $B_s \to D^0_{CP}\phi$ and three body $B_s(\bar{B}_s) \to D^0_{CP}KK$ decays.

## 29.1 Introduction

Apart from the direct searches at colliders, low energy observables in flavor physics play an essential role for an indirect search of NP; in this respect FCNC processes are important. The data from the decays of K, D and B mesons have so far been consistent with the Cabbibo-Kobayashi-Maskawa (CKM) paradigm of Standard Model (SM), however the flavor changing neutral current (FCNC) processes involving $b \to s$ transitions are expected to be sensitive to many sources of new physics (NP) since FCNC decays are rare (i. e. loop-suppressed) in the SM [1–3].

In light of this, it is particularly important to study $b \to s$ transitions and look for new-physics (NP) effects. Now, if NP is present in $\Delta B = 1$ $b \to s$ decays, it would be highly unnatural for it not to also affect the $\Delta B = 2$ transition, in particular $B^0_s$-$\bar{B}^0_s$ mixing. At the same time, we do hope wealth of data on $B_s$ system from LHCb.

In order to see where NP can enter, we briefly review the mixing. Effective Hamiltonian for $B_q - \bar{B_q}$ mixing

$$H_{eff} = \begin{pmatrix} M_{11q} - \frac{i}{2}\Gamma_{11q} & M_{12q} - \frac{i}{2}\Gamma_{12q} \\ M^*_{12q} - \frac{i}{2}\Gamma^*_{12q} & M_{11q} - \frac{i}{2}\Gamma_{11q} \end{pmatrix},$$

where $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$ correspond respectively to the dispersive and absorptive parts of the mass matrix. The off-diagonal elements, $M^s_{12} = M^{s*}_{21}$ and $\Gamma^s_{12} = \Gamma^{s*}_{21}$, are generated by $B^0_s$-$\bar{B}^0_s$ mixing.

We define

$$\Gamma_s \equiv \frac{\Gamma_H + \Gamma_L}{2}, \quad \Delta M_s \equiv M_H - M_L, \quad \Delta\Gamma_s \equiv \Gamma_L - \Gamma_H\ , \tag{29.1}$$

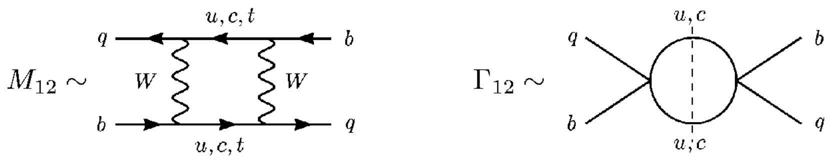


Figure 29.1: Diagrams contribute to $M_{12}$ and $\Gamma_{12}$.

where $L$ and $H$ indicate the light and heavy states, respectively. $M_{L,H}$ and $\Gamma_{L,H}$ are the masses and decay widths of the light and heavy mass eigenstates respectively. Mass difference and width difference can be calculated from the dispersive and absorptive part of the box diagram shown in 29.1

Expanding the mass eigenstates, we find, to a very good approximation [4],

$$\Delta M_s = 2|M_{12}^s| \,, \;\; \Delta\Gamma_s = 2|\Gamma_{12}^s|\cos\phi_s \,, \;\; \frac{q}{p} = e^{-2i\beta_s}\left[1 - \frac{a}{2}\right] \,, \tag{29.2}$$

where $\phi_s \equiv \arg(-M_{12}^s/\Gamma_{12}^s)$ is the CP phase in $\Delta B = 2$ transitions. In Eq. (29.2) the small expansion parameter $a$, the semileptonic asymmetry, is given by

$$a = a_{sl}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^s|}\sin\phi_s \,. \tag{29.3}$$

This is expected to be $\ll 1$, and hence can be neglected in the definition of $q/p$. The weak phase $2\beta_s$ appears in the indirect (mixing-induced) CP asymmetries.

The SM predictions of all these observables are given by [5]

$$\begin{aligned} \Delta M_s &= (17.3 \pm 2.6)ps^{-1}, \qquad \Delta\Gamma_s = (0.087 \pm 0.021)ps^{-1} \\ 2\beta_s &\approx 2^\circ, \qquad \phi_s = 0.22^\circ, \qquad a_{sl}^s = (1.9 \pm 0.3) \times 10^{-5} \end{aligned} \tag{29.4}$$

The present world averages are given by [6–10]

$$\begin{aligned} \Delta M_s &= (17.69 \pm 0.08)ps^{-1}, \qquad \Delta\Gamma_s = (0.103 \pm 0.014)ps^{-1} \\ 2\beta_s &= 0.14^{+0.11}_{-0.16}, \qquad a_{sl}^s = -0.0105 \pm 0.0064 \end{aligned} \tag{29.5}$$

Therefore, the present data still allow 20% to 30% CP-violating NP effects. There is no separate measurement on $\phi_s$ and it is not wise to consider $\phi_s = 2\beta_s$, as we can see from Eq. (29.4), even in the SM they are not equal. However, it is possible to constrain $\phi_s$ along with $\Gamma_{12}^s$ from the measurement of $\Delta\Gamma_s$ and semileptonic asymmetry $a_{sl}^s$. Combining Eqs. (29.2) and (29.3) we obtain

$$\begin{aligned} \tan\phi_s &= \frac{a_{sl}^s\,\Delta M_s}{\Delta\Gamma_s} = -1.80 \pm 1.12 \,, \\ |\Gamma_{12}^s| &= \frac{\sqrt{{\Delta\Gamma_s}^2 + {a_{sl}^s}^2\,{\Delta M_s}^2}}{2} = 0.106 \pm 0.051 \,. \end{aligned} \tag{29.6}$$

The constrained values of the phase $\phi_s$ and $|\Gamma_{12}^s|$ are consistent with the SM within the error bar, however, significant deviations can not be ruled out. Once we include $a_{sl}^s = (-1.81 \pm 1.06)\%$, the data provided by DØ from dimuon asymmetry measurement [11] , the situation will be further worsen with respect to SM predictions. Therefore, more precise measurements of $\Delta\Gamma_s$ and $a_{sl}^s$ are essential.

So far $2\beta_s$ seems to be SM like, however, there are facts to remember. Extraction of $2\beta_s$ from $B_s \to J/\psi M$ decays is theoretically clean, provided the subleading terms are assumed to vanish. In the next few years, with the LHCb we are entering the era of high precision physics. For example, the CP asymmetry $S_{\psi\phi}$ in $B_s^0 \to J/\psi\phi$ decay will be measured with 3% accuracy. Hence, subleading SM contributions will become important. On the other hand due to our poor understanding of low energy QCD it is extremely hard to estimate/calculate reliably the ratio of leading to the subleading contributions [12]. The problem lies with the evaluation of the hadronic matrix element. At the same time, the possibility of NP in $b \to c\bar{c}s$ decays can not be ruled out [13]. Therefore, it is worthwhile to look for a process in which NP in the decay can essentially be neglected, and permits the determination of $2\beta_s$ and $\Delta\Gamma_s$ without any ambiguity. In this regard, tree level $B_s$ decays via $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions may play an interesting role. In the following sections, we discuss the extraction of $2\beta_s$ and $\Delta\Gamma_s$ from two and three body $B_s$ decays.

## 29.2 Two body decays: $B_s^0(\bar{B}_s^0) \to D^0\phi, \bar{D}^0\phi$

We consider first the two body decays via $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions, and try to see what can we learn from such decays. Consider a final state $f$ to which both $B_s^0$ and $\bar{B}_s^0$ can decay, and the decay amplitudes are dominated by a single weak phase.

$$\frac{\Gamma(B_s^0(t) \to f) - \Gamma(\bar{B}_s^0(t) \to f)}{\Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)} = \frac{C\cos\Delta m_s t - S\sin\Delta m_s t}{\cosh(\Delta\Gamma_s t/2) - \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)} . \tag{29.7}$$

Therefore, the following interesting observables can be extracted

$$C \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda|^2}, \quad S \equiv \frac{2\,\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}, \quad \mathcal{A}_{\Delta\Gamma} \equiv \frac{2\,\mathrm{Re}\lambda_f}{1 + |\lambda_f|^2}, \tag{29.8}$$

where $\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = |\lambda_f| e^{-i(\phi_s^{mix} + \theta - \delta)}$, $\phi_s^{mix}$ is the mixing phase and $\theta - \delta = -Arg\left[\frac{\bar{A}_f}{A_f}\right]$. The weak and strong phase difference between the decay amplitudes $\bar{A}_f = \bar{B}_s^0 \to f$ and $A_f = B_s^0 \to f$ are given by $\theta$ and $\delta$ respectively. Similarly, for the final state $\bar{f}$ we get

$$\bar{S} \equiv \frac{2\,\mathrm{Im}\bar{\lambda}_f}{1 + |\bar{\lambda}_f|^2}, \quad \bar{\mathcal{A}}_{\Delta\Gamma} \equiv \frac{2\,\mathrm{Re}\bar{\lambda}_f}{1 + |\bar{\lambda}_f|^2}, \tag{29.9}$$

with $\bar{\lambda}_f \equiv \frac{p}{q}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} = \frac{1}{|\lambda_f|} e^{-i(\phi_s^{mix} + \theta + \delta)}$. The various combination of the these observables are useful to extract the CP phase.

In the SM, the amplitude of the $B_s^0 \to D^0\phi$ and $\bar{B}_s^0 \to D^0\phi$ decays are of the same order, hence, leads to interference effects between $B_s^0$-$\bar{B}_s^0$ mixing and decay process. By measuring the time dependence of the decays, one can obtain $S$, $\bar{S}$, $A_{\Delta\Gamma}$ and $\bar{A}_{\Delta\Gamma}$ as given in Eqs. (29.8) and (29.9), for detail see Ref. [14]. Using these observables we extract $\sin(2\beta_s+\gamma+\delta_\phi)$, $\sin(2\beta_s+\gamma-\delta_\phi)$, $\cos(2\beta_s+\gamma+\delta_\phi)$, $\cos(2\beta_s+\gamma-\delta_\phi)$, which allows us to obtain $2\beta_s+\gamma$ with a twofold ambiguity; similar information as in $B_s^0(\bar{B}_s^0)\to D_s^\pm K^\mp$ decays [15, 16].

The advantage of these decays is that there is a third decay which is related: $B_s^0(\bar{B}_s^0)\to D_{CP}^0\phi$, where $D_{CP}^0$ is a CP eigenstate (either CP-odd or CP-even). In our analysis we consider $D_{CP}^0$ as the CP-even superposition $(D^0+\bar{D}^0)/\sqrt{2}$. In this case, time dependent decay distributions allow to extract two more functions $\cos(\gamma+\delta_\phi)$ and $\cos(\gamma-\delta_\phi)$. Therefore, various algebraic combination of all these functions allow one to determine $\sin 2\beta_s$, $\cos 2\beta_s$, $\sin(2\beta_s+2\gamma)$, $\cos(2\beta_s+2\gamma)$. Hence, unambiguous determinations of $2\beta_s$ and $2\gamma$ is possible.

## 29.3 Three body $B_s^0(\bar{B}_s^0)\to D_{CP}^0K\bar{K}$ decays: Dalitz analysis

In the previous section we discussed two-body $\bar{b}\to\bar{c}u\bar{s}/\bar{b}\to\bar{u}c\bar{s}$ decays; in this section we examine the corresponding three-body decays. In recent years, various tests of the SM, as well as the extraction of weak phases, have been examined in the context of $B\to K\pi\pi$, $B\to K\bar{K}K$, $B\to\pi\bar{K}K$ and $B\to\pi\pi\pi$ decays [17, 18], which uses Dalitz-plot analyses. The extra piece of information available in $B_s$ decays, due to the sizeable lifetime difference $\Delta\Gamma_s$, can provide important insights into the CP violation studies of three body Dalitz analysis. The $B_s^0(\bar{B}_s^0)\to D_{CP}^0K\bar{K}$ decays receive a tree contribution. The CKM matrix elements of these decays are the same as in the corresponding two-body decay modes, and will therefore exhibit very similar time-dependent CP asymmetries.

In the following, we perform a time-dependent Dalitz-plot analysis of the $B_s^0(\bar{B}_s^0)\to D_{CP}^0K\bar{K}$ decays, which can decay either via intermediate resonances ($\phi$, $f_0$ etc.) or non-resonant contributions. This permits the measurement of each of the contributing amplitudes, as well as their relative phases. In the isobar model, the individual terms are interpreted as complex production amplitudes for two-body resonances, and one also includes a term describing the non-resonant component. The amplitude is then given by

$$\mathcal{A}(s^+,s^-)=\sum_j a_jF_j(s^+,s^-),\qquad \bar{\mathcal{A}}(s^+,s^-)=\sum_j \bar{a}_j\bar{F}_j(s^-,s^+) \tag{29.10}$$

where the sum is over all decay modes (resonant and non-resonant). Here, the $a_j$ are the complex coefficients describing the magnitudes and phases of different decay channels, while the $F_j(s_{12},s_{13})$ contain the strong dynamics. It takes different (known) forms for the various contributions.

The time-dependent decay rates for decay to the same final state $f$, are given by [14]

$$\begin{aligned}\Gamma(B_s^0(t)\to f) &\sim \frac{1}{2}e^{-\Gamma_s t}\Big[A_{ch}(s^+,s^-)\cosh(\Delta\Gamma_s t/2)-A_{sh}(s^+,s^-)\sinh(\Delta\Gamma_s t/2)\\ &\qquad + A_c(s^+,s^-)\cos(\Delta m_s t)-A_s(s^+,s^-)\sin(\Delta m_s t)\Big]\,,\\ \Gamma(\bar{B}_s^0(t)\to f) &\sim \frac{1}{2}e^{-\Gamma_s t}\Big[A_{ch}(s^-,s^+)\cosh(\Delta\Gamma_s t/2)-A_{sh}(s^-,s^+)\sinh(\Delta\Gamma_s t/2)\\ &\qquad - A_c(s^-,s^+)\cos(\Delta m_s t)+A_s(s^-,s^+)\sin(\Delta m_s t)\Big]\,. \qquad (29.11)\end{aligned}$$

Here

$$\begin{aligned}A_{ch}(s^+,s^-) &= |\mathcal{A}(s^+,s^-)|^2+|\bar{\mathcal{A}}(s^+,s^-)|^2\,,\\ A_c(s^+,s^-) &= |\mathcal{A}(s^+,s^-)|^2-|\bar{\mathcal{A}}(s^+,s^-)|^2\,,\\ A_{sh}(s^+,s^-) &= 2\mathrm{Re}\left(e^{-2i\beta_s}\bar{\mathcal{A}}(s^+,s^-)\mathcal{A}^*(s^+,s^-)\right)\,,\\ A_s(s^+,s^-) &= 2\mathrm{Im}\left(e^{-2i\beta_s}\bar{\mathcal{A}}(s^+,s^-)\mathcal{A}^*(s^+,s^-)\right)\,. \qquad (29.12)\end{aligned}$$

Maximum likelihood fit over the entire Dalitz plot, allows to extract the magnitudes and relative phases of the $a_j$ or $\bar{a}_j$.

As mentioned before the $B_s^0(\bar{B}_s^0)\to D_{CP}^0 K\bar{K}$ decays can proceed via various two body resonances, here for simplicity we consider only the interefernce of two such resonances. Maximum likelihood fit to the Dalitz-plot PDFs allows to extract $\tan\gamma$ without ambiguity from $A_c^{DKK}$ and $A_{ch}^{DKK}$,

$$\begin{aligned}A_c^{DKK} &= \sum_{i=\phi,f_0}\left[\left(|A_i|^2-|\bar{A}_i|^2\right)+2\mathrm{Re}(A_\phi A_{f_0}^*-\bar{A}_\phi\bar{A}_{f_0}^*)\right]\,,\\ A_{ch}^{DKK} &= \sum_{i=\phi,f_0}\left[\left(|A_i|^2+|\bar{A}_i|^2\right)+2\mathrm{Re}(A_\phi A_{f_0}^*+\bar{A}_\phi\bar{A}_{f_0}^*)\right]\,, \qquad (29.13)\end{aligned}$$

for detail see [14]. Hadronic uncertainties cancel, theoretically clean determination of the CKM angle $\gamma$ is possible.

From the interference of two resonances in $A_s^{DKK}$,

$$A_s^{DKK}=\mathrm{Im}\left[e^{-2i\beta_s}\mathcal{A}^*\bar{\mathcal{A}}\right]=\mathrm{Im}\left[e^{-2i\beta_s}(A_\phi^*\bar{A}_\phi+A_\phi^*\bar{A}_{f_0}+A_{f_0}^*\bar{A}_\phi+A_{f_0}^*\bar{A}_{f_0})\right]\,, \qquad (29.14)$$

we extract $\sin 2\beta_s$, $\sin(2\beta_s+\gamma\pm\delta_i)$, $\cos(2\beta_s+\gamma\pm\delta_i)$, $\sin(2\beta_s+2\gamma\pm\delta_{ij})$, $\cos(2\beta_s+2\gamma\pm\delta_{ij})$, where $i=\phi$ or $f_0$. In the above functions $\delta_i$ is the strong phase difference between the amplitudes of the $B_s$ and $\bar{B}_s$ decay to the final state $i$. From these trigonometric functions, it is straightforward to find expressions for $\tan 2\beta_s$ and $\tan\gamma$. We can extract $\sin 2\beta_s$ along with constraining $\tan 2\beta_s$, hence, an unambiguous determination of $2\beta_s$ is possible. The tagged analysis alone allows the extraction of $2\beta_s$ without ambiguity [14].

The time dependent untagged differential decay distribution is given by

$$\Gamma_{untagged}(D_{CP}^0 K^+K^-,t)=e^{-\Gamma_s t}\left[A_{ch}^{DKK}\cosh(\Delta\Gamma_s t/2)+A_{sh}^{DKK}\sinh(\Delta\Gamma_s t/2)\right]. \qquad (29.15)$$

For a single resonance, say $\phi$,

$$
\begin{aligned}
A_{ch}^{DKK} &= A_\phi^2 + \bar{A}_\phi^2, \\
A_{sh}^{DKK} &= \text{Re}\left[e^{-2i\beta_s}|C_2^\phi|^2|F_\phi|^2\{1 + {r_\phi}^2 e^{-2i\gamma} + r_\phi(e^{-i(\gamma+\delta_\phi)} + e^{-i(\gamma-\delta_\phi)})\}\right] \quad (29.16)
\end{aligned}
$$

$A_{ch}^{DKK}$ is fully known from the CP-averaged branching fraction of the intermediate resonance $\phi$. Fit to the tagged decay rate distribution determines: $2\beta_s$, $(2\beta_s + \gamma \pm \delta_\phi)$ and $\cos(2\beta_s + 2\gamma)$ without ambiguity, hence, $A_{sh}^{DKK}$ can be fully obtained Therefore, $\Delta\Gamma_s$ is the only unknown in the untagged decay rate distribution given in Eq. (29.15), it can be determined from the fit, for detail see [14].

## 29.4 Conclusion

We are entering a new era of high precision studies, the CP phase $2\beta_s$ and $\Delta\Gamma_s$ will be measured with better accuracy. Extraction of same observable from various processes are always encouraging, in particular from those modes which are theoretically clean, as was done in $B_d$ decays. In this regard, the tree level processes via $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions may play an interesting role. Combining tagged and untagged measurements of $B_s^0(\bar{B}_s^0) \to (D^0, \bar{D}^0, D_{CP}^0)\phi$ decays, we can extract $2\beta_s$ without any ambiguity. Time dependent Dalitz analysis of the $B_s^0(\bar{B}_s^0) \to D_{CP}^0 KK$ allows us to extract $2\beta_s$ (from tagged) and $\Delta\Gamma_s$ (from untagged) without any ambiguity. In addition, this processes allow a theoretically clean determination of the CKM angle $\gamma$.

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# Bibliography


[1] H.-Y. Cheng, C.-K. Chua, and A. Soni, Phys.Rev. **D72**, 094003 (2005), arXiv:hep-ph/0506268 [hep-ph]

[2] G. Buchalla, G. Hiller, Y. Nir, and G. Raz, JHEP **0509**, 074 (2005), arXiv:hep-ph/0503151 [hep-ph]

[3] E. Lunghi and A. Soni, JHEP **0908**, 051 (2009), arXiv:0903.5059 [hep-ph]

[4] I. Dunietz, R. Fleischer, and U. Nierste, Phys.Rev. **D63**, 114015 (2001), arXiv:hep-ph/0012219 [hep-ph]

[5] A. Lenz and U. Nierste(2011), arXiv:1102.4274 [hep-ph]

[6] T. Aaltonen *et al.* (CDF Collaboration), Phys.Rev. **D85**, 072002 (2012), arXiv:1112.1726 [hep-ex]

[7] V. M. Abazov *et al.* (D0 Collaboration), Phys.Rev. **D85**, 032006 (2012), arXiv:1109.3166 [hep-ex]

[8] R. Aaij *et al.* (LHCb Collaboration), Phys.Rev.Lett. **108**, 101803 (2012), arXiv:1112.3183 [hep-ex]

[9] Y. Amhis *et al.* (Heavy Flavor Averaging Group)(2012), arXiv:1207.1158 [hep-ex]

[10] Y. Xie, P. Clarke, G. Cowan, and F. Muheim, JHEP **0909**, 074 (2009), arXiv:0908.3627 [hep-ph]

[11] V. M. Abazov *et al.* (D0 Collaboration)(2012), arXiv:1208.5813 [hep-ex]

[12] M. Jung(2012), arXiv:1206.2050 [hep-ph]

[13] C.-W. Chiang, A. Datta, M. Duraisamy, D. London, M. Nagashima, *et al.*, JHEP **1004**, 031 (2010), arXiv:0910.2929 [hep-ph]

[14] S. Nandi and D. London, Phys.Rev. **D85**, 114015 (2012), arXiv:1108.5769 [hep-ph]

[15] R. Fleischer, Nucl.Phys. **B671**, 459 (2003), arXiv:hep-ph/0304027 [hep-ph]

[16] S. Nandi and U. Nierste, Phys.Rev. **D77**, 054010 (2008), arXiv:0801.0143 [hep-ph]

[17] M. Ciuchini, M. Pierini, and L. Silvestrini, Phys.Lett. **B645**, 201 (2007), arXiv:hep-ph/0602207 [hep-ph]

[18] M. Gronau, D. Pirjol, A. Soni, and J. Zupan, Phys.Rev. **D75**, 014002 (2007), arXiv:hep-ph/0608243 [hep-ph]