COMPUTATION OF COUPLING IMPEDANCE USING CAVITY CODES*

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Summary

Methods are presented for adapting existing cavity codes to the computation of the longitudinal and transverse coupling impedance of an obstacle of general shape above the cut-off of the beam pipe. The final result is given as an integral of the fields over the cavity or beam pipe surface only where the two are different. Numerical results for the longitudinal coupling impedance are compared with semi-analytic computation for a pillbox. In addition the "interference" between neighboring obstacles is explored as a function of frequency and obstacle spacing.

Introduction

In a recent paper[1] we showed that the longitudinal coupling impedance of a cavity with a beam pipe can be written as the sum of that for the beam pipe alone and a contribution from the cavity, which can be written as an integral of the fields over the surface of the cavity. A method for obtaining the fields and coupling impedance using an adaptation of the program SUPERFISH[2] was outlined. In the present paper we describe the numerical method in some detail, and present comparisons with previous calculations.

Field Analysis

We consider a beam pipe of cross sectional radius b and circumferential length $2\pi R$ in which an azimuthally symmetric cavity-like obstacle with dimensions small compared to R is located. The longitudinal coupling impedance is defined as[3]

$$Z_L(\omega) = -\int \vec{J}^* \cdot \vec{E}\, dv / |I_o|^2 \qquad (1)$$

where $\vec{E}$ is the electric field in the cavity/beam pipe combination due to driving current given by

$$J_z(r,z,t) = \begin{cases} (I_o/\pi a^2)\, e^{i\omega z/v} & , \ r < a \\ 0 & , \ r > a \end{cases} \qquad (2)$$

The factor $\exp(-i\omega t)$ has been omitted from all fields, currents and charges.

We write the fields in the cavity/beam pipe combination (denoted by subscript 2) as the sum of the fields for the beam pipe alone (denoted by subscript 1) and $\vec{e},\vec{h}$, the field increments due to the cavity. The fields $\vec{e},\vec{h}$ satisfy the homogeneous Maxwell equations

$$\nabla \times \vec{e} = i\omega\mu\vec{h} \quad , \quad \nabla \times \vec{h} = -i\omega\varepsilon\vec{e} \ , \qquad (3)$$

as well as the wall boundary condition

$$\vec{n}_2 \times \vec{e} = -\vec{n}_2 \times \vec{E}_1 \text{ on surface } S_2, \qquad (4)$$

where $\vec{n}_2$ is a unit vector normal to the cavity/beam pipe wall surface S_2.

Equations (3) and (4) represent an equivalent SUPERFISH problem, with specified frequency and boundary conditions. In our previous work[1] we assumed no losses, and as a result obtained a purely imaginary coupling impedance which reflected the resonant behavior of the entire cavity/beam pipe region above the cut-off frequency of the beam pipe. We therefore modified the program by introducing a small conductivity (imaginary dielectric constant) into the medium filling the beam pipe, corresponding to imposing outgoing boundary conditions on $\vec{e}$ and $\vec{h}$ at the cavity/beam pipe interface with no reflection. The result was a more realistic complex coupling impedance above cut-off, varying smoothly with frequency, as expected. Other aspects of the computational process may be worth mentioning: we assume a regular mesh with mesh lines z = constant in the beam pipe. This makes the numerical result independent of the values of z corresponding to the cavity/beam pipe interface. In addition, we perform the matrix inversion by Gaussian elimination, leaving the first and last rows to the end. In this way all block matrix inversions except the last are performed on real matrices.

Longitudinal Impedance

By using Maxwell's equations we showed[1] that the increment in coupling impedance due to the cavity could be written as

$$\Delta Z_L = \frac{1}{|I_o|^2} \int_{S_2 \neq S_1} dS\, \vec{n}_2 \times \vec{E}_1^* \cdot \vec{H}_2 \qquad (5)$$

where the surface integral is evaluated only over that part of the cavity which differs from the beam pipe. Further analysis shows that the contribution to Eq. (5) can be split into two parts by using

$$\vec{H}_2 = \vec{H}_1 + \vec{h} \qquad (6)$$

and that the contribution from $\vec{H}_1$ is imaginary, and inversely proportional to γ^2. In the relativistic limit, one then can write ΔZ_L as a line integral

$$\Delta Z_L \simeq -\frac{Z_o}{I_o} \int_{S_2 \neq S_1} r dr\, e^{-i\frac{\omega z}{c}} h_\phi(r,z), \qquad (7)$$

where we have used

$$E_{1r} = \frac{Z_o I_o}{2\pi r} e^{i\frac{\omega z}{c}} \qquad (8)$$

and where $Z_o = \sqrt{\mu/\varepsilon}$ is the impedance of free space.

Comparison with Previous Work

In a recent paper, Henke[4] calculates impedances for single pillboxes with side tubes using series expansions. We have used the present program for several of Henke's pillboxes and find excellent agreement of both the real and imaginary part of the coupling impedance. An example of this comparison is

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shown in Figure 1 for a pillbox cavity of length ℓ = b/20 and radius ρ = 1.1 b, where the complexity of the impedance is well duplicated even for a frequency as high as 7 times the cut-off of the beam pipe.

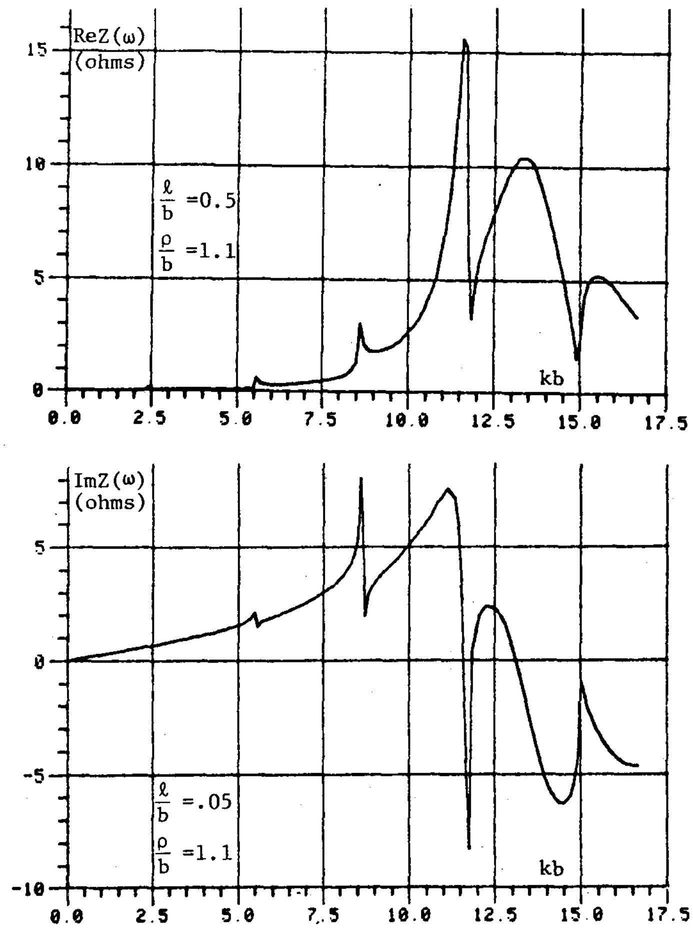

Fig. 1a. Real and imaginary part of impedance

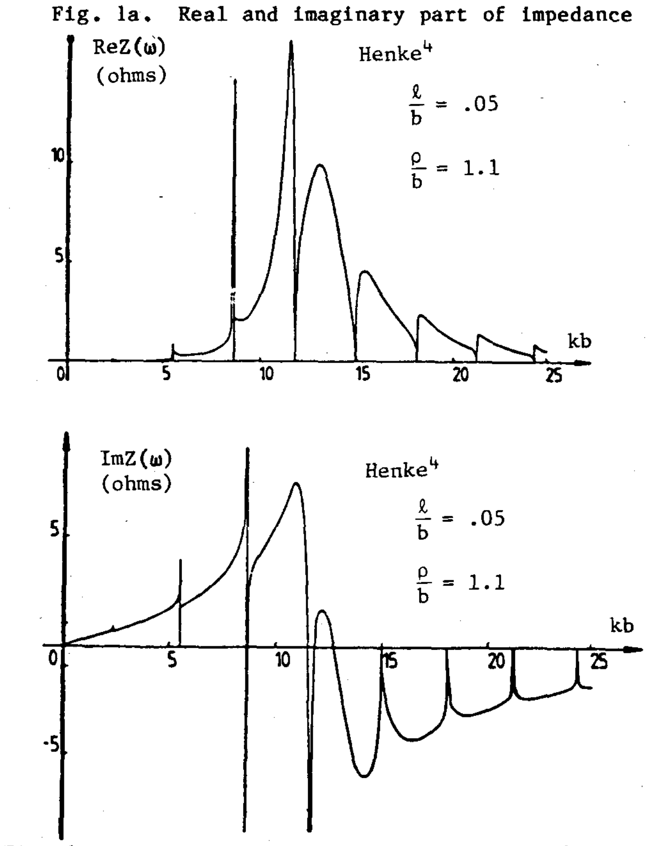

Fig. 1b. Real and imaginary part from Henke's paper[4]

Van Rienen and Weiland[5] use the program URMEL[6] to obtain the impedance of cavities with beam ports above cut-off. They employ an approximate outgoing boundary condition at the cavity/beam port interface, and obtain reasonably good agreement with Henke's calculations.[4] They also compared their results with those obtained by using TBCI[7] combined with Fast Fourier Transform (FFT), and conclude that the TBCI/FFT combination, which is now the primary tool for making impedance estimates for the SSC, gives only a "quite rough approximation" to the correct results for the PETRA cavity.

We have also obtained the impedance of a "bellows" for which Bisognano and Ng[8] did a TBCI/FFT computation using a Gaussian bunch. The agreement with our results for a point bunch shown in Figure 2 is reasonably good when one makes the frequency dependent correction for the bunch shape. (The Gaussian bunch reduction factor is about 0.8 at 21 GHz and 0.5 at 38 GHz.)

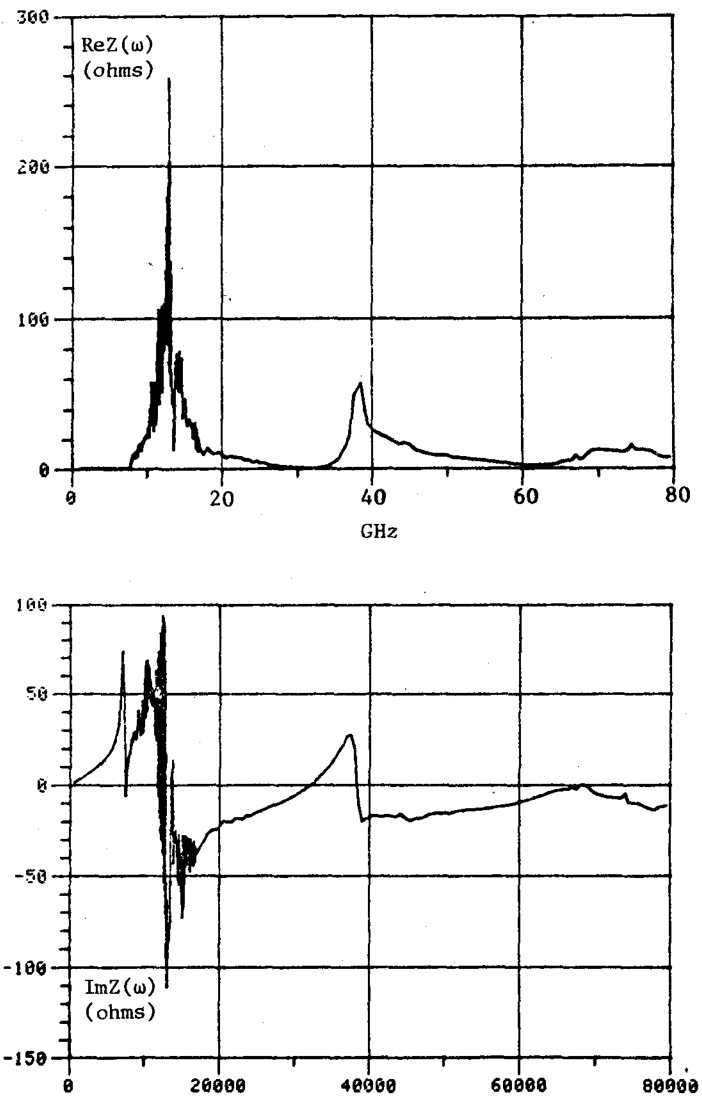

Fig. 2a. Real and imaginary part of impedance for a five "cavity" bellows with b = 1.5 cm, ρ/b = .1, L/b = .9 vs. frequency

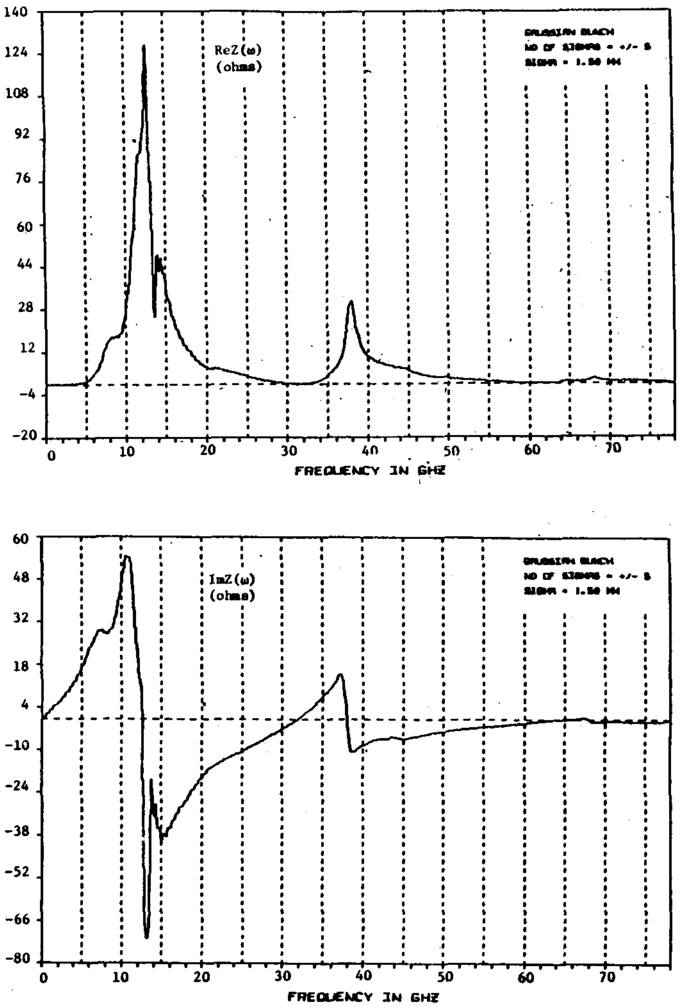

Fig. 2b. Real and imaginary part from Bisognano and Ng^8 vs. frequency for same cavity as in Fig. 2a.

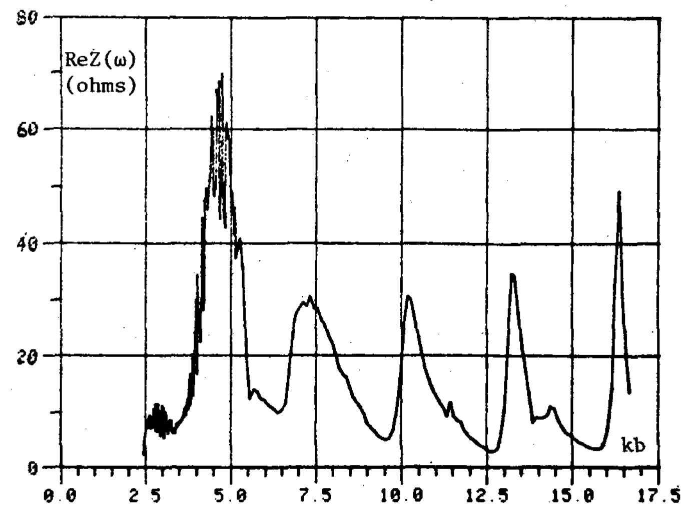

Fig. 3a. Real part of impedance; 1 cavity; $\rho = 4b/3$. $\ell = b$.

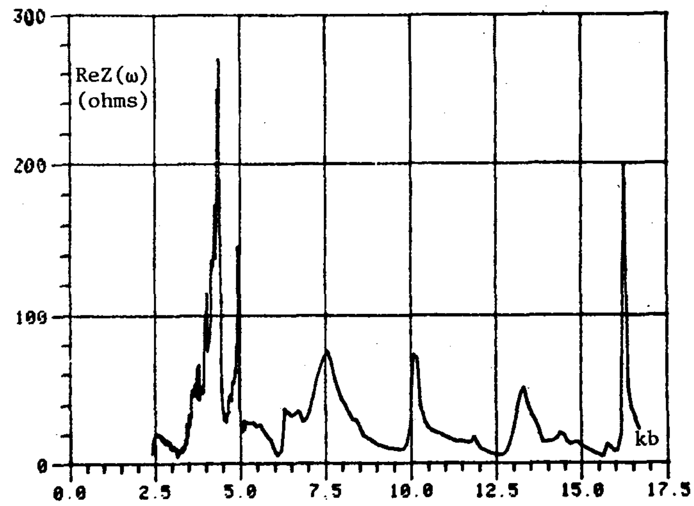

Fig. 3b. Real part of impedance; 2 cavities; $\rho = 4b/3$, $\ell = b$, $L = 3b/2$.

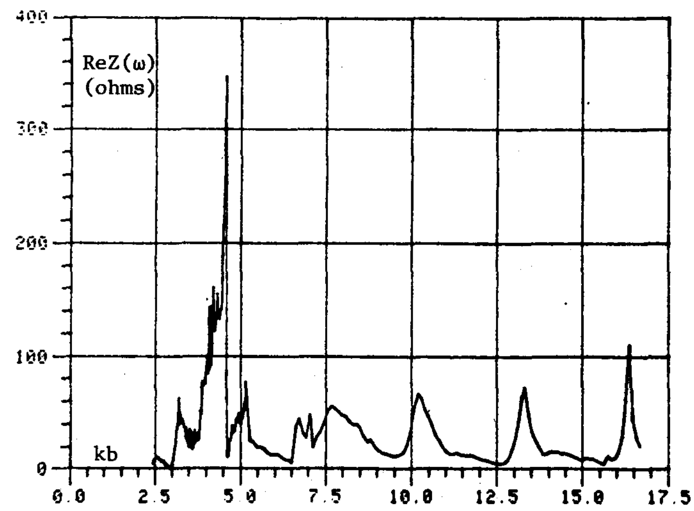

Fig. 3c. Real part of impedance; 2 cavities; $\rho = 4b/3$, $\ell = b$, $L = 2b$.

Numerical Results

In addition to the comparisons described in the previous section, we have calculated the impedance of two pillboxes as a function of frequency and the distance between the pillboxes. The purpose of this investigation is to explore the "interference" between separated obstacles in order to determine the conditions under which such impedances can be added. In these and other computations we use approximately 2000-3000 mesh points.

We have chosen each pillbox to have a length $\ell = b$ and a radius $\rho = 4\ b/3$. Figure 3 shows the impedance for a single cavity as well as for two cavities whose centers are separated by distances $L = 3\ b/2$ and $2\ b$. It appears that interference is important in the frequency range from 1 to 2 times the cut-off frequency. This work is continuing for cavities of different sizes and shapes.

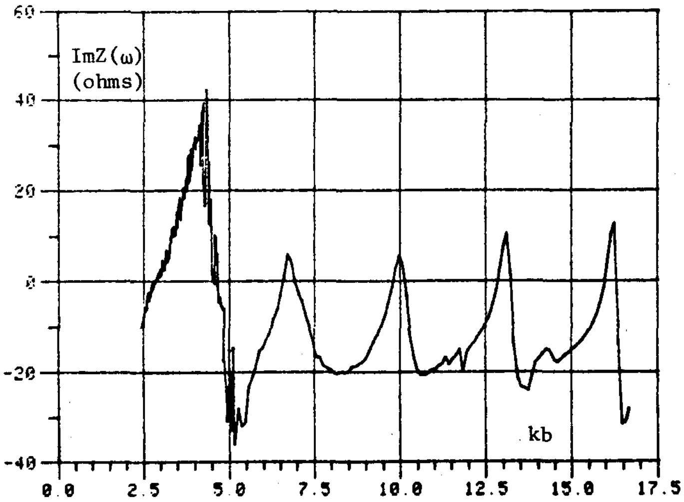

Fig. 3d. Imaginary part of impedance; 1 cavity; $\rho = 4b/3$, $\ell = b$.

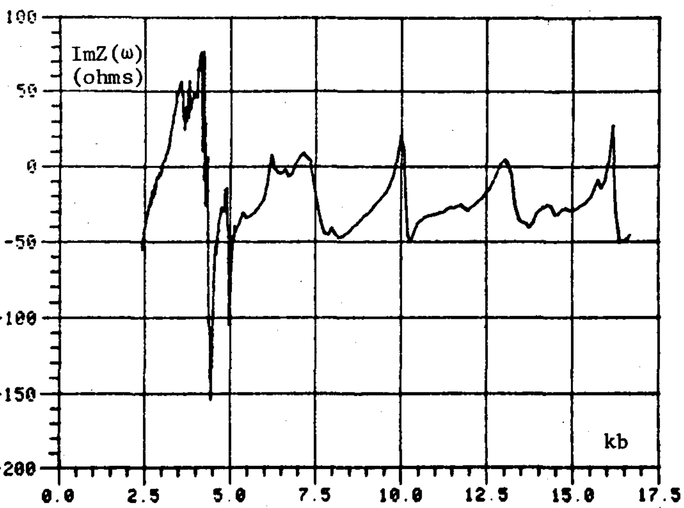

Fig. 3e. Imaginary part of impedance; 2 cavities, $\rho = 4b/3$, $\ell = b$, $L = 3b/2$.

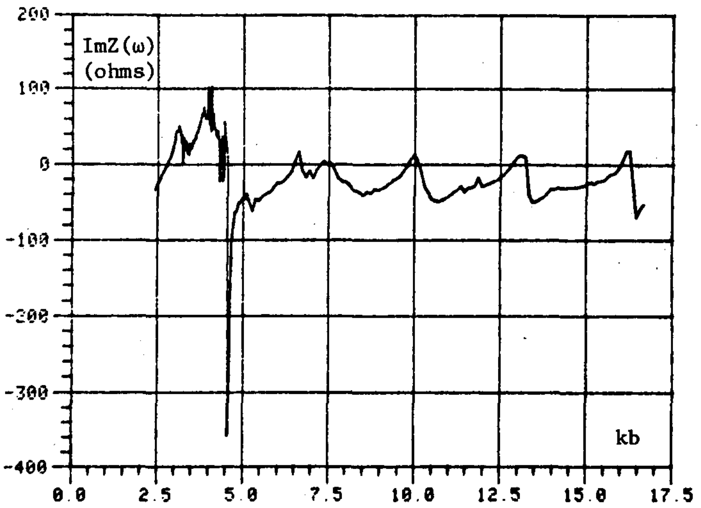

Fig. 3f. Imaginary part of impedance; 2 cavities, $\rho = 4b/3$, $\ell = b$, $L = 2b$.

Transverse Coupling Impedance

The conversion of the coupling impedance for an obstacle to an integral only over the surface of the obstacle works equally well for the transverse coupling impedance. Starting with the driving current[9]

$$J_z = \frac{I_1}{\pi a^2} e^{i\omega z/v} \cos\theta \, \delta(r-a), \tag{9}$$

one can use Maxwell's equations to show that the increment in coupling impedance is given by

$$\Delta Z_T^{(1)} = (v/\omega) \, \Delta Z_L^{(1)} ,$$

where

$$\Delta Z_L^{(1)} = \frac{v}{\omega |I_1|^2} \int_{S_2 \neq S_1} ds \, \vec{n}_2 \times \vec{E}^*{}_1 \cdot \vec{H}_2$$

$$= \frac{v}{\omega |I_1|^2} \int_{S_1 \neq S_2} ds \, \vec{n}_1 \times \vec{E}_2 \cdot \vec{H}_1^* . \tag{10}$$

Here the superscript (1) denotes the dipole mode.

The fields for the beam pipe alone (subscript 1) can readily be obtained outside the beam from the potentials (Lorentz gauge) in the relativistic limit:

$$\phi = - cA_z = - \frac{Z_o I_1}{2\pi b} \left(\frac{b}{r} - \frac{r}{b}\right) e^{ikz} \cos\theta \ , \ r > a \tag{11}$$

where b is the beam pipe radius.

The computation of the fields $\vec{h} = \vec{H}_2 - \vec{H}_1$ and $\vec{e} = \vec{E}_2 - \vec{E}_1$ requires a program which can handle azimuthally asymmetric modes. Work is now in progress on the code ULTRAFISH.[10] In addition, the URMEL codes are now being adapted by van Rienen and Weiland[5] for this application.

Acknowledgment

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References

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9. cf. Reference 3, p. 359.

10. Gluckstern, Holsinger, Halbach and Minerbo, Proceedings of the 1981 Linac Conference, Santa Fe, N. Mex., October 1981, p. 102.