STRANGE PARTICLE DECAYS AND THE

NATURE OF WEAK INTERACTIONS

by

Jógesh C. Pati

TN-60-1051a - Vol I

STRANGE PARTICLE DECAYS AND THE NATURE
OF WEAK INTERACTIONS

VOLUME I*.

by

Jogesh C. Pati

University of Maryland, College Park, Maryland

Physics Department Technical Report No. 193

September, 1960

UNIVERSITY OF MARYLAND
PHYSICS DEPARTMENT
COLLEGE PARK, MARYLAND

TN-60-1051a - Vol I

STRANGE PARTICLE DECAYS AND THE NATURE OF WEAK INTERACTIONS

VOLUME I*.

by

Jogesh C. Pati

University of Maryland, College Park, Maryland

Physics Department Technical Report No. 193

September, 1960

This is the first volume containing chapters I, II and III (Pages 1-141). The second volume consists of chapter IV and the appendices (Pages 142-281).

* This research was supported by the United States Air Force through the Air Force Office of Scientific Research and Development Command under Contract AF 49(638)-24. Reproduction in whole or part is permitted for any purpose of the United States Government.

ABSTRACT

Title of Thesis: Strange Particle Decays and the Nature of Weak Interactions

Jogesh Chandra Pati, Doctor of Philosophy, 1960

Thesis directed by: Dr. Joseph Sucher, Assistant Professor of Physics, University of Maryland.

In this thesis an attempt is made to provide a natural explanation of the approximate validity of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - rule as well as the slowness of the leptonic modes of strange particle - decays compared to the nonleptonic ones. The simplest scheme of weak interaction, namely the Gell-Mann-Dellaporta tetrahedron scheme, which introduces the primary weak interactions only among the leptons, the nucleons and the Λ -hyperon, is adopted. It is shown ih chapter II that for $\Lambda \rightarrow N + \pi$ -decays a <u>new</u> set of diagrams, satisfying the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - rule is much more important than the "usually" considered diagram, which contains appreciable amount of $|\Delta \underset{\sim}{I}| = 3/2$ transitions in addition to $|\Delta \underset{\sim}{I}| = \frac{1}{2}$- ones. This makes it easier to explain the <u>approximate validity</u> of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - rule. Furthermore, the inclusion of this new set of diagrams, which contribute only to nonleptonic modes provides a consistent explanation of the "slower" rate of the leptonic modes and the "faster" rates of the nonleptonic ones, if we associate the strangeness violating current with a weaker coupling constant than the strangeness conserving one.

The relative rates of the various K^+ and $K^0_{1,2}$ - decay modes are estimated in chapter III with the mechanism of Λ - decay developed in chapter II both by perturbation theory and by approximate form - factor calculations. It is shown that the inclusion of the new class of diagrams for $\Lambda \to N+\pi$ -decays (mentioned above) leads to improved agreement with experiments for all the relative rates of K-meson decay modes. We assume phenomenologically a slight damping of the matrix element (evaluated in the lowest order perturbation theory) for each pion emission from a closed baryon - antibaryon loop. This seems plausible due to the observed rate of $\pi \to \mu+\nu$ -decay. It is shown that, by including the new class of diagrams for $\Lambda \to N+\pi$ -decays one can explain the previously unexplained ratios $W(K^+ \to \pi^+ + \pi^0)/W(K^0_1 \to 2\pi)$ and $W(K^+ \to \pi^0 + e^+ + \nu)/W(K^0_1 \to 2\pi)$ etc. without the introduction of final state $\pi - \pi$ -interaction; one can also explain the fact that the final state in $K^+ \to \pi^+ + \pi^- + \pi^+$ -decay is predominantly summetric between the three pions. The nature of agreement obtained with experiments may indicate that the final state $\pi - \pi$ interaction is perhaps not as important as has been suggested by other workers.

We finally examine in chapter IV the possibility that the four fermion interactions may be mediated by a charged vector boson. It is pointed out that the immediate objection to such a possibility due to the slowness of $\mu \to e+\gamma$ -decay can be removed by assigning opposite lepton numbers to μ^- and e^- and adopting a "restricted" four component theory of the neutrino. The various possible effects of this nonlocality in the four fermion interactions are discussed. It is pointed out that accurate measurement of the pion spectrum in K_{e3} -decay may serve to distinguish between local and nonlocal Fermi-interactions. Furthermore the effect of the intermediate boson on the $\Lambda \to N+\pi$ -decays is examined. It is shown that the matrix element of the new class of diagrams (whose local limit is discussed in chapter II) is such that we can not explain the observed sign of the asymmetry parameter of $\Lambda \to p+\pi^-$ -decay in the framework of V-A-interaction, unless the intermediate boson is extremely heavy. This may be regarded as an evidence against the intermediate boson hypothesis. However the sign of the asymmetry parameter can be explained if we adopt V + A - interaction for Λ-decay.

ACKNOWLEDGEMENT

It is my humble pleasure to acknowledge my deep indebtedness to Professor Sadao Oneda for suggesting the investigation carried out in this thesis and for his continued guidance, help, collaboration and inspiration. I am most grateful to Professors George A. Snow, Joseph Sucher and John S. Toll for their kind encouragement, help and interest throughout my graduate study at the University of Maryland. It is my great pleasure to thank specially Professor Sucher for many comments, criticisms and suggestions regarding the work in this thesis. I am also grateful to Dr. B. Sakita at the University of Wisconsin for his kind interest and communications.

I wish to thank Mr. D. Morgan for assistance with numerical calculations. Finally I wish to acknowledge gratefully the financial support of the U.S. Air Force through the Air Force Office of Scientific Research under contract no. AF 49(638) - 24.

TABLE OF CONTENTS

Chapter				Page
ACKNOWLEDGEMENT				ii
List of Figures				vi
List of Tables				ix
I.	INTRODUCTION			1
	A.	Nature of Interactions of Elementary Particles		1
	B.	Experimental Foundation of Universal Four Fermion Interactions Between Nucleons and Leptons		5
		1.	Preliminary Discussion	5
		2.	Experiments on Invariance Properties in Weak Interactions	8
		3.	Experiments on Nature of β -Decay Interactions	10
		4.	Experiments on Nature of μ -Decay and μ -Capture Interactions	13
		5.	Experiments on the Nature of Neutrino	15
		6.	Experiments on Nature of Weak Interaction Constants	20
		7.	Conclusion	22
	C.	Theoretical Frame-Work of Universal Four Fermion Interaction		23
	D.	The Strange Particles		28
	E.	General Features of Strange Particle Decays		32
		1.	The $\lvert\Delta S\rvert = 1$ - Rule	36
		2.	The $\lvert\Delta \underset{\sim}{I}\rvert = \frac{1}{2}$ - Rule	38
		3.	The Slowness of Leptonic Modes	41

F. A List of Problems 45

II. THE DECAY OF THE Λ -HYPERON 47

Abstract 47

A. Scheme of Interaction 48

B. Preliminaries on Λ -Decay 54

C. Previous Work (The OMS-Analysis) and Successes 63

D. Needs for New Lock 77

E. A New Class of Diagrams 79

F. Evaluation of the Matrix Element of the New Diagram 94

G. Conclusion 104

III. THE DECAY OF THE K-MESON 106

Abstract 106

A. The Leptonic Decay Modes of K-Meson 107

B. The Non Leptonic Decay Modes of K-Meson 118

C. The Asymmetry in the Energy Distribution of $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$ -Decay 135

D. Concluding Remarks 140

IV. THE POSSIBLE EXISTENCE OF AN INTERMEDIATE VECTOR BOSON 142

Abstract 142

A. Speculation 144

B. General Effects of the Nonlocality in Four Fermion Interactions 151

C. Absence of $\mu \rightarrow e + \gamma$ -Decay 161

D. The Assignment of Lepton Numbers and the Nature of the Neutrino 165

E. Effects of Non-Locality on the Energy Spectra in K_{e3} ($K_{\mu 3}$) -Decays....... 178

F. Effects of Non-Locality on the Decay of the Λ -Hyperon.................... 188

APPENDIX

I. NOTATIONS... 196

II. THE ANGULAR DISTRIBUTION OF PIONS AND LONGITUDINAL POLARISATION OF NUCLEON IN THE DECAY OF Λ -HYPERON.................... 199

III. MATRIX ELEMENTS OF VARIOUS K-MESON-DECAY MODES.................................... 202

IV. DECAY RATES OF VARIOUS K-MESON-MODES............. 233

V. ENERGY SPECTRA IN $K_{\mu 3}$ AND K_{e3} -DECAYS FOR LOCAL AND NON LOCAL FOUR FERMION INTERACTION.. 262

VI. MATRIX ELEMENT OF THE SINGLE NEUTRON-INTERMEDIATE STATE DIAGRAM FOR $\Lambda \rightarrow N + \pi$ -DECAY WITH INTERMEDIATE VECTOR BOSON.......................... 272

SELECTED BIBLIOGRAPHY.................................... 279

List of Figures

Figure		Page
1.	The Tiomno-Wheeler Triangle for Universal Fermi Interaction	22
2.	Gell-Mann-Dellaporta Tetrahedron Scheme	49
3.	Decay of the Λ -Hyperon	57
4.	The usual lowest order diagram for $\Lambda \rightarrow N+\pi$ -Decays	67
5.	The usual lowest order diagram for $\Lambda \rightarrow N+\pi$ - decays with pionic and kaonic corrections inside the bubble	67
6.	The diagram for $\pi \rightarrow \mu+\nu$ -decay	70
7.	The lowest order π-N - intermediate state-diagram for $\Lambda \rightarrow N+\pi$ -decay	75
8.	The general π-N - intermediate state - diagram for $\Lambda \rightarrow N+\pi$ -decay	75
9.	Diagramatic Expansion of $\Lambda \rightarrow N+\pi$ - decay in terms of intermediate states	80
10.	The simplest single-neutron-intermediate state Feynman diagram for $\Lambda \rightarrow N+\pi$ -decay	88
11.	Diagram involving single pion-correction to fig. 10	89
12.	Virtual Exchanges of pions and kaons between Λ, p, $\bar{p}$ and n	89
13.1 13.2 13.3	Various Modifications of the ($p\Lambda$) and (np) - vertices due to strong virtual effects	90
14.	The single neutron-intermediate state diagram with corrections shown in fig. 13.1	94
15.	The lowest order diagram for $K^+ \rightarrow \mu^+ + \nu$ -decay	108
16.	The lowest order diagram for $K^+ \rightarrow \pi^0 + \mu^+(e^+) + \nu$ - decay	108

17. The lowest order diagram for $K^+ \to \pi^+ + \pi^- + \mu^+(e^+) + \nu$ - decay.......... 108

18.
19.
20.
21. Diagrams for $K \to 2\pi$ -decays.......... 121

22.
23. Diagrams for $K^+ \to \pi^+ + \pi^- + \pi^+$ -decay.......... 127

24. Lowest order diagram for $n \to p + e^- + \bar{\nu}$ - decay with local Fermi interaction.......... 151

25. Lowest order diagram for $n \to p + e^- + \bar{\nu}$ - decay with nonlocal Fermi interaction.......... 151

26. Lowest order diagram for $\Lambda \to p + e^- + \bar{\nu}$ - decay with intermediate boson.......... 153

27. Diagram for $\pi^+ \to \mu^+ + \nu$ -decay without intermediate boson.......... 155

28. Diagram for $\pi^+ \to \mu^+ + \nu$ -decay with intermediate boson.......... 155

29. Diagram for $\mu^- \to e^- + \nu + \bar{\nu}$ -decay with intermediate boson.......... 156

30. Lowest order Feynman diagram for $\mu \to e + \gamma$ - decay without intermediate boson.......... 161

31. Feynman diagram for $\mu \to e + \gamma$ -decay without intermediate boson and with two weak vertices.......... 161

32. Lowest order Feynman diagrams for $\mu \to e + \gamma$ - decay with intermediate boson.......... 162

33. The scheme for $\pi^- \to \mu^- \to e^-$ -chain with μ^- being a lepton.......... 168

34. The scheme for $\pi^- \to \mu^- \to e^-$ -chain with μ^- being an antilepton.......... 170

35. The assignments of two-component-theory and restricted four-component theory.......... 172

36. The diagram for K^+_{e3} -decay without intermediate boson.......... 178

37. The diagram for K^+_{e3} -decay with intermediate boson.. 178

38. Electron Energy Spectrum in K_{e3} -decay with local Fermi interaction........................ 184

39. Electron Energy Spectrum in K_{e3} -decay with nonlocal Fermi interaction........................ 185

40. Pion Energy Spectra in K_{e3} -decay with local and non local Fermi interactions.................. 187

41. The usual diagram for $\Lambda \to p + \pi^-$ -decay with intermediate boson................................ 188

42. The single-neutron-intermediate State diagram for $\Lambda \to N + \pi$ -decay with intermediate boson....... 188

43. Predicted muon spectrum in $K_{\mu 3}$ -decay with local Fermi interaction.............................. 269

List of Tables

Table		Page
I.	Isotopic spin and strangeness quantum numbers of strongly interacting particles in Gell-Mann-Nishijima-Scheme........................	31
II.	Decays of Strange Particles........................	33
III.	Various features of the $\lvert \Delta I \rvert = \frac{1}{2}$-rule..........	39
IV.	Branching Ratios of the leptonic modes of hyperon-decays..................................	42
V.	The Sakata Model..................................	52
VI.	The form-factor A_c of fig. 4 for few values of cut off λ	69
VII.	Rates for $\Lambda \to p + \pi^-$ -decay given by figs. 4 and 5..	73
VIII.	Comparison of the matrix elements of figs. 14 and 5......................................	100
IX.	Compilation of the main results of chapter III...	139
X.	Form factors for various K-meson decay-modes evaluated in the lowest order perturbation theory with cut off $\Lambda \simeq 1.8$ mp.....................	232
XI.	The values of Θ_1 and Θ_3 determining the matrix element of fig. 42 for $\Lambda \to N + \pi$ decay for a few values of cut off λ and mass of the intermediate boson m_B	278

Errata

1. Page 11, 5th line from bottom; γ is missing in front of ray.
   4th line; $\mathcal{H}_\gamma = -0.67 - 0.10$

2. Page 25, 4th line from top; change 'renorlisation' to 'renormalisation'.

3. Page 29, Eq. (11) should read

$$Q = I_z + \frac{N}{2} + \frac{S}{2}$$

4. Page 34, "Tiche" should be changed to "Ticho".

5. Page 36, $\frac{\Delta S}{\Delta Q} = -1$ at the bottom line should be changed to $\frac{\Delta S}{\Delta Q} = +1$

6. Page 37, 4th line from top; change ΔB to ΔN.

7. Page 41, 10th line from top; change (76) to (86).

8. Page 44, 2nd line from bottom, change "Oneida" to "Oneda"

9. Page 106, 7th line from bottom, change 3 to $\sqrt{3}$.

10. Page 182, 10th line from bottom, change Fig. 42 to 43.

11. Page 280, Add at the top; R. Jost "Eine Bemerkung Zum - Theorem" - Helv. Phys. Acta, 30, 409 (1957).

12. Where ever there appears "Gellmann", change to Gell-Mann.

Note: The author will appreciate his attention being drawn to any further errors and missing references.

CHAPTER I.

INTRODUCTION*

A. Nature of Interaction of Elementary Particles

The advent of new high-energy machines has revealed a host of new particles (the heavy mesons and the heavy baryons) with a number of "curious" properties. The experimental study and the theoretical understanding of their behaviour are of great current interest.

Prior to the discovery of these new particles, one knew the existence of the photon, the electron, the muon, the pions and the nucleons. The existence of the neutrino was known indirectly. These particles had already revealed the three basic types of forces (leaving aside gravitational forces) known to us so far, which in order of their strength are:

(1) The Strong Interactions:

A typical example is the pion-nucleon interactions, presumably proceeding through the virtual yukawa process $N \rightarrow N + \pi$. The strength of this process may be measured by the dimensionless coupling constant:

$$g_\pi^2/4\pi\hbar c \approx 15$$

* Parts of Chapters II and III of this thesis have been reported by the author in the University of Maryland Physics Department Technical Report Nos. 160 and 171 respectively with Drs. S. Oneda and B. Sakita as co-authors. The former is to be published in the July 1, 1960 issue of Physical Review and the latter in Nuclear Physics. Part of Chapter IV has been reported by the author in the University of Maryland Physics Department Technical Report No. 126 with Dr. S. Oneda as coauthor and has been published in Physical Review Letters 2, 516 (1959).

In general the strong interactions are characterised by Coupling Constants of the order of unity.

(2) The Electromagnetic Interactions:

A typical example is the virtual emission or absorption of a photon by any charged particle. The strength is characterized by the universal fine structure constant .

$$e^2/\hbar c \approx \frac{1}{137}$$

(3) The Weak-Interactions:

The typical examples are the interactions involved in β-decay and μ-decay. The strength of such interactions is characterized by:

$$f_F^{2}/(\hbar c)^2(\hbar/m_\pi c)^4 \approx 10^{-14}$$

At the moment the purely electromagnetic phenomena are very well understood in the framework of quantum electrodynamics. Any such attempt to have a profound understanding of the strong interactions is however frustrated, since the perturbation series in the S-matrix formulation becomes useless for coupling constants of the order of unity. The axiomatic approach of field theory (leading to dispersion-integrals) have to take recourse to a number of simplifying assumptions from the point of view of practicability. The difficulties in understanding the weak processes are even worse in a sense, since all the observed weak processes, (except the decay of the muon) involve strongly interacting particles, so that one has not only to search for the true nature of the bare weak-interactions but one must worry about handling the strong virtual processes occuring in the weak-decays.

The theories of both strong and weak-interactions have developed in very close analogy with the framework of quantum electrodynamics.

Yukawa postulated that the exchange of pions play the same role in nuclear forces as that of photons in electromagnetism. The theory of β -decay, first proposed by Fermi[1], is based on the idea that the decay takes place by the interaction of the nucleonic-current $-(\bar{p}O_i n)$ with the leptonic current $(\bar{e}O_i \nu)$ at a point (through the exchange of a boson of infinite mass[2], if one likes to say so), just as the electromagnetic currents of two charged particles interact with each other through the exchange of a photon.

Being guided by such analogies, the observation that the coupling constant of all elementary particles to the electromagnetic field is either zero or the universal parameter $|e|$ and not any fractions or multiples of it, is quite suggestive. It may indicate that all the elementary-particle interactions probably divide themselves into a few groups, and each group tries to maintain a hierarchy of universality. Such a viewpoint leads in the case of strong interactions to the hypotheses of "Global Symmetry"[3] for the pions and the "Cosmic Symmetry"[4] for the kaons and in case of weak-interactions to the proposition[5] of "Universal Four Fermion Interaction."

---

1 E. Fermi, Z. Phys. 88, 161 (1934).

2 The possibility that the weak-interactions are intermediated by a boson of finite mass is an interesting possibility and has yet to be checked by experiments. This will form the subject matter of the discussion in Chapter IV. However, as will be shown later, the effect of the existence of such an intermediate particle is negligible for processes like β-decay with low momentum transfer. So, for a while we will assume that the Four-Fermion Interactions are local.

3 M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

4 J. Sakurai, Phys. Rev. 113, 1679 (1959).

5 G. Puppi, Nuovo Cimento 5, 505 (1948).
O. Klein, Nature 161, 897 (1948).
J. Tiomno and J.A. Wheeler, Rev. Modern Phys. 21, 144 (1949).
Lee, Rosenbluth and Yang, Phys. Rev. 75, 905 (1949).

There will not be any occasion to discuss the symmetries of the strong interactions in the present work. However, since a good deal of subsequent discussion will rest heavily on a scheme of four fermion interactions, a brief discussion on the experimental and theoretical backgrounds of "Universal Fermi Interactions" will be given in the next two sections.

B. Experimental Foundation of Universal Four-Fermion Interactions Between Nucleons and Leptons

1. Preliminary Discussion

The three main weak processes involving nucleons and leptons are:

$$\text{(i)}\ n \rightarrow p + e^- + \bar{\nu}$$
$$\text{(ii)}\ \mu \rightarrow e + \nu + \bar{\nu}$$
$$\text{(iii)}\ \mu^- + p \rightarrow n + \nu$$

Even in the preCo^{60} experiment days these three processes had already revealed a number of similarities among themselves. First of all, these three processes involve four fermions directly. Secondly, it was recognised[5] that the coupling constants involved in all of these processes are $\approx 10^{-49}$ erg cm^3. Thus it was natural to extend the hypothesis of β-decay interaction into μ-decay and μ-capture.

Relativistic invariance prescribes five possible forms of interactions involving four fermions. For the above three processes they are:

$$n \rightarrow p+e^-+\bar{\nu} : \quad H_{\beta\text{-decay}} = \sum_i (\bar{p}O_i n)(\bar{e}\, O_i[C_i + C_i' i\gamma_5]\nu) \tag{1}$$

$$\mu^- \rightarrow e+\nu+\bar{\nu} : \quad H_{\mu\text{-decay}} = \sum_i (\bar{\nu}O_i\mu^-)(\bar{e}\, O_i[D_i + D_i' i\gamma_5]\nu) \tag{2}$$

$$\mu^-+p \rightarrow n+\nu : \quad H_{\mu\text{-capture}} = \sum_i (\bar{n}O_i p)(\bar{\nu}\, O_i[E_i + E_i' i\gamma_5]\mu^-) \tag{3}$$

Where the sum i goes over scalar, vector, tensor, axial vector and pseudoscalar forms of interactions with the corresponding O_i given by:

$$\begin{aligned} O_S &= I \\ O_V &= \gamma_\mu \\ O_T &= \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) \\ O_A &= -i\gamma_\mu\gamma_5 \\ O_P &= i\gamma_5 \end{aligned} \tag{4}$$

Thus the above three processes involve in general 30 unknown coupling constants. If we assume time-reversal invariance (hence CP-invariance by the TCP theorem) of weak-interactions, then these coupling constants are real. Further, if we could assume invariance under space-reflection P, then either all the primed or all the unprimed coupling constants are zero for each of the above three interactions.

Fortnuately enough the numerous experiments done in the past few years have greatly restricted the forms of the above interactions and the relationships between the various coupling constants.

In particular the various recent experiments on several aspects of β-decay, μ-decay, μ-capture and $\pi \to \mu \to e$-decays reveal that:

(1) These processes are non-invariant under space-reflection(P) and charge-conjugation(C). They are, however, consistent with the hypothesis of CP[6] invariance and T-invariance, thus satisfying the more general invariance property, the TCP-invariance.

(2) The four-fermion interactions involved in each[7] of the above mentioned processes has Vector-Axial Vector (V-A) form.

(3) The bare coupling constants for the vector and axial-vector parts are "presumably"[8] the same for any one of these processes.

---

[6] It was first proposed by Landau that "Combined Parity" (CP-Invariance) should be considered as the fundamental symmetry law of nature instead of C and P invariance seperately. See L. D' Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 405(1957), Soviet Phys. JETP 5, 336(1957). See also T. D. Lee and C. N. Yang, Phys. Rev. 104, 254(1956).

[7] The situation is not yet completely clear for μ-capture.

[8] One says "presumably", because whatever differences are found between the coupling constants, although small, are presumed to arise due to the effects of the strong virtual processes.

(4) The bare coupling constants for the processes (i), (ii) and (iii) listed before are "presumably" equal to each other, which implies a scheme of "Universal Weak Interactions" in processes involving nucleons and leptons.

(5) The antineutrino[9] involved in β^--decay is always right handed ($\frac{\vec{\sigma}_{\bar{\nu}} \cdot \vec{p}_{\bar{\nu}}}{|\vec{\sigma}_{\bar{\nu}}||\vec{p}_{\bar{\nu}}|} = +1$) and the neutrino involved in β^+-decay is always left handed ($\psi_\nu = + i\gamma_5 \psi_\nu$).

(6) If one assumes the usual assignment of lepton numbers (i.e. ν, e^-, μ^- are leptons and $\bar{\nu}$, e^+, μ^+ are anti-leptons) then neutrino appears in nature in only two-component form.

We will give below a list of some of the key-experiments[10] leading to the above conclusions:

---

[9] By definition the neutral massless spin-$\frac{1}{2}$-particle accompanying electron in β^--decay is called "Antineutrino" ($\bar{\nu}$). Hence the corresponding companion of positron is β^+-decay is called "Neutrino" (ν). Once we have defined neutrino and antineutrino as above, whether a $\bar{\nu}$ or ν accompanies μ^- in π^--decay, depends upon whether μ^- has the same leptonic charge as e^-, and whether leptonic charge is conserved.

[10] For a detailed review of the various key-experiments and their analysis, see C. S. Wu, Rev. Modern Phys. 31, 783(1959); E. J. Konopinski, Ann. Rev. Nuclear Sci. 9, 99(1959); Ya. Smorodinskii, Soviet Physics (USPEKHI) Vol. 2, (67), 1, 1(1959).

2. Experiments on Invariance Properties in Weak Interactions

(1) β -Asymmetry from Polarised Nuclei:

The first experiment that revealed the violation of P and C invariance in weak-interactions is the famous Co^{60} experiment performed by Wu, et.al.[11] The experiment essentially consisted of aligning the spins of Co^{60} nuclei along a certain direction and to determine that the electrons were emitted preferentially in the direction opposite to that of nuclear spin. Thus it showed that β-decay does not conserve parity and charge-conjugation-invariance.

(2) Experiments on $\pi \to \mu \to e$ Chains:

The e^- and e^+ emitted in μ^- and μ^+ decays are found[12] to be distributed asymmetrically about the direction of motion of the muon. In particular both $e^{\mp}$ are found[13] to be emitted perdominantly antiparallel to the $\mu^{\mp}$ momentum in the rest frame of the parent pions. Furthermore, the measurements[14] of circular polarisation of the γ-rays emitted by the high energy $e^{\mp}$ show that the high energy e^- is left handed and e^+ is right handed. Direct measurements of the helicities of $\mu^{\mp}$ have yet to yield conclusive[15] results.

---

[11] Wu, Ambler, Hayward, Hopes, and Hudson, Phys. Rev. 105, 1413(1957).

[12] Bordon, Verley and Lederman, Phys. Rev. Letters 2, 56(1956).

[13] J. I. Friedman and V. L. Telegdi, Phys. Rev. 105, 1681(1957). Garwin, Lederman and Weinrich, Phys. Rev. 105, 1415(1957).

[14] Marcq, Crowe and Haddock, Phys. Rev. 112, 2061(1958).

[15] A preliminary measurement by Love et.al. (Love, Marder, Nadelhaft, Siegel and Taylor, Phys. Rev. Letters 2, 107(1959)) consisted of detecting the asymmetry of β-decay of B^{12} produced by polarised muons via the reaction $\mu^- + C^{12} \to B^{12} + \nu$. Preliminary results indicated that negative muons are emitted with positive helicity. Continued measurements (Love et.al., Phys. Rev. Letters 4, 382(1960)) show however, that the results are not conclusive.

The fact that the $e^{\mp}$ are distributed asymmetrically with respect to the direction of motion of $\mu^{\mp}$ in the rest frame of the parent pions, together with the consequent deduction that muons must have been polarised in pion-decay implies that P and C are not conserved in both π-decay and μ-decay.

In fact, just from the observation of opposite helicities of high energy $e^{\mp}$ coming via muons from positive and negative pions, it follows that the involved weak-interactions are not invariant under charge conjugation.

It is easy to check; the fact that both $e^{\mp}$ are emitted predominantly antiparallel to the $\mu^{\mp}$ momentum with opposite helicities is consistent with CP-invariance[16].

(3) Experiments on Time-Refelection Invariance in β-decay:

As suggested by Jackson, Treiman[17] and Wyld, there have been experiments[18] to detect the coefficient D of the term $\frac{\langle\vec{J}\rangle \cdot \vec{p}_e \times \vec{p}_\nu}{(J)\, E_e E_\nu}$ in the distribution of the decay products of oriented nuclei with polarisation $\frac{\langle\vec{J}\rangle}{J}$.

The results show $D = -0.04 \pm 0.07$. This indicated that β-decay interaction is invariant under time-reflection.

---

[16] This can be checked by performing the following operations:

Under P: $\vec{p} \to -\vec{p}$; $\vec{\sigma} \to \vec{\sigma}$; Particle $\to$ Particle

Under C: $\vec{p} \to \vec{p}$, $\vec{\sigma} \to \vec{\sigma}$, Particle $\to$ Anti Particle

[17] Jackson, Treiman and Wyld, Phys. Rev. 106, 517(1957).

[18] Burgy, Krohn, Novey and Ringo, Phys. Rev. Letters 1, 324(1958).
Clark, Robson and Nathans, Phys. Rev. Letters 1, 100(1958).

3. Experiments on Nature of β-decay Interactions:

(1) Electron-Neutrino Angular Correlation Coefficients:

It used to be believed only a few years ago that β-decay interaction must be consisting of (S,T) combination rather than (V,A). A strong evidence for such belief came from the results of the old[19] He^6-(e-ν) angular correlation experiment. The experiment however has been repreated more carefully and has revealed completely different results. It has been found[20] that the angular correlation coefficient $\lambda = -0.39 \pm 0.02$, which certainly favors axial-vector coupling in Gamow-Teller transition. Similar experiments[21] on A^{35}, which involves primarily Fermi transition, strongly supports, at the moment, vector rather than scalar-interaction.

The e-ν angular correlation experiment in

$$_3Li^8 \rightarrow {}_4Be^8 + e^- + \bar{\nu}, \quad Be^8 \rightarrow 2\alpha$$

at California[22] Institute of Technology sets the upper limits of tensor mixture in G-T interaction to be less than 10%.

<u>Thus the (e-ν) angular correlation experiments strongly favor (V,A) combination for β-decay interaction.</u>

---

[19] B. N. Rustad and S. Ruby, Phys. Rev. <u>97</u>, 991(1955).

[20] Herrmannsfeldt, Burman, Stahelin, Allen and Braid, Phys. Rev. Letters <u>1</u>, 61(1958).

[21] Herrmannsfeldt, Maxson, Stahelin and Allen, Phys. Rev. <u>107</u>, 641(1957).

[22] Lauritsen, Barnes, Fowler and Lauritsen, Phys. Rev. Letters <u>1</u>, 61(1958). Barnes, Fowler, Girsenstein, Lauritsen and Nardberg, Phys. Rev. Letters <u>1</u>, 328(1958).

(2) Beta-decay of Polarised Neutrons:

Burgy[23] et. al. have studied the correlation coefficients A and B of $\vec{P}_e$ and $\vec{P}_\nu$ respectively with the spin of polarised neutron. They find:

$A = -0.11 \pm 0.02, \quad B = 0.88 \pm 0.15,$

from which they conclude that the <u>β^--decay-interaction is predominantly V-A and the helicity of the emitted antineutrino is positive.</u>

(3) Electron Capture in Eu^{*152}

Goldhaber[24] et. al. measured the circular polarisation of γ-rays emitted from $Sm^{*152}(1^-)$ which was produced via the k-capture of electron in $Eu^{*152}(0^-)$. The scheme of the reaction is as follows:

$$e^- + Eu^{*152}(0^-) \longrightarrow Sm^{*152}(1^-) + \nu$$
$$\hookrightarrow Sm^{152}(0^+) + \gamma$$

They observed that the helicity of the -ray is negative ($\mathcal{H}_\gamma = -0.67 = 0.10$) from which they conclude that:

<u>The neutrino emitted in electron-capture has negative helicity and hence the Gamow-Teller interaction in e^--capture is predominantly axial-vector and not tensor.</u>

---

23 Burgy, Krohn, Novey Ringo and Telegdi, Phys. Rev. <u>110</u>, 1214(1958).

24 Goldhaber, Grodzins and Sunyar, Phys. Rev. <u>109</u>, 1015(1958).

(4) Decay of pion to $(e+\nu)$ and $(\mu+\nu)$:

Present experiments (contrary to old ones) reveal[25] the following branching ratio for pion-decay:

$$R = \frac{W(\pi \to e+\nu)}{W(\pi \to \mu+\nu)} = (1.03 \pm 0.20) \times 10^{-4}$$

From the theoretical standpoint, since the pion decaying to $(e+\nu)$ or $(\mu+\nu)$ is a $0^- \to 0^+$ transition, only pseudoscalar and axial-vector interactions can contribute. The computed[26] values of R, taking radiative corrections into account, are 1.20×10^{-4} and 5.4 for the axial-vector and pseudoscalar interactions respectively. Hence the observed value of R proves:

<u>The pion-decay proceeds predominantly through axial-vector and not pseudo-scalar interaction.</u>

---

25 Anderson, Fujii, Miller and Tau, Phys. Rev. Letter 2, 53(1959). Further references can be found here.

26 The calculations involve divergent integrals in both $\pi \to e+\nu$ and $\pi \to \mu+\nu$-rates. But their ratio is finite. The theoretical value quoted above presumes the same strength at the weak vertices of $\pi \to \mu+\nu$ and $\pi \to e+\nu$-decays.

4. Experiments on Naure of μ-Decay and μ-Capture Interactions

(1) Asymmetry Coefficient in μ-Decay:

The angular distribution of the decay electrons with respect to the muon-momentum has been measured. From the observed asymmetry coefficient[27], the usual assignment of lepton numbers[28], and the assumption of two component theory of neutrino (i.e. ν is always left-handed and $\bar{\nu}$ right-handed), it follows that <u>μ-decay interaction has predominantly V-A-form with nearly equal strength for vector and axial-vector parts.</u>

(2) Hyperfine Effect in μ-Capture:

When μ^--mesons are captured by nuclei with non-zero spins the probabilities of capture from the two hyperfine μ^--mesic atom states are different if the relevant weak-interaction is spin-dependent. This would imply that the rate of appearance of the decay electrons coming from bound μ^- decay will not follow a simple exponential law as a function of time. It was shown by Telegdi[29] that, in particular a <u>negative curvature</u> in the plot of logarithm of the rate of appearance of the decay electrons as a function of time can occur only if the capture from the lower level of the hyperfine structure is greater than that from the upper level.

---

27 Bardon, Berley and Lederman, Phys. Rev. Letters 2, 56(1959).

28 The usual assignment of lepton numbers is: e^-, μ^- and ν are leptons; e^+, μ^+ and $\bar{\nu}$ are antileptons.

29 V.L. Telegdi, Phys. Rev. Letters 3, 59(1960). See also Bernstein, Lee, Yang and Primakoff, Phys. Rev. III, 313(1958).

A negative value of the curvature was, in fact, observed(30) by Lathrop, et. al. for μ^- capture in Al^{27}. Since magnetic moment of Al^{27} is positive, this would imply that the probability of μ^--capture is greater for antiparallel hyperfine spin state than parallel hyperfine-spin state, which in turn implies, assuming that the main part in μ^--capture in Al^{27} is played by one external proton, that the basic interaction for μ^--capture is V-A.

---

30 Lathrop, Lundy, Swanson, Telegdi, See A.I. Alikhanov Kiev-Conference Report (1959).

5. Experiments on the Nature of Neutrinos

There are several experiments[31], which set the upper limit on the mass of the neutrino to be less than 1/2000 me (me = mass of the electron), and they are all consistent with the mass of the neutrino being zero. We will assume for subsequent discussion that $m_\nu = 0$. The fact that the mass of the neutrino is zero permits one to postulate an invariance under the chirality transformation $\Psi_\nu \rightarrow \pm i\gamma_5 \Psi_\nu$ if one give up the hypothesis of invariance under space reflection P. Such an invariance will lead to the appearance of only left-handed (right-handed) neutrinos and right-handed (left-handed) antineutrinos in nature. Right at the time, when parity conservation was questioned in β-decay, such a possibility for "<u>handedness</u>" of neutrino was suggested by Landau[32] and Salam[33] and also by Lee and Yang. This is referred to as the "<u>Two-Component Theory of the Neutrino</u>."

We will first cite below a few typical experiments, which provide evidences for the fact that the neutrinos[34] involved in β^+-decay are left-handed and the antineutrinos involved in β^--decay are right-handed.

---

31 See for instance L.M. Langer and R.J.D. Moffat, Phys. Rev. 88, 689 (1952).

32 L. Landau, Nuclear Phys. 3, 127 (1959).

33 A. Salam, Nuovo Cimento 5, 299 (1957) and T.D. Lee and C. N. Yang Phys. Rev. 105, 1671 (1957).

34 For definition of neutrino (ν) and antineutrino ($\bar{\nu}$) see footnote 9.

(1) Capture Cross Section for the Antineutrinos ($\bar{\nu} + p \rightarrow n + e^{+}$):

By a detailed balancing method, it is easy to check that the cross section for the reaction $\bar{\nu} + p \rightarrow n + e^{+}$ is twice as big if the antineutrino have only one spin-state than if they have both. The observed[35] cross section of capture of antineutrino (emitted by neutrons in a reactor) in hydrogen is in agreement with the two-component theory of neutrino.

(2) Electron-Capture in Eu^{*152}

The electron-capture-experiment in Eu^{*152} by Goldhaber[24] et. al; described before, show that the neutrino emitted in electron-capture has negative helicity.

(3) Directional Aymmetry in CO^{60} Experiment:

The β^{-}-asymmetry[36] A in the decay of polarized CO^{60} which involves a transition $5^{+} \rightarrow 4^{+}$, was found[37] to be nearly -1 to within a systematic uncertainty not larger than 20%. This, together with the information from ($e - \nu$) angular correlation experiments (mentioned before) that β-decay interaction involves (V,A) combination, implies that:

$$C_A \approx C_A'$$

---

35 F. Raines and C.L. Cowan, Phys. Rev. __92__, 830(1953).

36 The distribution of β^{-} in the decay of polarised CO^{60} mucleus can be put in the form $1 - A\cos\Theta\left(\frac{v}{c}\right)\frac{\langle J_z\rangle}{J}$, where Θ denotes the angle between the spin ($\vec{J}$) of CO^{60} nucleus and the direction of motion of β^{-}, v denotes the velocity of the electron.

37 Wu, Ambler, Hayward, Hoppes and Hudson, Phys. Rev. __105__, 1413(1957).

which shows that the axial vector part in β-decay interaction is invariant under the transformation $\Psi_\nu \to + i\gamma_5 \Psi_\nu$, hence follows the two-component structure of neutrino.

Thus from the above experiments one can conclude that <u>the neutrinos (ν) involved in β^+-decay are left-handed and antineutrinos ($\bar{\nu}$) involved in β^--decay are right-handed.</u>

To say whether only left-handed neutrinos and right-handed antineutrinos exist in nature, one has to examine the other neutrino involving reactions like μ-capture, piondecay and muon-decay, etc. At the moment, the experiments are consistent with either of the following two possibilities:

(i) Only left-handed neutrinos and right-handed antineutrinos exist in nature; e^-, μ^- and ν are leptons with lepton no. $l = +1$; e^+ μ^+ and $\bar{\nu}$ are antileptons with lepton no. $l = -1$ and lepton number is conserved in all reactions.

(ii)[38] The neutrino has four components like any other Dirac particle, e^-, μ^+ and ν are leptons, while e^+, μ^- and $\bar{\nu}$ are antileptons, lepton number is conserved in all reactions, and the weak interaction hamiltonian is such that the neutrinos associated with e^- and μ^+ have opposite helicities.

We will mention in chapter IV some experimental means of distinguishing between the above two possibilities.

---

38 This possibility, which we will discuss in chapter IV, has been pointed out by a number of people particularly in connection with the forbiddenness of $\mu \to e + \gamma$ -decay in the framework of intermediate vector boson theory for four fermion interactions. See for instance, S. Oneda and J.C. Pati, Phys. Rev. Lett. 2, 516 (1959). E.M. Lipmanov (See report by R.E. Marshak at Kiev, 1959); T.D. Lee and C.N. Yang Phys. Rev. (Tobe published).

However the discussion in chapters II and III will be the same for either of the above two possibilities. <u>So we will assume for the purpose of discussion in the next two chapters that (i) holds.</u>

In order to see how (i) is consistent with processes involving μ- decay and μ- capture, we will examine the $\pi \rightarrow \mu \rightarrow e$ chain experiments and the value of the michel-parameter ρ.

If (i) holds; then in $\pi^- \rightarrow \mu^- + \bar{\nu}$ decay, μ^- and $\bar{\nu}$ should both be right-handed in the rest frame of the decaying pion in order to conserve both linear and angular momentum. At the high-energy end of the β^- spectrum the neutrino and antineutrino are emitted together in the opposite direction from the β^- particle. Since neutrinos have two-components, therefore ν and $\bar{\nu}$ emitted together with high-energy e^- cancel each other's spin. Therefore the high-energy β^- carries the angular momentum of the right-handed parent μ^-. Since β^- are found[13] to be emitted predominantly antiparallel to the muon-momentum in the rest frame of the pion, therefore the high-energy β^- is expected to be left-handed. Correspondingly the high-energy β^+ coming from $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ chain is expected to be right-handed. These are confirmed by experiments mentioned before.[14]

Furthermore, if (i) holds, then in μ-decay a neutrino and an antineutrino with opposite helicities should be emitted. The michel[39] parameter ρ which characterises the electron spectrum in muon-decay, is 3/4 for such a situation. In the framework of two-component theory of neutrino $\rho = 0$, if two neutrinos or two antineutrinos are emitted in muon decay. The recently observed values[40] of ρ varies between 0.72 to 0.82, which are consistent with the value

39 L. Michel, Proc. Phys. Soc. A63, 514 (and) 1371 (1950).

40 See A.I. Alikhanov, Kiev Conference Report(1959) - Table I.

3/4. Thus if one assumes the two-component theory of neutrino, with usual assignment of lepton numbers (i.e. e^-, μ^- and ν are leptons), then the observed value of ρ confirms the hypothesis of lepton number conservation and vice versa.

6. Experiments On Nature of Weak Interaction Constants.

It has been pointed out in the previous subsection that β-decay interactions gives rise to only left-handed neutrinos and right-handed antineutrinos. This implies ,

$$C_V = C_V'$$
$$C_A = C_A'$$

where the C_i's have been defined in eq. (1). The vector-coupling constant in β-decay has been[41] determined from the decay rate of O^{14}, which yields, (in the units; $\hbar = c = 1$)

$$\sqrt{2}\, C_V = (1.01 \pm 0.01) \times 10^{-5}/m_p^2 \qquad (5)$$

The ratio C_A/C_V has been determined principally by two methods. By comparing the neutron half life[42] (11.7 ± 0.3 min.) with the O^{14} ft value[43], one gets:

$$\frac{|C_A|^2}{|C_V|^2} = 1.42 \pm 0.08$$

From the asymmetry in the distribution of electrons from polarised neutrons[44], one obtains:

$$\frac{|C_A|^2}{|C_V|^2} = 1.56 \pm 0.14$$

By comparing O^{14} decay rate with the muon lifetime it turns out that

$$C_V \approx D_V \qquad (\text{within } 2\%)$$

---

[41] Broomley, Almquist, Gove, Litherland, Paul and Ferguson, Phys. Rev. 105, 957(1957).

[42] Sosnovskij Spivak, Prokofiev, Kutikov and Dobrynin, Proceedings of the CERN Conference On High Energy Physics, 1958.

[43] J. B. Gerhart, Phys. Rev. 109, 897(1958).

[44] Burgy, Krohn, Novey, Ringo and Teledgi, Phys. Rev. 110, 1214(1958).

Similar comparison between β-decay and μ-capture coupling constants is hard to make due to the uncertainities in the evaluation of the nuclear matrix elements. However, the CERN-group[45] (among others) has measured the probability of a μ^--meson capture in a C^{12}-nucleus with the production of a B^{12} nucleus and that of the transition of B^{12} to C^{12} by β-decay. They found the ratio of these probabilities to be equal to 312 ± 18, from which they conclude

$$\delta^2 \frac{|E_A|^2}{|C_A|^2} \approx 0.72 \pm 0.05$$

where δ^2 denotes the ratio of matrix elements of μ^--meson capture and β-decay involved in the above mentioned transitions. According to the calculations of Fujii and Primakoff[46] $\delta^2 = 0.612$. This shows that the value of C_A and E_A are in agreement with the hypothesis of strict equality of bare coupling constants of β-decay and μ-capture.

An indirect check on the equality of β-decay and μ-capture[47] coupling constants comes by comparing the observed and the calculated values of the ratio of the rates of $\pi \to e + \nu$ and $\pi \to \mu + \nu$-decays. The observed value[25] of the ratio is, as mentioned before, $(1.03 \pm 0.20) \times 10^{-4}$ and the calculated value for (V-A)-interaction is 1.20×10^{-4}. This close agreement is consistent with the hypothesis of universal coupling for β-decay and μ-capture.

---

[45] Burgman, Ficher, Leontic, Lundby, Meunier, Street and Teja, Phys. Rev. Letters 1, 469(1958).

[46] A. Fujii and H. Primakoff, Nuovo Cimento 12, 327(1959).

[47] Here one must assume that $\pi \to \mu + \nu$ and $\pi \to e + \nu$ decays involve μ-capture and β-decay respectively as virtual processes.

7. Conclusion

From the experiments discussed above, it can thus be concluded that all the present experiments involving weak-interactions of nucleons and leptons are consistent with a scheme of Universal V-A Interaction

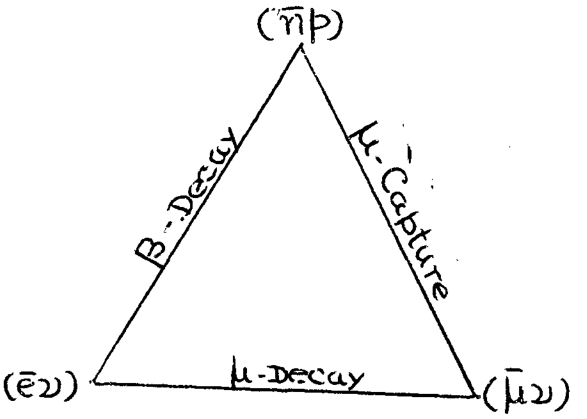


Fig.-1

with identical strength in all the processes. The situation can therefore very well be pictured in terms of a triangle (originally suggested by Tiomno and Wheeler [48]) drawn in fig. 1, with the currents ($\bar{n}p$), ($\bar{e}\nu$) and ($\bar{\mu}\nu$) placed at its three vertices. Experiments suggest that the three legs of this triangle, giving rise to the three main processes: (i) β-decay, (ii) μ - decay and (iii) μ -capture, possess beautiful symmetry in form and strength.

---

[48] J. Tiomno and J. A. Wheeler, Rev. Modern Phys. 21, 144(1949).

C. Theoretical Frame-Work of Universal V-A-Four-Fermion Interaction

The idea that the fundamental weak-interactions should be represented by a universal V-A law was proposed by Sudarshau and Marshak(49), independently by Feynman and Gellmann(50) and also by Sakurai(51). The first authors chose the point of view that the covariant four-fermion interactions should be invariant under the chirality transformation $\psi \rightarrow i\gamma_5\psi$. Feynmann and Gellmann liked to represent all spin-$\frac{1}{2}$ particles by two-component pauli spinnors satisfying a second order differential equation with the additional requirement that the interaction hamiltonian contains no gradients of these spinnors. Sakurai preferred to adopt the point of view that the four-fermion hamiltonian should be invariant under seperate reversal of the sign of the mass in the Dirac equation for each fermion. It is rather attractive that all these principles lead rather elegantly and essentially(52) uniquely to the interaction density:

$$\mathfrak{f}_{F}/\sqrt{2}\left[\left\{\bar{A}\gamma_{\alpha}(1+i\gamma_5)B\right\}^{+}\left\{\bar{C}\gamma_{\alpha}(1+i\gamma_5)D\right\}+H.C.\right] \quad (6)$$

where $\mathfrak{f}_F$ denotes the coupling constant and A, B, C, D are the field operators of four Dirac particles.

49E. C. G. Sudarshan and R. E. Marshak, Report to Padua Venice Conference, Mesons Newly Discovered Particles (Italy, September 1957) and Phys. Rev. 109, 1806(1958).

50R. P. Feynman and M. Gellmann, Phys. Rev. 109, 193(1958).

51J. J. Sakurai, Nuovo Cimento 7, 649(1958).

52To choose between V-A and V+A-law, one has to appeal to the experimental fact that the electrons are emitted in β-decay with left hand polarisation.

In order to write the total weak interaction hamiltonian-density in a form analogous to electromagnetism, one is tempted to conjecture that, it has the form

$$H\ \text{weak} = J^{*}_{\mu} J_{\mu} + \text{H.C.} \tag{7}$$

where J_{μ} is a sum of charged[53] positive-chiral-currents:

$$J_{\mu} = \frac{\sqrt{f_F}}{2^{1/4}} \left\{ \bar{e}\,\gamma_{\alpha}(1+i\gamma_5)\nu + \bar{\mu}\,\gamma_{\alpha}(1+i\gamma_5)\nu + \bar{n}\,\gamma_{\alpha}(1+i\gamma_5)p + \cdots \right\} \tag{8}$$

This idea of a charged chiral current coupled to itself has certain new consequences. In addition to the usual phenomena of β-decay, μ-decay, μ-capture and related processes it predicts, for example, electron neutrino scattering with a cross-section $\approx 10^{-45}$ cm^2 (nearly equal to the cross section of absorption of antineutrinos by nuclei) and a parity non-conserving nuclear force arising from the (np)(pn) term of the weak-coupling, which will lead to an anplitude $\approx 10^{-7}$ times that of the parity conserving term. These possibilities may be checked eventually by experiments.

There is one puzzling feature of the universal scheme yet to be discussed. Starting from a universal scheme of bare four-fermion interactions: it is rather puzzling as to why the observed coupling constants for β-decay should agree so well with those of μ-decay. The decay of the muon offers an example of pure weak-interaction, except for electrodynamic corrections, which are finite and have been taken into account[54]. On the other hand β-decay coupling constants are expected to be modified due to the strong

---

[53] The possibility of neutral currents is not so attractive, since from the absence of processes like $\mu \to e+e+e$, $K \to \pi+\nu+\bar{\nu}$, the presence of neutral leptonic currents is not permitted.

[54] T. Kinoshita and A. Sirlin, Phys. Rev. 107, 593(1957).
S. Berman, Phys. Rev. 112, 267(1958).

virtual effects between the nucleons. However, the axial-vector coupling constant in β-decay is only about 25% larger than that of μ-decay and the vector-coupling constant in β-decay (evaluated from O^{14} rate) agrees with the μ-decay coupling constant with 2 o/o. The renorlisation, only by a factor ≈ 1.25, in case of the former is hard to understand at the moment. It is even more striking that the latter agrees so well indeed.

Struck by the above observation, Feynman and Gellmann(55) (see also earlier work by Gerstein and Zeldovich)(56) have put forward the hypothesis that the vector part of the strangeness conserving current should satisfy the conservation law: (except for electromagnetic corrections)

$$\frac{\partial \overset{V}{J_\mu}(\Delta S=0)}{\partial x^\mu} = 0 \tag{9}$$

So that, including the interaction of the nucleons with pions, hyperons and K-mesons, $\overset{V}{J_\mu}(\Delta S=0)$ has the form:

$$\begin{aligned}\overset{V}{J_\mu}(\Delta S=0) = \frac{\sqrt{f_F}}{2^{1/4}} \Big[& \bar{n}\gamma_\mu p + \sqrt{2}\left(\frac{\partial \pi^+}{\partial x^\mu}\pi^0 - \pi^+\frac{\partial \pi^0}{\partial x^\mu}\right) \\ & + \bar{\Xi}^-\gamma_\mu \Xi^0 + \sqrt{2}\left(\bar{\Sigma}^-\gamma_\mu \Sigma^0 - \bar{\Sigma}^0\gamma_\mu \Sigma^+\right) \\ & + \left(\partial K^+/\partial x^\mu \, \overset{*}{K}{}^0 - K^+ \, \partial \overset{*}{K}{}^0/\partial x^\mu\right) \Big]\end{aligned} \tag{10}$$

The virtue of such a conserved vector current is that it guarantees non-renormalisation of the vector— Coupling constant in β-decay, just as the conservation of the electromagnetic current explains why all charged elementary particles have the same charge $\pm e$.

---

55 R. P. Feynman and M. Gellmann, Phys. Rev. 109, 193(1958).

56 S. S. Gerstein and J. B. Zeldovich, Zhur. Eksptl i Theoret. Fiz, 29, 698(1955).

The occurrence of the additional terms in the above equation has consequences, which are in principle, observable. The simplest is the β -decay of pions $\pi^{+}\to\pi^{0}+e^{+}+\nu$. However, the branching ratio compared to the $\pi^{+}\to\mu^{+}+\nu$-mode is nearly $1.0 \pm 0.2 \times 10^{-8}$, which is too small to be detected at present. The other consequences are in the β-decay-spectra (57) for example of B^{12} and N^{12} and also in the β-α and β-γ-correlations (58), etc. The experiments are not yet accurate enough to judge the truth of the theoretical predictions.

To sum up the situation with regard to the weak-interactions of nucleons and leptons: the success of the universal V-A theory has been enormous indeed. Its intial disagreements with the He^{6} recoil experiment and the $\pi\to e+\nu$ experiment have been removed by redoing the experiments. Subsequent experiments have confirmed the universal V-A theory more and more. These are, of course, problems yet to be understood. For example, one would like to know the truth of the hypothesis of "conserved vector-current". One would also like to understand(59) quantitatively why the axial vector part in β-decay is renormalised by a factor ≈ 1.2. It is also of interest to know whether the four-fermion interactions are strictly local, or whether they are intermediated by a charge vector boson.

---

[57] M. Gellmann, Phys. Rev. 111, 312(1958)

[58] J. Bernstein and R. R. Lewis, Phys. Rev. 112, 232(1958).

[59] The static model applied to the axial-vector current predicts that its strength should be reduced by renormalisation effects. However, it is generally believed that nucleon pair effects may account for the observed strength.

One would next ask; Does the same theory hold when extended to hyperons?

In the next section we will briefly review the present situation concerning the strange particles.

D. The Strange Particles

The heavy mesons (the K-mesons) and the heavy baryons (the hyperons) were produced in high-energy nuclear collisions with cross sections of the order of millibarns. This implied that the K-mesons and the hyperons must interact strongly with the nucleons and the pions. On the other hand the decays of these particles to systems of pions and nucleons exhibited life times of the order of 10^{-10} sec., extremely slow on the nuclear scale. This strange relationship between their production and decay behaviour led to their designation as "The Strange Particles".

A single interaction (like $\Lambda p \pi^-$) with a large momentum dependence could probably account for the strong Λ-production and the slow decay-process, since the decay process involves low momentum while the momentum in the energetic collisions for production are large. However this possibility is excluded by the observation that the Λ-life time is not sensibly althered by the presence of neighboring nucleons, although pions of high momenta are available in their clouds. Thus it was believed that the production and the decay processes of the "Strange Particles" must proceed through distinct interactions.

This led Pais[60] to propose the hypothesis of associated production of strange particles, according to which the processes involving at least two strange particles, as in production ($\pi^- + p \rightarrow \Lambda_o + K^o$), are fast and those involving only one, as in the decay processes ($\Lambda \rightarrow p + \pi^-$) are slow.

60A. Pais, Phys. Rev. 86, 663, (1952).

It was realised[61] that in order to understand why processes which are observed do occur and those which are not observed do not, a new quantum number "S" is needed in addition to the isotopic spin I for each particle, which would be linearly related to the charge Q, baryonic number N and Z component of isotopic spin (I_z) such that $\Delta I_z \neq 0$; if and only if $\Delta S \neq 0$. The selection rule, which then decides whether the process is governed by strong or weak interactions is:

$\Delta S = 0 \Rightarrow$ The process is governed by strong and electromagnetic interactions only[62].

$\Delta S \neq 0 \Rightarrow$ The process must involve weak interaction.

S is called the strangeness quantum number and is determined by:

$$Q = I_z - \frac{N}{2} - \frac{S}{2} \tag{11}$$

The usual scheme, known as the Gellmann-Nishijima Scheme, of assignment of isotopic spin and the grouping of the various particles into isotopic multiplets is shown in Table I. A word of caution, however, is in order. Such a grouping into isotopic multiplets, apart from being based on the observed mass-spectrum, of course presumes that the particles belonging to the same multiplet have identical space-time properties, like spin

---

[61] M. Gellmann, Phys. Rev. 92, 833(1953).
T. Nakano and K. Nishijima, Prog. Theoret. Phys. (Kyoto) 10, 581(1953).
K. Nishijima, Prog. Theoret. Phys. (Kyoto) 13, 285(1955).
R. G. Sachs, Phys. Rev. 99, 1573(1955).

[62] Provided the process is not forbidden by other conservation laws like conservation of charge, angular momentum, parity or total isotopic-spin, etc.

and parity etc. If it turns out to be false, one may have to alter the Gellmann-Nishijima scheme. For instance, as pointed out by Pais[63] if the relative parity of K^- and K^0 are found to be odd, one may have to go to a representation in four dimensional isotopic spin space in order to maintain the usual assignment of S (Table I), which is consistent with experiments.

---

[63] A. Pais, Phys. Rev. 112, 624(1958).

Table I

Isotopic Spin and Strangeness Quantum Numbers of the Strongly Interacting Particles in the Gell-Mann- Nishijima scheme.

ISOTOPIC GROUPING	I_z	I	S	N
$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$	1 0 -1	1	0	0
$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$	½ -½	½	1	0
$\begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$	½ -½	½	-1	0
$\begin{pmatrix} p \\ n \end{pmatrix}$	½ -½	½	0	1
Λ	0	0	-1	1
$\begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}$	1 0 -1	1	-1	1
$\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$	½ -½	½	-2	1

E. General Features of Strange Particle Decays

Table II gives the list of familiar decays of the K-mesons and the hyperons with the respective branching ratios and life times. Only K^+ $(S = +1)$ and not $K^-(S = -1)$ decay modes have been listed, since the decay properties of K^- meson are known in much less detail than those of K^+ meson. The K^- mesons, when stopped in matter, are usually captured by the nucleus through strong interactions before having a chance to decay (a fact which bears its cause to the strangeness quantum number of K^- being -1). The few registered K^- events are essentially decays in flight and within the limits of experimental errors the observed K^-[64] and K^+ decay modes do satisfy the particle-antiparticle-relationship prescribed by TCP invariance; i.e. equalities of masses, life times and branching ratios, etc.

The general features of the strange-particle decays may be said to be the following:

(a) Their life times are of the order of $10^{-8} \sim 10^{-10}$ sec., which implies that the strength of the interactions, responsible for their decays must be weak, i.e. of the same order of magnitude as in β-decay or μ-decay.

(b) The strange particle decays exhibit the same parity violating characteristics as the decays of nonstrange particles like neutron or muon. The well known $(\Theta-\tau)$ paradox was the starting point of the Lee-Yang investigation of the break down of reflection invariance in weak interactions. A more direct evidence of break down of parity in strange particle decays comes from the observation of up-down asymmetry in the distribution of decay products of polarised Λ $(\Lambda \rightarrow N+\pi)$ and Σ^+ $(\Sigma^+ \rightarrow p+\pi^0)$,

---

[64] Freden, Gilbert and White, Phys. Rev. 118, 564(1960).

Table II

Decays of Strange Particles*

Particle	Mass (Mev)	Mean life time (Sec)	Decay Modes	Branching Ratios (%)
K^+	494 ± 0.2	$(1.224 \pm 0.013)\times 10^{-8}$	$\mu^+ + \nu$	59 ± 2
			$\pi^0 + \mu^+ + \nu$	4.0 ± 0.77
			$\pi^0 + e^+ + \nu$	4.19 ± 0.42
			$\pi^+ + \pi^0$	25.6 ± 1.7
			$\pi^+ + \pi^- + \pi^+$	5.66 ± 0.30
			$\pi^+ + \pi^0 + \pi^0$	1.70 ± 0.32
K_1^0	497.1 ± 0.8	$(0.99 \pm 0.07)\times 10^{-10}$	$\pi^+ + \pi^-$	73 ± 11
			$\pi^0 + \pi^0$	27 ± 11
K_2^0		$(8 \pm 3)\times 10^{-8}$	$\pi^\pm + \mu^\mp \mp \nu$	~ 40
			$\pi^\pm + e^\mp \mp \nu$	~ 40
			$\pi^+ + \pi^- + \pi^0$	~ 15
			$\pi^0 + \pi^0 + \pi^0$	?
Λ^0	1115.2 ± 0.14	$(2.60 \pm 0.15)\times 10^{-10}$	$p + \pi^-$	62.7 ± 3.1
			$n + \pi^0$	43 ± 1.4
Σ^+	1189.4 ± 0.25	$(0.83 \pm 0.05)\times 10^{-10}$	$p + \pi^0$	~ 50
			$n + \pi^+$	~ 50
Σ^0	1190.5 ± 1	$< 10^{-11}$	$\Lambda^0 + \gamma$	
Σ^-	1196.5 ± 0.5	$(1.72 \pm 0.17)\times 10^{-10}$	$n + \pi^-$	
Ξ^0	1311 ± 8	$\sim 10^{-10}$	$\Lambda^0 + \pi^0$	
Ξ^-	1318.7 ± 2.0	$\sim 1.8\times 10^{-10}$	$\Lambda^0 + \pi^-$	

* See next page for foot note

* The data presented in this table haven been compiled from (i) M. Gell-Mann and A.H. Rosenfeld - Ann. Rev. Nuclear Sci. 7, 407 (1957), (ii) L. Okun - Ann. Rev. Nuclear Sci. 9, 61 (1959), and (iii) Lectures on "Strange Particle Physics" - CERN-59-35. The data obtained by more recent experiments are nearly the same as those quoted in the above three references, except for a considerable change in the values of $K_1^0 \to 2\pi^0$ and $K_1^0 \to \pi^+ + \pi^-$ - branching ratios, which we have taken from Crawford, Cresti, Douglass, Good, Kalbfleisch, Stevenson and Tiche - Phys. Rev. Lett. 2, 266 (1959). The Leptonic modes of hyperon decays have not been listed in the above table. They are given separately in Table IV.

Since these two general features of strange particle decays are identical to those of the weak processes involving nucleons and leptons, and since the latter are very well represented by an universal scheme, symbolised by the triangle in fig. 1, it is certainly attractive to hope that nature believes in simplicity and universality and thus one may probably extend the triangle (fig. 1) to a tetrahedron or a polyhedron, involving hyperons, which would provide a mechanism for strange particle decays. This was first proposed by Gellmann[65] and Dallaporta[66], who suggested for simplicity that only the (Λ p) vertex be added to the triangle involving nucleons and leptons.

An extension of the theoretical proposals of Sudarshan and Marshak, or Feynman and Gellmann or Sakurai would of course imply an universal V-A form of four fermion interaction involving hyperons also. It is consequently of great interest to establish the truth in such an extension. Unfortunately experimental information regarding the strange-particle decays is much poorer than that involving weak-decays of nucleons and leptons. It is not yet possible to do angular correlation experiments in Λ-beta-decay as in nuclear β -decay, since only two events of Λ-beta-decay have been observed so far. Hence a direct determination of the nature of the interaction involved in strange particle decays is not possible at the moment. One way to judge the validity of the theory may, of course, be to compare the various theoretical results (based on some form of interactions) with the corresponding experimental ones. Such arguments, however, cannot be <u>conclusive</u>; since not only is there a scope for several <u>alternatives</u> regarding the scheme of interaction (i.e. the structure of J_μ), but also we do not have a reliable way to tackle the strong virtual effects.

---

[65] M. Gellman, Proceedings of the 6th Annual Rochester Conference on High Energy Physics (1956)

[66] N. Dallaporta, Nuovo Cimento 1, 962(1953)
G. Costa and N. Dallaporta, Nuovo Cimento 2, 519(1955)

One, therefore, has to try to obtain insight into the structure of the interaction by looking for any consistent regularities, like symmetry principles, selection rules, correlations between different types of decay modes etc. in strange particle decays. All the observed freatures of strange particle decays seem to exhibit three such regularities:

1. The $|\Delta S| = 1$-Rule

All the observed decays of strange-particles seem to involve change of strangeness by one unit only. One has not found so far any event of $\Xi \rightarrow N + \pi$-decay ($\Delta S = \pm 2$), although the Q value for this process is rather large. The measurement[67] of the mass difference between K_1^o, and K_2^o shows that it is $\sim 10^5$ ev, which means (if one believes the experiment) that the transition $K^o \rightarrow \bar{K}^o$ (or $K_1^o \rightarrow K_2^o$) is second order in weak-interaction and not first order. These results seem to indicate (although the number of Ξ-events is too few at the moment to draw definite conclusion) that $|\Delta S| = 1$ is probably a good selection rule for strange particle decays, considered as first order processes in weak interactions.

This suggests that currents involving $|\Delta S| = 2$ (like $(\bar{\Xi}^- n)$) do not enter into J_μ [eq (8)]. Admitting charged[53] currents only, one may add to J_μ such currents as $(\bar{P}\Lambda)$, $(\bar{n}\,\Sigma^-)$, $(\bar{\Xi}^o \Xi^-)$ etc. (satisfying $\Delta S/\Delta Q = +1$) and currents such as $(\bar{\Sigma}^+ n)$, $(\bar{\Xi}^o \Sigma^-)$ etc. (satisfying $\Delta S/\Delta Q = -1$) However, if one likes to maintain the form of the interaction ($J_\mu^* J_\mu$) then the coupling between currents satisfying $\Delta S/\Delta Q = +1$ and $\frac{\Delta S}{\Delta Q} = -1$ would lead to a net change of strangeness by two units in the first order of weak interaction. Since the presence of $\frac{\Delta S}{\Delta Q} = -1$ currents is required due to

---

[67] Boldt, Caldwell and Pal, Phys. Rev. Letters 1, 150(1958)
Muller et.al., Phys. Rev. Letters 4, 418(1960).

observation of processes such as $\Lambda \rightarrow p + e^- + \bar{\nu}$, this suggests[68] that one should reject the presence of any current in J_μ satisfying $\frac{\Delta S}{\Delta Q} = -1$.

From eq.(11)

$$\Delta Q = \Delta I_z + \frac{\Delta S}{2} \qquad (\text{Since } \Delta B = 0)$$

Hence;

$$\frac{\Delta S}{\Delta Q} = +1 \quad (\quad \text{assuming } |\Delta Q| = 1) \Rightarrow \Delta I_z = \pm \tfrac{1}{2}$$

While;

$$\frac{\Delta S}{\Delta Q} = -1 \quad (\quad \text{assuming } |\Delta Q| = 1) \Rightarrow \Delta I_z = \pm \tfrac{3}{2} \qquad (12)$$

Hence from (12), a strangeness violating current which behaves as a spinor in isotopic spin sapce (we will call such a current, following the terminology of Okubo[69] et.al, an $I = \frac{1}{2}$-current) will automatically satisfy $\Delta S/\Delta Q = +1$.

There are direct consequences of the hypothesis of such a $I = \frac{1}{2}$-current particularly for leptonic modes, which can be tested. It predicts for example:

$$\begin{aligned} W(K_1^0 \rightarrow \pi^{\pm} + \mu^{\mp} \mp \nu) &= W(K_2^0 \rightarrow \pi^{\pm} + \mu^{\mp} \mp \nu) \\ &= 2W(K^+ \rightarrow \pi^0 + \mu^+ + \nu) \end{aligned} \qquad (13)$$

There exist similar relation for electron modes. It also predicts:

$$\Sigma^+ \not\rightarrow n + e^+ + \nu$$

Both of these seem to be consistent with present experiments[70], [71].

---

68 This is consistent with absence of any $\Sigma^+ \rightarrow n + e^+ + \nu$ decay so far. However absence of any $\Sigma^- \rightarrow n + e^- + \bar{\nu}$-decay also makes the situation a little ambiguous. It is to be noted, that if $\Sigma^+ \rightarrow n + e^+ + \nu$- decay is observed, then one has to allow currents such as $(\bar{\Sigma} + n)$ in J_μ. However, one may still forbid processes involving $|\Delta S| = 2$ by decoupling $\frac{\Delta S}{\Delta Q} = +1$-currents from $\frac{\Delta S}{\Delta Q} = -1$-currents.

69 Okubo, Marshak, Sudarshan, Teutsch and Weinberg, Phys. Rev. 112, 665(1958)

70 Crawford e.al., Phys. Rev. Letters 2, 361(1959)

71 Leitner et.al., Bull. American Phys. Soc. Series II, Vol. 4, 356(1959).

2. The $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-Rule

An interesting feature of strange-particle decays is the observation that they seem to proceed through $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-channel, where $\Delta \underset{\sim}{I}$ denotes the vector difference between the isotopic spins of the final and the initial systems. The suggestion[72] of the approximate validity of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule stemmed initially from the observation that the rate of $K^+ \to 2\pi$ is anomalously slow compared to that of $K_1^0 \to 2\pi$. At the moment there seems to be increasingly more evidence in favour of the approximate validity of the above rule.

The branching ratios of $K \to 2\pi$, $K^+ \to 3\pi$, $K_2^0 \to 3\pi$, $\Lambda \to N + \pi$, $\Sigma^{\pm} \to N + \pi$ and the asymmetry parameters of Λ and $\Sigma^{\pm}$-decays seem to be consistent with the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule. Except for the decay rates and asymmetry parameters in $\Sigma^{\pm} \to N + \pi$-decays[73], the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule is sufficient to guarantee all the above mentioned relations concerning the various branching ratios and asymmetry parameters etc. <u>However it is not necessary for all of them.</u> In Table III, we give some of the theoretical predictions of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule and the corresponding experimental observations.

It is clear that the experimental facts show that if the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule is at all meaningful for all the strange particle decays, it cannot be a strict rule. Otherwise $K^+ \to 2\pi$ would be an unobserved process. However an approximate $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule (presumably with a minor superposition of $|\Delta \underset{\sim}{I}| = \frac{3}{2}$ component, nearly 5% in amplitude) certainly seems to be in agreement with present experiments. This may suggest that:

---

[72] M. Gell-Mann and A. Pais, Proceedings of the International Conference on High Energy Physics (pergamon, London, 1955)

[73] For $\Sigma^{\pm} \to N + \pi$-decays, one has to assume a certain amount of parity violation (corresponding to S*-P-interference-term) in one of the three modes of $\Sigma^{\pm} \to N + \pi$-decays. Then the rest is prescribed by $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule. (See for instance, M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407(1957))

Table III

Various Features of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-Rule*

No.	Principle	Expected Values	Observed Values
1	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$ and nearly 5% $\|\Delta \underset{\sim}{I}\| = 3/2$	$\frac{W(K^{+} \to 2\pi)}{W(K_1^0 \to 2\pi)} \approx \frac{1}{500}$	$\sim \frac{1}{500}$
2	Same as above	$\frac{W(K_1^0 \to 2\pi^0)}{W(K_1^0 \to 2\pi)} = R$, where $0.28 < R < 0.38$	(76) 0.27 ± 0.11
3	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$	$\frac{W(K^{\pm} \to \pi^{\pm} + \pi^0 + \pi^0)^{**}}{W(K^{+} \to \pi^{+} + \pi^{-} + \pi^{+})} \approx \frac{1}{4}(1.3)$	(77) 0.32 ± 0.05
4	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$	$\frac{W(K_2^0 \to 3\pi^0)}{W(K_2^0 \to 3\pi)} \approx \frac{3}{5}$	?
5	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$	$\frac{W(K_2^0 \to \pi^{+} + \pi^{-} + \pi^0)}{W(K^{+} \to 3\pi)} \approx \frac{2}{5}$	(76) consistent with expected value
6	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$	$\frac{W(\Lambda \to p + \pi^{-})}{W(\Lambda \to N + \pi)} \approx \frac{2}{3}$	(76) 0.627 ± 0.031
7	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$	$\frac{{}^{***}\alpha(\Lambda \to n + \pi^0)}{\alpha(\Lambda \to p + \pi^{-})} = 1$	(78) $+1.22 \pm 0.24$
8	Strict $\|\Delta \underset{\sim}{I}\| = \frac{1}{2}$ with specific amount of parity violation in one of $\Sigma^{\pm} \to N + \pi^{-}$ decay modes(80)	$W(\Sigma^{+} \to p + \pi^0) : W(\Sigma^{+} \to n + \pi^{+}) : W(\Sigma^{-} \to n + \pi^{-}) = 1 : 1 : 1$	$\approx 1 : 1 : 1$ (79)
		$\alpha_{\Sigma^{\pm} \to n + \pi^{\pm}} \approx 0$ $\|\alpha_{\Sigma^{+} \to p + \pi^0}\| \approx 1$	(81) $\alpha_{\Sigma^{+} \to n + \pi^{+}} \overline{P} = 0.02 \pm 0.07$ $\alpha_{\Sigma^{-} \to n + \pi^{-}} \overline{P} = 0.02 \pm 0.05$ $\alpha_{\Sigma^{+} \to p + \pi^0} \overline{P} = 0.70 \pm 0.30$

* See next page for foot note

* In the above table (2), (3), (4), (6), (7), and (8) are consistent, but do not imply the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule.

** The factor $\approx$ 1.3 arises due to difference in phase space factor of $\tau^{+\prime}$ and τ^{+} -modes. We have not inserted such factors in the other ratios.

*** α denotes the pion - asymmetry parameter in the corresponding decay process (see Appendix II).

[76] Crawford et al. Phys. Rev. Lett. 2, 266(1959)

[77] Birge et al. Nuovo Cimento 6, 478 (1957)

[78] Cronin et al. Bulletin American Phys. Soc. Series II, Vol. 5, 11 (1960)

[79] See table in M. Gell-Mann and A.H. Rosenfeld - Ann. Rev. Nuclear Sci., 7, 407 (1957)

[80] M. Gell-Mann and A.H. Rosenfeld - Ann. Rev. Nuclear Sci., 7, 407 (1957)

[81] Cool et al. - Phys. Rev. 114, 912 (1959)

(i) Either, the decay-interaction hamiltonian of strange-particles transforms in the isotopic spin space as a spherical tensor of rank $\frac{1}{2}$, but the deviations from the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule take place to the desired extent by some possible mechanism. One[74] possible mechanism may be the nature of symmetry and strength involved in "very" and "moderately" strong interactions. Another alternative[75] may be the infludence of final state interactions in addition to electromagnetic ones.

(ii) or, the decay-interaction does not satisfy the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule. However, in the decays of strange particles, the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-component predominates by some possible mechanism. A possible[76] means is the special importance of a set of diagrams, which satisfy the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule.

It is the second alternative that we shall be concerned with in the present work (see Chapter II).

3. The Slowness of Leptonic Modes

The leptonic modes of strange particle decays seem to be rarer than predicted by the simplest extension of universal fermi interaction, by an order of magnitude.

So far only two decays of $(\Lambda \rightarrow P + e^- + \bar{\nu})$ and no leptonic decays of $\Sigma^{\pm}$ have been observed. Altogether, nearly 1500 Λ's and 2000 Σ's have been produced by high energy pions and slow K^- in the hydrogen bubble chamber and decayed under conditions, which would have permitted efficient detection of leptonic decays. Table IV gives the experimental upper limits and the theoretical predictions for leptonic decays of hyperons. These show that the observed rates are smaller than the universal rates by an order of magnitude.

---

[74] See for instance, M. Gell-Mann, Rev. Modern Phys. 31, 834(1959)

[75] M. L. Good and W. G. Holladay, Phys. Rev. Letters 4, 138(1960)

Table IV

Branching Ratios of Leptonic Decays of Hyperons*

Leptonic Mode	Berkely Data (%)	World Data (%)	Theoretical Prediction of UFI (%)
$\Lambda \to p + e^- + \bar{\nu}$	0.3	0.2	1.6
$\Lambda \to p + \mu^- + \bar{\nu}$	<0.1	0.1	0.24
$\Sigma^- \to n + \mu^- + \bar{\nu}$	<0.4	0.2	2.5
$\Sigma^- \to n + e^- + \bar{\nu}$	<0.4	0.2	5.8
$\Sigma^+ \to n + e^+ + \nu$	<0.6	0.4	–
$\Sigma^+ \to n + \mu^+ + \nu$	<0.5	0.3	–
$\Sigma^- \to n + \left\{ \begin{matrix} e^- \\ \mu^- \end{matrix} \right\} + \bar{\nu}$	<0.1	–	–
$\Sigma^+ \to \Lambda + \left\{ \begin{matrix} e^+ \\ \mu^+ \end{matrix} \right\} + \nu$	<0.2	–	–

* The above table has been taken from D.A. Glaser - Kiev Conference Report (1959)

The observed slowness of the leptonic decays of hyperons is also consistent with the rates of the leptonic decay modes of K-meson. The absolute transition rate of $K_{\mu 2}$ ($W_{K_{\mu 2}} \approx 4.8 \times 10^7$ sec^{-1}) is nearly equal to that of $\pi_{\mu 2}$ ($W_{\pi_{\mu 2}} \approx 4.0 \times 10^7$ sec^{-1}), in spite of the bigger phase space in case of the former. This might, of course, be due to the smallness of the K-meson baryon coupling constant, compared to the pion baryon coupling constant. However, adopting a plausible value $g_K^2 / g_\pi^2 \approx \frac{1}{10}$, a disperion theoretic calculation (similar to Goldberger Trieman's Calculation of $\pi \to \mu + \nu$) by Sakita[82] and Albright[83] shows that the observed rate of $K_{\mu 2}$ can be explained only by adopting a smaller coupling constant for the weak vertex (by a factor of 5 nearly) than the universal strength. Roughly the same conclusion is reached by the recent dispersion theoretic calculation of K_{e3} and $K_{\mu 3}$ rates by Sawyer[84]. It is pertinent to notice that only the axial vector part contributes to $K_{\mu 2}$ and only the vector part to $K_{\mu 3}$ or K_{e3}; assuming K meson parity relative to (hyperon, nucleon) system is odd. (For even parity the reverse situation holds). Hence we have an indication for weaker strength of both the vector and the axial vector part of the strangeness violating vertex than the universal strength.

The slowness of the leptonic decays of the hyperons is not likely to arise as an accident from damping due to strong virtual effects (these effects are quite small in $n \to p + e^- + \bar{\nu}$ decay), since[85] no V.A interference term appears in the rate and the damping must occur for both V and A interactions seperately. Moreover the damping must exist to nearly the same extent in Λ, Σ and K-meson decays.

---

[82] B. Sakita, Phys. Rev. 114, 1650(1959)

[83] C. H. Albright, Phys. Rev. 114, 1648(1959)

[84] R. F. Sawyer, Phys. Rev. (To be published)

[85] S. Weinberg, Phys. Rev. 115, 481(1959)

If one accepts that the discrepancy between the universal rates and the observed rates of the leptonic modes of strange particle decays is real, and is not solely due to renormalisation effects, then, probably one should associate the strangeness violating current with a weaker coupling constant than the strangeness-conserning one. Then one may adopt the point of view[86] that the strength of the weak coupling constant does in fact depend upon the change in strangeness. This would serve to explain the observed rates of the leptonic modes of strange particle decays.

Can one then still explain consistently the observed rates of the non-leptonic modes of strange-particle decays?

This will form the subject matter of our discussion in Chapter II.

---

[86] See for instance, S. Oneida, J. C. Pati and B. Sakita (Phys. Rev. July 1, 1960 - to be published)

F. A List of Problems

The foregoing discussion raises the following interesting problems:

(1) Does the universal V-A four-fermion interactions, established so well for the nucleons and the leptons extend also to the hyperons in form and strength?

(2) Should one treat all the hyperons as equally basic (subject to suitable selection rules) in the primary four-fermion interactions, or should one single out one hyperon (say the Λ) as basic for the primary weak-interactions, like in the Sakata[87] model?

(3) Can one explain the approximate validity of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule, borne out experimentally in the various decays of the strange-particles?

(4) If one assumes that the strangeness violating current may be associated with a weaker coupling constant than the strangeness conserving one, as is indicated by the slowness of the leptonic modes of the strange-particle decays; can one still* explain the observed rates of the nonleptonic modes?

In particular; does there exist a correlation between the slowness of the leptonic modes and the explanation of the approximate validity of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule?

(5) Last, but not the least, are the four-fermion interactions strictly local, or are they intermediated by a charged vector boson?

---

[87] S. Sakata, Prog. Theoret. Phys. (Kyoto), <u>16</u>, 686(1956).

* The rates of non-leptonic modes, like those of $\Lambda \to N + \pi$-decays, calculated on the basis of the usual set of diagrams (Fig. 4), with a <u>universal</u> weak coupling constant are nearly half the observed values.

Closely related with the above question, (due to the slowness[88] of the $\mu \to e + \gamma$ -decay) is[89] the question:

Is the μ^- a lepton like e^-, or is it an antilepton?

Correspondingly, is the massless neutrino to be represented by a two component theory or a "restricted" (to be explained later) four-component theory?

The work in this thesis will be largely concerned with the above five problems. We assume the simplest scheme of weak-interaction, namely the Gell-Mann-Dellaporta-tetrahedron scheme, which introduces the primary weak interactions only among the leptons, the nucleons and the Λ -hyperon (more or less in the spirit of the Sakata model) and try to pursue the possible successes of this scheme. We discuss the mechanism of Λ -hyperon decay in Chapter II in connection with problems (3) and (4) and show a natural way of explaining the approximate $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule as well as the slowness of the leptonic modes of strange-particle decays compared to the non-leptonic ones. The various modes of K-meson-decays are discussed in chapter III, and their relative rates are compared with experiments. It is shown that the mechansim of Λ -decay, developed in chapter II, leads to improved agreement with experiments for all the relative rates of K-meson decay modes. We finally discuss in chapter IV the possibility that the four fermion interactions may be mediated by a charged vector boson (Problem (5)).

88 See for instance Davis and Zipf, Phys. Rev. Letters 2, 211 (1959).

89 G. Feinberg, Phys. Rev. 110, 1482 (1958).
S. Oneda and J.C. Pati, Phys. Rev. Letters 2, 516 (1959).

CHAPTER II

THE DECAY OF THE Λ-HYPERON

Abstract

In this chapter the decay of the Λ -hyperon will be discussed in the framework of the V-A interaction extended to the Gell-Mann - Dellaporta tetrahedron scheme. It is shown that a new class of diagrams, which contribute only to non-leptonic processes like $\Lambda \to N+\pi$ - decays etc. and which satisfy the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - rule are much more important than the usually considered diagrams containing appreciable amount of $|\Delta \underset{\sim}{I}| = \frac{3}{2}$ - transitions in addition to $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - part. This makes it easier to explain the approximate validity of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule and also leads to a consistent explanation of the slower rates of leptonic modes and the faster rates of nonleptonic modes of strange particle decays, provided one associates the strangeness violating current with a weaker coupling constant than the strangeness conserving ones.

A. Scheme of Interaction

The weak-interaction phenomena involving nucleons and leptons seem to be well understood in terms of an universal V-A four-fermion interaction involving the self-coupling of a charged positive chiral current with itself. The extension of this scheme to hyperons is by no means clear. One does not know whether to treat all the hyperons as equally basic (subject to selection rules) for the primary weak-interactions. It is also not clear whether the universality should extend in form and strength to the hyperons. One does not also know, whether to include or excude the neutral[91] baryonic currents whose existence has been conjectured by many.

Under these circumstances, undoubtedly one has to keep an open mind regarding the various possibilities and speculations, until a completely reliable method of judging their validity becomes available. In this thesis we consider, more or less in the spirit of Sakata-model, the simplest possibility, namely the Tetra-hedron-Scheme[92], which introduces the primary weak interactions only between leptons, nucleons and the Λ-hyperon and pursue the possible successes of this scheme.

---

91 It has been recognised by many people that, to construct a four-fermion interaction which satisfies the strict $|\Delta I| = 1/2$-rule, one needs to introduce neutral baryonic currents in addition to charged currents (See M. Gell-Mann, Rev. Mod. Phys. 31, 834, 1959). It is also known that there are no immediate objections to introducing such neutral baryonic currents. However due to the absence of processes like $\mu \rightarrow e+e+e$, $K \rightarrow \pi+e+e$ and $K \rightarrow \pi+\nu+\bar{\nu}$ etc., one ought to exclude neutral leptonic currents. This is however rather unaesthetic, since the interaction would then contain neutral baryonic currents, but no neutral leptonic currents.

92 M. Gell-Mann, Proceedings of the 6th Annual Rochester Conference on High-Energy Physics, 1956. N. Dallaporta, Nuovo Cimento 1, 962(1953)

We will assume in the present chapter that the four-fermion interactions are local. (The effect of non-locality in four-fermion interactions on Λ-decay will be investigated in chapter IV). We will also assume that the slowness of the leptonic modes of strange-particle decays is not solely due to renormalisation effects. The consistency of this slowness (See chapter IE) in the β-decays of Λ and Σ^- as well as in $K_{\mu 2}$, $K_{\mu 3}$ and K_{e3}-decays seem to justify this assumption. Thus we will adopt the point of view[93] that <u>the strength of the weak-interaction does in fact depend upon the change in strangeness</u>.

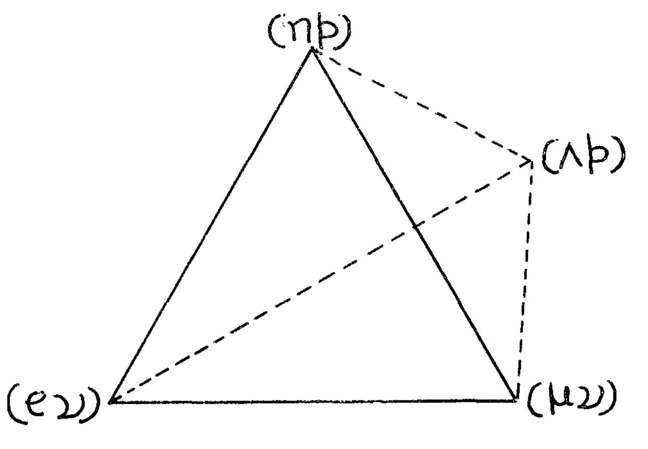


Fig. 2

---

93 In this sense we are giving up the concept of strict universality. However, this may be compared with the situation in strong interactions, where the K-mesons seem to possess a slightly weaker coupling constant than π-mesons.

So we will associate the (Λp) - vertex with a coupling constant f' (Referring to the dotted lines in Fig. 2) different form f, which is associated with the other three vertices of the tetrahedron (Referring to the solid lines in Fig. 2). ((f' is expected to be smaller than f to explain the slowness of leptonic modes)). We will assume further that the weak interaction current J_α is composed of charged positive chiral currents only.[94] The weak-interaction, thus has the form:

$$H_{weak} = J_\alpha J_\alpha^* + H.C. \tag{15}$$

where J_α has the unique form:

$$J_\alpha = f\left[\bar{\nu}\gamma_\alpha(1+i\gamma_5)e + \bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu + \bar{p}\gamma_\alpha(1+i\gamma_5)n + \frac{f'}{f}\bar{p}\gamma_\alpha(1+i\gamma_5)\Lambda\right] \tag{16}$$

The coupling constant f is related to the usual fermi-coupling constant f_F ($f_F m_p^2 \approx 10^{-5}$) by $f^2 = f_F/\sqrt{2}$.

Perhaps it is worthwhile to point out some of the immediate virtues of the above choice of J_α .

(i) First of all it is probably the simplest choice, which can, at least qualitatively, explain all the observed weak processes. It is also consistent with the known invariance properties (Invariance under proper ortho-chronous Lorentz transformation, CP and T-invariance) and conservation laws(conservation of baryons and leptons

---

94 By definition a currect composed oftwo Fermi-particle-field-operators A and B is said to have positive chiral form, if it has the structure $\bar{A}\gamma_\alpha(1+i\gamma_5)B$, or its hermitean conjugate. We <u>confine</u> ourselves to only those charged current for which $|Q_A - Q_B| = 1$.

as well as electric-charge) in weak interactions. It is noninvariant under C and P.

(ii) If one adopts the view-point of the sakata model for strongly interacting Particle, which considers only the proton, the neutron and the Λ-hyperon as truely elementary and the other strongly interacting baryons and mesons as composite (See Table -V); and also if one admits only charged positive chiral currents, then the above choice of the interaction is unique.

(iii) The interaction satisfies the "$I=\frac{1}{2}$-current" rule for the strangeness violating current (mentioned earlier. See Chapter IE) and hence the $\frac{\Delta S}{\Delta Q}=+1$-rule[95]. In so far as no $\Sigma^+\rightarrow n+e^++\nu$-decay has been observed so far and the predicted relationships (See Chapter IE) between the rates of K^+_{e3} and K^0_{e3} or $K^+_{\mu 3}$ and $K^0_{\mu 3}$ are consistent with present experiments, the above rule seems to be a rather good choice.

(iv) Within the framework of the Sakata-model, the vector part of the strangeness conserving current is automatically conserved[96] without the addition of any pionic or Kaonic-currents etc. This would imply in the limit of low-momentum transfer[97] to the emitted lepton pair,

---

95
It is to be noted that within the frame-work of Sakata-model it is impossible to obtain $\Delta S/\Delta Q=-1$ currents. Hence occurrence of $\Sigma^+\rightarrow n+e^++\nu$-decay or any other evidence for $\Delta S/\Delta Q=-1$-currents will be rather fatal to Sakata model.

96
L. B. Okun, Report to CERN Ann. Intern. Conf. High-Energy Phys. Geveva, Switzerland, (1958).

97
For the case, where momentum transfer is large, one can conclude that the form-factors of the strangeness conserving weak vector interaction, resulting from strong virtual processes should be the same as the Isotopic-vector form factors of electromagnetic interaction, whose current also satisfies the conservation law.

Table V

The Sakata Model*

Structure of π, K, Σ and Ξ

MESONS	S	I	BARYONS	S	I
$\pi^+ = p\bar{n}$	0	1	$\Sigma^+ = p\bar{n}\Lambda$	-1	1
$\pi^- = \bar{p}n$	0	1	$\Sigma^- = \bar{p}n\Lambda$	-1	1
$\pi^0 = \frac{1}{\sqrt{2}}(p\bar{p}-n\bar{n})$	0	1	$\Sigma^0 = \frac{1}{\sqrt{2}}(p\bar{p}-n\bar{n})\Lambda$	-1	1
$K^- = \bar{p}\Lambda$	-1	½	$\Xi^- = \bar{p}\Lambda\Lambda$	-2	½
$\bar{K^0} = \bar{n}\Lambda$	-1	½	$\Xi^0 = \bar{n}\Lambda\Lambda$	-2	½
$K^+ = p\bar{\Lambda}$	+1	½			
$K^0 = n\bar{\Lambda}$	+1	½			

* Table taken from L. Okun, Annual Rev. Nuclear Sci. 9, 61(1959)

S = Strangeness; I = Isotopic Spin

that the coupling constant for the strangeness conserving vector-interaction should not be renormalised under the influence of strong interactions. This is consistent with the experiments on μ-decay and β-decay of O^{14}.

B. Preliminaries on Λ-Decay

It has been established[98] rather conclusively at the moment that the spin of the Λ-hyperon is $\frac{1}{2}$. The most common modes of decays for the Λ-hyperon are:

$$\Lambda \rightarrow p + \pi^-$$
$$\Lambda \rightarrow n + \pi^0$$

In addition, out of nearly 1500 Λ-decays, two leptonic-decays of Λ-hyperon ($\Lambda \rightarrow p + e^- + \bar{\nu}$) have been observed. We will confine our discussion for a while to the various feature of the above two non-leptonic decay modes.

Since the parent particle has spin $\frac{1}{2}$, the final pion-nucleon system, which is produced by the decay of Λ-hyperon, must necessarily be in S or p-state in the rest frame of the decaying Λ-hyperon. (A simultaneous presence of both S and p states is possible if reflection invariance fails to hold in the decay mechanism of Λ). Each of these S and p states can in turn be composed of a certain linear combination of $I = 1/2$ and $3/2$ -states, where I denotes the total isotopic-spin of the final π-N system. Hence there are four different channels for the decay of Λ to π-N system.

Let us denote the amplitudes for the decays into the four possible channels as follows:

---

98 Crawford, Cresti, Good, Stevenson and Ticho, Phys. Rev. Lett. **2**, 114 (1959).

Final state of π-N system	Amplitude
$^2S_{1/2}$; $I = \frac{1}{2}$	S_1
$^2S_{1/2}$, $I = 3/2$	S_3
$^2P_{1/2}$, $I = 1/2$	P_1
$^2P_{1/2}$, $I = 3/2$	P_3

(17)

Using the standard Clebsch-Gordon coefficients[99] the expansions of $|p\pi^-\rangle$ and $|n\pi^0\rangle$- systems in isotopic-spin space are given by:

$$|p\pi^-\rangle = \sqrt{1/3}\; Y_{3/2}^{-1/2} - \sqrt{2/3}\; Y_{1/2}^{-1/2}$$

$$|n\pi^0\rangle = \sqrt{2/3}\; Y_{3/2}^{-1/2} + \sqrt{1/3}\; Y_{1/2}^{-1/2} \qquad (18)$$

where Y_J^M behaves like a spherical tensor $T(JM)$ under rotation.

Denoting the net transition-operators for the decays $\Lambda \rightarrow p+\pi^-$ and $\Lambda \rightarrow n+\pi^0$ by t^- and t^0 respectively, we have form (17) and (18):

$$t_- = \left\{-\sqrt{2/3}\; S_1 + \sqrt{1/3}\; S_3\right\} + \left\{-\sqrt{2/3}\; P_1 + \sqrt{1/3}\; P_3\right\} \vec{\sigma}\cdot\hat{P}_\pi$$

$$t_0 = \left\{\sqrt{1/3}\, S_1 + \sqrt{2/3}\, S_3\right\} + \left\{\sqrt{1/3}\, P_1 + \sqrt{2/3}\; P_3\right\} \vec{\sigma}\cdot\hat{P}_\pi \qquad (19)$$

---

99 See "Theoretical Nuclear Physics" by J.M. Blatt and V.F. Weisskopf (John Wiley and Sons).

where $\vec{p}_\pi$ denotes a unit vector along the momentum of the pion and $\vec{\sigma}$ denotes the Pauli spin operator. S_1, S_3, P_1 and P_3 are in general complex numbers and are functions of $\vec{p}_\pi^{\,2}$, which is fixed for the two-body decay mode.

The decay rates, given by (19) are (See Appendix II):

$$W(\Lambda \to p+\pi^-) \propto \left\{ \tfrac{2}{3}|S_1|^2 + \tfrac{1}{3}|S_3|^2 - 2\sqrt{\tfrac{2}{9}}\,\mathrm{Re}(S_1^* S_3) \right\} + \left\{ \tfrac{2}{3}|P_1|^2 + \tfrac{1}{3}|P_3|^2 - 2\sqrt{\tfrac{2}{9}}\,\mathrm{Re}(P_1^* P_3) \right\} \tag{20}$$

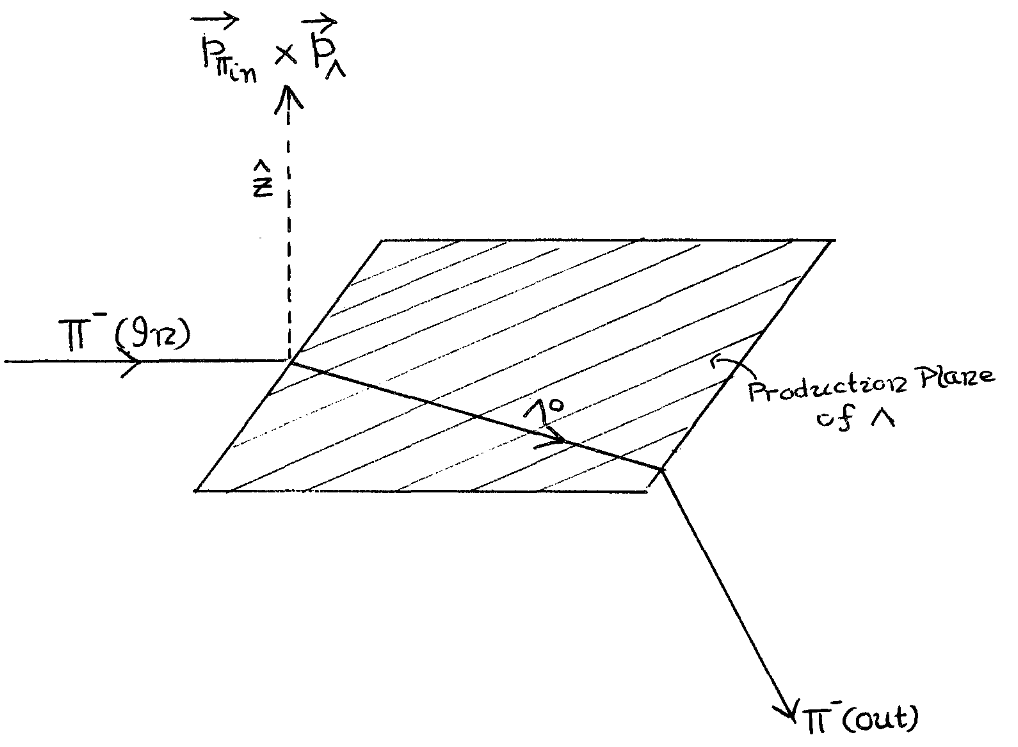
$$W(\Lambda \to n+\pi^0) \propto \left\{ \tfrac{1}{3}|S_1|^2 + \tfrac{2}{3}|S_3|^2 + 2\sqrt{\tfrac{2}{9}}\,\mathrm{Re}(S_1^* S_3) \right\} + \left\{ \tfrac{1}{3}|P_1|^2 + \tfrac{2}{3}|P_3|^2 + 2\sqrt{\tfrac{2}{9}}\,\mathrm{Re}(P_1^* P_3) \right\} \tag{21}$$

If the Λ-hyperon is polarised in the production mechanism and if the final π-N system is produced as a mixture of S and P states, it is easy to check that there will be an up-down asymmetry in the distribution of the decay pions (consequently in the distribution of the decay nucleons) with respect to the plane of production of the Λ-hyperon.

Let us denote the angular distribution of the decay-pions by $W(\xi)d\xi$; where

$$\xi = \cos\theta$$

θ being the angle between the momentum-vector of the decay-pion and the Z-axis, which is chosen along the normal to the plane of production of the Λ-hyperon. (For the case, in which Λ is produced by the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$, the Z-axis is chosen along $\vec{p}_{\pi_{in}} \times \vec{p}_{\Lambda}$ See, Fig. 3 , where $\vec{p}_{\pi_{in}}$ and $\vec{p}_{\Lambda}$ denote the momentum-vectors of the incident pion and the Λ-hyperon respectively).



(fig. 3)

The final π-N system, originating from the decay of a Λ with spin-projection m ($m = \pm 1/2$) on the Z-axis can be written as a linear combination of S and P-states as follows:

$$\Psi_{\frac{1}{2},m}(\pi N) = A\, S_{1/2,m} + B\, P_{1/2,m} \tag{22}$$

It is then easy to show (See Appendix II) that the angular distribution of the decay pions is proportional to:

$$W(\xi) = 1 + \alpha P \cos\theta \tag{23}$$

where,
$$\alpha = \frac{2\,\mathrm{Re}(A^{*}B)}{|A|^2 + |B|^2} \tag{24}$$

and P denotes the polarisation[100] of the Λ-hyperon, with the Z-axis chosen as the axis of quantisation for the spin of Λ.

---

100 It may be remarked that if parity is conserved in the production mechanism of Λ-hyperon, $\pi^- + p \to \Lambda_0 + K^0$, which involves unpolarised proton target, then Λ can not have any component of polarisation in its plane of production. It can only be polarised in a direction normal to the production plane.

α is called the "PION - ASYMMETRY - PARAMETER". Let us denote these parameters for the charged and neutral-modes of Λ-decay by α_- and α_0 respectively. By eqs. (19),(22) and (24) we have:

$$\alpha_- = \frac{2\,\mathrm{Re}\left\{(S_3-\sqrt{2}S_1)^*(P_3-\sqrt{2}P_1)\right\}}{|S_3-\sqrt{2}S_1|^2+|P_3-\sqrt{2}P_1|^2} \tag{25}$$

$$\alpha_0 = \frac{2\,\mathrm{Re}\left\{(S_1+\sqrt{2}S_3)^*(P_1+\sqrt{2}P_3)\right\}}{|S_1+\sqrt{2}S_3|^2+|P_1+\sqrt{2}P_3|^2}$$

It is clear that, since the polarisation may in general be a function of the angle of production of the Λ-hyperon (although a slowly varying function) we should replace P in eq. (23) by $\bar{P}$, where $\bar{P}$ denotes the hyperon-polarisation averaged over production angles. Obviously by studying the distribution of the pions with respect to the Z-axis, which has been chosen along the normal to the production-plane of the hyperon, one can only measure the product $\alpha\bar{P}$. Since $|\bar{P}|\leqslant 1$, this will enable us to assign a lower limit to the magnitude of α. Such measurements in the hydrogen-bubble-chamber in Berkely[101] yeild the result

$$\alpha_-\bar{P} = 0.73 \pm 0.14 \tag{26}$$

Hence $$\alpha_- \geqslant 0.73 \pm 0.14$$

101
D.A. Glaser Kiev Conference Report (1959)

Since $\bar{P}$ is unknown, one can not measure the sign of α'_-, nor can one determine its exact value by the above experiments. Another method for measuring α'_- is based on the fact that the longitudinal polarisation of the protons from the decay of <u>unpolarised</u> Λ 's at rest is equal to $-\alpha_-$ (See Appendix II). If the Λ -hyperons decay in flight with respect to the laboratory system, then the decay protons possess a transverse component of polarisation in the laboratory system, so that the sign of α'_- can be determined by noting the left-right asymmetry in the distribution of the protons, when they are scattered against nuclei. Such measurements have recently been carried out[102] in the propane bubble chanber in Berkely by injecting high-energy K -beam into the chamber. From 37 favorable cases of the scattering of protons from Λ-decay it is concluded that:

$$\alpha_- > 0 \tag{27}$$

i.e. the proton has negative helicity.[103]

As regards the asymmetry-parameter α_c of the $\Lambda \to n + \pi^0$-decay mode, a measurement has been done recently by Cronin[104] et. al.

102
R.W. Birge and W.B. Fowler - Bull. Am. Phys. Soc. Series II, vol. 4, No. <u>6</u>, 355(1959).

103
This confirms the result of previous measurement by Boldt, Bridge, Caldwell and Pal, Phys. Rev.Lett. <u>1</u>, 256 (1958).

104
Cronin, Cork, Kerth, Wenzel and Cool, Bull Am. Phys. Soc. Series II, vol. <u>5</u>, No. 11 (1960).

They produced Λ -hyperons by 1 $\frac{BeV}{c}\pi^+$ in the reaction $\pi^+ + d \rightarrow K^+ + \Lambda_0 + p$. The asymmetries in the charged and neutral decay modes of Λ's were measured by counter-arrangements, which detected separately charged pions and γ's from π^0's. Their measurements yield

$$\alpha_- \bar{P} = +0.46 \pm 0.04$$

$$\alpha_0 \bar{P} = +0.56 \pm 0.10$$

Hence one obtains:

$$\alpha_- / \alpha_0 = +1.22 \pm 0.24 \qquad (28)$$

Apart from the rather difficult measurements of the asymmetry parameters in Λ-decay, a pertinent fact is its branching ratio. Measurements by Crawford[105] et. al. in hydrogen bubble chamber yield:

$$R_{\Lambda_-} = \frac{W(\Lambda \rightarrow p + \pi^-)}{All\ \Lambda} = 0.627 \pm 0.031$$

$$R_{\Lambda_0} = \frac{W(\Lambda \rightarrow n + \pi^0)}{All\ \Lambda} = 0.43 \pm 0.014$$

(29)

Measurements by other groups of workers also confirm this result.

---

105 Crawford, Cresti, Douglas, Good, Kalbfleish, Stevenson, Sticho, Phys. Rev. Lett. 2, 266 (1959).

Finally it is of interest, that only two B-decays of the Λ-hyperon have been observed[106] out of nearly 1500 Λ's produced. This leads to a branching ratio:

$$\frac{W(\Lambda \rightarrow p + e^- + \bar{\nu})}{\text{All } \Lambda} \approx 0.3\ \% \tag{30}$$

To sum up; the facts that one would like to explain about Λ - decay are mainly the following:

(1) $$\frac{W(\Lambda \rightarrow p + \pi^-)}{W(\Lambda \rightarrow n + \pi^0)} = 0.627 \pm 0.031 \tag{31}$$

(2) $$\alpha_- \geqslant 0.73 \pm 0.14 \tag{32}$$

(3) $$\alpha_- / \alpha_0 = +1.22 \pm 0.24 \tag{28}$$

(4) $$W(\Lambda \rightarrow N + \pi) \approx 0.38 \times 10^{10} \text{ sec}^{-1} \tag{33}$$

(5) $$\frac{W(\Lambda \rightarrow p + e^- + \bar{\nu})_{\text{observed}}}{W(\Lambda \rightarrow p + e^- + \bar{\nu})_{UFI}} \approx \frac{1}{6} \tag{34}$$

---

106 Crawford, Cresti, Good, Stevenson, and Ticho, Phys. Rev. Lett. 2, 506 (1959).

C. Previous Work (The OMS-Analysis) and Successes

The decay of the Λ-hyperon has been discussed by a number of workers, in particular by Okubo, Marshak and Sudarshan(107) (referred to as OMS-Analysis). In this section we will briefly outline the main content of their work and its successes. In the next section we will point out the short-comings of their view-point and a possible remedy.

Experimental value for the branching ratio of charged and neutral modes of Λ-decay is given by eq. (31). Adopting the value

$$\frac{W(\Lambda \to p+\pi^-)}{W(\Lambda \to p+\pi^-)+W(\Lambda \to n+\pi^0)} = \frac{2}{3} \qquad (35)$$

we have from eq. (20) and (21) the following most general condition for the validity of eq. (35):

$$2\sqrt{2}\ \mathrm{Re}\,(S_1^* S_3 + P_1^* P_3) = -(|S_3|^2 + |P_3|^2) \qquad (36)$$

S_1, S_3, P_1 and P_3 are in general complex numbers of unknown phases. However, if we assume invariance of the theory under time reversal or under CP, then their phases are determined(108) in terms of pion-nucleon phase shifts and we can write:

---

107 S. Okubo, R.E. Marshak and E.C.G. Sudarshan, Phys. Rev. 113, 944 (1959) Referred to hereafter as OMS

108 F. Coester Phys. Rev 89, 619 (1953); E. Fermi, Nuovo Cimento 2, Suppl. 1, 54 (1955); M. Gell-Mann and K.M. Watson, Ann. Rev. Nuclear Science 5, 219 (1954).

$$S_1 = \pm |S_1| e^{i\delta_1} \quad , \quad P_1 = \pm |P_1| e^{i\delta_{11}}$$

$$S_3 = \pm |S_3| e^{i\delta_3} \quad , \quad P_3 = \pm |P_3| e^{i\delta_{33}} \qquad (37)$$

where the δ's denote the usual pion-nucleon phase shifts in the usual[109] notation at the energy available for Λ-decay ($\approx$ 37 mev.). At such energies the δ's are quite small, so that they may be all set equal to zero. Then S_1, S_3, P_1 and P_3 can be treated as real numbers, and eq. (36) becomes:

$$2\sqrt{2}\,(S_1 S_3 + P_1 P_3) = -(S_3^2 + P_3^2) \qquad (38)$$

If the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule holds, then

$$S_3 = P_3 = 0 \qquad (39)$$

In this case eq. (38) is satisfied most trivially. It is furthermore clear from eq. (25) that eq. (39) will imply $\alpha^{-}/\alpha_0 = +1$; as is required by experiments (eq. (28)).

Okubo, Marshak and Sudarshan have pointed out however that eq. (39) is not the only solution of eq. (38). In otherwords $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule is sufficient but not necessary to guarantee eq. (38).

---

109 'Mesons and Fields' - Vol II by H. Bethe and F. De Hoffmann (Row, Peterson and Company).

There exists another possible solution:

$$S_3 = -2\sqrt{2}\, S_1 \qquad , \quad P_3 = -2\sqrt{2}\, P_1 \qquad (40)$$

It is interesting that eq. (40) not only implies eq. (38), but also leads to the desired value for the ratio of the asymmetry parameters ; $\alpha_- / \alpha_0 = +1$.

Let us considernow, what does our[109] choice of the interaction for Λ-decay $\{(\bar{p}\Lambda)(\bar{n}p) + H.C.\}$ tell us about the isotopic spin selection rule. The tensor decomposition of the above interaction in isotopic spin space is as follows:

$$\begin{aligned} H_\Lambda &\propto [(\bar{p}\Lambda)(\bar{n}p) + H.C.] \\ &= \frac{1}{3}\left[\{2(\bar{p}\Lambda)(\bar{n}p) - (\bar{n}\Lambda)(\bar{n}n)\} + H.C.\right] \\ &\quad + \frac{1}{3}\left[\{(\bar{p}\Lambda)(\bar{n}p) + (\bar{n}\Lambda)(\bar{n}n)\} + H.C.\right] \end{aligned} \qquad (41)$$

(The γ-matrices leading to V-A-interaction have been suppressed for convenience)

---

109 It is to be noted that the choice $\{(\bar{p}\Lambda)(\bar{n}p) + H.C.\}$ adopted in the present work for the decay interaction of Λ-hyperon is the same as that of OMS.

The term inside the first square-bracket on RHS of eq. (41) transforms as an $I = 3/2$ — entity under rotation in iso-space, while that inside the second square bracket transforms as an iso-spinor.

Thus the choice of the interaction given by eq. (41) is obviously expected to violate the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-selection rule. It leads to appreciable amount of $|\Delta \underset{\sim}{I}| = \frac{3}{2}$- transitions in addition to $|\Delta \underset{\sim}{I}| = \frac{1}{2}$- ones. OMS have shown however that this need not be disturbing from the experimental stand-point, at least as far as Λ -decay is concerned. They have pointed out that the interaction (41) leads in the lowest order (fig. 4) of weak and strong interaction to eq. (40), and hence explains the observed branching ratio as well as the ratio of the asymmetry parameters of the charged and neutral modes of Λ-decay. The same diagram further leads to the right sign and magnitude of the asymmetry-parameter and explains reasonably the absolute rate for $\Lambda \to N+\pi$ decays if one assumes universal strength for weak interactions. We will give below a brief discussion on the contribution from the lowest order diagram and its successes.

The Lowest Order Diagram for $\Lambda \to N + \pi$ - decays:

The lowest order diagram for $\Lambda \to N+\pi$ - decay is shown in fig. 4. Fig. 5 has the same structure as fig. 4, except for the strong virtual processes in side the loop included in the former. In fact it is equivalent to fig. 4 in all respects (i.e. it gives the same asymmetry parameter branching-ratio and possesses the same isotopic transformation property as fig. 4 except for a constant scale-factor, which is material only for the absolute decay-rate. Thus when we refer to "the lowest-order diagram" we will often bear in mind fig. 5 rather than fig. 4.

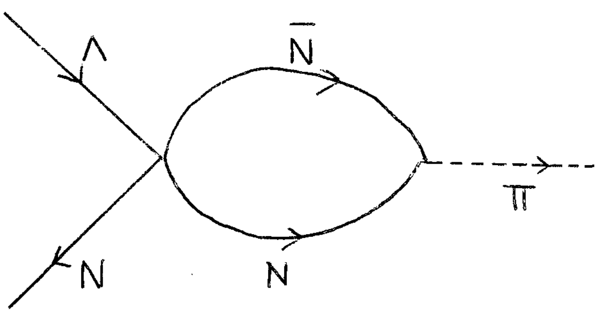


Fig. 4

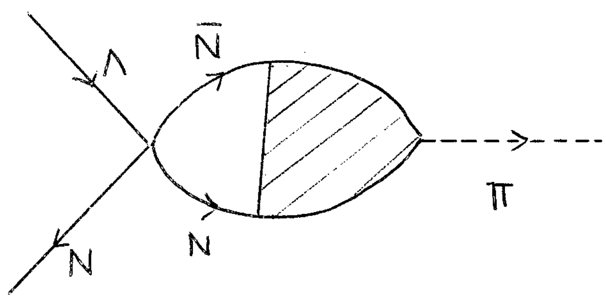


Fig. 5

By Lorentz-Invariance, the matrix elements of $\Lambda \to p + \pi^-$ and $\Lambda \to n + \pi^0$ -decays represented by Fig. 5 must have the form:

$$M(\Lambda \to p + \pi^-) = (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_{\pi^-})(f f' \sqrt{2} g_\pi)^{m_p}_{\Lambda} A \, \mathbb{P}_{\pi\mu} \bar{p} \gamma_\mu (1 + i\gamma_5) \Lambda \tag{42}$$

$$M(\Lambda \to n + \pi^0) = (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_n - \mathbb{P}_{\pi^0})(f f' g_\pi)^{m_p}_{\Lambda} A' \, \mathbb{P}_{\pi\mu} \bar{n} \gamma_\mu (1 + i\gamma_5) \Lambda \tag{43}$$

g_π denotes the renormalised pion-nucleon coupling constant $(g_\pi^2/4\pi \approx 15)$. A and A' are dimensionless-scalars representing the contribution of the black-box in fig. 5, and are functions only of $\mathbb{P}^2_{\pi^-}$ and $\mathbb{P}^2_{\pi^0}$ respectively; using the charge independence of pion-nucleon interactions we have:

$$A = A' \tag{44}$$

Let A_0 represent the corresponding contribution of the reduced diagram (fig. 4) without radiative corrections inside the bubble.

Clearly eqs. (42) and (43) yield the desired 2:1 branching ratio (eq. (31)) for the charged and neutral modes of $\Lambda \to N + \pi$ - decays. Also from eqs. (42) and (43) the asymmetry parameters for the corresponding decay modes are obtained to be:[110]

$$\alpha_- = \alpha_0 \approx +0.89 \tag{45}$$

The problem of evaluating the contribution from fig. 5 to the absolute rate of $\Lambda \to N + \pi$- decays can be solved, if we could evaluate A reliably. For the strictly lowest order diagram (fig.4); A_0 is logarithmically divergent, but can be evaluated readily by using a Feynman cutoff λ. We will first sketch below the evaluation of A_0. Since there is no way of evaluating A reliably, we will subsequently evaluate it from the known rate of $\pi \to \mu + \nu$-decay.

---

110 S.A. Bludman, Phys. Rev. 115, 468 (1959).

Evaluation of A_0

Denoting the internal momentum of Fig. 4 by $\mathbb{K}$, and comparing with eq. (42), we have:

$$m_p A_0 \mathbb{P}_{\pi\mu} = \frac{(-1)}{(2\pi)^4} \int d^4\mathbb{K}\, \mathrm{Tr}\left[\gamma_\mu (1+i\gamma_5)(\not{\mathbb{K}} - m_n)^{-1}\, i\gamma_5 \{(\not{\mathbb{K}} - \not{\mathbb{P}}_\pi) - m_p\}^{-1}\right.$$
$$\times \left(-\lambda^2 / \mathbb{K}^2 - \lambda^2\right)$$

$$= \frac{(-1)}{(2\pi)^4} \int d^4\mathbb{K} \frac{m_p \mathbb{K}_\mu}{(\mathbb{K}^2 - m_p^2)\left[(\mathbb{K} + \mathbb{P}_\pi)^2 - m_p^2\right]} \left(-\lambda^2 / \mathbb{K}^2 - \lambda^2\right) \tag{46}$$

This yields:

$$A_0 \approx \frac{1}{4\pi^2 i} \left(\lambda^2 / \lambda^2 - m_p^2\right)\left[\lambda^2 / \lambda^2 - m_p^2 \log_e \lambda^2 / m_p^2 - 1\right] \tag{47}$$

The values of A_0 for a few values of λ are tabulated below:

TABLE - VI

$\lambda \longrightarrow$	m_p	$1.5\, m_p$	$1.8\, m_p$	$2\, m_p$
$4\pi^2 i A_0 \longrightarrow$	0.50	0.792	1.01	1.13

Evaluation of A

As mentioned before, we will estimate A (the contribution of the black-box in fig. 5) from the known rate of $\pi \to \mu + \nu$ decay. It is to be noted that, apart from electromagnetic corrections the pion-decay matrix element (fig. 6)

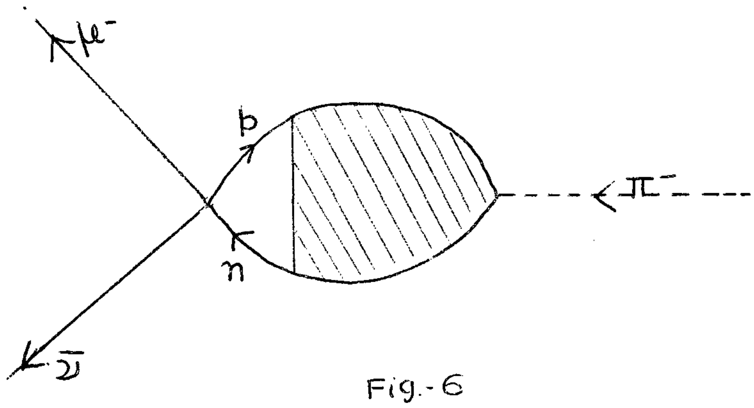


Fig.- 6

is rigorously given by:

$$M(\pi \to \mu + \nu) = (2\pi)^4 \delta^4(\mathbb{P}_{\pi^-} - \mathbb{P}_{\mu^-} - \mathbb{P}_{\bar{\nu}})(f^2 \sqrt{2}\, g_\pi) m_p B \, \mathbb{P}_{\pi\mu} \bar{\mu} \gamma_\mu (1 + i\gamma_5) \nu \tag{48}$$

Comparing figures 5 and 6, we may put

$$A \approx B$$

The value of B can be evaluated from the observed rate of $\pi \to \mu + \nu$-decay, which, thus yields:

$$4\pi^2 i A \approx 0.2954 \tag{49}$$

Absolute Rate for $\Lambda \to N + \pi$ -decays

For convenience, we shall define a parameter G_Λ in terms of A. Let us denote the effective decay-interaction for $\Lambda \to p + \pi^-$ decay (Fig. 5) by:

$$\frac{G_\Lambda}{m_\pi \sqrt{2}} \partial_\alpha \varphi_\pi \bar{p} \gamma_\alpha (1 + i\gamma_5) \Lambda \qquad (50)$$

The matrix element of fig. 5 for $\Lambda \to p + \pi^-$-decay is then given by

$$(2\pi)^4 \delta^4 (p_\Lambda - p_p - p_{\pi^-}) \left(G_\Lambda / m_\pi \sqrt{2} \right) p_{\pi\mu} \bar{p} \gamma_\mu (1 + i\gamma_5) \Lambda \qquad (51)$$

Comparing eqs. (51) and (42), we have:

$$G_\Lambda = \frac{(2 g_\pi f f' m_p^2)(m_\pi/m_p) A}{(2\pi)^4}$$

$$= \frac{(f'/f)(\sqrt{2} g_\pi f_F m_p^2)(m_\pi/m_p) A}{(2\pi)^4} \qquad (52)$$

where we have used $f^2 = f_F/\sqrt{2}$. Putting $f_F m_p^2 = 10^{-5}$ we have:

$$G_\Lambda^2 / 4\pi = (f'/f)^2 (42.60 \times 10^{-15}) |A|^2 \qquad (53)$$

The rate for $\Lambda \to p + \pi^-$-decay, calculated from the matrix-element (51) is given by:[110]

$$W(\Lambda \to p + \pi^-) = \frac{G_\Lambda^2}{4\pi} \frac{(m_p + E_p)}{2} \left[\langle \gamma_\mu \rangle^2 + \langle \gamma_\mu i \gamma_5 \rangle^2 \right] \frac{q}{m_\Lambda} \quad (54)$$

E_p is the total energy and q the magnitude of the momentum of the proton in the rest-frame of the parent-hyperon. It is easy to check from the kinematics involved in $\Lambda \to p + \pi^-$-decay that

$$E_p \approx 941 \text{ Mev}$$

and $$q/m_\Lambda \approx 0.0896 \quad (55)$$

Using the values of $\langle \gamma_\mu \rangle^2$ and $\langle \gamma_\mu i \gamma_5 \rangle^2$ given by Bludman[110], and using tabel VI, eqs. (49), (53), (54) and (55), the rates of $\Lambda \to p + \pi^-$ decay obtained from fig. 4 (for different values of the cut off λ) and fig. 5 are as given in table VII.

---

110 S.A. Bludman - Phys. Rev. 115, 468 (1960).

Table VII*

Figure	λ	$(f/f')^2 W(\Lambda \to p+\pi^-)$ (calculated) Number Per Sec.	$W(\Lambda \to p+\pi^-)$ (observed) Number Per Sec
4	m_p	0.302×10^{10}	$\approx 0.25 \times 10^{10}$
	$1.5\ m_p$	0.757×10^{10}	
	$1.8\ m_p$	1.231×10^{10}	
	$2.0\ m_p$	1.544×10^{10}	
5	Estimated by $\pi \to \mu+\nu$-Rate	0.104×10^{10}	

* The rate of $\Lambda \to n+\pi^0$ -decay is half that of $\Lambda \to p+\pi^-$ -decay.

Thus it is clear from the above table that if we adopt universal strength for all the vertices of the tetrahedron (fig. 2), i.e. if we put $f' = f$), fig. 4 as well as fig. 5, can explain the observed rate of $\Lambda \to p+\pi^-$-decay reasonably well (within a factor $\approx 1-5$).

So it appears that the merits of fig. 4 and 5 to explain the various observed features of Λ-decay are surprisingly many:

(i) It explains the desired 2 : 1 branching ratio for $\Lambda \to p + \pi^-$ and $\Lambda \to n + \pi^0$ - modes.

(ii) It explains the observed signs and magnitudes of the asymmetry parameters of the $\Lambda \to N + \pi$-decays.

(iii) If one assumes strictly universal coupling (i.e. $f' = f$), then one can also explain the absolute rate of $\Lambda \to N + \pi$-decays.

The perturbation calculation for fig. 4 does serve to explain (i) and (ii) independently of the choice of the cut-off, while as far as (iii) is concerned, it tends to yield slightly higher rates (within a factor ≈ 5) than the observed rate for reasonable choice of the cut-off ($\lambda \approx m_p \leftrightarrow 1.8 m_p$). This is expected since the dispersion theoretic calculation by Goldberger and Treiman[111] on $\pi \to \mu + \nu$-decay exhibits that the virtual strong processes inside the loop of fig. 4 will lead to a damping of the matrix element. <u>Henceforth we will refer to only fig. 5 as representative of the class of diagrams given by fig. 4 and 5.</u>

Being motivated by the above successes of fig. 5 whose matrix element can be put in the factorised form:

$$\langle p\pi^- | (\bar{p}\Lambda)(\bar{n}p) | \Lambda \rangle \to \langle p | (\bar{p}\Lambda) | \Lambda \rangle \langle \pi^- | (\bar{n}p) | 0 \rangle \quad (56)$$

---

111 M.L. Goldberger and S.B. Treiman Phys. Rev. 110, 1178, (1958).

OMS have been tempted to speculate that the above factorisation is in fact a good approximation for Λ-decay, i.e. fig. 5 is in fact the dominant diagram for $\Lambda \rightarrow N + \pi$ -decays.

In order to strengthen their speculations they have considered

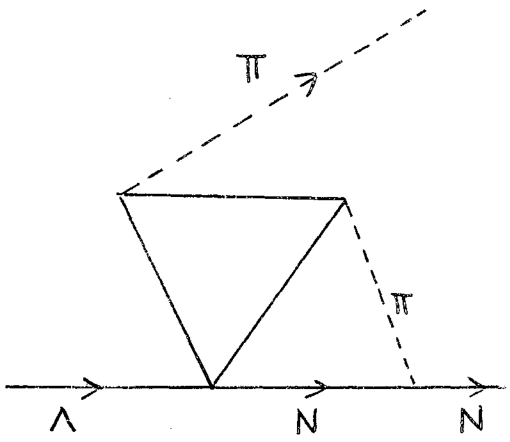


Fig. 7

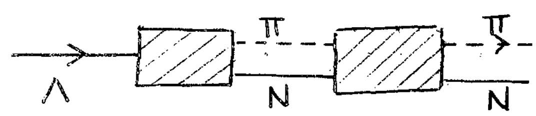


Fig. 8

Some special classes of higher order diagrams (Fig. 7 or more generally Fig. 8) in perturbation-theory as well as by dispersion techniques. In the latter case they have treated the Λ-field in perturbation and have included only the π-N intermediate state (Fig. 8) for the evaluation of the absorptive part of the decay transition amplitude. They find that the inclusion of such complications in addition to the "Born-approximation" (Fig. 5) does not lead to any essentially different result, as far as the observed features of Λ-decay are concerned, although the ratio of S and P wave amplitudes does get altered by such improvements over the Born-term.

It is at this point that one nees to be rather critical about the above mentioned speculation of OMS. In spite of the remarkable successes of the class of diagrams given by Fig. 5 to explain the observed features of $\Lambda \rightarrow N + \pi$ -decays, it will be shown in the next section that there are a number of draw-backs in regarding Fig. 5 as the dominant class of diagrams for $\Lambda \rightarrow N + \pi$ - decays, so that the virtues of Fig. 5 are to be regarded as accidental and one needs a "<u>new look</u>" for the mechanism of $\Lambda \rightarrow N + \pi$ -decays.

D. Needs for New Look

The drawbacks of considering the class of diagrams given by fig. 5, as the dominant diagrams for $\Lambda \rightarrow N + \pi$ -decays are the following:

(i) The diagrams shown in fig. 5, stronly violate the $|\Delta I| = \frac{1}{2}$-rule. They lead to appreciable amount of $|\Delta I| = 3/2$ -transitions in addition to $|\Delta I| = \frac{1}{2}$ -ones. Hence, even though they do serve to explain the observed features of $\Lambda \rightarrow N+\pi$ -decays, (for which, it is sufficient but not necessary to adopt $|\Delta I| = \frac{1}{2}$ -rule), they fail to explain the observed features of $|\Delta I| = \frac{1}{2}$ -rule in K-meson decays (See I.E) if one presumes that the K-meson decays proceed through $\Lambda \rightarrow N+\pi$-decays as virtual processes. In fact fig. 5 will lead to comparable rates for $K^+ \rightarrow 2\pi$ and $K_1^0 \rightarrow 2\pi$ -decays, where as the observed rates are in the ratio 1:500. There is then no way[112] to explain this big ratio, unless one is inclined to believe in an unreasonably large suppression of $|\Delta I| = 3/2$-part compared to $|\Delta I| = \frac{1}{2}$ part by higher order corrections.

(ii) If we accept fig. 5 as the main mechanism for $\Lambda \rightarrow N+\pi$ -decays then it has been shown (See Table VII) that the observed rates can be explained reasonably by putting $f' \approx f$.

---

112 Okubo and Marshak (Phys. Rev. 100, 1809 1955) have suggested that adopting fig. 5 as the dominant mechanism of $\Lambda \rightarrow N+\pi$ -decays, the extremely slow rate of $K^+ \rightarrow 2\pi$ compared to that of $K_1^0 \rightarrow 2\pi$ can be explained, if there exists a strong attractive final state π-π interaction in $T = 0$, $J = 0$ state, so that $K_1^0 \rightarrow 2\pi$ gets enhanced over $K^+ \rightarrow 2\pi$. Apart from the fact that it is rather artificial to expect such a large effect due to final state interaction (leading to a factor 100 in the decay rate), there seems to be an objection to the above hypothesis from consideration of the effect of the same final state interaction on the rate of K_{e4} -decay (See Chadan & Oneda - Phys. Rev. - to be published).

However with this choice of f' we run into difficulties in explaining the slowness of the leptonic modes of strange particle decays, so that, accepting fig. 5 as the dominant diagram for $\Lambda \rightarrow N + \pi^-$ decays there does not seem to be anyway to explain consistenly the observed rates of the leptonic as well as non-leptonic modes of strange particle decays, unless we attribute this dicrepancy solely to renormalisation effects.

The above two reasons strongly suggest that fig. 5, may not be the dominant class of diagrams for $\Lambda \rightarrow N + \pi^-$ decays and that <u>one must look for a new class of diagrams, which are missing so far from the discussion of Λ-decay</u>. These diagrams should be such that:

(a) They are much more important than fig. 5.

(b) They satisfy the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ -rule and hence, because of (a) serve to explain the <u>approximate</u> validity of the $|\Delta \underset{\sim}{I}| = 1/2$ -rule in strange particle decays.

(c) They contribute only to the non-leptonic modes in the first order of weak-interaction. Hence, due to (a), they serve to explain consistantly the slower rates of the leptonic modes along with the faster rates of the non-leptonic modes, if one associates the strangeness violating current with a weaker coupling constant than the strangeness conserving ones ($f' < f$).

E. A New Class of Diagrams

1. Diagramatic Expansion in Terms of Intermediate States

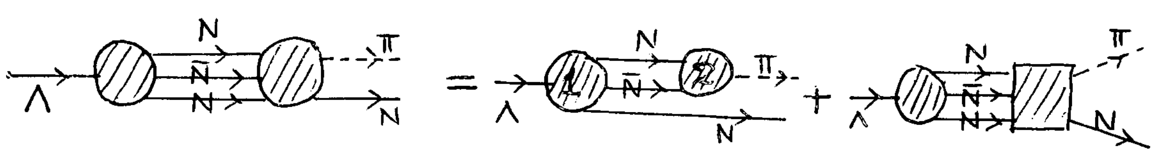
The diagrams, that we are looking for are readily obtained by asking the following question. What are the possible intermediate states that are relevant for the decay $\Lambda \to N+\pi$, subject to the necessary conservation laws. (i.e. conservation of charge and baryon number for all interactions and conservation of isotopic spin and strangeness only for strong interactions)?

Let us try to answer the above question by expanding the black box of the mechanism for $\Lambda \to N+\pi$ -decay in terms of intermediates states in order of increasing mass. The expansion can be represented diagramatically as shown in fig. 9.

The usual diagram (Fig. 4 and 5) discussed before clearly belongs[113] to the class shown in the fourth diagram on the right hand side of (57) with $(N\bar{N}N)$ intermediate state. However the <u>lowest mass-intermediate state is just a single neutron intermediate state</u> (the 1st term on the right side of (57)).

---

113 The fourth diagram on the right hand side of (57) can be expanded further as follows:



It is the first diagram on the right hand side of the above expansion, which can be identified with fig. 5 if we consider box-1 as a point. The square box on the right hand side represents effects in which there is <u>no</u> spectator nucleon in the final $(\pi\ N)$-production process.

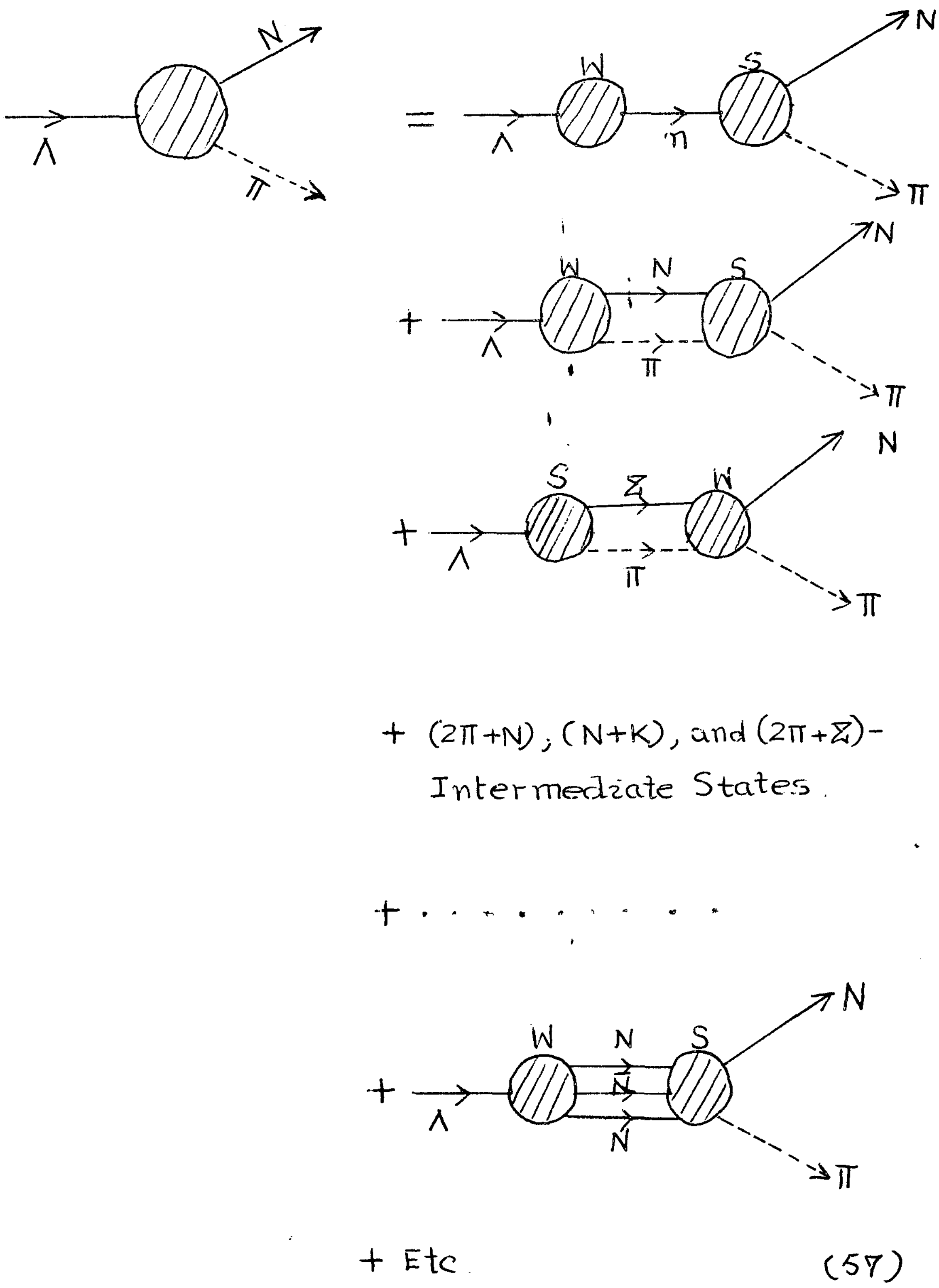


Fig.-9

The box S contains only strong interactions. The box W contains weak interaction only once in addition to strong interactions.

It is furthermore interesting that such class of diagrams do satisfy the criteria (b) and (c) mentioned before in Section D, namely, they satisfy the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ -rule strictly and contribute only to non-leptonic modes in the first order of weak interaction. Hence it is pertinent to examine the importance of these class of diagrams compared to the class shown in fig. 5.

Next to the single-neutron intermediate state are the (πN) and then the $(\pi \Sigma)$ intermediate states. The contribution from the (πN)-intermediate state, as mentioned before is expected to be small owing to the smallness of the relevant (πN)-phase shifts. The analysis of OMS more or less leads to the same conclusion. In the present work we will omit any further consideration of the (πN)-intermediate state. We shall also omit any discussion of $(\pi \Sigma)$ or any other higher mass intermediate states, mainly for reasons of simplicity.

2. A Dispersion Theoretic Discussion

The fact that the single-neutron intermediate state is expected to be significant also follows by appealing to the usual dispersion theoretic approach. In the present work no attempt will be made to pursue a detailed dispersion theoretic investigation. However much insight can be gained by a preliminary discussion, which we will present below following OMS.

In the dispersion theoretic approach, as is well known, one can, following the standard reduction technique of LSZ[114] (based on general assumptions such as validity of the asymptotic behavior of field operators in the time component of their arguments; the correct transformation properties of the operators under inhomogenous Lorentz group etc.) convert the particles in the state-vectors into field-operators, so that the transition amplitudes can be expressed in terms of field operators. If one wishes they may be completely reduced to vacuum expectation values of appropriate products of field operators.

The S-matrix element in terms of Heisenberg field operators for the decay $\Lambda \to p + \pi^-$, considered in the first order of weak interaction can be written as:

$$
\begin{aligned}
S(\Lambda \to p+\pi^-) &\propto \langle p\pi^- | \int H_{weak}(y)\, d^4y | \Lambda \rangle \\
&= \langle p\pi^- | \int e^{-iP.y} H_\omega(0)\, e^{iP.y} d^4y | \Lambda \rangle \\
&= (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_\pi) \langle p\pi^- | H_\omega(0) | \Lambda \rangle \qquad (58)
\end{aligned}
$$

---

114 Lehmann, Symanzik and Zimmerman Nuovo Cimento 1, 205, (1955).

$H_\omega(0)$ denotes the weak interaction hamiltonian density taken at the origin. Dropping the factor $(2\pi)^4\delta^4(\mathbb{P}_\Lambda-\mathbb{P}_p-\mathbb{P}_\pi)$ and using the above mentioned reduction technique, the transition amplitude can be written as:

$$t(\Lambda\to p+\pi^-) \propto \langle p\pi^- | H_\omega(0)|\Lambda\rangle$$
$$\propto \bar{u}_p \int d^4x\, e^{-i\mathbb{P}_p\cdot x}\, D_x\langle\pi^-|T\left(\psi_p(x), H_\omega(0)\right)|\Lambda\rangle \qquad (59)$$

where $\quad D_x \equiv -i\gamma^\mu\partial_\mu + m_p \qquad (60)$

so that $\quad D_x\psi_p(x) = J_p(x) \qquad (61)$

T denotes the wick-time-ordering operator. It is easy to check that

$$T\left(\psi_p(x)H_\omega(0)\right) \equiv \Theta(x)\left[\psi_p(x), H_\omega(0)\right]_+ + H_\omega(0)\psi_p(x) \qquad (62)$$

where $\Theta(x) = 1$ for $x_0 > 0$ and $= 0$ for $x_0 < 0$. The second term on the righthand side does not contribute to <u>physical Λ-decay</u>, since inserting the second term on right hand side of (62) into (59) we have:

$$\bar{u}_p \int d^4x\, e^{-i\mathbb{P}_p\cdot x}\langle\pi^-|H_\omega(0)J_p(x)|\Lambda\rangle$$
$$= \bar{u}_p \int d^4x\, e^{-i\mathbb{P}_p\cdot x}\sum_n\langle\pi^-|H_\omega(0)|n\rangle\langle n|J_p(x)|\Lambda\rangle$$
$$= \bar{u}_p\sum_n\langle\pi^-|H_\omega(0)|n\rangle\langle n|J_p(0)|\Lambda\rangle\,\delta^4(\mathbb{P}_p+\mathbb{P}_n-\mathbb{P}_\Lambda) \qquad (63)$$

The intermediate states contributing to the sum in (63) should have 4-momentum $P_n = P_\Lambda - P_p = P_\pi$, so that $P_n^2 = m_\pi^2$. There exists however no intermediate state, satisfying $P_n^2 = m_\pi^2$ and having strangeness -1, so that $\langle n | J_p(0) | \Lambda \rangle = 0$ for $P_n^2 = m_\pi^2$. Hence the second term on the right-hand side of (62) does not contribute to physical Λ -decay and can be dropped.

Inserting the first term on the right hand side of (62) into (59) and using (61), we have:

$$t(\Lambda \to p + \pi^-) \propto \bar{u}_p \int d^4x \, e^{-iP_p \cdot x} \langle \pi^- | \{ -i\gamma^0 \delta(x_0) [\psi_p(x), H_\omega(0)] + \Theta(x_0) [J_p(x), H_\omega(0)] \} | \Lambda \rangle \tag{64}$$

The first-term involving the equal time commutator can be identified with the Born-term. In fact if we treat the Λ -field in perturbation, so that we may express the physical state $|\Lambda\rangle$ as $b^+_\Lambda(P_\Lambda)|0\rangle$ then using the usual commutation relations[115] it is easy to check that we can express the equal time commutator-term as follows:

$$\bar{u}_p \int d^4x \, e^{-iP_p \cdot x} \langle \pi^- | -i\gamma^0 \delta(x_0) [\psi_p(x), H_\omega(0)] | \Lambda \rangle = \frac{(ff') \sqrt{m_\Lambda / E_\Lambda}}{(2\pi)^{3/2}} \bar{u}_p \gamma_\mu (1 + i\gamma_5) u_\Lambda \langle \pi^- | (-i) \bar{\psi}_n \gamma_\mu (1 + i\gamma_5) \psi_p(0) | 0 \rangle \tag{65}$$

---

115 See "Mesons and Fields" Vol I by Schweber, Bethe and De Hoffman (Row, Peterson and Company)

clearly, the right-hand side of (65) can be identified as the matrix element of fig. 5.

Writing $\Theta(x_0) = \frac{1+\epsilon(x_0)}{2}$ where $\epsilon(x_0) = +1$ for $x_0 > 0$ and -1 for $x_0 < 0$, we can break up t into the Born-term (B), the dispersive (real), and the absorptive (imaginary) parts, i.e.

$$t = B + D + iA \tag{66}$$

where;

$$B \propto \bar{u}_p \int d^4x \, e^{-i\mathbb{P}_p \cdot x} \langle \pi | \frac{1}{i} \gamma^0 \delta(x_0) [\psi_p(x), H_\omega(0)] | \Lambda \rangle \tag{67}$$

$$D \propto \bar{u}_p \int d^4x \, e^{-i\mathbb{P}_p \cdot x} \langle \pi | \frac{\epsilon(x_0)}{2} [J_p(x), H_\omega(0)] | \Lambda \rangle \tag{68}$$

$$A \propto \bar{u}_p \int d^4x \, e^{-i\mathbb{P}_p \cdot x} \langle \pi | \frac{1}{2i} [J_p(x), H_\omega(0)] | \Lambda \rangle \tag{69}$$

The transition amplitude can be considered as a funtion of the invariant $\mathbb{P}_\pi \cdot \mathbb{P}_p = \nu$ (Say). If the dispersive and absorptive parts are "well behaved" in the upper half complex-plane, then by virtue of the causal properties of the transition amplitude, its real and imaginary parts can be expressed as hilbert transforms of each other, so that one has the usual dispersion relation:

$$\mathrm{Re}\left(t(\nu) - B(\nu)\right) + i\,\mathrm{Im}\,t(\nu) = \frac{1}{\pi} \int_{-\infty}^{+\infty} d\nu' \, \frac{\mathrm{Im}\left(t(\nu') - B(\nu')\right)}{\nu' - \nu - i\epsilon}$$

or,
$$t(\nu) = B(\nu) + \frac{1}{\pi} \int_{-\infty}^{+\infty} d\nu' \, \frac{A(\nu')}{\nu' - \nu - i\epsilon} \tag{70}$$

Thus to evaluate t, one needs to evaluate $A(\nu')$ for all values of ν' on the real-axis. $A(\nu')$ is evaluated in the usual way by inserting in it a complete set of states. Inserting such a set into (69), noting that the $H_\omega(0)J_p(x)$-term does not contribute for reasons mentioned before, and treating the Λ-field in perturbation we have:

$$A \propto \frac{1}{2i}\bar{u}_p \int d^4x\, e^{-i p_p\cdot x} \sum_n \{\langle \pi^- | e^{-ip\cdot x} J_p(0) e^{ip\cdot x} | n\rangle$$

$$\langle n | H_\omega(0) | \Lambda\rangle\}$$

$$= \frac{1}{2i}\frac{\sqrt{m_\Lambda/E_\Lambda}}{(2\pi)^{3/2}}\bar{u}_p \Big[\sum_n \langle \pi^- | J_p(0) | n\rangle\langle n | \bar{J}_\Lambda(0) | 0\rangle$$

$$\delta^4(p_p + p_\pi - p_n)\Big] u_\Lambda \tag{71}$$

where $$\bar{J}_\Lambda(0) = \left[\frac{\delta H_\omega(x)}{\delta \psi_\Lambda(x)}\right]_{x=0} \tag{72}$$

The lowest mass intermediate state contributing to the sum in (71) is clearly a SINGLE-NEUTRON INTERMEDIATE-STATE. The next one is a (π-N)-intermediate state, and so on. Thus again we arrive at the same situation as we discussed before in terms of fig. 9. We can not have the ($\Sigma\pi$)-intermediate state (as drawn in fig. 9) in the right-hand side of (71), since we are treating Λ-field in lowest order perturbation theory .

From now on, we will not have any further discussion of the role of the single neutron-intermediate state from the point of view of dispersion theory.

3. The Single Neutron Intermediate State

The simplest Feynman diagram involving the single-neutron intermediate state is shown in fig. (10). This diagram involves no virtual emission and reabsorption of pions or kaons during the transformation of Λ to n.

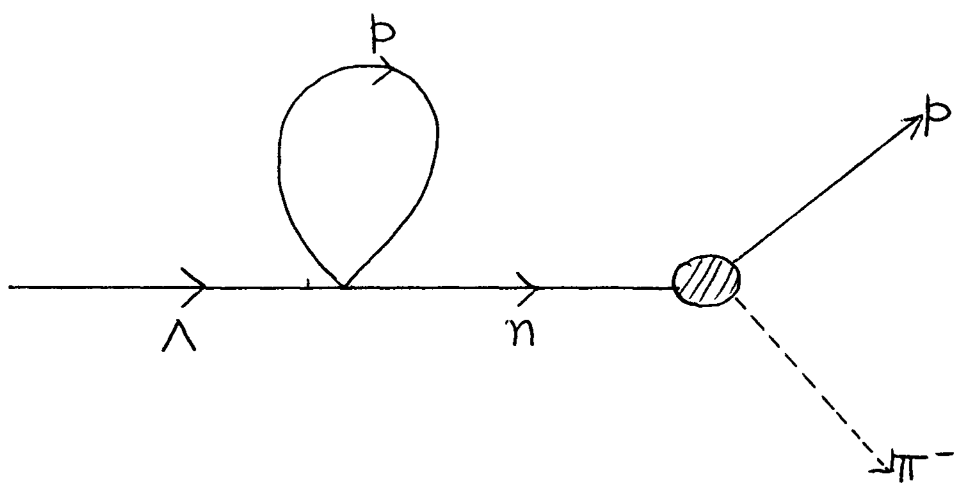


Fig. 10

It turns out however, as will be clear from the discussion in the next section, that the bare four-fermion interaction having the form $\{(\bar{p}\gamma_\mu(1\pm i\gamma_5)\Lambda)(\bar{n}\gamma_\mu(1\pm i\gamma_5)p)+H.c.\}$ with identical strengths for vector and axial vector-parts does not give rise to such a transformation of Λ to n. However if we introduce the virtual exchange of pions and kaons between the four fermions taking part in the four-fermion interaction, the above interaction does lead to the transformation of Λ to n.

This is checked most simply by letting the proton and the anti-proton, created by the four fermion interaction annihilate each other to a π^0 and letting the π^0 be absorbed by the neutron also created by the four-fermion interaction (See fig. (11)). Thus fig. (11) is the simplest Feynman diagram involving a single-neutron intermediate state which has a non-vanishing matrix element.

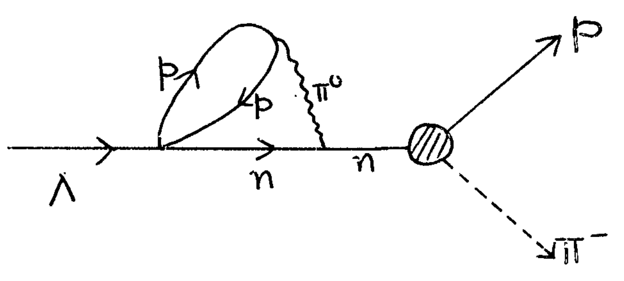


Fig. (11)

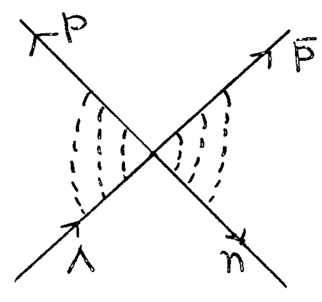


Fig. (12)

In general the transformation $\Lambda \to$ n via the weak four fermion interaction involves the exchange of virtual pions and kaons between the four fermions Λ, P, $\bar{P}$, and n in a complicated way. However, in order to make the problem tractable, we will consider such exhanges <u>only</u> between the Λ and P-lines and between the n and $\bar{P}$ lines, as shown in fig. (12). This will amount to modifying the bare ($p\Lambda$) and ($n\bar{p}$) - vertices independently of each other.

The corresponding Feynman diagram for $\Lambda \rightarrow n$ - transformation is given by fig. (13-1). We are hereby omitting considerations of diagrams of the type shown in figs. (13-2) and (13-3) for $\Lambda \rightarrow n$-transformations. The effect of such corrections is hard to estimate. However, as long as they do not cancel the effect of fig. (13-1), the

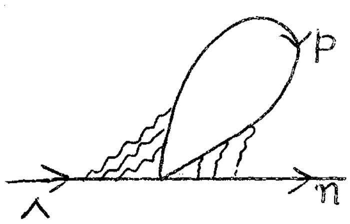


Fig. (13-1)

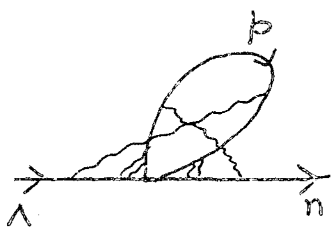


Fig. (13-2)

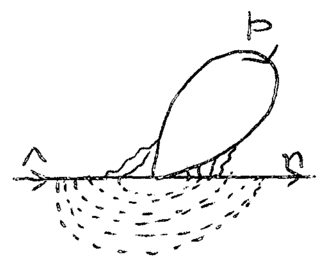
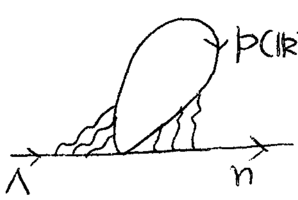


Fig. (13-3)

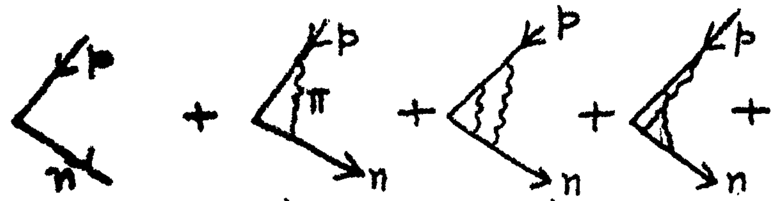
qualitative nature of our conclusion, to be discussed in the next two sections, will not be altered.

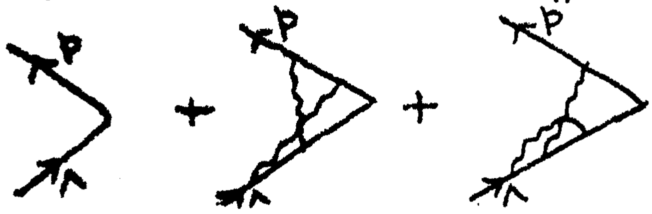
The matrix elements of fig. (13-1) can clearly be represented by:



$$\propto \bar{u}_n(\mathbb{P}_\Lambda)\left[\int d^4\mathbb{k}\; \Gamma_\mu(\mathbb{k},\mathbb{P}_\Lambda)\,(\mathbb{k}-m_p)^{-1}\,\Gamma'_\mu(\mathbb{P}_\Lambda,\mathbb{k})\right]u_\Lambda(\mathbb{P}_\Lambda) \qquad (73)$$

Here Γ_μ and Γ'_μ are modified vertex functions, defined symbolically by:

$\Gamma_\mu =$  + etc. ....

$\Gamma'_\mu =$  + etc. ....

where the first terms on the right hand sides of the above expansion of Γ_μ and Γ'_μ correspond to the bare vertices $f\gamma_\mu(1+i\gamma_5)$ and $f'\gamma_\mu(1+i\gamma_5)$ respectively.

By invariance arguments the modified vertex functions Γ_μ and Γ'_μ in general have the form:

$$\Gamma_\mu(p, p_\lambda) = f\left[a_1\gamma_\mu + a_2\sigma_{\mu\nu}q_\nu + ia_3q_\mu + b_1\gamma_\mu i\gamma_5 + b_2\sigma_{\mu\nu}i\gamma_5 q_\nu + ib_3 i\gamma_5 q_\mu\right] \tag{74}$$

$$\Gamma'_\mu(p_\lambda, p) = f'\left[c_1\gamma_\mu + c_2\sigma_{\mu\nu}q_\nu + c_3 iq_\mu + d_1\gamma_\mu i\gamma_5 + d_2\sigma_{\mu\nu}i\gamma_5 q_\nu + id_3 i\gamma_5 q_\mu\right] \tag{75}$$

where $q_\mu = p_\lambda - p$ = four momentum transfer at the repective vertices. The a_i, b_i, c_i and d_i are invariant functions of q_μ and are real, if time reversal invariance holds.

In the limit of very low momentum transfer, as in β-decay of neutron, the terms involving q_μ do not contribute. The β-decay experiments further reveal[116] that for low momentum transfers:

$$a_1(0) \approx 1 \;, \quad b_1(0) \approx 1.25 \tag{76}$$

For the purpose of orientation, since we have no reliable way of dealing with strong-interactions so as to be able to evaluate the above form-factors, <u>we will assume that in case of Λ-decay, the form factors $a_2, a_3, b_2, b_3, c_2, c_3, d_2$ and d_3 are small and negligible</u>.

Thus we will assume, more or less phenomenologically, that the (ΛP) and (Pn)-vertices in fig. (13-1) may be replaced[117] by:

$$\begin{aligned} (\Lambda p) &\longrightarrow f\,\bar{p}\,\gamma_\alpha (1 + iA\gamma_5)\,\Lambda \\ (pn) &\longrightarrow f\,\bar{n}\,\gamma_\alpha (1 + iB\gamma_5)\,p \end{aligned} \tag{77}$$

---

[116] To conclude $a_1(0) \approx 1$ from experiments, one assumes the same bare coupling constants for β-decay as for μ-decay.

[117] It should be noted that such a phenomenological method of taking account of the strong virtual effects at the vertices is similar in spirit to Feynman and Speisman's method (Phys. Rev. 94, 500, 1954) of evaluating neutron-proton mass difference by introducing the magnetic moment term.

We have chosen the bare coupling constant - f to be the coefficient for the (Pn)-vertex by assuming conservation of the vector part of the $(\bar{n}P)$ current. We could have chosen a constant f" different from the bare coupling constant f' for the (ΛP)-vertex. However, this is not material for our discussion, since we are going to choose f' anyway to explain the observed rate of Λ-decay. In this sense f' may be thought of as a renormalised coupling constant. Moreover, it is also not impossible to expect that the vector part of $(\bar{P}\Lambda)$- current may approximately be conserved.

F. Evaluation of the Matrix-Element of the New Diagram

Under the approximations mentioned in the previous section the matrix-element for fig. 14 is given[118] by:

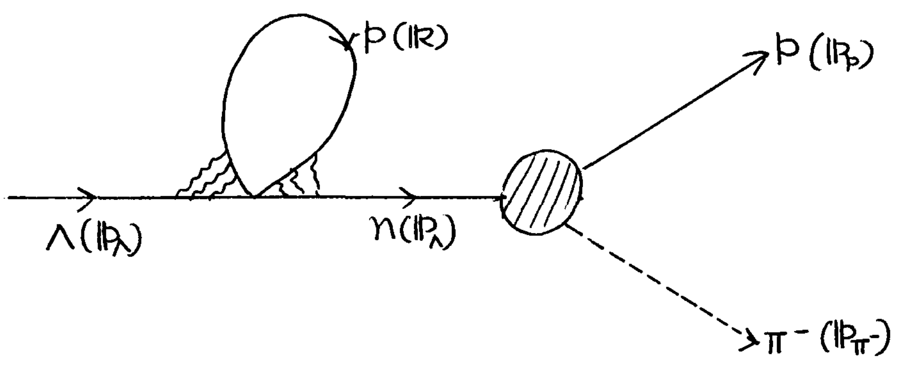


Fig. 14

$$\mathcal{N}(\Lambda \to p + \pi^-) = \frac{f f' \sqrt{2} g_\pi}{1} \delta^4(p_\Lambda - p_p - p_{\pi^-}) \bar{u}_p i \gamma_5 (\not{p}_\Lambda - m_n)^{-1} \int d^4 k$$
$$\left[\gamma_\alpha (1 + iB\gamma_5)(\not{k} - m_p)^{-1} \gamma_\alpha (1 + iA\gamma_5)\right] u_\Lambda \qquad (78)$$

---

118 As a <u>convention</u> we will denote throughout this work the matrix-elements of diagrams involving fig. 14 by N to distinguish them from those involving fig. 5, which we will denote always by M.

We have assumed pseudo-scalar pion nucleon coupling which accouhts for $\sqrt{2}g_\pi i\gamma_5$ in the above matrix element. To evaluate N we need to evaluate the integral:

$$I \equiv \int d^4 k \; \gamma_\alpha (1+iB\gamma_5)(\not{k}-m_p)^{-1}\gamma_\alpha(1+iA\gamma_5)$$

$$= \int d^4 k \; \frac{[4m_p\{(1-AB)+i(A-B)\gamma_5\}-2\{(1+AB)-i(B+A)\gamma_5\}\not{k}]}{(k^2-m_p^2)}$$

(79)

The $\not{k}$-term in (79) does not contribute due to symmetrical integration over k. The first term in (79) involves the integral which is quadratically divergent. Hence to evaluate it, we introduce a covariant quadratic cut off factor $(-\lambda^2/k^2-\lambda^2)^2$. Thus using the Standard Feynman - techniques, we have

$$I = 4m_p\{(1-AB)+i(A-B)\gamma_5\}\int d^4 k \,(k^2-m_p^2)^{-1}(-\lambda^2/k^2-\lambda^2)^2$$

$$= 4m_p\{(1-AB)+i(A-B)\gamma_5\}\int_0^1 2x\,dx\int d^4 k \frac{1}{[k^2-(\lambda^2-m_p^2)x-m_p^2]^3}(\lambda^4)$$

$$= 4m_p\{(1-AB)+i(A-B)\gamma_5\}\int_0^1 2x \frac{(-i\pi^2)}{2\{(\lambda^2-m_p^2)x+m_p^2\}}dx\,(\lambda^4)$$

$$= 4m_p\{(1-AB)+i(A-B)\gamma_5\}(-i\pi^2)(\lambda^4/\lambda^2-m_p^2)[1-m_p^2/\lambda^2-m_p^2\log_e \lambda^2/m_p^2]$$

(80)

Thus by eqs. (78), (79) and (80)

$$N(\Lambda \to p+\pi^-) = (f f' \sqrt{2}\, g_\pi)\, \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_{\pi^-}) \left\{ 4 m_p (-i\pi^2) \left(\lambda^4 / \lambda^2 - m_p^2 \right) \right.$$

$$\left. \left[1 - m_p^2 / \lambda^2 - m_p^2 \log_e \lambda^2 / m_p^2 \right] \bar{u}_p i \gamma_5 (\not{\mathbb{P}}_\Lambda - m_n)^{-1} \left\{ (1-AB) + i(A-B)\gamma_5 \right\} u_\Lambda \right. \tag{81}$$

Using $\mathbb{P}_\Lambda = \mathbb{P}_p + \mathbb{P}_\pi$, and the Dirac equation for the free field $\bar{u}_p(\not{\mathbb{P}}_p - M_p) = 0$, and neglecting proton neutron mass-difference, we have:

$$N(\Lambda \to p+\pi^-) = (-i)\sqrt{2}\, g_\pi\, \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_{\pi^-}) \left\{ (f f' m_p^2)(2\pi)^2 \frac{\lambda^4}{m_p^2(\lambda^2 - m_p^2)} m_p \right.$$

$$\left. \left[1 - m_p^2 / \lambda^2 - m_p^2 \log_e \lambda^2 / m_p^2 \right] \right\} (m_\Lambda^2 - m_n^2)^{-1} \bar{u}_p \not{\mathbb{P}}_\pi \left[(B-A) + i(AB-1)\gamma_5 \right] u_\Lambda \tag{82}$$

The magnitude of N depends on the choice of A and B. In particular if we neglect the pionic and kaonic corrections at the $(\bar{P}\Lambda)$ and $(\bar{n}\, p)$ vertices, then $A = B = 1$, in which case (as mentioned before) N vanishes. However, as will be shown below, the inclusion of pionic and kaonic corrections at the above two vertices leads to a rather large magnitude of $N(\Lambda \to p+\pi^-)$ compared to that of $M(\Lambda \to p+\pi^-)$ (matrix element of fig. 5)

There is no reliable way of calculating magnitude of the above mentioned corrections from first principles. We shall therefore estimate their effect phenomenologically as follows: We shall assume that the bare-vertex $f\, \bar{n}\, \gamma_\alpha(1+i\gamma_5)p$ in fig. 14 is modified due to the above corrections to $f\, \bar{n}\, \gamma_\alpha(1+i\, 1.25\gamma_5)p$ as in β-decay, (We here assume[119] the conservation of the vector-part of the ($\bar{n}p$)-current), i.e. we will assume:

$$B = +1.25 \tag{83}$$

The question of modification of the bare vertex $f'\,\bar{p}\gamma_\alpha(1+i\gamma_5)\Lambda$ in fig. 14, due to strong virtual effects is also problematic. In this case, we do not even have any experimental information on the modification of the above vertex in the β-decay of Λ-hyperon, as we do for the modification of ($\bar{n}p$)-vertex in the β-decay of neutron. However, as mentioned before, we have tentatively assumed that the above vertex is modified to $f'\,\bar{p}\gamma_\alpha(1+iA\gamma_5)\Lambda$ by the pionic and Kaonic corrections.

The question which comes up next is: What value should one choose for A? Fortunately, it turns out that with the choice of B given by (83), the choice of A is rather limited, if one requires that the calculation also yields correctly the sign and magnitude of the asymmetry parameter of $\Lambda \to p + \pi^-$-decay, ($\alpha_- \geqslant 0.73 \pm 0.14$). From eq. (82), it follows that the asymmetry given by the matrix element N is:

$$\alpha_- = \frac{2\,\mathrm{Re}\left[(B-A)^*(AB-1)\langle\gamma_\mu\rangle\langle\gamma_\mu i\gamma_5\rangle\right]}{|B-A|^2\langle\gamma_\mu\rangle^2 + |AB-1|^2\langle\gamma_\mu i\gamma_5\rangle^2} \tag{84}$$

119
As mentioned before, such a conservation is automatically quaranteed in the framework of Sakata model for strongly interacting particles without invoking the pionic and Kaonic currents etc.

Taking $B = +1.25$, it is easy to check that in order to obtain the desired sign and magnitude of the asymmetry parameter of $\Lambda \to p + \pi^-$ decay from (84) the choice of A is restricted[120] to the values:

$$+0.93 < A < 1.17 \qquad (85)$$

Eq. (85) shows that, <u>within the frame work of this calculation</u>, the modification of the axial-vector part due to the strong virtual effects in the $(P\Lambda)$-vertex can not be very large. It is even smaller than the corresponding modification at the $(\bar{n}p)$-vertex, which also is rather small (25%). In as much as the $(\bar{p}\Lambda)$-vertex can be modified only through exchange of at least two pions or two Kaons, while a single pion-exchange can lead to a modification of the $(\bar{n}p)$-vertex; the renormalisation effect at the $(\bar{p}\Lambda)$-vertex may be expected to be smaller than that at the $(\bar{n}p)$-vertex. It is therefore comforting that the restriction (85) imposed on the choice of A by considering the experimental value (sign and magnitude) of α_-' does confirm the above expectations.

For the purpose of further computations it is therefore reasonable to choose:[121]

$$A \approx 1 \qquad (86)$$

---

120 In as much as we are going to show that the contribution of fig. 14 is much more important than that of fig. 5, it is necessary that (84) should yield the right sign and magnitude of the asymmetry parameter for the sake of consistency.

121 Our discussion will not sensitively depend upon any choice of A between the limits given by eq. (85).

Such a choice for A is equivalent to treating the Λ-field in lowest order perturbation theory (as OMS do).

With the above mentioned choice of A, we have B-A $=$ AB-1. Hence eq. (82) can be written as:

$$\mathcal{N}(\Lambda \to p+\pi^-) = (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_{\pi^-})(-i)(\sqrt{2}\, g_\pi)\, \rho\, m_p / m_\Lambda^2 - m_n^2$$

$$\bar{u}_p \not{P}_\pi (1+i\gamma_5)\, u_\Lambda \tag{87}$$

where ,

$$\boxed{\rho = \frac{(f f' m_p^2)(B-A)}{(2\pi)^2} \left\{ \frac{\lambda^4}{m_p^2(\lambda^2 - m_p^2)} \right\} \left\{ 1 - \frac{m_p^2}{\lambda^2 - m_p^2} \log_e \lambda^2/m_p^2 \right\}} \tag{88}$$

It is easy to see that the matrix-element (87) can be attributed to an <u>effective</u> $(\bar{n}\Lambda)$-interaction given by:

$$\mathcal{H}_{n\Lambda}^{\text{effective}} = \rho\, m_p \left[\bar{n}(1+i\gamma_5)\Lambda + \text{H.C.} \right] \tag{89}$$

Comparison of Matrix-Elements of Fig. 5 and Fig. 14

From eqs. (51) and (87) the absolute square of the ratio of the matrix elements of figs. 14 and 5 is given by:

$$\left|\frac{N(\Lambda\to p+\pi^-)}{M(\Lambda\to p+\pi^-)}\right|^2 = \left|\frac{\sqrt{2}\, g_\pi (m_\Lambda^2 - m_n^2)^{-1} \rho\, m_p}{G_\Lambda (\sqrt{2}\, m_\pi)^{-1}}\right|^2 , \tag{90}$$

Putting $G_\Lambda^2/4\pi = 3.67\times 10^{-15} (f/f')^2$ [by eqs. (49) and (53)] we have: from (90)

$$\left|\frac{N(\Lambda\to p+\pi^-)}{M(\Lambda\to p+\pi^-)}\right|^2 \approx (2.12\times 10^{15})(f/f')^2 \rho^2 \tag{91}$$

We give below in Table-VIII a few values of ρ (calculated from eq. (88)) and the corresponding values of the absolute square of the ratio of the matrix elements given above for a few values of the cut off λ .

TABLE - VIII

λ	$\rho(f/f')$	$\left\lvert\frac{N(\Lambda\to p+\pi^-)}{M(\Lambda\to p+\pi^-)}\right\rvert^2$
m_Λ	3.54×10^{-8}	2.66
$1.5 m_p$	6.37×10^{-8}	8.68
$1.8 m_p$	9.974×10^{-8}	21.12
$2\ m_p$	12.84×10^{-8}	35.01

Thus, the table-VIII shows that the class of diagrams of the type shown in fig. 14 are considerably more important than that shown in fig. 5 for a reasonable choice[122] of the cut off. Thus fig. does in fact satisfy all the three criteria ((a),(b) and (c)) laid down in II.D page - 78).

Since the class of diagrams shown in fig. 14 leads to pure $|\Delta I| = \frac{1}{2}$ -transitions and is <u>much</u> <u>more</u> <u>important</u> than the class shown in fig. 5, which leads to a mixture of $|\Delta I| = 1/2$ and 3/2-transitions; the inclusion of both fig. 5 and fig. 14 as relevant mechanism for $\Lambda \rightarrow N+\pi$ -decays will certainly make it easier to explain the approximate validity of the $|\Delta I| = \frac{1}{2}$ - rule in strange particle-decays; the small deviations from the $|\Delta I| = \frac{1}{2}$ rule (for example the occurrence of $K^{+} \rightarrow \pi^{+} + \pi^{0}$ - decay etc.) can be attributed to fig. 5.

Moreover fig. 5 alone leads to a decay rate for $\Lambda \rightarrow N+\pi$, which is nearly half the observed value, if one assumes strictly universal strength (i.e. $f = f'$) for all weak interactions (Table VIII). Therefore, one can now explain consistently the observed rate of $\Lambda \rightarrow N+\pi$ -decays, as well as the fact that the leptonic modes of strange particle decays are nearly an order of magnitude smaller than the universal rate by including both fig. 5 and 14 for $\Lambda \rightarrow N+\pi$-decays and choosing:

---

122 It may be remarked that a choic of cut off around 1.5mp serves to explain the proton neutron mass differenc (See Ref. 117). A similar choice of cut off in the present case reveals the marked importance of fig. 14 over fig. 5.

$$f'^2 \approx f^2/10 \tag{92}$$

and

$$\lambda \approx 1.8 m_p \tag{93}$$

The above choice of leads to a value $\approx$ 21 for the ratio $|N(\Lambda \to p+\pi^-)/M(\Lambda \to p+\pi^-)|^2$ (See table VIII) and the corresponding value of ρ is:

$$\boxed{\rho = 9.974 \times 10^{-8}} \tag{94}$$

For the purpose of further discussion (K-meson-decays etc.), we shall adopt consistently the value of the cut off $\lambda \approx 1.8 m_p$ and the value of ρ given correspondingly by eq. (94) although (as will be seen later), the results are not highly sensitive to the choice of the cut off.

Before passing on to the chapter on K-mesons decays, it is worth mentioning that, although the inclusion of fig. 14 makes it easier to explain the approximate $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ rule, the branching ratio of Λ-decay still deviates from the desired $2:1$ value due to the interference of the matrix-elements of fig. 5 and 14.

This is a rather serious situation in our treatment, if we adhere to the numbers (for the matrix-elements of fig. 5 and 14), which we have calculated. However, since our method of **evaluation** of the relative magnitudes of $|\Delta \underset{\sim}{I}| = 1/2$ and 3/2 parts coming from fig. 5 is rather poor, it maybe hoped that the higher order corrections can lead to a suppression of $|\Delta \underset{\sim}{I}| = 3/2$ part in fig. 5. So as to give the desired branching ratio.

G. Conclusion

The calculations in the present chapter reveal that the class of diagrams shown in fig. 14 are more important than the usually considered diagram (fig. 5). This important class of diagrams leads to pure $|\Delta \underset{\sim}{I}| = 1/2$-transitions in contrast to the usual class of diagrams (fig. 5), which contains a mixture of $|\Delta \underset{\sim}{I}| = 1/2$ and 3/2 parts. This makes it easier to explain the approximate validity of the $|\Delta \underset{\sim}{I}| = 1/2$-rule. Moreover this class of diagrams contributes only to non-leptonic modes in the first order of weak interaction and hence provides a consistent explanation of the rates of leptonic and non-leptonic modes of strange particle decays, with a choice $f'^2 \approx f^2/10$.

At least one may say this much: there may be still deeper reasons for the validity of the $|\Delta \underset{\sim}{I}| = 1/2$-rule and the slowness of the leptonic modes. The true form of the weak-interaction may be different from and more complicated than what has been adopted in the present discussion. However the class of diagrams of the type shown in fig. 14 can <u>not</u> be ignored as compared to the class of diagrams given by fig. 5.

Before concluding, it is worth mentioning that it would be useful to try to examine the validity of the present approach by more rigorous methods. It would be worthwhile to study carefully the structure of the $(\bar{p}\Lambda)$ and $(\bar{n}p)$-vertices in fig. 14.

One may also try to evaluate the contribution of the single neutron intermediate state (as in fig. 14) to the absorptive part of the transition amplitude with suitable approximations and evaluate the net transition-amplitude by dispersion relations.

It might be hoped that a qualitative and indirect check on the present approach could be obtained by calculating the relative rates of the various K-meson decay-modes with the aid of Λ -decay mechanism developed here and comparing with experiment. This will be our task in the next chapter.

CHAPTER III

THE DECAY OF THE K-MESON

Abstract

In this chapter, an attempt is made to explain the branching ratios of the various leptonic and non-leptonic modes of K-meson decays in the framework of the V-A four fermion interaction, extended to the tetrahedron scheme. The non-leptonic modes have been investigated with the inclusion of the new class of diagrams for $\Lambda \rightarrow N+\pi$ -decays, which has been discussed in chapter II. The final state pion-pion interaction has been neglected. It is shown that all the relative rates (involving ratios between different decay rates) can be explained reasonably well (at least as far as the order of magnitude is concerned); if we assume consistently that the matrix element for any process, evaluated by lowest order perturvation theory, should be damped by a factor ≈ 3, whenever a pion is emitted from a closed baryon-antibaryon loop. The observed features of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ - rule in K-meson decays become easy to understand as a result of the inclusion of the new class of diagrams for $\Lambda \rightarrow N+\pi$-decays. It is also possible to explain the fact that the final state in $K^{+} \rightarrow \pi^{+}+\pi^{-}+\pi^{+}$ - decay is predominantly symmetric between the three pions.

A. The Leptonic Decay Modes of K-Meson

So far two main leptonic decay modes of the K-meson have been observed:

$$(1)\quad K_{\mu 2}\ ;\quad K^{\pm} \rightarrow \mu^{\pm} \pm \nu$$

$$(2)\quad K_{\mu 3}(K_{e3}):\ K^{\pm} \rightarrow \pi^{0} + \mu^{\pm}(e^{\pm}) \pm \nu$$

$$K^{0}_{1,2} \rightarrow \pi^{\mp} + \mu^{\pm}(e^{\pm}) \pm \nu$$

In addition to discussing these two modes, we will also discuss the $K_{\mu 4}$ and K_{e4}-modes:

$$(3)\quad K_{\mu 4}(K_{e4}):\ K^{\pm} \rightarrow \pi^{+} + \pi^{-} + \mu^{\pm}(e^{\pm}) \pm \nu$$

$$\rightarrow \pi^{0} + \pi^{0} + \mu^{\pm}(e^{\pm}) \pm \nu$$

$$K^{0}_{1,2} \rightarrow \pi^{\mp} + \pi^{0} + \mu^{\pm}(e^{\pm}) \pm \nu$$

The relevant Feynman diagrams (in lowest order perturbation theory) for these three modes of K^{-}-meson decay have been given in figs. 15, 16 and 17 respectively. To write down the general forms of the matrix elements for these processes, we shall adopt the contact V-A four fermion interaction, discussed in chapter II (eqs. (15) and (16)). Then the lepton-vertex (without electromagnetic corrections) is given by:

$$f f' \bar{\nu} \gamma_{\alpha} (1 + i\gamma_5) \mu(e)$$

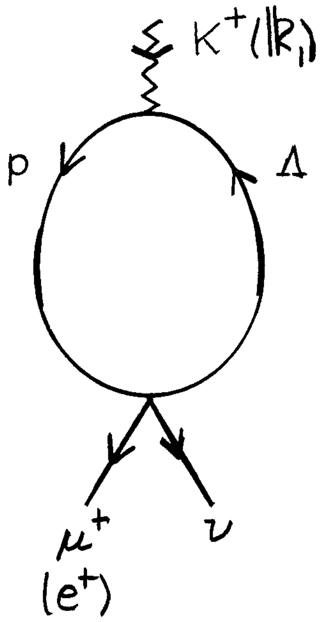


FIG. - 15

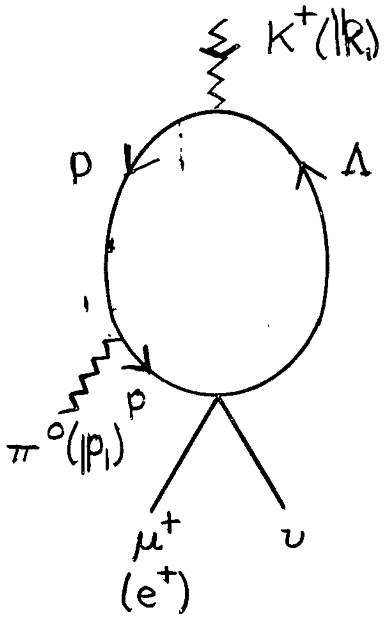


FIG. - 16

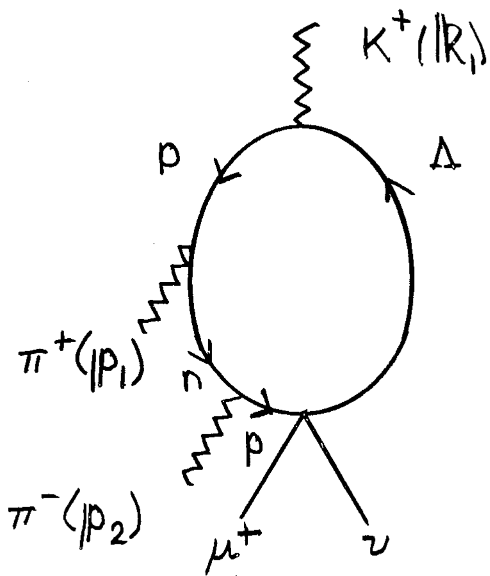


Fig - 17

The general forms of the matrix-elements of $K^{+}_{\mu 2}$, $K^{+}_{\mu 3}$ and $K^{+}_{\mu 4}$-decays, as prescribed by Lorentz - invariance, are therefore given by:

$$M(K^{+}\to \mu^{+}+\nu) = (2\pi)^{4}\,\delta^{4}(\mathbb{K}_{1}-\ell)(g_{K}ff')\,m_{p}\,I\,\mathbb{K}_{1\alpha}\,\bar{\nu}\,\gamma_{\alpha}(1+i\gamma_{5})\,\mu \tag{95}$$

$$M(K^{+}\to \pi^{0}+\mu^{+}+\nu) = (2\pi)^{4}\,\delta^{4}(\mathbb{K}_{1}-\mathbb{P}_{\pi}-\ell)(g_{K}g_{\pi}ff')(J_{1}\,\mathbb{K}_{1\alpha}+J_{2}\,\ell_{\alpha})\,\bar{\nu}\,\gamma_{\alpha}(1+i\gamma_{5})\,\mu \tag{96}$$

$$M(K^{+}\to \pi^{+}+\pi^{-}+\mu^{+}+\nu) = (2\pi)^{4}\,\delta^{4}(\mathbb{K}_{1}-\mathbb{P}_{1}-\mathbb{P}_{2}-\ell)\,[g_{K}(\sqrt{2}\,g_{\pi})^{2}ff']\,(1/m_{p})$$
$$\left[K_{1}\mathbb{P}_{1\alpha}+K_{2}\mathbb{P}_{2\alpha}+K_{3}\ell_{\alpha}+K_{4}\,\epsilon_{\alpha\beta\gamma\delta}\frac{\mathbb{P}_{1\beta}\mathbb{P}_{2\gamma}\ell_{\delta}}{m_{p}^{2}}\right]$$
$$\bar{\nu}\,\gamma_{\alpha}(1+i\gamma_{5})\,\mu \tag{97}$$

where by convention $\mathbb{K}_1$ denotes the 4-momentum of the incoming K-meson; $\mathbb{P}_i$ the 4-momentum of the ith emitted pion[123] (See figs. 16 and 17), and ℓ_α the sum of the 4-momenta of the emitted lepton and anti-lepton. g_K denotes the K-meson coupling constant to the (Λ-N)-system and g_π the usual pion-nucleon coupling constant. We have kept the factor involving g_K, g_π and ff' outside in order to compare with perturbation calculation results. The quantities I, J_1, J_2, K_1, K_2, K_3, and K_4 are form-factors[124] denoting the contributions of the intermediate black-boxes in the above decay processes.

---

[123] For example, for $K^{+}\to\pi^{+}+\pi^{-}+\mu^{+}+\nu$ -decay, $\mathbb{P}_1$ and $\mathbb{P}_2$ denote the 4-momenta of π^{+} and π^{-} respectively (See fig. 17)

[124] Assuming time-reversal-invariance, J_1/J_2 and the ratio of any two of K_1, K_2, K_3, and K_4 are real.

They are dimensionless scalars and presumably slowly varying functions of $\mathbb{P}_i \cdot \mathbb{P}_j / M^2$ and $\mathbb{P}_i \cdot \mathbb{P}_j / M^2$, where M is of the order of baryon-mass.[125] It is reasonable to neglect the dependence of the form factors on the energies of the decay pions for the range of mementa involved in the above decay-processes ($\overset{+}{K}_{\mu 3}$ and $\overset{+}{K}_{\mu 4}$ - modes). This assertion is explicitly exhibited by the results of the lowest order perturbation calculation, which will be presented in the following discussion and in Appendix III.

The matrix-elements for $\overset{+}{K}_{e2}$, $\overset{+}{K}_{e3}$ and $\overset{+}{K}_{e4}$-decays have exactly the same forms as eqs. (95), (96) and (97) respectively with μ being replaced by e. Clearly, in case of decays involving electron (of either sign), the contribution of the J_2 and K_3-terms may be neglected owing to the smallness of the mass of the electron. The contribution of the K_4-term in $K_{\mu 4}$ and K_{e4} -decays will also be neglected, since it is two orders of magnitude smaller than the other terms (in powers of mp).

The decay rates for the above processes (assuming the form-factors are independent of the energies of the decay products) are given by Appendix IV, eqs. (A-92), (A-104), (A-105), (A-106) and (A-107) :

$$W(\overset{+}{K} \to \overset{+}{\mu} + \nu) = \frac{m_\mu^2 m_K}{4\pi} \left(\frac{m_K^2 - m_\mu^2}{m_K^2} \right)^2 (g_K f f')^2 (m_p^2 |I|^2) \tag{98}$$

$$W(\overset{+}{K} \to \overset{o}{\pi} + \overset{+}{\mu} + \nu) = \frac{m_\pi^5 (m_K/m_\pi)}{(2\pi)^3} (g_K g_\pi f f')^2 \Big[0.431 |J_1|^2 + 0.042 |J_2|^2 + 0.089 (\overset{*}{J_1} J_2 + J_1 \overset{*}{J_2}) \Big] \tag{99}$$

---

125 This is due to the fact that the intermediate particles in the above decays are baryons and anti-baryons (with mass at least equal to nucleonic mass). So that, the characteristic interaction lengths for the above decays are of the order of baryon compton wavelength.

$$W(K^{+} \to \pi^{0} + e^{+} + \nu) = \frac{m_{\pi}^{5}(m_K/m_\pi)}{(2\pi)^3} (g_K g_\pi f f')^2 [0.541\, |J_1|^2] \quad (100)$$

$$W(K^{0} \to \pi^{-} + \mu^{+} + \nu) = \frac{m_{\pi}^{5}(m_K/m_\pi)}{(2\pi)^3} (g_K \sqrt{2}\, g_\pi f f')^2 [0.359\, |J_1|^2$$

$$+ 0.074\,(J_1^{*} J_2 + J_1 J_2^{*}) + 0.034\, |J_2|^2] \quad (101)$$

$$W(K^{0} \to \pi^{-} + e^{+} + \nu) = \frac{m_{\pi}^{5}(m_K/m_\pi)}{(2\pi)^3} (g_K \sqrt{2}\, g_\pi f f')^2 [0.456\, |J_1|^2] \quad (102)$$

It is to be noticed that the decay rates of $K^{+}_{\mu 3}$ and K^{+}_{e3} are slower than those of $K^{0}_{\mu 3}$ and K^{0}_{e3} respectively by a factor of 2 (apart from small differenced due to the mass difference of the participating particles). This is due to the choice of our interaction $ff'(\bar{\Lambda} p)(\bar{\nu} \ell)$, which satisfies the $I = \frac{1}{2}$ - current rule[126] mentioned before (chapter I.E). For our purposes, we take the form-factors for K^0-decays to be the same as the corresponding ones for K^{+}-decays. This amounts to neglecting the (K^{+}, K^0) and (π^{+}, π^{0}) mass differences compared to nucleonic mass for the values of the form-factors. However, we have included these mass differences in the above expressions for the calculation of the phase-space in the above decays. This is what gives rise to the slightly different expressions for K^{+} and K^0 -decay rates (compare with the earlier references[127]).

---

126 Okubo, Marshak, Sudarshan, Teutsch & Weinberg - Phys. Rev. 112, 665(1958).

127 S. Oneda -Nuclear Phys. 9, 476(1959); A. Fujii & M. Kawaguchi- Phys. Rev. 113, 1156(1959); R. Gatto-Phys. Rev. 111, 1426(1958); R.F. Streater & J.C. Taylor - Nuclear Phys. 7, 276(1958).

From eqs. (98) and (100), putting $g^2_{\pi}/4\pi = 15$, we get

$$R_1 \equiv \frac{W(K^+ \to \pi^0 + e^+ + \nu)}{W(K^+ \to \mu^+ + \nu)} \approx \frac{1}{5.2} \frac{|J_1|^2}{|I|^2} \tag{103}$$

The perturbation calculation (See Appendix III, Table - X) yields;

$$J_1 \approx \frac{1}{4\pi^2 i} (0.91)$$

$$I \approx \frac{1}{4\pi^2 i} (1.02)$$

for cut off Λ = 1.8mp. Putting these values of J_1, and I in eq. (103), we get

$$R_1 \approx \frac{1}{6.5} \tag{104}$$

The observed value (See Table II) of $R_1 \approx \frac{1}{15}$. Thus, since the cutoff dependences for the two processes $K^+_{\mu 2}$ and K^+_{e3} are nearly the same (logarithmic), so that the ratio R_1 is rather insensitive to the choice of the cut-off; the discrepancy by a factor $\approx$ 3 between the observed and the estimated ratio of the rates of the above two processes may be attributed to the fact, that the structure of the black boxes for the two processes differ only by the emission of an additional pion from the baryon-anti-baryon loop in the K^+_{e3} - decay as compared to the $K^+_{\mu 2}$ -decay. This suggests that we assume phenomenologically, that the matrix-element for any[128] process involving emission of pion from a closed baryon-anti-baryon loop is damped by a factor $\approx \sqrt{3}$. It is worthwhile to point out that a perturbation calculation of $\pi \to \mu + \nu$-decay yields a decay rate, bigger than the observed value by a factor ≈ 11 (for cut off $\approx$ 1.8 mp), which therefore seems to indicate the plausibility of the above assumption. A dispersion theoretic treatment[129] of the nucleon anti-nucleon loop involved in $\pi \to \mu + \nu$-decay does in fact exhibit such damping.

In the subsequent discussion, we will denote the effect of this damping of the matrix element due to emission of each pion fro a closed baryon-anti-baryon loop by putting $\alpha \mathcal{D} g_\pi$ instead of g_π as the effective coupling constant for each pion-vertex.

Tentatively we assume,

$$\mathcal{D}\, g_\pi \approx \frac{g_\pi}{\sqrt{3}} \qquad (105)$$

for all processes.

It is now our task to examine, how consistently can we explain all the relative rates of the various leptonic and nonleptonic mode of $K^{\pm}$ and $K^0_{1,2}$-decays with the above assumption of damping and

---

128 The damping is, of course, expected to depend upon the energy release in a process. We, however, assume for our purposes the effective damping to be nearly a constant.

129 M.L. Goldberger and S.B. Treiman - Phys. Rev. 110, 1178(1958).

the inclusion of fig. 14 in addition to fig. 5 as the relevant diagrams for Λ-decay. Before entering into the discussion of the non-leptonic decay modes, we first give blow the relative rates of $K^+_{\mu 3}$ and K^+_{e4} -modes compared to K^+_{e3} -decay.

By eqs. (99) and (100), we have

$$R_2 \equiv \frac{W(K^+ \to \pi^0 + \mu^+ + \nu)}{W(K^+ \to \pi^0 + e^+ + \nu)} \approx \frac{0.794 |J_1|^2 + 0.077 |J_2|^2 + 0.164 (J_1^* J_2 + J_1 J_2^*)}{|J_1|^2} \tag{106}$$

If $J_1 \approx J_2$, then $R_2 \approx 1.20$ or 0.54 depending upon $J_1/J_2 =$ -1. By actual perturbation calculation (See Appendix III, Table X),

$$J_1 \approx \frac{1}{4\pi^2 \ell} (0.91)$$

$$J_2 \approx -\frac{1}{4\pi^2 \ell} (0.58)$$

for cut off $\Lambda \approx 1.8$ mp. These values of J_1 and J_2 yield,

$$R_2 \approx \frac{1}{1.62} \tag{107}$$

This is in fair agreement with the experimental numbers (See Table II) $(4.0 \pm 0.77)\%$ and $(4.19 \pm 0.42)\%$ of $K^+_{\mu 3}$ and K^+_{e3} -decays respectively. It is to be noted that the value of R_2 is also not sensitive to the choice of the cut off.

The amplitude for the K^+_{e4} -decay-mode is convergent in perturbation calculation. The leading terms in powers of baryon-mass for K_1, K_2, and K_3 are given in Appendix -III, eq. (A-84).

Looking at these expressions, the K_2-term (being proportional to $(m_p - m_\Lambda)$ may be neglected compared to the K_1-term. The decay-rate of $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ is then given by [See Appendix IV, eq. (A-134)]:

$$W(K^+ \to \pi^+ + \pi^- + e^+ + \nu) \approx \frac{[g_K(\sqrt{2}\mathcal{D} g_\pi)^2 f f']^2}{64\pi^5 m_p^2} m_\pi^7 \{0.02|K_1|^2\} \tag{108}$$

In the above expression, the damping factor $\mathcal{D}$ appears due to the assumption mentioned above. For $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$-decay, P_1 and P_2- terms occur in the combination $(K_1 + K_2)(P_1 + P_2)/\sqrt{2}$ in the matrix-element due to the symmetry between the two final neutral pions. The decay rate is therefore given by [See Appendix IV, eq. (A-136)]:

$$W(K^+ \to \pi^0 + \pi^0 + e^+ + \nu) \approx \frac{[g_K(\mathcal{D} g_\pi)^2 f f']^2 m_\pi^7}{64\pi^5 m_p^2} \left\{0.067\left|\frac{K_1+K_2}{\sqrt{2}}\right|^2\right\} \tag{109}$$

By eqs. (100) and (108), we have:

$$R_3 \equiv \frac{W(K^+ \to \pi^+ + \pi^- + e^+ + \nu)}{W(K^+ \to \pi^0 + e^+ + \nu)} \approx \mathcal{D}^2 (2.13 \times 10^{-3}) \frac{|K_1|^2}{|J_1|^2} \tag{110}$$

Perturbation calculation yields (See Appendix III, Table - X)

$$J_1 \approx \frac{1}{4\pi^2 i}(0.91) \quad \text{for } \Lambda = 1.8 m_p$$

$$K_1 \approx -\frac{1}{4\pi^2 i}(0.56)$$

Thus, we get from eq. (110)

$$R_3 \approx 2.42 \times 10^{-4} \tag{111}$$

Furthermore, from eqs. (108) and (109) we get:

$$R_4 \equiv \frac{W(K^+ \rightarrow \pi^0 + \pi^0 + e^+ + \nu)}{W(K^+ \rightarrow \pi^+ + \pi^- + e^+ + \nu)} \approx 0.42 \tag{112}$$

The predicted ratio R_3 given by eq. (111) is smaller by an order of magnitude than the number given by Chadan and Oneda[130] This arises due to the introduction of the damping of the matrix-element due to each pion-emission and a somewhat higher choice of the cut off adopted in the present discussion. At this point, it is perhaps worthwhile to point out the following;

Chadan and Oneda[131], adopting a value of $R_3 \approx 10^{-3}$, have recently pointed out that the hypothesis of strong final state π-π-interaction in T = 0, J = 0 state, suggested by Okubo and Marshak[132], could lead to an enhanced value of $R_3 \approx 10^{-1}$. This would tend to disfavour the above hypothesis, because no K^+_{e4} - event has been found so far. However, in the present scheme, the enhanced value of R_3 is $\approx 10^{-2}$. Hence, the absence of any K^+_{e4}-event in future (with accumulation of more K^+_{e3} -events) may serve to rule out the above hypothesis regarding final state interaction.

---

130 K. Chadan and S. Oneda - Phys. Rev. Lett. 3, 292 (1959)

131 K. Chadan and S. Oneda - Phys. Rev. (To be published)

132 S. Okubo and R.E. Marshak - Phys. Rev. 100, 1809(1955).

This completes our discussion of the leptonic modes of the K-meson decays. We have mostly discussed the relative rates of $K^{\pm}$-meson decays. Those for K^o(or $K^o_{1,2}$) - decays are however easily obtained from the results given in the present section with the aid of appropriate symmetry properties. They have not been discussed here partly because of lack of data.

B. The Non Leptonic Decay Modes of K-Meson

The non leptonic decay modes of the K-meson involve the mechanism for $\Lambda \rightarrow N+\pi$ -decays, as developed in chapter II. So we have to include two possible diagrams (figs. 5 and 13.1) for Λ-transformation involving change of strangeness.

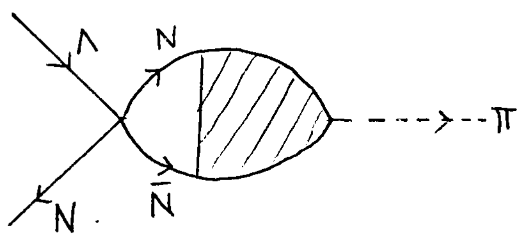


Fig. 5

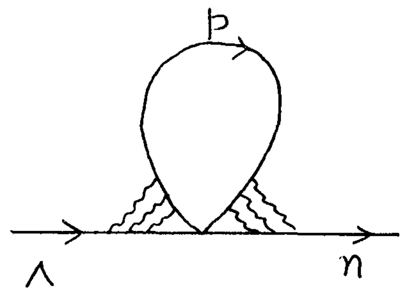


Fig. 13.1

It may be recalled that the effective interaction for the transformations shown if figs. 5 and 13.1 are given by (see eqs. (50) and (89)):

Fig. 5:
$$\frac{G_\Lambda}{m_\pi\sqrt{2}}\,\partial_\alpha \phi_\pi \left[\bar{p}+\frac{1}{\sqrt{2}}\bar{n}\right]\gamma_\alpha(1+i\gamma_5)\Lambda \tag{50}$$

Fig. 13.1:
$$\rho\, m_p \left[\bar{n}(1+i\gamma_5)\Lambda + \text{H.C.}\right] \tag{89}$$

where the value of G_{Λ}, evaluated from the observed rate of $\pi \to \mu + \nu$-decay [See eqs. (49) and (53)] is :

$$G_{\Lambda}^2/4\pi \approx 3.67 \times 10^{-15} (f'/f)^2 \quad (113)$$

and that of ρ, evaluated for cut off $\lambda = 1.8 m_p$ [See Table VIII] is:

$$\rho \approx 9.974 \times 10^{-8} (f'/f)^2 \quad (114)$$

The discussion of $K \to 2\pi$ and $K \to 3\pi$-decays will rest solely on the effective interactions (50) and (89) for Λ-transformation. We will first discuss the $K \to 2\pi$-decays.

(1) <u>The $K \to 2\pi$-Decays</u>

There are three decays belonging to this group:

1) $K^{\pm} \to \pi^{\pm} + \pi^{0}$
2) $K_1^{0} \to \pi^{+} + \pi^{-}$
3) $K_1^{0} \to \pi^{0} + \pi^{0}$

It is well known that the first process is forbidden by a strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule, while the second and the third processes can proceed through both $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ and 3/2-channels. Thus only fig. 5 plays a role for the first process, while both figs. 5 and 13.1 contribute to the second and the third processes.

The relevant[133] Feynman diagrams (for the processes (1) and (2) only) are therefore given by figs. 18, 20 and 21.

As a <u>convention</u>, we shall denote the matrix elements of diagrams involving fig. 5 by M and those involving fig. 13.1 by N.

Remembering that in these 2π-decay modes $E_{\pi 1} \approx E_{\pi_2} \approx m_K/2$ (In the rest frame of the K-meson), the matrix-elements for the first two processes are given by:

$$M(\overset{+}{K} \to \overset{+}{\pi} + \overset{0}{\pi}) \approx (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2)(2)\left[g_K(\mathcal{D}g_\pi)\frac{G_\Lambda}{m_\pi\sqrt{2}}\right](L_1 m_K^2/2 + L_2 m_\pi^2)$$

$$\approx (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2)(2)\left[g_K(\mathcal{D}g_\pi) G_\Lambda/m_\pi\sqrt{2}\right] m_\pi^2 (6.3 L_1 + L_2)$$

(115)

$$M(\overset{0}{K_1} \to \overset{+}{\pi} + \overset{-}{\pi}) \approx (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2)\sqrt{2}\left[g_K(\sqrt{2}\mathcal{D}g_\pi) G_\Lambda/m_\pi\sqrt{2}\right] m_\pi^2 (6.3 L_1 + L_2)$$

(116)

$$N(\overset{0}{K_1} \to \overset{+}{\pi} + \overset{-}{\pi}) = (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2)\sqrt{2}\left[g_K(\sqrt{2}\mathcal{D}g_\pi)^2 \rho\, m_p\right] \mathfrak{M}$$

(117)

The factor (2) in $M(\overset{+}{K} \to \overset{+}{\pi} + \overset{0}{\pi})$ is due to the possibility of interchange in the mechanism of π^+ and π^0 -emissions (See fig. 18).

---

133 We are neglecting the contribution of diagrams of the type shown in fig. 19. In the present choice of the weak-interactions, these diagrams, however, are known to yield vanishing contribution without pionic corrections between the Successive bubbles. This is because the matrix-element for fig. 20 can be represented by $\langle 2\pi|(\bar{p}n)|0\rangle\langle 0|(\bar{\Lambda}p)|K^+\rangle$, which vanishes, since $(\bar{p}n)$ behaves as an iso-vector, while the 2π -system can not be in I = 1-state by symmetry requirements. It is likely that their contribution even with pioic corrections between bubbles is of the same order of magnitude as that of fig. 18, and hence may be omitted from the present discussion (See eq. (119)).

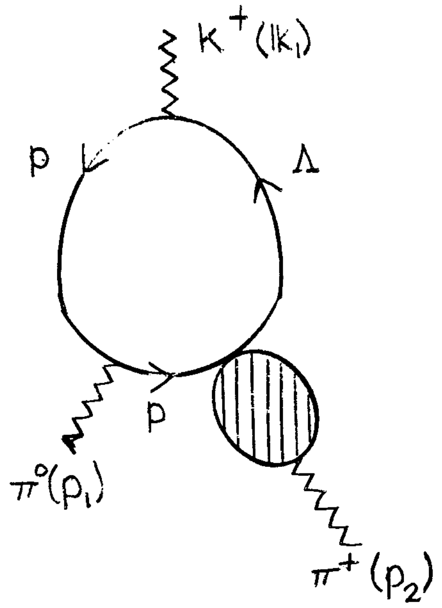


FIG. 18

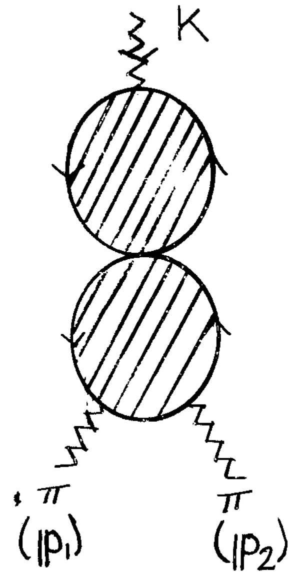


FIG. 19

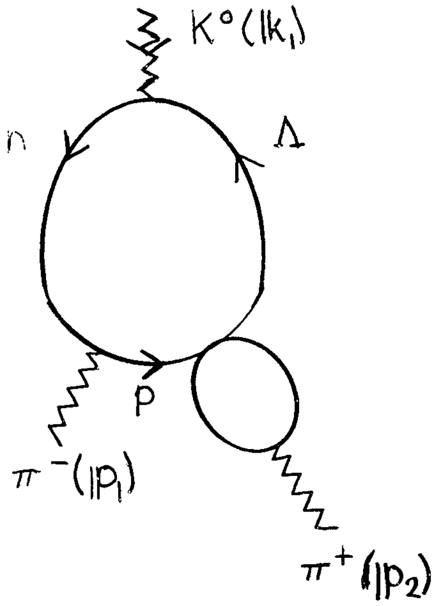


FIG. 20

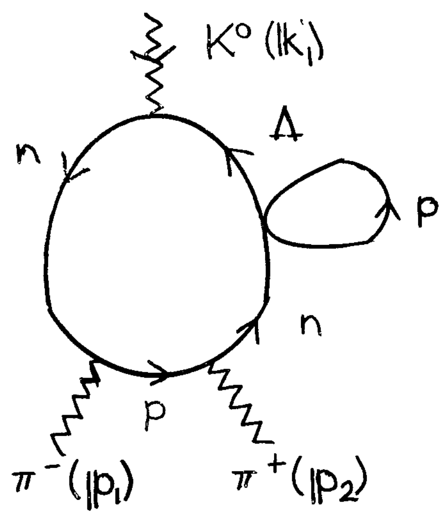


FIG. 21

The factor ($\sqrt{2}$) in $M(K_1^0 \to \pi^+ + \pi^-)$ and $N(K_1^0 \to \pi^+ + \pi^-)$ is due to the definition: $K_1^0 \equiv (K_0 + \bar{K}^0)/\sqrt{2}$. L_1, L_2 and $\mathfrak{M}$ are dimensionless scalars, denoting the contribution of the respective black-boxes. Since the pion emergies are fixed for two-pion-decay modes, L_1, L_2 and $\mathfrak{M}$ are function only of m_K^2 and m_π^2 and hence constants. Comparing fig. 16 with fig. 18; clearly in the lowest order perturbation calculation;

$$L_1 = J_1 \quad , \text{ and } L_2 = J_2$$

where J_1 and J_2 denote the form-factors for $K^+_{\mu 3}$ -decay (See eq. (96)).

By eqs. (113), (114), (115) and (117), we have:

$$\left| \frac{M(K^+ \to \pi^+ + \pi^0)}{N(K_1^0 \to \pi^+ + \pi^-)} \right| \approx \frac{1}{(\mathcal{S})\, 86.73} \left| \frac{6.3 L_1 + L_2}{\mathfrak{M}} \right| \tag{118}$$

If we assume for the purpose of a crude estimate that $|L_1| = |L_2| = \mathfrak{M}$, then the ratio

$$\left| \frac{M(K^+ \to \pi^+ + \pi^0)}{N(K_1^0 \to \pi^+ + \pi^-)} \right| \approx \frac{1}{(\mathcal{S})\, 11.88} \text{ or, } \frac{1}{\mathcal{S}\,(16.36)}$$

depending upon $L_1/L_2 = +1$ or -1. By perturbation calculation (See Appendix III, Table X), we have:

$$L_1 \approx \frac{1}{4\pi^2} i\,(0.91)$$

$$L_2 \approx -\frac{1}{4\pi^2} i\,(0.58)$$

$$\mathfrak{M} \approx +\frac{1}{4\pi^2} i\,(0.86)$$

for cut off Λ = 1.8 mp. If we use these values of L_1, L_2 and $\mathfrak{M}$ we obtain:

$$R_5 \equiv \left| \frac{M(\overset{+}{K} \rightarrow \overset{+}{\pi} + \overset{o}{\pi})}{N(K_1^o \rightarrow \overset{+}{\pi} + \overset{-}{\pi})} \right|^2 \approx \frac{1}{\mathcal{D}^2 (211.3)} \tag{119}$$

The value of R_5 is fairly insensitive to the choice of the cut off Λ, since both figs. 18 and 21 lead to logarithmically divergent amplitudes. It may be noticed from table VIII and eq. (119) that the ratio of the absolute squares of the matrix elements of diagrams involving figs. 5 and fig. 13.1 respectively is diminished by a factor $\approx$ 10 in K-meson-decays, as compared to Λ-decay. This therefore leads to a rather satisfactory explanation of the $|\Delta I| = \frac{1}{2}$ $\frac{1}{2}$-rule in K-meson decays. In fact, as shown by eq. (119), without damping the matrix element for each pion emission (i.e. if we put $\mathcal{D} \approx 1$), we obtain;

$$\frac{W(\overset{+}{K} \rightarrow \overset{+}{\pi} + \overset{o}{\pi})}{W(K_1^o \rightarrow \pi^+ + \pi^-)} \approx \frac{1}{200}$$

At this point, it is worth remembering that since the $\overset{+}{K} \rightarrow \overset{+}{\pi} + \overset{o}{\pi}$ - decay proceeds through $|\Delta I|$ = 3/2-channel, the evaluation of the matrix element of fig. 18 by equating the shaded loop to that occuring in the $\pi \rightarrow \mu + \nu$ -decay may be a little ambiguous. In fact, as mentioned already (See remark at the end of Chapter II.F), it may be hoped that the higher order effects can lead to a suppression of $|\Delta \underset{\sim}{I}| =$ 3/2-part as compared to the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-part in fig. 5 in order to explain the desired branching ratio of $\Lambda \rightarrow N + \pi$ -decays. This Suppression could therefore be expected to compensate for the damping factor δ in the denominator of eq. (119) which would provide a fair explanation of the observed ratio (See Table II) :

$$\left.\frac{W(K^+ \rightarrow \pi^+ + \pi^0)}{W(K_1^0 \rightarrow \pi^+ + \pi^-)}\right|_{\text{observed}} \approx \frac{1}{300}$$

The other interesting feature of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule in $K \rightarrow 2\pi$ -decays is the value of the ratio:

$$R_6 \equiv \frac{W(K_1^0 \rightarrow \pi^0 + \pi^0)}{W(K_1^0 \rightarrow 2\pi)} \tag{120}$$

It is easy to check by inspecting fig. 20 and 21 that:

$$\begin{aligned} M(K_1^0 \rightarrow \pi^+ + \pi^-) &= -\sqrt{2}\, M(K_1^0 \rightarrow 2\pi^0) \\ N(K_1^0 \rightarrow \pi^+ + \pi^-) &= +\sqrt{2}\, N(K_1^0 \rightarrow 2\pi^0) \end{aligned} \tag{121}$$

where M and N refer to matrix elements of diagrams of the type shown in fig. 20 and 21 respectively for $K \rightarrow 2\pi$ -decays. The above relations are based on charge independence of strong interactions, which we assume throughout the present work. By eqs. (119), (120) and (121), we have:

$$\begin{aligned} R_6 &= \frac{|M(K_1^0 \rightarrow 2\pi^0) + N(K_1^0 \rightarrow 2\pi^0)|^2}{|M(K_1^0 \rightarrow \pi^+ + \pi^-) + N(K_1^0 \rightarrow \pi^+ + \pi^-)|^2 + |M(K_1^0 \rightarrow 2\pi^0) + N(K_1^0 \rightarrow 2\pi^0)|^2} \\ &= 0.24, \text{ or } 0.44 \end{aligned} \tag{122}$$

where the alternative values depend upon the phase of M relative to N. In perturbation calculation, the phases of $M(K_1^0 \rightarrow \pi^+ + \pi^-)$ and $N(K_1^0 \rightarrow \pi^+ + \pi^-)$ differ by 180°, which corresponds to

$$R_6 \approx 0.24 \qquad (122\tfrac{1}{2})$$

This value of R_6 includes the damping factor $\mathcal{D}$ and is in good agreement with the observed value (See Table II):

$$R_6\Big|_{\text{observed}} = 0.27 \pm 0.11$$

If the damping factor $\mathcal{D}$ in this case is compensated (as mentioned before) by the suppression of the $|\Delta I| =$ 3/2-part in the matrix element of diagrams involving fig. 5, then the theoretical value for R_6 will tend more closely to 0.33, which is the value given by the strict $|\Delta I| =$ $\frac{1}{2}$-rule.

(2) <u>The $K \rightarrow 3\pi$ -Decays</u>

There are four decays belonging to this group:

(1) $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$
(2) $K^+ \rightarrow \pi^+ + \pi^0 + \pi^0$
(3) $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$
(4) $K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0$

All of these four decays can proceed through both $|\Delta I| = \frac{1}{2}$ and 3/2-channels.

Two relevant[134] Feynman diagrams for the first process have been given in figs. 22 and 23. The corresponding matrix elements for these two diagrams, with the necessary symmetrisation between the two identical positively charged pions, are respectively given by:

$$M(\overset{+}{K}\to\overset{+}{\pi}+\overset{-}{\pi}+\overset{+}{\pi}) = \frac{(2\pi)^4\delta^4(\mathbb{K}_1-\mathbb{P}_1-\mathbb{P}_2-\mathbb{P}_3)}{m_p}\left[g_K(\sqrt{2}\mathcal{D}g_\pi)^2 G_A/m_\pi\sqrt{2}\right]$$

$$\left[\frac{2N_1\,\mathbb{P}_1\cdot\mathbb{P}_3 + N_2\,\mathbb{P}_2\,(\mathbb{P}_1+\mathbb{P}_3) + N_3(\mathbb{P}_1^2+\mathbb{P}_3^2)}{\sqrt{2}}\right] \tag{123}$$

$$N(\overset{+}{K}\to\overset{+}{\pi}+\overset{-}{\pi}+\overset{+}{\pi}) = \frac{(2\pi)^4\delta^4(\mathbb{K}_1-\mathbb{P}_1-\mathbb{P}_2-\mathbb{P}_3)}{m_p}\left[g_K(\sqrt{2}\mathcal{D}g_\pi)^3\rho\, m_p\right](\sqrt{2}\,O) \tag{124}$$

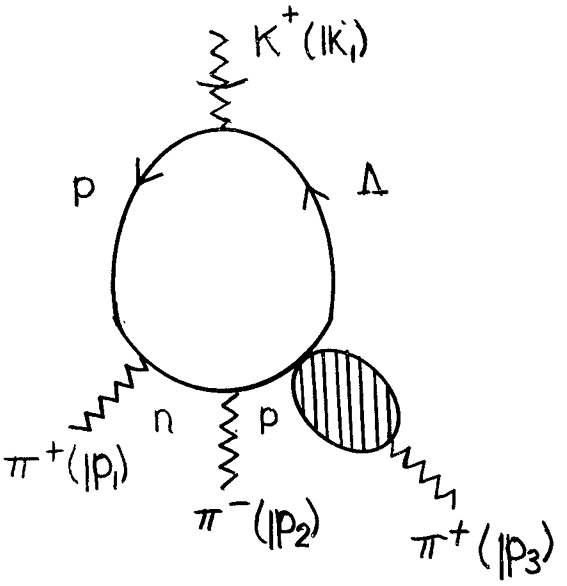
where, $\mathbb{K}_1$, $\mathbb{P}_1$, $\mathbb{P}_2$ and $\mathbb{P}_3$ denote the 4-momenta of the K-meson, π_1^+, π^- and π_2^+ -meson. The form-factors N_1, N_2, N_3 and O are dimensionless scalars denoting the contribution of the black-boxes in the corresponding processes. Comparing fig. 17 and 22, it is clear that in lowest order perturbation calculation:

$$N_1 = K_1 \;, \quad N_2 = K_2 \;, \text{ and } \; N_3 = K_3 \tag{125}$$

where K_1, K_2 and K_3 denote the form factors for $K^+_{\mu 4}$-decay [See eq. (97).]

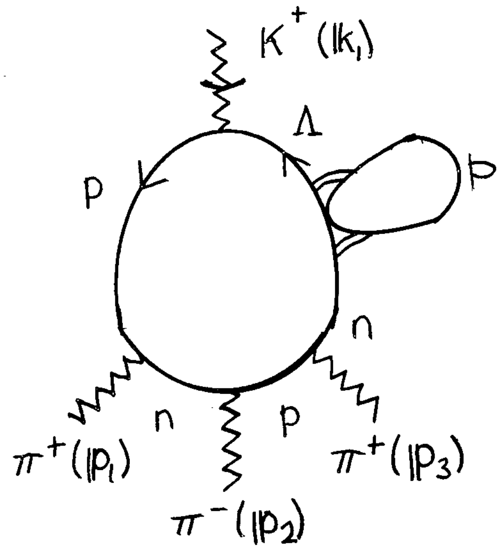
---

134 We omit a discussion of other possible diagrams which contribute to τ^+-decay in the present work. It is reasonable to expect that their contribution to the decay rates will be of the same order as that of fig. 22 and since fig. 22 has a negligible contribution to the decay rate, compared to fig. 23 See eq. (127)), this omission is justifiable.



$K^+(\mathbb{k}_1)$
p
Λ
n
p
$\pi^+(\mathbb{p}_1)$
$\pi^-(\mathbb{p}_2)$
$\pi^+(\mathbb{p}_3)$

FIG.-22



$K^+(\mathbb{k}_1)$
Λ
p
P
n
n
p
$\pi^+(\mathbb{p}_1)$
$\pi^+(\mathbb{p}_3)$
$\pi^-(\mathbb{p}_2)$

FIG.-23

From eqs. (123) and (124) we have;

$$\left|\frac{N(K^+\rightarrow \pi^+ + \pi^- + \pi^+)}{M(K^+\rightarrow \pi^+ + \pi^- + \pi^+)}\right| \approx 25.52 \left|\frac{m_p m_\pi \sigma}{2N_1|P_1||P_3| + N_2|P_2|(|P_1|+|P_3|) + N_3(|P_1|^2+|P_3|^2)}\right|^2 \tag{126}$$

If, for a crude estimate, we assume $N_1 = N_2 = N_3 = \sigma$, and use a rough upper bound for the denominator of (126) by taking into account the nature of kinematics involved in τ^+ -decay, the the above ratio is still greater than 15. In perturbation calculation, the N_2 term in the denominator is proportional to $(m_\Lambda - m_p)$ [See Appendix III, Table - X], so it can be dropped. With the values of N_1, N_3 and σ given in Appendix III, Table X , it can be checked[135] that;

$$\left|\frac{N(K^+\rightarrow \pi^+ + \pi^- + \pi^+)}{M(K^+\rightarrow \pi^+ + \pi^- + \pi^+)}\right| > 23 \tag{127}$$

Thus in the following discussion of $K\rightarrow 3\pi$ -decays, <u>the contribution of diagrams of the type shown in fig. 22 will be neglected</u>.

---

[135] We use the fact that in τ^+ -decay the maximum of $\frac{|P_1|\cdot|P_3|}{m_\pi} \approx 22.7$.

Comparison of $K^+ \to \pi^+ + \pi^- + \pi^+$ and $K_1^0 \to \pi^+ + \pi^-$ Decay Rates

For the $K_1^0 \to \pi^+ + \pi^-$ -decay rate, it is enough to consider the contribution of fig. 21 only, since fig. 20 gives a rather small contribution [See eq. (119)]. The decay-rates for $K_1^0 \to \pi^+ + \pi^-$ and $K^+ \to \pi^+ + \pi^- + \pi^+$, in the rest frame of the K-meson are given by [See Appendix IV eqs. (A-138) and (A-145)]:

$$W(K_1^0 \to \pi^+ + \pi^-) = \frac{\sqrt{1-4(m_{\pi^\pm}/m_K)^2}}{16\pi m_K} \left| N(K_1^0 \to \pi^+ + \pi^-) \right|^2$$

$$\approx \frac{2\left[g_K(\sqrt{2}\mathcal{D}g_\pi)^2 \rho m_p\right]^2}{16\pi m_{\pi^+}} \left[0.23 |\mathfrak{M}|^2\right] \tag{128}$$

$$W(K^+ \to \pi^+ + \pi^- + \pi^+) \approx \frac{\left[g_K(\sqrt{2}\mathcal{D}g_\pi)^3 \rho m_p\right]^2}{128\pi^3 m_p^2} \left(\frac{m_\pi^2}{m_K}\right) \left\{0.32 |\mathcal{O}|^2\right\} \tag{129}$$

By eqs. (128) and (129) we have

$$R_7 \equiv \frac{W(K^+ \to \pi^+ + \pi^- + \pi^+)}{W(K_1^0 \to \pi^+ + \pi^-)} \approx \frac{1}{150} \frac{|\mathcal{O}|^2}{|\mathfrak{M}|^2} \tag{130}$$

By perturbation calculation See Appendix III, Table X ,

$$\mathcal{O} \approx -\frac{1}{4\pi^2 i}(0.59)$$

$$\mathfrak{M} \approx +\frac{1}{4\pi^2 i}(0.86) \quad \text{for cutoff } \Lambda = 1.8 m_p$$

Putting these values of σ and $\mathfrak{M}$, we have

$$R_7 \approx \frac{1}{320} \tag{131}$$

This may be compared with the observed value [See Table II]

$$R_7 \Big|_{observed} \approx \frac{1}{1500}$$

Thus, in this case there seems to be a discrepancy by a factor of 5 between the observed and the estimated ratios. This, however, can not be regarded as a serious discrepancy, particularly considering the fact that the calculated amplitude of $K_1^0 \to \pi^+ + \pi^-$ decay is cut-off dependent, while that of $K^+ \to \pi^+ + \pi^- + \pi^+$ decay is convergent. Moreover the above ratio also depends upon the value of the effective damping in the emission of a pion from a closed loop. Although this damping is expected to be dependent on the energy release in a process, we have assumed that it is a constant ($\mathcal{D} \approx \sqrt{1/3}$) for both 2π and 3π -decays. It is also expected that the inclusion of final state $\pi - \pi$ - interactions, neglected here, may help resolve the discrepancy.

<u>The $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule in $K \to 3\pi$ -decays</u>

There are three features of the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule, assuming a totally symmetric wave function in the final state, in $K \to 3\pi$-decays:

$$\text{(a)} \quad \frac{W(K^+ \to \pi^+ + \pi^0 + \pi^0)}{W(K^+ \to \pi^+ + \pi^- + \pi^+)} \approx \frac{1}{4}(1.3)$$

$$\text{(b)} \quad \frac{W(K_2^0 \to 3\pi^0)}{W(K_2^0 \to \pi^+ + \pi^- + \pi^0)} \approx \frac{3}{2}$$

$$\text{(c)} \quad \frac{W(K_2^0 \to \pi^+ + \pi^- + \pi^0)}{W(K^+ \to 3\pi)} \approx \frac{2}{5}$$

The factor (1.3) on the right hand side of (a) arises from the phase-space difference between τ^+ and $\tau^{+\prime}$-modes. Similar factors have not been put into (b) and (c). The features (a) and (b) follow for any linear combination of $|\Delta I| = \frac{1}{2}$ and 3/2-transitions. However (c) tests[136] the validity of the $|\Delta I| = \frac{1}{2}$-rule. Out of these three features, present experiments (See Table III) do not bear any information on (b). However, for (a) the observed[77] ratio 0.32 ± 0.05 agrees well with the theoretical value 0.325. The single event of $K_2^0 \to \pi^+ + \pi^- + \pi^0$ observed by Crawford et al.[76] is also consistent with the expected ratio (c).

Since the matrix-element of the pure $|\Delta I| = \frac{1}{2}$-diagram (fig. 23) is [See eq. (127)] considerably bigger in absolute magnitude than the other class of diagrams (fig. 22), which contains both

---

136 R.H. Dalitz - Rev. Mod. Phys. 31, 823(1959); A. Pais & S.B. Treiman - Phys. Rev. 106, 1106(1957); Okubo, Marshak & Sudarshan - Phys. Rev. Lett. 2, 12(1959); M. Gell-Mann & A.H. Rosenfeld - Ann. Rev. Nuclear Science 1, 407(1957).

77 Birge et al. - Nuovo Cimento 6, 478 (1957)

76 Crawford et al. - Phys. Rev. Lett. 2, 266(1959)

$|\Delta \underset{\sim}{I}| = \frac{1}{2}$ and 3/2-parts, it is clear that the features (a), (b) and (c) are explained quite well in the present scheme.

Comparison of K^{+}_{e3} and $K^{o}_{1} \rightarrow \pi^{+} + \pi^{-}$ -Decay Rates

So far, we have not compared the rate of a leptonic mode with that of a non leptonic mode. For this purpose we will choose (for reasons which will become clear) a particularly suitable one, namely the rate of K^{+}_{e3} -mode compared to that of $K^{o}_{1} \rightarrow 2\pi$ -mode.

By eqs. (100) and (128), we have:

$$R_8 \equiv \frac{W(K^{+} \rightarrow \pi^{o} + e^{+} + \nu)}{W(K^{o}_{1} \rightarrow \pi^{+} + \pi^{-})} \approx 1.71 \times 10^{-4} \frac{|J_1|^2}{|\mathfrak{M}|^2} \tag{132}$$

By perturbation calculation [See Appendix III, Table - X]

$$J_1 \approx \frac{1}{4\pi^2 i}(0.91)$$

$$\mathfrak{M} \approx \frac{1}{4\pi^2 i}(0.86)$$

for cut off $\Lambda = 1.8$ mp. With these values,

$$R_8 \approx 1.91 \times 10^{-4} \tag{133}$$

Hence, by eq. (122')

$$R_9 \equiv \frac{W(K^{+} \rightarrow \pi^{o} + e^{+} + \nu)}{W(K^{o}_{1} \rightarrow 2\pi)} \approx 1.45 \times 10^{-4} \tag{134}$$

By applying the $I = \frac{1}{2}$ -current-rule[137] (so that, $W(K^+ \to \pi^0 + e^+ + \nu)$ $= \frac{1}{2} W(K^0 \to \pi^- + e^+ + \nu)$), we have:

$$R_{10} \equiv \frac{W(K^0 \to \pi^- + e^+ + \nu)}{W(K_1^0 \to 2\pi)} \approx 2.9 \times 10^{-4} \tag{135}$$

This may be compared with the observed[138] value

$$R_{10}\Big|_{\text{observed}} \approx 6 \times 10^{-4}$$

It is pertinent to remark that, both K^+_{e3} and $K_1^0 \to 2\pi$ -matrix elements are cut off dependent in nearly the same manner (logarithmic divergence) and in the former only $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-transition occurs, while in the latter the contribution of the $|\Delta \underset{\sim}{I}| = 3/2$ part (coming from fig. 20) can be neglected compared to the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ part (coming from figs. 20 and 21); therefore one does not have to worry about the ambiguity in the suppression of the $|\Delta \underset{\sim}{I}| = 3/2$-part by higher order corrections so that the comparison of the calculated rates of K^+_{e3} and $K_1^0 \to 2\pi$-decays with experiments is particularly suitable for judging the validity of our approach.

---

137 Recent observation by Crawford et al. Phys. Rev. Lett. 2, 361 (1959) on the three-body leptonic decays of K_1^0 and K_2^0 is in agree- with the $I = \frac{1}{2}$-current rule. (See ch. I.E)

138 The observed value of R_{10} is obtained indirectly by noting the rate of K^+_{e3} -decay and applying the result of the $I = \frac{1}{2}$ -current rule. See for instance, S. Oneda.- Nuclear Physics 9, 476(1959)

The fair agreement[139] (within a factor ≈ 2) obtained with experiment in this case seems to justify our stand point.

---

[139] It may be noted that on the basis of previous work [See for instance S Oneda, Nuclear Phys. 9, 476 (1959)] in which only fig. 5 was included as the relevant mechanism for $\Lambda \to N + \pi$-decays, it was very hard to understand the observed value of R_{10}. The inclusion of fig. 13.1 however, serves to remove this difficulty.

C. The Asymmetry In The Energy Distribution of $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$ -Decay

It is known[140],[141] that the final state in $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$ - decay is predominantly symmetric between the three pions with a small mixture of nonsymmetric states. We shall briefly show that the proposed new mechnism of non leptonic decays (inclusion of fig. 13.1) makes it easier to explain this fact.

By just looking at the structures of the "lowest" - order Feynman diagrams shown in figs. 22 and 23 for $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$ - decay, we may expect that fig. 23 (involving the new mechanism) would tend to favour more totally symmetric states than fig. 22 (involving the usual mechanism). This is explicitly borne out by the results of the perturbation theoretic calculations in which the leading term in the matrix-element of fig. 23 [See eqs. (124) and (A - 84)] is independent of the pion energies, while the leading term in the matrix-element of fig. 22 [See eqs. (123) and (A-84) involves the invariant scalar products $\mathbb{P}_1 \cdot \mathbb{P}_3$, $\mathbb{P}_2 \cdot \mathbb{P}_3$ and $\mathbb{P}_1 \cdot \mathbb{P}_2$, which give rise to non symmetric states. We shall now estimate the magnitude of the non symmetric states comeing from the leading term of the matrix-element of fig. 22.

Following Dalitz[140] we choose as two independent variables;

---

140 R.H. Dalitz - Phil. Mag. <u>44</u>, 1068 (1953) and Proc. Phys. Soc. A <u>69</u>, 527 (1956)

141 M. Gell-Mann and A.H. Rosenfeld - Ann. Rev. Nuclear Science <u>7</u>, 407 (1957)

$$x \equiv \frac{\sqrt{3}\,(T_1 - T_3)}{Q}$$

$$y \equiv \frac{(3T_2 - Q)}{Q} \tag{136}$$

wher T_1 and T_3 denote the kinetic energies of the two π^+ and T_3 the kinetic energy of the π^-, while Q denotes the Q value in the τ^+-decay ($Q \approx$ 75.11 Mev), so that

$$Q = T_1 + T_2 + T_3 \tag{137}$$

By eq. (123) the matrix element of fig. 22 is proportional to:

$$M(K^+ \to \pi^+ + \pi^- + \pi^+) \propto \left[2N_1 \mathbb{P}_1 \cdot \mathbb{P}_3 + N_2 \mathbb{P}_2 \cdot (\mathbb{P}_1 + \mathbb{P}_3) + N_3 (\mathbb{P}_1^2 + \mathbb{P}_3^2)\right]$$

$$= \left[N_1 (m_K^2 - 2m_K E_\pi - m_\pi^2) + N_2 (m_K E_\pi - m_\pi^2) + 2N_3 m_\pi^2\right]$$

$$= \left\{ m_K^2 N_1 + m_\pi^2 (2N_3 - N_1 - N_2) + m_K (m_\pi + Q/3)(N_2 - 2N_1) \right\}$$

$$[1 + Ay] \tag{138}$$

where

$$A = \frac{(N_2 - 2N_1) m_K Q/3}{m_K^2 N_1 + m_\pi^2 (2N_3 - N_1 - N_2) + m_K (m_\pi + Q/3)(N_2 - 2N_1)} \tag{139}$$

Substituting the cut off independent values of N_1, N_2 and N_3 given by perturbation theory [See Appendix III, Table - $\overline{\underline{X}}$], we obtain

$$A \approx -1/3.77 \tag{140}$$

Clearly the term 1 in eq. (138) leads to a final state, which is totally symmetric between the three pions, while the A-term does not. A good fit to the data on τ^+-decay, obtained from the Dalitz plot [(140),(141)] is given by

$$A \sim \frac{1}{10} \qquad (141)$$

Thus fig. 22 leads to non symmetric states, whose intensity is an order of magnitude larger [(142)] than the observed value and gives a wrong sign of A. However, as discussed in section B, the contribution of fig. 23 for τ^+-decay is considerably larger than that of fig. 22 [See eq. (127)], and the leading term of the matrix element of fig. 23, being independent of the pion energies, leads to totally symmetric states.

The nonsymmetric terms arise from the second order terms (in powers of $1/m_p$ or $1/m_n$) of the matrix element of fig. 23 and the first order terms of the matrix elements of diagrams other than fig. 23 (one of the typical ones being fig. 22). Although, a reliable estimate of the above mentioned contributions to the non symmetric term is hard to make, these together with the leading term of the matrix element of fig. 23 and some influence of the final state pion-pion interactions in an appropriate [(143)] state in τ^+-decay

---

142
This may be regarded as an evidence against taking diagrams of the type of fig. 5 (which enters into fig. 22) as the main mechanism of the nonleptonic-modes of strange-particle decays.

143
We do not mean the same mechanism of final state π-π interaction as was suggested by Good and Holladay Phys. Rev. Lett. 4, 138(1960) so as to explain the observed value of the ratio $W(K^+ \to 2\pi)/W(K_1^0 \to 2\pi)$;

may be expected to yield the observed asymmetry in τ^{+}-decay. An attempt to explain the energy spectra of the pions in τ^{+}-decay on the basis of final state interactions alone has been made by Thomas and Holladay[144].

---

[144] B.S. Thomas and W.G. Holladay - Phys. Rev. 115, 1329 (1959)

Table IA*

Compilation of the Main Results of Chapter III

	Experimental	No damping $\mathcal{D}^2 = 1$	$\mathcal{D}^2 = 1/3$	$\mathcal{D}^2 = \frac{1}{3}$ and suppression of $\lvert\Delta I\rvert = 3/2$-part
$R_1 \equiv \frac{W(K^+ \to \pi^0 + e^+ + \nu)}{W(K^+ \to \mu^+ + \nu)}$	$\frac{1}{15}$	$\frac{1}{6.5}$	$\frac{1}{19.5}$	$\frac{1}{19.5}$
$R_2 \equiv \frac{W(K^+ \to \pi^0 + \mu^+ + \nu)}{W(K^+ \to \pi^0 + e^+ + \nu)}$	1	$\frac{1}{1.62}$	$\frac{1}{1.62}$	$\frac{1}{1.62}$
$R_3 \equiv \frac{W(K^+ \to \pi^+ + \pi^- + e^+ + \nu)}{W(K^+ \to \pi^0 + e^+ + \nu)}$	?	7.26×10^{-4}	2.42×10^{-4}	2.42×10^{-4}
$R_5 \equiv \frac{W(K^+ \to \pi^+ + \pi^0)}{W(K_1^0 \to \pi^+ + \pi^-)}$	$\frac{1}{300}$	$\frac{1}{210}$	$\frac{1}{70}$	$\frac{1}{250}$
$R_6 \equiv \frac{W(K_1^0 \to 2\pi^0)}{W(K_1^0 \to 2\pi)}$	0.27 ± 0.11	0.24	0.24	0.27
$R_7 \equiv \frac{W(K^+ \to \pi^+ + \pi^+ + \pi^-)}{W(K_1^0 \to 2\pi)}$	$\frac{1}{1500}$	$\frac{1}{106}$	$\frac{1}{320}$	$\frac{1}{320}$
$R_{10} \equiv \frac{W(K^0 \to \pi^- + e^+ + \nu)}{W(K_1^0 \to 2\pi)}$	6×10^{-4}	0.96×10^{-4}	2.9×10^{-4}	2.9×10^{-4}

* The theoretical results in the last three columns are based on perturbation theory calculations with cut off $\Lambda = 1.8$ mp. $\mathcal{D}$ denotes the damping of the matrix element for each pion emission from a baryon antibaryon loop. In the last column, we choose the suppression of $\lvert\Delta I\rvert = 3/2$-part in the matrix element of diagrams involving fig. 5, so as to obtain $R_5 \approx \frac{1}{250}$, in spite of taking $\mathcal{D}^2 = \frac{1}{3}$

D. Concluding Remarks

The discussion in the present chapter is based larely on perturbation theoretic calculations. However, we feel that the essential qualitative features of the present discussion involving <u>ratios</u> of the decay rates of various processesses are treated reasonably by such calculations. If we admit this point of view, the present investigation of K-meson decay brings out the following points:

The various relative rates of K^{+} and $K^{0}_{1,2}$ -decay modes seem to be explained (Table - IX) reasonably well in the scheme of Λ-decay developed in chapter II. The different features of the $|\Delta I| = \frac{1}{2}$-rule in K-meson decays can also be understood in the present scheme rather successfully. These therefore may be regarded as some of the attractive features of the tetrahedron scheme(145) of four fermion interactions. Furthermore from the nature of results obtained here, it may be remarked that the final state pion-pion interactions in S-state, suggested by others(146) need not be very important in order to explain the observed decay rates of $K^{+} \rightarrow 2\pi$ and $K^{0}_{1} \rightarrow 2\pi$.

145

If the tetrahedron scheme turns out to be the true feature of weak interactions, it may lend evidence in favor of the Sakata model of elementary particles. However, there could be some experimental checks. For instance, as mentioned before, any evidence for the presence of $|\Delta S/\Delta Q| = -1$ -currents, like the occurrence of $\Sigma^{+} \rightarrow n + e^{+} + \nu$-decay etc. will be rather fatal to Sakata model.

146

Okubo & Marshak (Phys. Rev. 100, 1809, 1955), starting from an interaction containing appreciable amount of $|\Delta I| = 3/2$ -part suggested the possibility of a strong final state π-π- interaction in T = 0, J = 0-state (leading to an enhancement by a factor $\approx$ 100 in the rate of $K^{0}_{1} \rightarrow 2\pi$ - decay) in order to explain the observed value of the ratio $W(K^{+} \rightarrow 2\pi)/W(K^{0}_{1} \rightarrow 2\pi)$. Recently Good and Holladay,(143) starting from a strict $|\Delta I| = \frac{1}{2}$-weak interaction have indicated the possible explanation of the above ratio by electromagnetic corrections and strong final state π-π interaction in T = 2, J = 0 state.

The necessity of damping the matrix element by a factor $\approx\sqrt{3}$ for emission of each pion from a closed baryon-anti baryon loop (Sec. A and B to correlate the rates of processes involving different number of pion emissions also seems to be rather suggestive. The fact that this damping is roughly consistent with the observed rate of $\pi \to \mu + \nu$ -decay indicates the plausibility of the assumption. In addition, it is rather striking that one gets reasonable agreement with experiment in particular for the ratio $W(K^+ \to \pi^0 + e^+ + \nu)/W(K_1^0 \to 2\pi)$ and for the reasons mentioned at the end of Sec. B, this could be taken as an indication of the validity of the gross features of our approach. Finally, we may conclude that the minor discrepancies (within a factor of 5, See Table IX) for some of the ratios of decay rates, obtained in the present attempt may be explained by the slight effect of the final state π -π -interaction and the dependence of the damping factor $\mathfrak{D}$ of the matrix-elements on the energy release in the corresponding processes.

TN-60-1051b - Vol II

STRANGE PARTICLE DECAYS AND THE NATURE OF WEAK INTERACTIONS

VOLUME II*

by

Jogesh C. Pati

University of Maryland, College Park, Maryland

Physics Department Technical Report No. 193

September, 1960

UNIVERSITY OF MARYLAND PHYSICS DEPARTMENT

COLLEGE PARK, MARYLAND

TN-60-1051b - Vol II

STRANGE PARTICLE DECAYS AND THE NATURE OF WEAK INTERACTIONS

VOLUME II*

by

Jogesh C. Pati

University of Maryland, College Park, Maryland

Physics Department Technical Report No. 193

September, 1960

This is the second volume containing chapters IV and the appendices (Pages 142-281). The first volume contains the first three chapters (Pages 1-141).

* This research was supported by the United States Air Force through the Air Force Office of Scientific Research and Development Command under Contract AF49(638)-24. Reproduction in whole or part is permitted for any purpose of the United State Government.

TABLE OF CONTENTS

Chapter				Page
ACKNOWLEDGEMENT				ii
List of Figures				vi
List of Tables				ix
I.	INTRODUCTION			1
	A.	Nature of Interactions of Elementary Particles		1
	B.	Experimental Foundation of Universal Four Fermion Interactions Between Nucleons and Leptons		5
		1.	Preliminary Discussion	5
		2.	Experiments on Invariance Properties in Weak Interactions	8
		3.	Experiments on Nature of β -Decay Interactions	10
		4.	Experiments on Nature of μ -Decay and μ -Capture Interactions	13
		5.	Experiments on the Nature of Neutrino	15
		6.	Experiments on Nature of Weak Interaction Constants	20
		7.	Conclusion	22
	C.	Theoretical Frame-Work of Universal Four Fermion Interaction		23
	D.	The Strange Particles		28
	E.	General Features of Strange Particle Decays		32
		1.	The $\lvert \Delta S \rvert = 1$ - Rule	36
		2.	The $\lvert \Delta \underset{\sim}{I} \rvert = \frac{1}{2}$ - Rule	38
		3.	The Slowness of Leptonic Modes	41

F. A List of Problems 45

II. THE DECAY OF THE Λ -HYPERON 47

Abstract 47

A. Scheme of Interaction 48

B. Preliminaries on Λ -Decay 54

C. Previous Work (The OMS-Analysis) and Successes 63

D. Needs for New Look 77

E. A New Class of Diagrams 79

F. Evaluation of the Matrix Element of the New Diagram 94

G. Conclusion 104

III. THE DECAY OF THE K-MESON 106

Abstract 106

A. The Leptonic Decay Modes of K-Meson 107

B. The Non Leptonic Decay Modes of K-Meson 118

C. The Asymmetry in the Energy Distribution of $K^+ \rightarrow \pi^+ + \pi^- + \pi^+$ -Decay 135

D. Concluding Remarks 140

IV. THE POSSIBLE EXISTENCE OF AN INTERMEDIATE VECTOR BOSON 142

Abstract 142

A. Speculation 144

B. General Effects of the Nonlocality in Four Fermion Interactions 151

C. Absence of $\mu \rightarrow e + \gamma$ -Decay 161

D. The Assignment of Lepton Numbers and the Nature of the Neutrino 165

E. Effects of Non-Locality on the Energy Spectra in K_{e3} ($K_{\mu 3}$) -Decays........ 178

F. Effects of Non-Locality on the Decay of the Λ -Hyperon.................... 188

APPENDIX

I. NOTATIONS.. 196

II. THE ANGULAR DISTRIBUTION OF PIONS AND LONGITUDINAL POLARISATION OF NUCLEON IN THE DECAY OF Λ -HYPERON...................... 199

III. MATRIX ELEMENTS OF VARIOUS K-MESON-DECAY MODES...................................... 202

IV. DECAY RATES OF VARIOUS K-MESON-MODES.............. 233

V. ENERGY SPECTRA IN $K_{\mu 3}$ AND K_{e3} -DECAYS FOR LOCAL AND NON LOCAL FOUR FERMION INTERACTION.................................. 262

VI. MATRIX ELEMENT OF THE SINGLE NEUTRON-INTERMEDIATE STATE DIAGRAM FOR $\Lambda \rightarrow N+\pi$ -DECAY WITH INTERMEDIATE VECTOR BOSON......................... 272

SELECTED BIBLIOGRAPHY..................................... 279

CHAPTER IV

THE POSSIBLE EXISTENCE OF AN INTERMEDIATE VECTOR BOSON

Abstract

In this chapter the possibility that the four-fermion interaction is mediated by a charged vector boson is examined. It is pointed out that the immediate objection to such a possibility (due to slowness of the $\mu \rightarrow e + \gamma$-decay) can be removed by assigning opposite lepton numbers to μ^- and e^- and adopting a "restricted" four-component theory of the neutrino. It is also pointed out that the effects of the non-locality (though small, if the mass of the vector-boson is greater than that of K-meson) on the michel parameter and life-time of μ-decay, are consistent with present experiments; though more accurate experiments are desirable. The energy spectra of e, μ and the pion in K_{e3} and $K_{\mu 3}$-decays are calculated for both local and non-local interactions with the assumption that the form-factors are nearly constant over the range of variation in the pion-energy. It is pointed out that an accurate measurement of the pionenergy-spectrum in these decays could serve to distinguish between local and non-local interactions.

The effect of non-locality is also investigated in $\Lambda \rightarrow N + \pi$-decays. The contribution to the matrix-element of a class of diagrams (whose local limit has been discussed in chapter II) satisfying the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule is estimated, with the help of plausible assumptions. It is found, as in the case of local-interactions,

these diagrams are much more important than the usually considered diagram, which contains appreciable amount of $|\Delta I| = 3/2$-transitions in addition to $|\Delta I| = \frac{1}{2}$-ones. However, if one assumes the universal V-A form of interactions, one can not explain the observed sign of the asymmetry-parameter in $\Lambda \rightarrow p + \pi^-$-decay for reasonably finite values of the mass of the intermediate boson (less than two times the mass of the proton, say). This may be regarded as evidence against the charged vector meson. However, the sign of the asymmetry parameter could be explained if one adopts V+A-interaction for Λ-decay.

A. Speculation

The discussion of the various weak processes has so far been based on a local (contact) V-A four-fermion interaction, which presumes that all the four fermions interact at the same space time point. At this stage we may compare the weak interactions with the two other familiar elementary particle-interactions (leaving aside gravitational forces), i.e. (i) Electromagnetic and (ii) meson-baryon interactions. The first one is well understood in terms of an interaction density of the form $e\bar{\Psi}(x)\gamma_\mu\Psi(x)A_\mu(x)$ which involves two fermion fields and one boson field interacting at the same space-time point (Yukawa type). The second one also is presumably of Yukawa type having the form $\bar{B}'(x)\,\mathcal{O}\,B(x)\begin{Bmatrix}\pi(x)\\ K(x)\end{Bmatrix}$. It is natural to ask, whether all "fundamental" interactions occurring in nature are in fact of the Yukawa-type. In particular, one is then tempted to conjecture that the four-fermion interactions may not be primary. They may be arising as second order processes out of a primary interaction, which is Yukawa-type. This will involve interactions between two fermions and one boson, not yet found. To carry vector and axial-vector interactions this unobserved boson must have spin 1. Hence its field-operator $\mathbb{B}_\mu$ should transform as a 4-vector in space-time.

There exists another attractive reason for assuming[147] the

---

147 The possibility that the weak-interactions are mediated by a boson has been considered by many authors, starting with H. Yukawa, Proc. Phys.- Math. Soc., Japan 17, 48(1935); J. Schwinger; Ann. Phys. 2, 407(1957), T.D. Lee & C.N. Yang Phys. Rev. 108, 1611(1957); R.P. Feynman & M. Gell-Mann Phys. Rev. 109, 193(1958); Y. Tanikawa, Prog. Theo. Phys. Japan 3, 338(1948); Y. Tanikawa & S. Watanabe Phys. Rev. 113, 1344(1959)

existence of such an intermediate boson. This has to do with the absence of processes such as $\mu \to e+e+e, K \to e+e, \mu+\mu$ etc., which suggests that the weak-interaction current J_α is presumably composed of charged currents only[148]. If so, this could have its' origin in the fact that the weak-interaction current J_α interacts with itself through the intermediary of a vector boson (just as the electromagnetic current j_α interacts with itself through the photon), which is charged, and hence J_α is necessarily composed of charged currents only. The non-locality will therefore extend over the Compton wavelength of the vector boson.

The present chapter will be concerned with some of the consequences of such an intermediary for four fermion interactions.

Denoting the vector-boson-field by B_α, the primary weak-interaction leading to the observed weak-processes is then assumed to be:

$$H_{weak} = J_\alpha B_\alpha + \text{H.C.} \tag{142}$$

where

$$J_\alpha = g\,[\bar{\nu}\gamma_\alpha(1+i\gamma_5)e + \bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu + \bar{p}\gamma_\alpha(1+i\gamma_5)n + g'/g\;\bar{p}\gamma_\alpha(1+i\gamma_5)\Lambda] \tag{143}$$

we have written here g and g' instead of f and f' (compare eqs. (15) and (16)), since it will be shown that they are related by:

148 As mentioned before the absence of such processes specifically imply absence of neutral leptonic currents only. There is no compelling reason, however, to exclude neutral baryonic currents.

$$\left.\begin{aligned} g^2/m_B^2 &\approx f^2 = f_F/\sqrt{2} \\ gg'/m_B^2 &\approx ff' \end{aligned}\right\} \qquad (144)$$

where m_B denotes the mass of the vector-boson, and f_F is the usual fermi coupling constant ($f_F m_p^2 \approx 10^{-5}$). Thus processes which involve B-meson coupling only <u>once</u> can be called '<u>Semi Weak</u>'. Their transition probabilities (subject to energy-momentum conservation) are faster than the usual weak processes, but slower than the strong processes. While the usual weak-processes are nearly 10^{-12} times slower than the strong processes, the semi weak-processes are expected to be nearly 10^{-6} times slower than the strong ones.

It is easy to check that the observed mass spectrum of elementary particles require,

$$m_B > m_K \qquad (145)$$

If m_B were less than m_k, then K-meson will immediately decay into

$$\begin{aligned} K^+ &\rightarrow B^+ + \gamma \\ &\rightarrow B^+ + \pi^0 \qquad \text{(etc.)} \end{aligned}$$

within a period much shorter than the observed lifetime of K^+. It is further easy to check that condition (145) also simultaneously guarantees the observed mass spectrum of the particles heavier than the K-meson due to baryon number and energy-momentum conservation.

For instance, by (145) the following decays are then forbidden by energy-momentum conservation:

$$p \rightarrow B^+ + \text{(Any system with charge zero and baryon number 1.)}$$

$$\Lambda \rightarrow B^- + p$$

$$\Sigma^- \rightarrow \Lambda + B^-$$

etc.

Production of $B^{\pm}$ - Mesons

The $B^{\pm}$-mesons can be produced by a variety of reactions, such as:

(1) $\nu + Z \rightarrow \mu^-(e^-) + B^+ + Z^*$

(2) $n + Z \rightarrow p + B^- + Z^*$

(3) $\pi^+ + p \rightarrow B^+ + p$

(4) $\gamma + Z \rightarrow B^+ + B^- + Z^*$ etc.

where Z denotes a nucleus with charge Z; Z^* an excited state. All of the above reactions are allowed for incident particle-energies above the respective thresholds. It has been pointed out by Lee and Yang(149) that high energy neutrinos ($E_\nu \gg 2$ Bev) are expected to yield cross sections of the order of 10^{-35} cm^2 (for Z = 26) for the process (1). Thus such experiments may be feasible in the near future.

149 T.D. Lee and C.N. Yang Phys. Rev. Lett. 4, 307 (1960)

Nucleon-Nucleon (process (2)) and pion-nucleon collisions (process (3)) are also expected to lead to production of $B^{\pm}$. However, since they have to compete against the possible strong processes, they are expected to occur only at a rate of one in about a million interactions.

The fourth process involves pair production of $B^{\pm}$, when a nucleus is present to absorb the recoil momentum. The cross section for such pair production above threshold will be roughly $(m_e/m_B)^2$ times that of $e^{\pm}$-pair production. The subsequent rapid decay of $B^{\pm}$ to $(\mu+\nu)$ and $(e+\nu)$ might lead to detection of the $\mu^{\pm}$ and $e^{\pm}$ or (μ, e)-pair.

Decays of $B^{\pm}$ - Mesons

The B-meson can decay in a variety of ways:

$$B^+ \longrightarrow \begin{cases} 1.\ e^+ + \nu \\ 2.\ \mu^+ + \nu \\ 3.\ \pi^+ + \pi^0 \\ 4.\ \pi^0 + e^+ + \nu \\ 5.\ \pi^0 + \mu^+ + \nu \\ 6.\ K^+ + \pi^0 \\ 7.\ K^+ + \gamma \end{cases} \qquad \text{etc.}$$

The sixth mode is allowed if ,

$$m_B > m_{K^+} + m_{\pi^0}$$

All the other modes, however, are allowed by energy-momentum conservation if

$$m_B \gtrsim m_K$$

The rate for $B \to \mu(e) + \nu$-decay is given by[150]:

$$\frac{1}{\tau(B\to\mu+\nu)} = \frac{2g^2}{4\pi}\left(\frac{m_B}{2}\right)\left(1-m_\mu^2/m_B^2\right)^2\left[1-\tfrac{1}{3}\left(1-m_\mu^2/m_B^2\right)\right]$$

$$= \frac{f_F m_B^3}{\sqrt{2}\cdot 4\pi}\left(1-m_\mu^2/m_B^2\right)^2\left[1-\tfrac{1}{3}\left(1-m_\mu^2/m_B^2\right)\right] \tag{146}$$

For $m_B \gtrsim m_K$;

$$\frac{1}{\tau(B\to e+\nu)} \sim \frac{1}{\tau(B\to\mu+\nu)} \gtrsim 8\times 10^{16}\ \mathrm{sec}^{-1}$$

or,

$$\boxed{\tau(B) \lesssim 1.25\times 10^{-17}\ \mathrm{sec.}} \tag{147}$$

Hence its lifetime is too short to observe it directly by its track in emulsion (for instance). However, as pointed out by Lee and Yang[151]; one can infer their existence by processes, such as apparent lepton production or K-meson production in π-N-collisions, i.e.

$$\pi^+ + p \to B^+ + p \to \mu^+(e^+) + \nu + p$$
$$\to K^+ + \pi^0 + p \qquad \text{etc.}$$

---

150 S. Oneda and A. Wakasa, Nuclear Physics **1**, 445(1956)

151 T.D. Lee and C.N. Yang "Implications of the Intermediate Boson Basis of the Weak Interactions. (Preprint) - Phys. Rev. (To be published)

Above the threshold for $B^{\pm}$ production, these processes should occur with a probability $\sim 10^{-6}$ times that of the strong processes, while their probabilities are $\sim 10^{-12}$ times that of the strong processes, if the weak interactions are local. Hence, detection[152] of such processes may throw some light on the existence of the B-meson.

---

[152] For a fuller discussion of the feasibility of detection of such processes, See Ref. (151)

B. General Effects of the Non-Locality in Four Fermion Interactions

1. The β-decay of the Neutron

The lowest order Feynman diagrams for $n \to p + e^- + \bar{\nu}$ -decay with and without the intermediate boson are given by figs. 24 and 25 respectively.

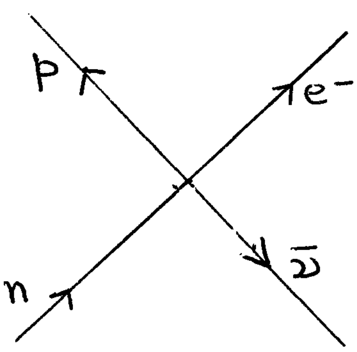


Fig. 24

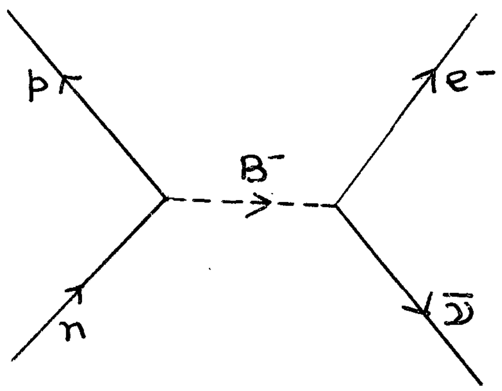


Fig. 25

The matrix elements of figs. 24 and 25, calculated from the local Fermi-interaction (eq.(15)) and the Yukawa type interaction (eq. (142)) respectively are given by:

$$M(\text{fig.-}24) = (2\pi)^4 \delta^4(p_n - p_p - p_e - p_{\bar{\nu}})(-i) f^2 (\bar{p}\gamma_\alpha(1+i\gamma_5)n)$$
$$(\bar{e}\gamma_\alpha(1+i\gamma_5)\nu) \tag{146}$$

$$M(\text{fig.-25}) = (2\pi)^4 \delta^4 (\mathbb{p}_n - \mathbb{p}_p - \mathbb{p}_e - \mathbb{p}_{\bar\nu}) (-i)^3 g^2 (\bar p \gamma_\alpha (1+i\gamma_5) n)$$

$$\frac{g_{\alpha\beta} - q_\alpha q_\beta / m_B^2}{q^2 - m_B^2} \; \bar e \gamma_\beta (1+i\gamma_5) \nu \tag{147}$$

where q denotes the momentum transfer at the (pn)-vertex, i.e.

$$q = \mathbb{p}_n - \mathbb{p}_p \tag{148}$$

Clearly, for β-decay q^2 is very small. In particular, since $m_B \gtrsim m_K$,

$$q^2 << m_B^2$$

Furthermore, if we neglect the mass of the electron, eq.(147) reduces to,

$$M(\text{fig.-25}) = (2\pi)^4 \delta^4 (\mathbb{p}_n - \mathbb{p}_p - \mathbb{p}_e - \mathbb{p}_{\bar\nu}) (-i)(g^2/m_B^2)$$

$$(\bar p \gamma_\alpha (1+i\gamma_5) n)(\bar e \gamma_\alpha (1+i\gamma_5) \nu) \tag{149}$$

Since β-decay experiments are consistent with putting

$$f^2 = \frac{f_F}{\sqrt{2}} \approx \frac{10^{-5} m_p^{-2}}{\sqrt{2}}$$

in eq. (146), by comparing eqs. (146) and (147) we must have; as mentioned before;

$$\boxed{\frac{g^2}{m_B^2} \approx f^2 = \frac{f_F}{\sqrt{2}}} \tag{150}$$

It is clear from eq. (147) that for processes like β-decay with low momentum transfers the non-locality in four-fermion interactions has very little effect.

2. The β-decay of Λ-Hyperon

The lowest order Feynman diagram for $\Lambda \rightarrow p + e^- + \bar{\nu}$ -decay involving the intermediate boson is shown in fig. 26.

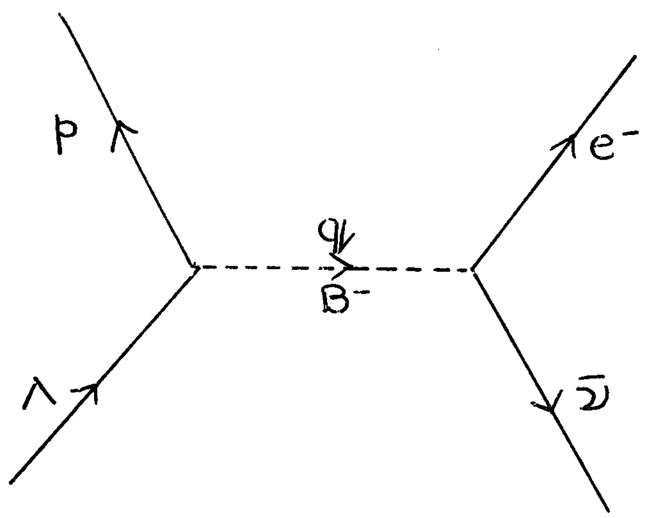


Fig. 26

The matrix element of fig. 26 is given by;

$$M(\text{fig-26}) = (2\pi)^4 \delta^4(p_\Lambda - p_p - p_{e^-} - p_{\bar{\nu}})(-i)^3 g g' \bar{p} \gamma_\alpha (1+i\gamma_5) \Lambda$$

$$\frac{g_{\alpha\beta} - q_\alpha q_\beta / m_B^2}{q^2 - m_B^2} \; \bar{e} \gamma_\beta (1+i\gamma_5) \nu \qquad (151)$$

If we neglect the mass of the electron, the $q_\alpha q_\beta$ -term in the boson propagator can be dropped. The $g_{\alpha\beta}$-term coincides with the matrix element for local four fermion interactions except for the factor,

$$\frac{-gg'}{q^2 - m_B^2}$$

instead of ff'. Replacing $\frac{gg'}{m_B^2}$ by ff' (analogous to eq. (150)), we have;

$$\frac{-gg'}{q^2 - m_B^2} = -\frac{gg'}{(p_\Lambda - p_p)^2 - m_B^2}$$

$$\leqslant \frac{gg'}{m_B^2}\,\frac{m_B^2}{m_B^2 - (m_\Lambda - m_p)^2}$$

$$\leqslant ff'\ (1.14) \qquad (\text{For } m_B \gtrsim m_K)$$

Hence for the β-decay of Λ-hyperon, there will be an additional factor $\approx$ 1.3 in the decay rate for non-local interaction as compared to the local interaction.

3. The $\pi \rightarrow \mu + \nu$ and $K \rightarrow \mu + \nu$-decays

The Feynman diagrams for $\pi \rightarrow \mu + \nu$-decay with and without the vector boson are shown in figs. 27 and 28 respectively:

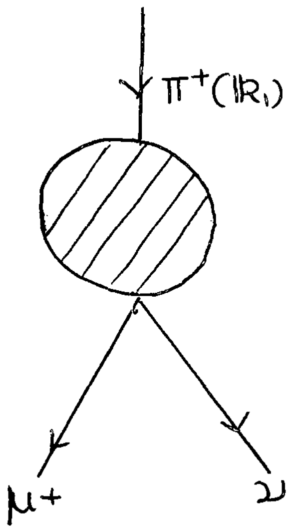


Fig. 27

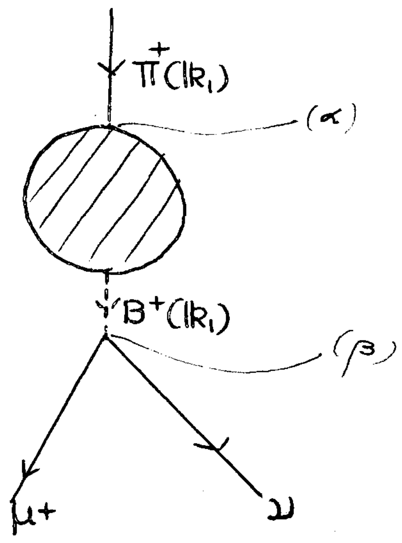


Fig. 28

The matrix elements for figs. 27 and 28 are given by:

$$M(\text{fig.-27}) = (2\pi)^4 \delta^4(k_1 - p_{\mu+} - p_\nu)\, f^2 F(k_1^2)\, k_{1\alpha}\, \bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu \tag{152}$$

$$M(\text{fig.-28}) = (2\pi)^4 \delta^4(k_1 - p_{\mu+} - p_\nu)(-g^2) F(k_1^2)\, k_{1\alpha} \frac{g_{\alpha\beta} - k_{1\alpha}k_{1\beta}/m_B^2}{k_1^2 - m_B^2}\, \bar{\nu}\gamma_\beta(1+i\gamma_5)\mu \tag{153}$$

where $F(k_1^2)$ is same scalar function of k_1^2 in both the matrix elements. Using $k_1^2 = m_\pi^2$, it is easy to check that eq. (153) reduces to;

$$M(\text{fig.-28}) = (2\pi)^4 \delta^4(k_1 - p_{\mu+} - p_\nu)(g^2/m_B^2)\, F(k_1^2)\, k_{1\alpha}\, \bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu \tag{154}$$

If we replace g^2/m_B^2 by f^2 (eq. (150)), there is no effect of non-locality in $\pi \to \mu + \nu$-decay . Same is the case for $K \to \mu + \nu$-decay.

4. The $\mu \to e + \nu + \bar{\nu}$ -decay

This decay has some favourable features to test the existence of intermediate boson indirectly, firstly because, the momentum transfer involved at the $(\mu\nu)$-vertex is rather big and secondly because the conclusions drawn theoretically are fairly unambiguous, since the $\mu \to e + \nu + \bar{\nu}$ -decay involves no strongly interacting particles. The lowest order Feynman diagram for the above decay is shown in fig. 29.

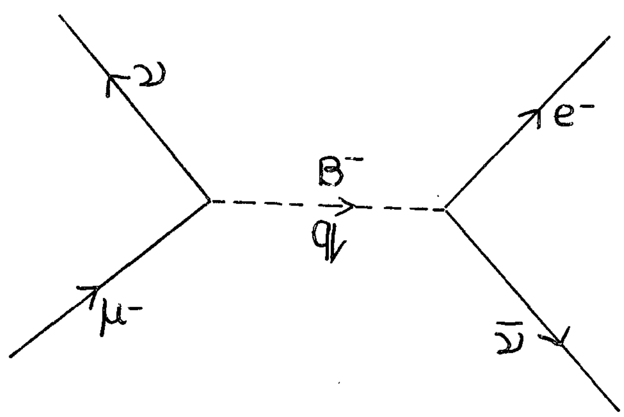


Fig. 29

The matrix element of fig. 29 is given by:

$$M(\text{fig.}\,29) = (2\pi)^4 \delta^4(p_{\mu^-} - p_e - p_\nu - p_{\bar{\nu}})\,(-i)^3 g^2\, \bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu \;\frac{g_{\alpha\beta} - q_\alpha q_\beta/m_B^2}{q^2 - m_B^2}\;\bar{e}\,\gamma_\beta(1+i\gamma_5)\nu \tag{155}$$

It is well known that for contact four-fermion interaction the electron-spectrum in μ -decay is characterised by a single parameter ρ, called the Michel parameter[153], and has the form,

$$W(p_e) \propto p_e^2 \left[(p_e^{max} - p_e) + 2/9 \, \rho \, (4p_e - 3p_e^{max}) \right] \tag{156}$$

In the framework of the two-component theory of the neutrino (See chapter I), it is easy to check that,

$$\begin{aligned} \rho &= 0 \quad ; && \text{If } (\nu+\nu) \text{ or } (\bar{\nu}+\bar{\nu}) \text{ are emitted with } e^- \\ &= 0.75 \, , && \text{If } (\bar{\nu}+\nu) \text{ are emitted with } e^- \end{aligned} \tag{157}$$

It has been pointed out by Lee and Yang[154] that the energy-spectrum of the electron will be altered if one adopts a non-local four-fermion interaction. It is no longer of Michel form. However, if it is approximated in the spirit of the least square fit, the effective Michel-parameter, obtained from eq. (155) is given by[154];

$$\rho \approx 0.75 + \frac{1}{3} (m_\mu / m_B)^2 \tag{158}$$

---

153 L. Michel - Proc. Phys. Soc. A63, 514 (and) 1371 (1950).

154 T.D. Lee and C.N. Yang - Phys. Rev. 108, 1611 (1957).

If we take $m_B \approx m_K$, eq. (158) gives

$$\rho \approx 0.765 \tag{159}$$

The recently observed value of ρ (corrected for electromagnetic corrections) is[155]:

$$\rho = 0.78 \pm 0.025 \tag{160}$$

which is not in contradiction with the value given by eq. (159).

Furthermore, the life-time computed from eq. (155), is given by[154]:

$$\tau = \tau_0 \left\{ 1 - 3/5 \, (m_\mu / m_B)^2 \right\} \tag{161}$$

where τ_0 is the life-time for contact fermi-interaction.

The theoretical value of τ_0 is obtained by assuming universal fermi interaction. If we obtain the usual fermi coupling constant f_F from the O^{14} -rate (after including the radiative corrections), then the theoretical value of the muon-life-time is given by[156]:

$$(\tau_0)_{\text{Theor.}} = (2.31 \pm 0.05) \times 10^{-6} \text{ Sec.} \tag{162}$$

155
R.J. Plano - Physical Review (To be published). For results of previous experiments, see report by A.I. Alikhanov - Kiev Conference, 1959.

156
This is the value given by Kinoshita and Sirlin (Phys. Rev. **113**, 1652(1959)), which includes radiative corrections in muon decay and is based on an O^{14} half-life of (72.5 ± 0.5) sec. and a β^+ end-point kinetic energy of 1810 ± 8 Kev. With a recently measured end-point energy of 1800 ± 6.5 Kev, the predicted muon life-time becomes $\tau_0 = (2.26 \pm 0.05) \times 10^{-6}$ sec. In this connection, see Durand, Landovitz and Marr - Phys. Rev. Lett. **4**, 620(1960).

The observed values of muon life-time are;

$$\tau_\mu = (2.22 \pm 0.02) \times 10^{-6} \text{ sec.} \qquad \text{(Ref. - 157)}$$

$$= (2.261 \pm 0.007) \times 10^{-6} \text{ sec.} \qquad \text{(Ref. - 158)}$$

$$= (2.20 \pm 0.015) \times 10^{-6} \text{ sec.} \qquad \text{(Ref. - 159)}$$

$$= (2.211 \pm 0.003) \times 10^{-6} \text{ sec.} \qquad \text{(Ref. - 160)}$$

$$(163)$$

If we accept the first, third or the fourth values of τ_μ, there seems to be a slight discrepancy between τ_μ (observed) and (τ_0) theoretical.

To obtain (τ_μ) theoretical in the frame work of vector-boson, one has to take into account the non-local effect in β-decay, as well as μ-decay with appropriate radiative corrections. It has been shown that the non-local effect in β-decay is completely negligible because of low momentum transfer. The non-local effect in muon-decay alters the life-time as given by eq. (161). However the inclusion of radiative corrections to muon-decay in the frame-work of the vector boson is rather complicated and has not yet been done.

---

[157] Bell and Hincks - Phys. Rev. 84, 1243(1951).

[158] Swanson et al. - Phys. Rev. Lett. 2, 430(1959).

[159] Fischer et al. - Phys. Rev. Lett. 3, 349(1959).

[160] Reiter et al. - Post dead line paper, Washington APS meeting, 1960.

If we neglect the change in the radiative corrections due to finiteness of m_B $(m_B \neq \infty)$, then by eq. (161) and (162) we obtain (for $m_B \approx m_K$);

$$(\tau_\mu)_{\text{Theor.}} = (2.25 \pm 0.05) \times 10^{-6} \text{ sec.} \tag{164}$$

which is in the right direction to maintain the universality of the coupling constant. It is to be noted that if we accept the predicted value $\tau_0 = (2.26 \pm 0.05) \times 10^{-6}$ sec. based on recent data (See foot note - 156), then the inclusion of non-local effect yields (by eq. (161)) (τ_μ) theor. $= (2.20 \pm 0.05) \times 10^{-6}$ sec. which is in excellent agreement with the first, third and fourth values listed in eq. (163). It is, of course, necessary to treat the radiative corrections correctly (without assuming $m_B \rightarrow \infty$) in order to get more precise information.

Thus, it may be concluded that the observed values of the Michel-parameter and the muon life-time are consistent with the hypothesis of an intermediate vector boson. More refined experiments are certainly desirable to draw somewhat more definite conclusions.

C. Absence of $\mu \to e+\gamma$ -Decay

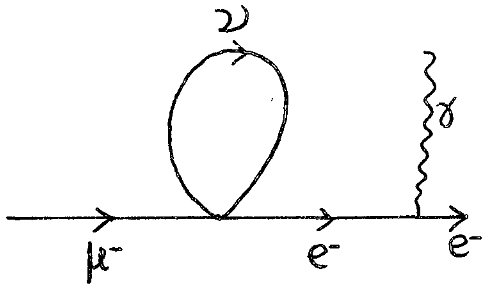


Fig. 30

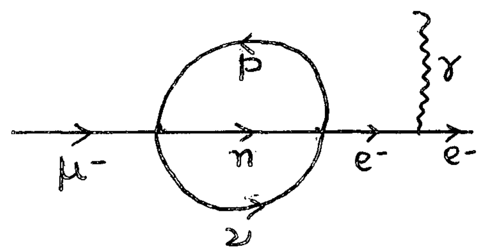


Fig. 31

For contact four-fermion interactions, the $\mu \to e+\gamma$-decay is forbidden[161] in the first order of weak interactions, since fig. 30 has a vanishing matrix element. The lowest order diagram, which has non-vanishing matrix element, involves two weak vertices, as in fig. 31. This is consistent with the observed[162] branching ratio of $\mu \to e+\gamma$ -decay;

$$R \equiv \frac{W(\mu \to e+\gamma)}{W(\mu \to e+\nu+\bar{\nu})} < 2\times 10^{-6} \tag{165}$$

161
We assume the principle of minimal electromagnetic interaction, which forbids the direct μ-e-γ -interaction. For a general theorem on the elimination of contact muon-electron interactions, See Cabibbo, Gatto and Zemach - Nuovo Cim. 16, 168(1959). See also Cabibbo and Gatto - Phys. Rev. 116, 1134(1959) and Feinberg, Kabir and Weinberg - Phys. Rev. Lett. 3, 527(1959).

162
Berley et al. - Phys. Rev. Lett. 2, 357(1959).

However, the situation is altered, when one presumes that the weak interactions involve an intermediate boson. The relevant lowest order Feynman diagrams for $\mu \to e + \gamma$ -decay are then given by figs. 32 (a), 32 (b), and 32 (c).

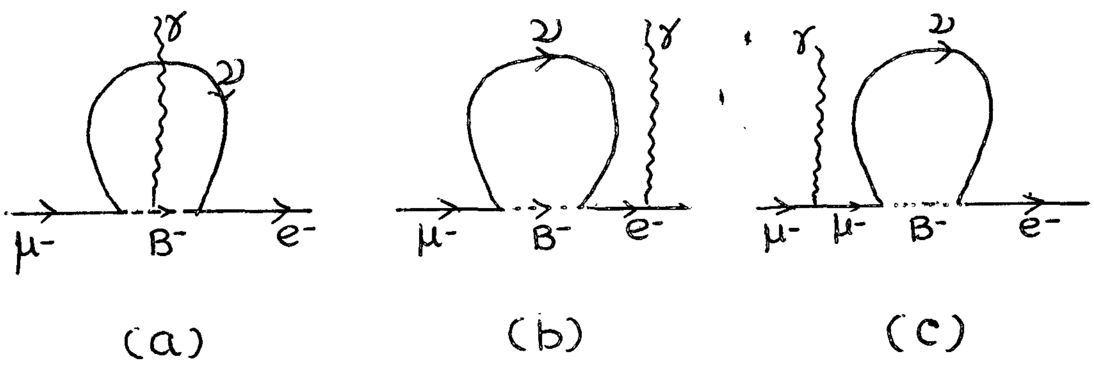


Fig. 32

Each of figures (a), (b) and (c), shown above, has a non-vanishing matrix element. These diagrams, however are of second order in the interaction $J_\alpha B_\alpha$ (eq.(142)) and hence first order in the usual weak-interaction $J^*_\mu J_\mu$ (eq.(15)). Therefore they lead to a rather large branching ratio for $\mu \to e + \gamma$-decay in contradicrion with experiment. The amplitude, calculated[163] from figs. (a), (b) and (c) is logarithmically divergent. The branching ratio R turns out to be a function of the ratio of the cut-off Λ and the intermediate boson mass m_B. For a choice of $\Lambda = m_B$, one obtains,

$$R \approx 10^{-4} \tag{166}$$

which is a hundred times larger than the experimental upper limit.

Meyer and Salzman[163] show that, one can not explain the experimental upper limit of R, unless one takes a ratio $\Lambda/m_B \lesssim \frac{1}{5}$. This is a rather unreasonable choice, unless the B-meson is extremely heavy.

Although, the results are not completely conclusive, being dependent on a choice of the cut off, it may be concluded that in the lowest order perturbation theory, the experimental upper limit of R is in contradiction with the "Conventional " vector boson theory (for reasonable choice of the cut-off), unless[164] the vector boson is extremely massive.

This may be taken as evidence against the vector boson hypothesis, if we believe the calculations. However, it is now pertinent to ask, is it possible that the vector boson -hypothesis is true, but the $\mu \to e + \gamma$ -decay is still forbidden by some <u>selection rule</u>?

The only selection rule, that could be pertinent in this case is the lepton number conservation. If μ^- and e^- possess opposite lepton numbers, the $\mu \to e + \gamma$ -decay is strictly forbidden in the scheme of either local or non-local interactions, if we require conservation of lepton number.

163
G. Feinberg - Phys. Rev. 110, 1482(1958).
P. Meyer and G. Salzman - Nuovo Cim. 14, 1310(1959).

164
An alternate possibility, rather unlikely, is to assume an anomalous magnetic moment of the vector boson, which serves to explain the low upper limit of R for relatively small values of m_B. See M.E. Ebel and F.J. Ernst - Nuovo Cimento 15, 173(1960).

The assignment of opposite lepton number to μ^- and e^-, however, leads to restriction on the nature of the neutrino. In particular, one has to abandon the conventional two-component theory of the neutrino in order to explain the facts, observed so far. This will be the subject of discussion in the next section.

D. The Assignment of Lepton Numbers and the Nature of the Neutrino

In the "usual" scheme, one adopts the assignment that e^-, μ^- and ν are leptons, while e^+, μ^+ and $\bar{\nu}$ are anti-leptons. By definition, the symbol ν denotes the neutral massless spin - $\frac{1}{2}$ particle emitted in the β^+ -decay of a nucleus. We call it "neutrino". Hence the anti-neutrino, denoted by $\bar{\nu}$, is the corresponding particle accompanying e^- in the β^- -decay of a nucleus (including the neutron). Experimentally it has been established (See Chapter I, B) that;

ν emitted in β^+-decay is left handed

and, $\bar{\nu}$ emitted in β^--decay is right handed

For convenience of discussion, we shall reserve for a short while the symbols ν and $\bar{\nu}$ to the neutrino-like particles emitted in $\beta^{\pm}$-decays only. Let us denote the neutrino-like particles associated with the muons (of either sign) by ω and $\bar{\omega}$. By convention let ω denote the companion of μ^+ in the two-body decay of π^+. Hence, $\bar{\omega}$ denotes the companion of μ^- in π^--decay; thus

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \omega \\ \pi^- &\rightarrow \mu^- + \bar{\omega} \end{aligned} \tag{167}$$

We would assume that the same neutral massless spin - $\frac{1}{2}$ particle is involved (apart from particle-antiparticle distinction) in μ -capture or in the decay of pions to muons as in β-decay. In other words, we omit the possibility that ω and ν , in spite of the same mass, charge and spin, may be <u>completely</u>[165] different entities. Under this assumption, and the assumption of lepton number conservation; whether[166]

$$\omega = \nu\ , \text{ or } \nu_c \tag{168}$$

depends upon whether μ^- is a lepton like e^- or anti-lepton like e^+.

The facts known so far about the $\pi \to \mu \to e$ -chain are [See chapter I, B];

(i) The high energy $e^{\pm}$ are emitted predominantly anti-parallel to $\mu^{\pm}$ -momentum.

(ii) The helicities of high energy $e^{\pm}$ (neglecting their mass) are ± 1.

(iii) The Michel parameter ρ in μ -decay is ~ 0.75.

165
By "completely different entities, we mean some inherent or intrinsic difference, which is not the same as particle-antiparticle distinction, i.e. it involves two <u>different</u> field operators for the two entities.

166
By ν_c , we mean the charge conjugate of ν . As an operator it stands for the charge-conjugate of ν , i.e. $\nu_c = C(\bar{\nu})^T$, where $C(\gamma^\mu)^T C^{-1} = -\gamma^\mu$, $C^T = -C$ and $C^\dagger = C^{-1} = -C^*$. As a particle-symbol, it denotes $\bar{\nu}$ (anti-neutrino). Thus ν_c , standing on the right hand side of the arrow for any reaction denotes creation of anti-neutrino; while standing on the left hand side, it denotes absorption of anti-neutrino. Accordingly $\bar{\nu}_c$, as particle-symbol, just denotes neutrino (ν).

An attempt[167] to measure directly the helicity of μ^- (hence of $\bar{\omega}$) from π^--decay has not yielded conclusive result.

We shall discuss the relationship between ω and ν, as required by the above known experimental facts under the alternative assumptions:

(I) μ^- is a lepton like e^-

(II) μ^- is an anti-lepton like e^+

In either case, it is to be remembered that we can not let the two neutrino-like particles emitted in muon decay to be identical in <u>all</u> respects because of (iii).

(I) <u>μ^- is a lepton like e^-</u>

Then the $\pi^- \to \mu^- \to e^-$-decay scheme is denoted by,

$$\pi^- \to \mu^- + \bar{\omega}$$
$$\mu^- \to e^- + \bar{\nu} + \omega$$

In this case, since μ^- is a lepton,

$$\omega = \nu$$

by lepton number conservation. Since the helicity of μ^- is not known, the helicity assignments for the high energy e^-, consistent with (i), (ii) and (iii) for the two alternative values of μ^--helicity are shown in figs. 33(a) and (b) respectively.

167 Love et al. - Phys. Rev. Lett. 4, 382 (1960).

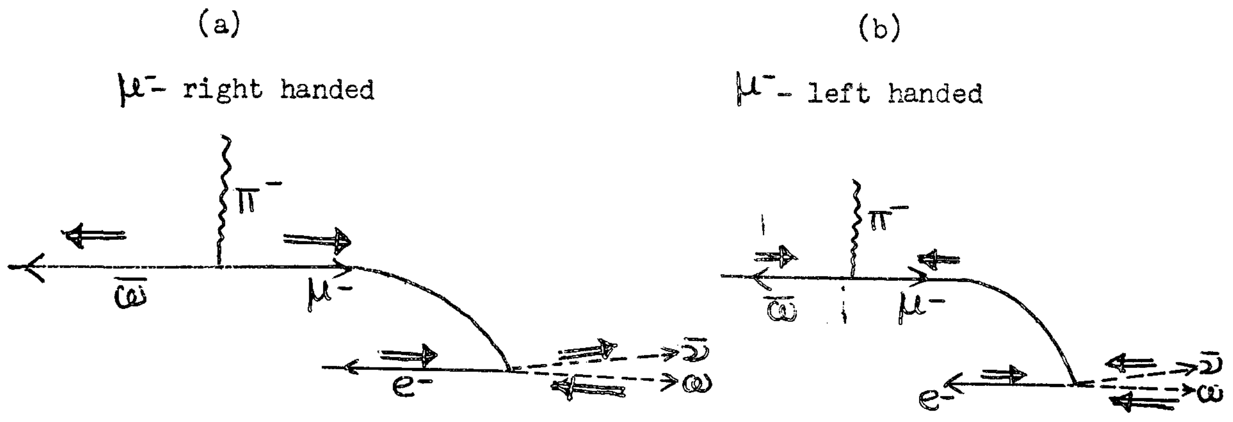


Fig. 33

If fig. 33(a) describes the phenomenon correctly, then it simply implies the validity of the two-component theory of the neutrino (since in the present case $\omega = \nu$), i.e. only left handed neutrinos and right handed anti-neutrinos occur in nature. The phenomenon is consistent with the universal V-A four fermion interaction.

If, however, fig. 33(b) is the true description of the process then one must adopt the four-component theory of the neutrino, since $\bar{\nu}$ is R.H.[168] in β^--decay and L.H. in μ-decay. Furthemore, it is clear from the helicity assignments in fig-33b that the $(e\nu)$-current must appear with opposite chiralities in β-decay and μ-decay interactions; so also the $(\mu\omega)$-current must appear in μ-decay and μ-capture interactions (presumably playing a role in pion-decay process) with opposite chiralities.

---

168 From here on we will often use the abbreviations R.H. and L.H. for "right handed" and "left handed" respectively.

Although this possibility can not be ruled out[169] unless μ^--helicity is found to be positive, it lacks theoretical appeal.

Thus if we assign the same lepton number to μ^- as to e^-, the observed facts strongly indicate the two component nature of the neutrino (Possibility (a)). Conversely, it is easy to check that if we assume the two-component nature of the neutrino, then μ^- must necessarily be considered as a lepton like e^-. The reason is as follows:

Let us assume the two-component theory of the neutrino. Let us suppose that μ^- is an anti-lepton like e^+. Then the μ^--decay must yield two anti-neutrinos alone with electron in order to conserve lepton number. By the two-component theory of the neutrino, the two anti-neutrinos must have the same helicity. At the high energy end of the electron spectrum, they travel parallel to each other. By Fermi statistics the matrix element for such a configuration of the final state is zero. This is in contradiction with the observed value of ρ ($\approx$ 0.75), which implies a non-zero intensity at the high energy end of the electron spectrum. Hence, we have proved the statement made above.

(II) μ^- is an anti-lepton like e^+

The $\pi^- \to \mu^- \to e^-$ -decay scheme is again denoted by;

$$\pi^- \to \mu^- + \bar{\omega}$$
$$\mu^- \to e^- + \bar{\nu} + \omega$$

---

169 In this connection, see S.P. Rosen Phys. Rev. Lett. 4, 613(1960).

Since, μ^- is an anti-lepton we must have,

$$\omega = \nu_c$$

by lepton number conservation. In this case, the helicity assignments in the $\pi^- \to \mu^- \to e^-$ chain for high energy e^-, consistent with (i), (ii) and (iii), for the two possible helicities of μ^- are given by figs. 34(a) and 34(b) respectively.

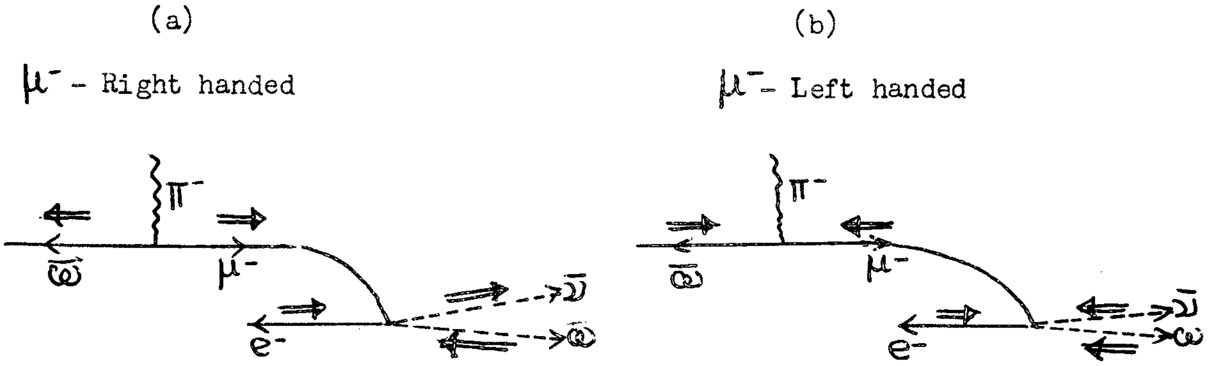


Fig. 34

If fig. 34(a) describes the phenomenon correctly, then one has to adopt a four-component theory of the neutrino, since one of the anti-neutrinos, denoted by $\omega (= \nu_c)$ emitted in μ^--decay is left handed and the other right handed. The $\bar{\omega}$ emitted in π^--decay (presumably involving μ-capture reaction) along with μ^- is correspondingly right handed. This, therefore, permits us to associate consistently only left handed neutrinos with e^- (both in β-decay and μ-decay interactions) and only right handed neutrinos with μ^+ (both in μ-capture and μ-decay interactions). This[170] differs from the

[170] J. Schwinger, Annals of Phys. 2, 407(1957), K. Nishijima, Phys. Rev. 108, 907(1958); M. Konuma, Nuclear Phys. 5, 504(1958).

usual two-component theory of the neutrino only by the fact that $\overline{\mu^-}\gamma_\alpha(1+i\gamma_5)\nu$ -current entering into any interaction is replaced by $\overline{\mu^-}\gamma_\alpha(1+i\gamma_5)\omega$, where $\omega = \nu_c$. Thus, this is a form of twin neutrino theory, where one twin is associated with e^-, the other with μ^+. Because of the restriction on the nature of helicities of the neutrinos, associated with electrons and muons, we shall refer to such a four component theory as "<u>The Restricted Four Component Theory of the Neutrino</u>".

If fig. 34 (b) is the true description of the event, then also one must adopt a four component theory of the neutrino, since $\bar{\nu}$ is R.H. in β^--decay, but L.H. in μ^--decay. Analogous to the situation in fig. 33 (b), however, the $(e\nu)$-current must enter with opposite chiralities into β-decay and μ-decay interaction; so also the $(\mu\omega)$-current must enter into μ-decay and μ-capture-interactions with opposite chiralities.

Thus the descriptions provided by either fig. 33 (b) or 34 (b) leads to rather theoretically unappealing interactions. Hence, we will ignore the possibility that μ^- amy be L.H., i.e. we <u>assume</u> that the future helicity measurements of μ^- will show that it is R.H.

Thus we are left with either of the following two possibilities [See fig. 35].

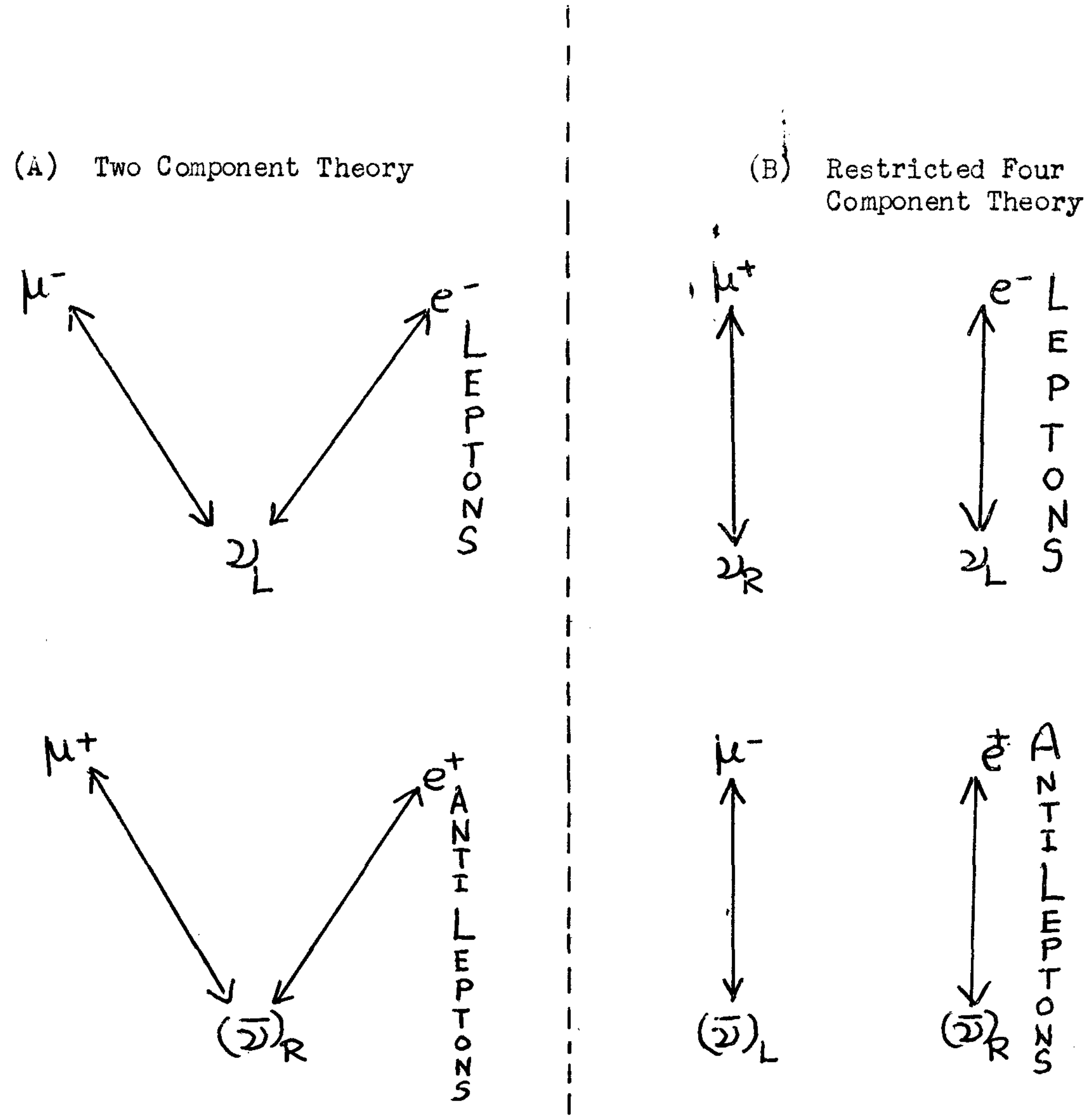

(A) Two Component Theory
(B) Restricted Four Component Theory
μ⁻
e⁻
LEPTONS
ν_L
μ⁺
e⁺
ANTI LEPTONS
(ν̄)_R
μ⁺
e⁻
LEPTONS
ν_R
ν_L
μ⁻
e⁺
ANTI LEPTONS
(ν̄)_L
(ν̄)_R

Fig. 35

(A) Two Component Theory of the Neutrino with μ^- Being a Lepton

The weak interaction current subject to lepton number conservation can in this case be written as:

$$J_\mu = g[\bar{n}\gamma_\mu(1+i\gamma_5)p+\bar{e}\gamma_\mu(1+i\gamma_5)\nu+\bar{\mu}\gamma_\mu(1+i\gamma_5)\nu + \cdots\cdots] \tag{169}$$

So that the interaction $(J_\mu B_\mu + H.C.)$ or $(J_\mu J_\mu^* + H.C.)$ is invariant under the trans-formation :

$$\nu \to i\gamma_5\nu, \quad \mu \to \mu, \quad e \to e \tag{170}$$

and the $\pi^- \to \mu^- \to e^-$ -chain is represented by:

$$\begin{aligned} \pi^- &\to \mu^- + (\bar{\nu})_R \\ \mu^- &\to e^- + \nu_L + (\bar{\nu})_R \end{aligned} \tag{171}$$

where the subscripts L and R denote[171] the helicities of the corresponding particles.

(B) The Restricted Four Component Theory of the Neutrino with μ^- Being an Anti-lepton:

The weak interaction current, subject to lepton number conservation can, in this case be written as;

$$J_\mu = g[\bar{n}\gamma_\mu(1+i\gamma_5)p+\bar{e}\gamma_\mu(1+i\gamma_5)\nu+\bar{\mu}\gamma_\mu(1+i\gamma_5)\omega + \cdots]$$

---

171 L and R denote L.H. and R.H. helicities respectively.

where , $$\omega = \nu_C \tag{172}$$

So that the interaction ($J_\mu B_\mu$ + H.C.) or ($J_\mu J^*_\mu$ + H.C.) is invariant under the transformation[172]

$$\nu \to i\gamma_5 \nu \ ; \ \mu \to -\mu \ ; \ e \to e \tag{173}$$

and the $\pi^- \to \mu^- \to e^-$ - chain is represented by;

$$\begin{aligned} \pi^- &\to \mu^- + \nu_R \\ \mu^- &\to e^- + (\bar{\nu})_R + (\bar{\nu})_L \end{aligned} \tag{174}$$

By using the properties[173] of the charge conjugate operator ν_C , it is easy to see from eqs. (169) and (172) that both (A) and (B) lead to the same results as far as the decay electrons are concerned (i.e. their helicities, Michel Parameter, Correlation of $\vec{P}_e$ with $\vec{P}_\mu$ etc.), and hence can not be distinguished from each other on the basis of present experiments.

It may appear that the possibility involving the association of different types of neutrinos with μ^- and e^- is probably an unnecessary complication. It, however, possesses some attractive features. Firstly it would imply that processes involving an odd number of μ-mesons and an odd number of electrons (of either sign) are forbidden (by lepton no. Conservation). For example the following transitions are forbidden under scheme B either

---

172 K. Nishijima - Phys. Rev. 108, 907 (1958)

173 See "Theory of Elementary Particles" Chapter IV, by P. Roman, North Holland Publishing Company.

in the framework of local or non-local four fermion interaction.

$$\text{(i)} \quad \mu \to e + \gamma$$
$$\text{(ii)} \quad K \to \mu + e$$
$$\text{(iii)} \quad \mu + p \to e + p \quad \text{etc.}$$

Thus one can explain the absence of $\mu \to e + \gamma$, even in the framework of vector-boson-theory for four fermion interactions, if one adopts B. Thus possibility B is quite welcome; since the vector boson hypothesis is rather appealing from the theoretical standpoint.

Secondly, the possibility B also has some implications with regard to the fact that the μ^- and e^- possess such apparent similarity in the nature of their interactions and yet have very different masses. It is generally believed that the physical masses of the particles can be attributed to their interactions. This is consistent with the observation that the leptons, having no strong interactions, occupy the lowest positions in the mass spectrum of the Fermi particles. From this point of view the $(\mu - e)$ -mass difference has been a mystery[174] for a long time. It might suggest that the μ^- and e^- may possess some inherently different quantum numbers,[175] for example, they may differ in their leptonic charge or lepton number, consequently they are associated with

---

174 One way out could, of course, be to assume that their bare masses are different from each other.

175 Some workers have tried to introduce the concept of strangeness for muon. See Gamba, Marshak and Okubo "Note on A Symmetry in Weak Interactions" (Preprint).

different types of neutrinos (possibility B) This does not, of course, explain in any way why their masses are different, but may be consistent with the indication that there may exist some intrinsic difference between the μ^- and the e^-.

In view of the above discussion, it appears that possibility B is rather appealing and hence it is worth investigating, how one may distinguish between A and B.

Distinction Between (A) and (B)

Clearly, in order to distinguish experimentally between (A) and (B), one has to test whether the neutrinos associated with the muons are capable of inducing the same reactions that can be induced by electron-associated neutrinos. This can be done, for example, by testing whether the $\bar{\omega}$ -particles emitted along with μ^- in π^--decay are capable of undergoing absorption via the reaction:

$$\bar{\omega} + p \rightarrow e^+ + n \qquad (175)$$

clearly, the above reaction can take place if (A) holds, while, it is forbidden if (B) is true.

High energy neutrino experiments to test if the above reaction goes or not, may be feasible[176] in the near future.

---

[176] For an interesting discussion of the feasibility of the above experiment, see B. Pontecorvo, J. Exptl. Theor. Phys. (U.S.S.R.) **37**, 1751 (1959).

Since the rest of the discussion in this chapter is concerned with the effects of the intermediate vector boson, which are independent of the choice (A) or (B), we will discuss in the frame work (A) for convenience.

E. Effects of Nonlocality on the Energy Spectra in K_{e3} ($K_{\mu 3}$) - Decays

We discussed before in Sec. B, the effect of nonlocal four fermion interaction on $\pi \to \mu(e) + \nu$ and $K \to \mu(e) + \nu$-decays and found that the results are the same as those for local four fermion interaction except that g^2/m_B^2 replaces $f^2 (= f_F/2)$ and gg'/m^2_B replaces ff'.

In this chapter, we wish to consider the energy spectra of the pions and the heavy leptons (e or μ) in K_{e3} ($K_{\mu 3}$)-decays for both local and nonlocal four fermion interactions. It is pointed out that accurate determination of the <u>pion</u> energy spectrum may serve to distinguish between the two types of interactions. The energy spectrum of e(μ) is however not so sensitive to the choice of the interaction (local or nonlocal) mainly because of kinematical reasons.

The K^+_{e3} ($K^+_{\mu 3}$) -decay process for local and nonlocal fermi interactions is represented by figs. 36 and 37 respectively.

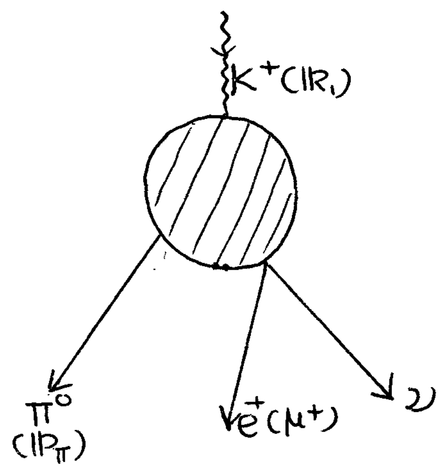


Fig. 36

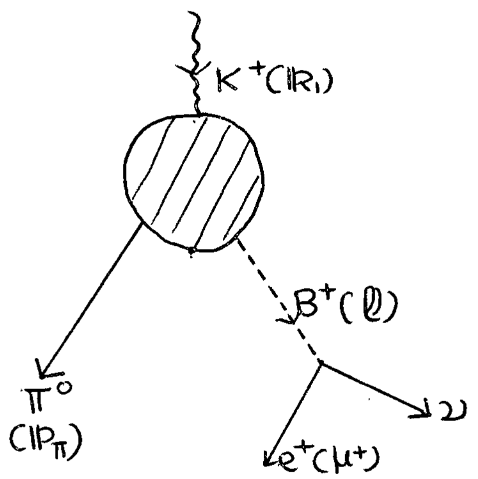


Fig. 37

The matrix elements of figs. 36 and 37, considering the muon-mode, are given by:

$$M(\text{fig-36}) = K\,(2\pi)^4\delta^4(k_1 - p_\pi - p_{\mu^+} - p_\nu)\,[J_1 k_{1\alpha} + J_2 \ell_\alpha]$$
$$\bar{\nu}\,\gamma_\alpha(1+i\gamma_5)\mu \tag{176}$$

$$M(\text{fig-37}) = F\,(2\pi)^4\delta^4(k_1 - p_\pi - p_{\mu^+} - p_\nu)\,[J_1 k_{1\alpha} + J_2 \ell_\alpha]$$
$$\frac{g_{\alpha\beta} - \ell_\alpha \ell_\beta / m_B^2}{\ell^2 - m_B^2}\;\bar{\nu}\gamma_\beta(1+i\gamma_5)\mu$$

$$= F\,(2\pi)^4\delta^4(k_1 - p_\pi - p_{\mu^+} - p_\nu)\,[J_1 k_{1\beta} + J_2' \ell_\beta]$$
$$\bar{\nu}\,\gamma_\beta(1+i\gamma_5)\,\mu \tag{177}$$

where,

$$K = g_K g_\pi f f'$$
$$F = g_K g_\pi g g'$$
$$\ell = p_{\mu^+} + p_\nu$$
and
$$J_2' = J_2 - J_1 (k_1 \cdot \ell)/m_B^2 - J_2 \ell^2/m_B^2 \tag{178}$$

J_1 and J_2 are scalar functions of $(\mathbb{k}_1 \cdot \mathbb{P}_\pi)/m_p^2$ and denote the contributions of the baryon-antibaryon loops, occuring inside the black boxes in figs. 36 and 37. The ratio J_1/J_2 is real by time reversal invariance.

We neglect the dependence of J_1, J_2 and J_2' on the pion energy, as in chapter III (See ch. III. A). This amounts to replacing the black boxes in figs. 36 and 37 by points. This is not so unreasonable, since the nonlocality due to the back boxes involving baryon-antibaryon pair is expected to be rather small; that due to the B-meson, however, will be taken into account (Since m_B may be roughly equal[177] to m_K)

The matrix elements for K_{e3}-decay are obtained by replacing μ by e in eqs. (176) and (177).

The energy spectra of the muon and that of the pion in $K_{\mu 3}$-decay have been obtained for both local and nonlocal four fermion interactions in Appendix V [eqs. (A-154), (A-160), (A-162) and (A-163)].

The corresponding expressions for electrons and pions for K_{e3}-decay are immediately obtained from the above mentioned equations by replacing m_μ by m_e. Since terms involving J_2 and J_2' in the energy spectra involve m_μ^2, we may drop such terms for the K_{e3}-mode by neglecting the mass of the electron.

177 For $m_B \gg m_p$, one simply obtains the results of the local fermi interaction.

The electron and pion-energy spectra for K_{e3} -decay are then given by:

$$\overset{\text{Local}}{W}(E_e)\,dE_e \propto \left[\eta_1(2m_K E_e - 2E_e^2 - a/2) + \eta_2(m_K - 2E_e)\right] dE_e \tag{179}$$

$$\overset{\text{Non Local}}{W}(E_e)\,dE_e \propto \Big[\left\{m_K^2(2m_K E_e - 2E_e^2 - a/2) + b/2\, m_K(m_K - 2E_e)\right\}\chi_1$$
$$+ m_K^2(m_K - 2E_e)\chi_2\Big]\, dE_e \tag{180}$$

$$\overset{\text{Local}}{W}(E_\pi)\,dE_\pi \propto (E_\pi^2 - m_\pi^2)^{3/2} \tag{181}$$

$$\overset{\text{Non Local}}{W}(E_\pi)\,dE_\pi \propto \frac{(E_\pi^2 - m_\pi^2)^{3/2}}{(b - 2m_K E_\pi)^2} \tag{182}$$

where, (for K_{e3} - mode)

$$\eta_1 = \frac{(\mathcal{L} - m_\pi^2)\,E_e}{\mathcal{L}} \quad ; \qquad a = m_K^2 + m_\pi^2$$

$$\eta_2 = \frac{(\mathcal{L}^2 - m_\pi^4)\,E_e(m_K - E_e)}{2\mathcal{L}^2} \quad ; \qquad b = m_K^2 + m_\pi^2 - m_B^2$$

$$\mathcal{L} = m_K^2 - 2m_K E_e$$

$$\chi_1 \equiv \frac{(\alpha - m_\pi^2) E_e}{\alpha \left\{ b - m_K/\alpha \, (m_K - E_e)(\alpha + m_\pi^2) \right\}^2 - \frac{(\alpha - m_\pi^2) E_e m_K^2}{2}}$$

$$\chi_2 \equiv \frac{1}{4 m_K^2} \log_e \frac{b - m_K/\alpha \left\{ (m_K - E_e)(\alpha + m_\pi^2) + E_e(\alpha - m_\pi^2) \right\}}{b - m_K/\alpha \left\{ (m_K - E_e)(\alpha + m_\pi^2) - E_e(\alpha - m_\pi^2) \right\}}$$

(183)

Since the J_2 or J_2' terms do not contribute (neglecting m_e) to K_{e3} -decay, we do not have to know the value of J_1 to evaluate the energy spectra of K_{e3} -decay. From this point of view the theoretical predictions of the energy spectra in K_{e3} -decay are somewhat more reliable than those in $K_{\mu 3}$-decay. We will therefore try to distinguish between local and nonlocal four fermion interactions on the basis of energy-spectra of K_{e3} -decays only. In Appendix-V, we have given a predicted spectrum of the muon (fig. 42) emitted in $K_{\mu 3}$-decay for contact four fermion interaction by adopting a typical value of J_1/J_2, obtained in the lowest order perturbation (with cut off $\Lambda = 1.8$ mp). The shape of the muon spectrum for nonlocal four fermion interaction is not expected to be very different from that due to local interaction (as in the case of electron).

The electron energy spectra for both local[178] [eq. (179)]

178 Furuichi, Kodama, Ogawa, Sugawara, Wakasa and Yonezawa - Prog. Theor. Phys. 17, 89 (1957).

and nonlocal[179] [eq. (180)] interactions are shown in figs. 38 and 39 respectively. In the latter case, the mass of the B-meson has been tentatively chosen to be $\sqrt{m_K^2+m_\pi^2}$. A comparison of the theoretical curves of either model with the available data[180] (fig. 38) shows a very poor agreement. However, the experimental data are subject to considerable uncertainty and are not reliable at present.

It is to be noted that the theoretically predicted electron spectra are nearly the same for either model of weak interactions (local or nonlocal) in spite of the apparently different expressions [eqs. (179 and (180)] . This arises partly due to the fact that the pion energy is a slowly varying function of the electron energy and attains nearly its maximum value for a number of different kinematical configurations. It is easy to check that the maximum possible values of E_e and E_π in K^+_{e3} decay are nearly 228.5 and 265.4 Mev. respectively. The value of E_π is maximum when E_e is minimum, corresponding to $\vec{p}_e = 0$ and it has nearly the maximum value when E_e is maximum, corresponding to $\vec{p}_\nu = 0$. For intermediate values of E_e, E_π takes nearly maximum value for different configurations. For example, for $E_e \approx (E_e)_{max}/2 \approx 114.25$ Mev, E_π varies between 169 and 265 Mev approximately. This nature of variation of E_π with respect to E_e partly masks the effect of the nonlocality, as is exhibited by the curves in figs. 38 and 39. Thus the electron energy-spectrum is not suitable to distinguish between

---

179 S. Oneda & J.C. Pati - Phys. Rev. Lett. 2, 516 (1959)

180 Birge et al. Nuovo Cimento, 4, 834 (1956)
Alexander et al. Nuovo Cimento, 6, 478 (1957).

Fig. 38

Electron Energy Spectra in K_{e3} -Decay with Local Four-Fermion Interactions [eq. (179)] . The histograms are based on the available data (Ref. 180). The theoretical curve is the same as that given in Ref. 178. The above curve has been reproduced from R.H. Dalitz - Rev. Mod. Phys. - 31, 823 (1959).

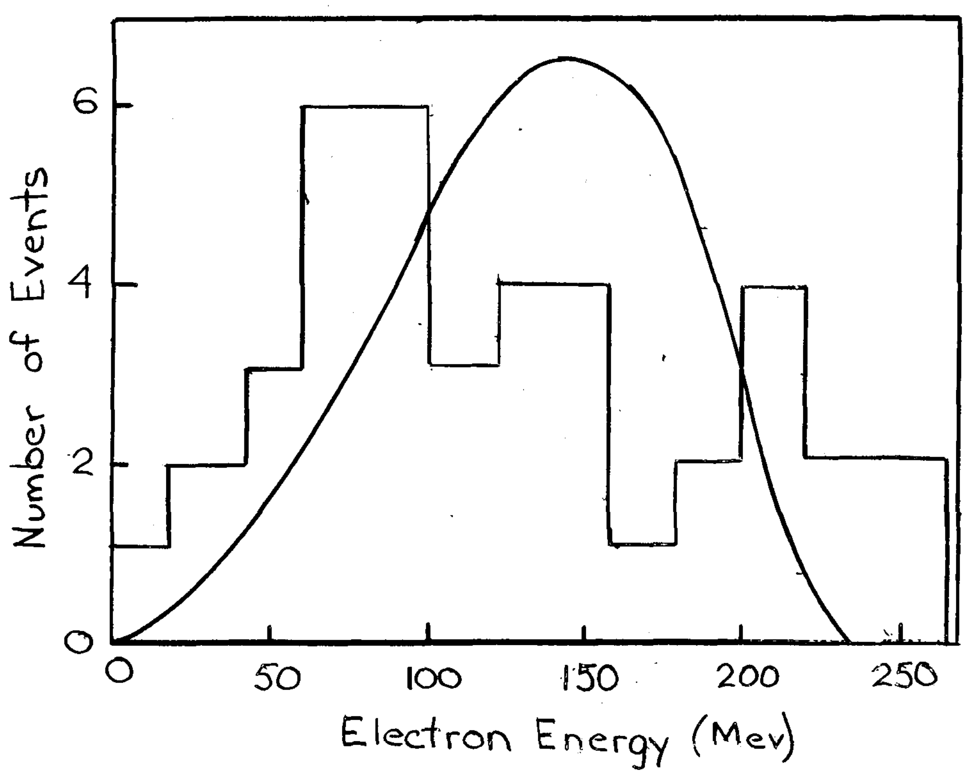
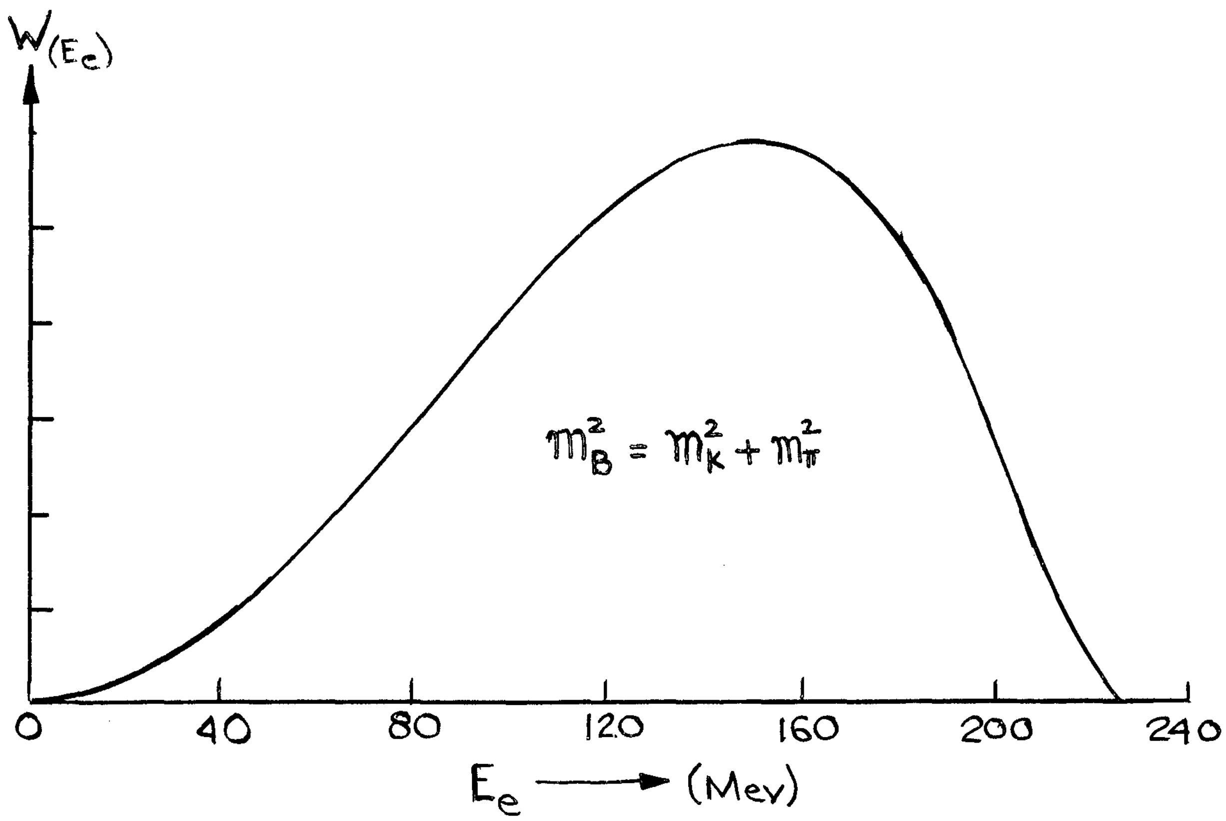


Fig. 39

Electron Energy Spectrum in K_{e3} -Decay with nonlocal four fermion interactions [eq. (180)]. Compare with the curve for local four-fermion interactions [fig. 38].



the local and nonlocal interactions.

The pion-energy spectra for local [eq. (181)] and nonlocal [eq. (182)] four fermion interactions have been plotted in fig. 40, where we have tentatively chosen

$$m_B = \sqrt{m_K^2 + m_\pi^2}$$

It is to be noted that there is a rather rapid variation in the slope of the spectrum at the lower end of the pion-energy for local interactions compared to non-local ones. Thus, it can be hoped that a careful study of the energy spectrum of the decay pions may help to decide whether four fermion interactions are local or nonlocal.

It may be remarked that, under the approximation that J_1 is independent of E_π, the energy spectra of $K^\pm \to \pi^0 + e^\pm + \nu$, $K_2^0 \to \pi^\mp + e^\pm \pm \nu$ and $K_1^0 \to \pi^\mp + e^\pm \pm \nu$ would be the same except for the small effect of the mass difference of the participating particles. In general, if the dependence of J_1 on E_π is marked, these spectra may be different from each other. However, if, as is the case for our choice of the interaction, the strangeness violating current behaves as an iso-spinor, then these spectra should be the same[181], irrespective of the dependence of J_1 on E_π, likewise for the muon modes.

---

181 Okubo, Marshak, Sudarshan, Teutsch and Weinberg - Phys. Rev. 112, 665 (1958).

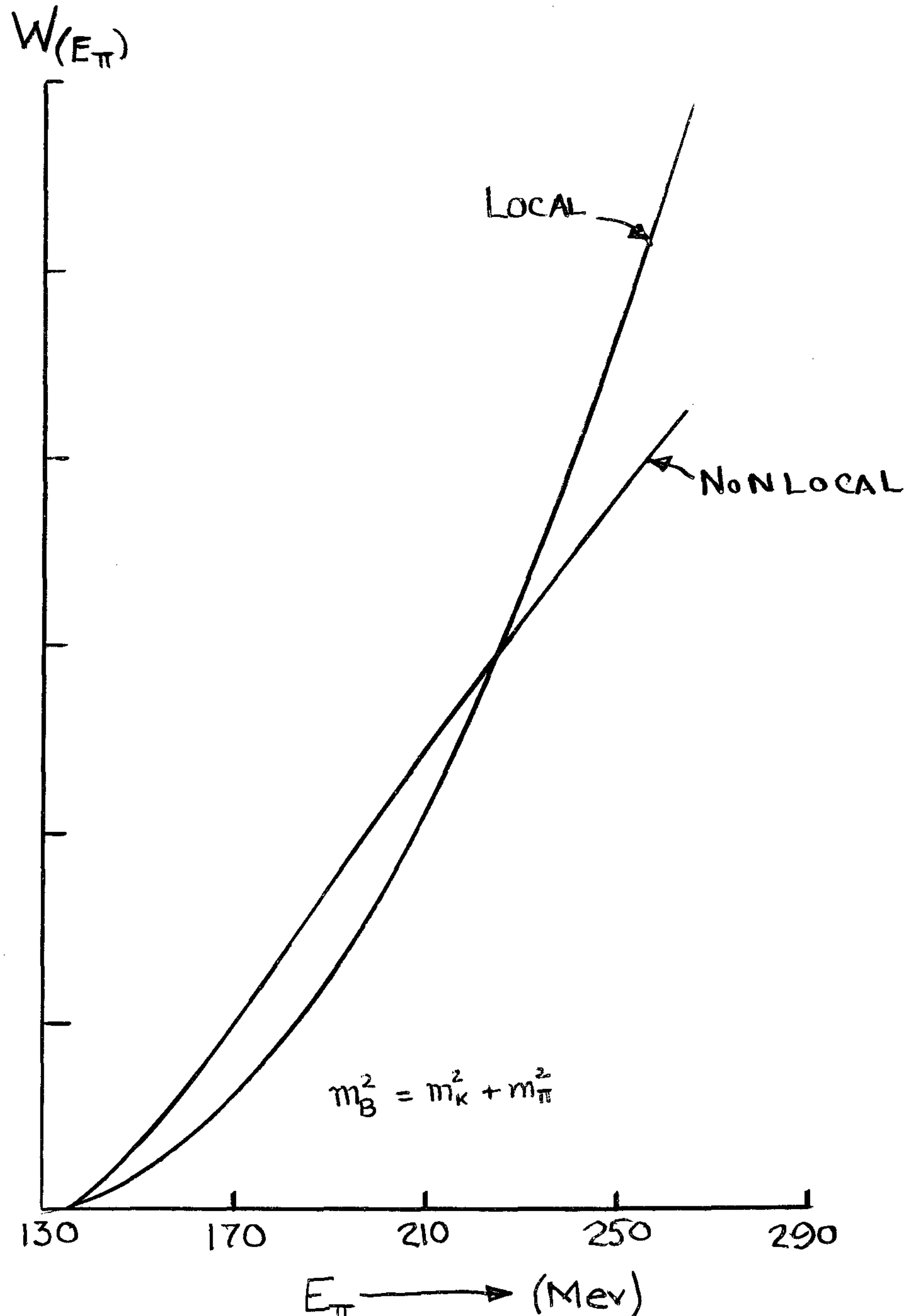


Fig. 40

Pion Energy Spectrum in K_{e3} -decay given by eqs. (181) and (182) . They are normalised to give the same area.

F. Effects of Non-Locality on the Decay of the Λ-Hyperon

The decay of the Λ-hyperon has been discussed in detail in chapter II on the basis of contact V-A four fermion interactions. It has been shown that a new class of diagrams (fig. 14) involving single-neutron intermediate state and satisfying the $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule is much more important than the usually considered class of diagrams (fig. 5), which involves a mixture of $|\Delta \underset{\sim}{I}| = \frac{1}{2}$ and 3/2 transitions. It will be our task in the present section to examine the contributions of the same two classes of diagrams with the introduction of the intermediate vector boson. It will be shown that the introduction of the non-locality in the four fermion interactions <u>increases</u> the importance of the <u>new</u> class of diagrams compared to the usual one. However, with reasonably finite values of mB, we can not explain (under the approximation discussed in chapter II E) the observed asymmetry parameter of Λ-decay if we stick to <u>positive</u>[182] bare chiral currents only.

The Feynman diagrams invoving vector boson and corresponding to figs. 5 and 14 of local fermi interactions are given by figs. 41 and 42 respectively.

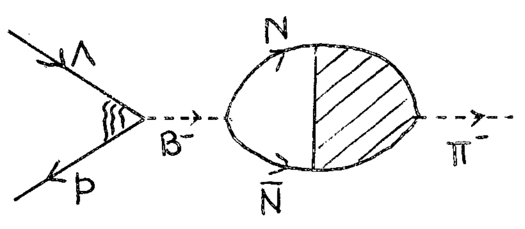


Fig. 41

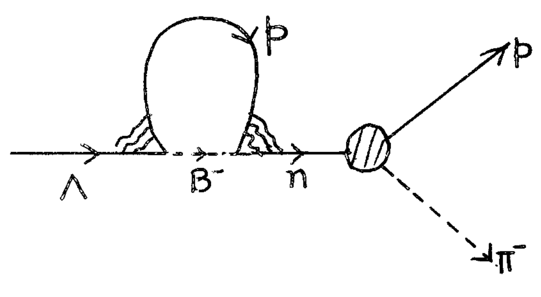


Fig. 42

---
182 We have defined previously currents of positive chirality to have the form $\bar{A}\gamma_\alpha(1+i\gamma_5)B$.

Comparing with eq. (42), the matrix element of fig. 41 is given by:[183]

$$M(\text{fig-41}) = (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_\pi)(-gg'\sqrt{2}g_\pi)\, m_p \mathbf{A}$$

$$\mathbb{P}_{\pi\alpha}\left[\frac{g_{\alpha\beta} - \mathbb{P}_{\pi\alpha}\mathbb{P}_{\pi\beta}/m_B^2}{\mathbb{P}_\pi^2 - m_B^2}\right] \bar{p}\gamma_\beta(1+iA\gamma_5)\Lambda$$

$$= (2\pi)^4 \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_\pi)(gg'/m_B^2)(\sqrt{2}g_\pi)\, m_p \mathbf{A}$$

$$\mathbb{P}_{\pi\beta}\, \bar{p}\gamma_\beta(1+iA\gamma_5)\Lambda \tag{184}$$

Thus there is no difference between the results of local and non-local fermi-interactions as far as fig. 41 is concerned, except for the replacement of gg'/m_B^2 by ff'.

Assuming the same modification at the (Λp) and (pn) vertices in fig. 42 as adopted for local fermi interactions [See chapter II E eq. (77)] ; the matrix element of fig. 42 has been derived in Appendix VI. By eq. (A-178), the matrix element of fig. 42 is given by:

$$N(\text{fig-42}) = \delta^4(\mathbb{P}_\Lambda - \mathbb{P}_p - \mathbb{P}_\pi)(gg'/m_B^2)\, i\pi^2\, m_n m_p^3 \left(\lambda^2/\lambda^2 - m_p^2\right)^2$$

$$\bar{p}\, \not{\mathbb{P}}_\pi\, \mathbf{C}(1+i\beta\gamma_5)\Lambda \tag{185}$$

---

183 Notice the distinction between $\mathbf{A}$ and A. The former is a form factor denoting the contribution of the nucleon antinucleon loop (fig. 41) and the latter denotes the modification of the ($p\Lambda$) vertex due to strong interactions [See chapter II E]. The value of $\mathbf{A}$ obtained from the known rate of $\pi \to \mu + \nu$-decay is given by: $4\pi^2 \mathfrak{r} \mathbf{A} \approx 0.2934$ [See chapter II, eq. (49)].

where ;

$$\alpha = (A-B)\Theta_1 - (A+B)(m_\Lambda/m_p)\Theta_3$$

$$\beta = (1-AB)\Theta_1 + (1+AB)(m_\Lambda/m_p)\Theta_3 \qquad (186)$$

The quantities Θ_1 and Θ_3 are expressions involving certain integrals and are functions of m_Λ, m_p, m_B and λ, where λ denotes the Feynman cut off parameter. They have been defined by eq. (A-176) and their values for a few values of m_B and λ have been tabulated in Table XI (Appendix VI). The quantities A and B denote modifications of the (Λp) and (pn)-vertices respectively in fig. 42. [See eq. 77] . As in chapter II, we choose tentatively, $B \approx +1.25$ and $A \approx 1$.

From eqs. (184) and (185) we then obtain;

$$R \equiv \left| \frac{N\,(\text{fig.-}42)}{M\,(\text{fig.-}41)} \right|^2 \approx 11 \qquad (\lambda = m_\Lambda,\ m_B \approx m_p)$$

$$\approx 89 \qquad (\lambda = 1.5 m_p,\ m_B \approx m_p) \qquad (187)$$

These therefore show that, even in case of non-local four fermion-interactions the contribution of fig. 42 is much larger than that of fig. 41.

In fact, the ratio R is much larger for non-local[184] fermi interactions with reasonably finite values of m_B (m_B less than 3 mp for example) than for local ones [compare eq. (187) and Table VIII]. Thus, in this respect, the qualitive nature of the explanation of the approximate $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule and the slowness of the leptonic modes compared to the nonleptonic ones of strange particle decays remain the <u>same</u> for nonlocal fermi interactions (m_B finite) as for local ones ($m_B \rightarrow \infty$).

<u>Study of the Sign and Magnitude of the Asymmetry Parameter of Λ -Decay</u>

The observed sign of the asymmetry parameter α_- (defined in chapter II B) demands that α and β in eq. (185) should have the same sign. The observed magnitude of α_- ($\alpha_- \geqslant 0.73 \pm 0.14$) also imposes restrictions on the possible values of β/α. The values of α and β depend on the choice of A and B and also on the choice of λ and mB; the latter determines the values of Θ_1 and Θ_3 [Table XI].

From Table XI, or from the defining equations of Θ_1 and Θ_3 [eq. (A-176)] it is easy to check that the variations of Θ_1 and Θ_3 as functions of m_B for any reasonable choice of the cut off λ is roughly as follows: Θ_1 and Θ_3 are comparable to each other in magnitude for values of m_B around nucleon's mass, the former having positive values only and the latter only negative ones.

---

184 It is easy to check from the values of Θ_1 and Θ_3 given in Table XI, that the said ratio is a rather sensitive function of the choice of the cut off λ for non-local interactions. It is not so sensitive, however, to the choice of m_B.

Θ_1 is an increasing function of mB and reaches a finite positive limit as $mB \to \infty$, which determines the contribution of fig. 42 in the local limit; Θ_3, on the other hand, decreases[185] in magnitude with the increase in mB and reaches zero limit from negative values as $mB \to \infty$. Thus the Θ_3-term does not contribute in the local limit.

It can be checked with the help of Table XI, and eq. (186), that with our choice of A and B, the above mentioned behaviour of Θ_1 and Θ_3 is such that;

$$\beta/\alpha < 0$$

for reasonably finite values of m_B (m_B less than 2mp, for example). Thus we find that, with our choice of the renormalisation effects, $A = +1$, $B = +1.25$, which corresponds to a choice of positive bare chiral currents only, there exists <u>no way</u> to explain the observed sign of the asymmetry parameter of Λ-decay for reasonably finite values of m_B. The choice of λ hardly alters the above situation.

In order to have some orientation one may try the following four possible choices of the relative signs of A and B, while considering the asymmetry parameter of Λ-decay.

$$A = -1;\begin{cases} B = +1.25 & \text{(a)} \\ B = -1.25 & \text{(b)} \end{cases}$$

---

185 For some choices of λ (for example $\lambda = 1.5$mp), however, Θ_3 increases in magnitude only over a small range of values of mB attaining a maximum and then decreases again (See Table XI)

$$A = +1, \begin{cases} B = +1.25 & \text{(c)} \\ B = -1.25 & \text{(d)} \end{cases}$$

From Table XI, with $\lambda = m_\Lambda$ (for instance), it is easy to check that:

(a) Yields the wrong sign of the asymmetry parameter for all values of B.

(b) Yields the desired sign and magnitude of the asymmetry parameter as well as a large value of the ratio R for reasonably finite values of m_B (m_B less than 2mp, say)

(c) does not yield the desired sign of the asymmetry parameter for reasonably finite values of m_B (m_B less than 2mp for example).

(d) Yields the desired asymmetry parameter for Λ - decay for almost all values of m_B.

This situation is not altered for a choice of $\lambda = 1.5$ mp or higher values of λ, except for an increase in the absolute decay-rate.

Thus, if we insist on positive bare chiral currents only (corresponding to V-A interaction), then for the present choice of renormalisation effects at the (Λp) and (pn) -vertices, there exists no way[186] to explain the observed sign and magnitude of the

186 It is to be noticed that, in the Sakata model, this statement holds, even if we start with a primary interaction satisfying the strict $|\Delta \underset{\sim}{I}| = \frac{1}{2}$-rule, with the addition of neutral baryonic currents (hence neutral vector bosons), as recently proposed for instance by T.D. Lee and C.N. Yang (Phys. Rev. - To be published).

asymmetry parameter of Λ -decay, unless the B-meson is extremely heavy. This in a certain sense may be taken as an evidence <u>against</u> the intermediate charged vector boson, and is in parallel with the conclusion drawn from the analysis of $\mu \rightarrow e + \gamma$ -decay (sec -C) in the frame work of the two-component theory of the neutrino.

However, if we insist on the idea of the intermediate boson, then the following possibility may be added, provided we abandon the universal V-A-form of weak interactions. As mentioned above, the choices (b) or (d) can explain the desired asymmetry parameter simultaneously yielding a large value of the ratio R for reasonably finite values of mB. The former would correspond to a choice of negative chiral form [i.e. of the form $\bar{A}\gamma_\alpha(1-i\gamma_5)B$] for both ($\bar{p}\Lambda$) and ($\bar{n}p$) -currents entering into Λ -decay interaction, and the latter would correspond to positive chiral form for ($\bar{p}\Lambda$) current and negative chiral form for ($\bar{n}p$) -current.

Either of the above two possibilities may suggest the existenee of two charged vector bosons, one mediating the strangeness conserving processes (B) and the other, the strangeness violating ohes (B'). The latter must be associated with a weaker coupling constant than the former and should couple the ($\bar{\Lambda}p$) - vertex to the ($\bar{n}p$), ($\bar{e}\nu$) and ($\bar{\mu}\omega$)- vertices. It may be noted that although in this scheme, ($\bar{n}p$) and ($\bar{\Lambda}p$) should couple with B' as given by either (b) or (d); ($\bar{\mu}\omega$) must couple with B' in the same way as with B (positive chiral form) due to the observed[187] similarity of the

187 Coombes et al. - Phys. Rev. <u>108</u>, 1348 (1957)

asymmetry parameters in $\pi \to \mu \to e$ and $K \to \mu \to e$ -chains and the indication of V-A interaction in μ-capture (see chapter I B). This may therefore suggest that one couple $(\bar{e}\nu)$ to B' also in positive chiral form. These possibilities however have little symmetry and hence little aesthetic appeal.

APPENDIX I

Notations

Although the notations and some of the useful relations, often ~~used~~ used in this thesis are very commonly used in the literature and text books, we will list some of them in this appendix for the sake of convenience and completeness. These are nearly the same as those used in Mesons and Fields - Vol. I by Schweber, Bethe and De Hoffman.

Three vectors and four-vectors:

Three vectors are denoted by an arrow over a light face letter ($\vec{x}, \vec{p}$ etc.). Their magnitudes are denoted by the same letter without arrow (x, p etc.). Four vectors are denoted by letters with double lines ($\mathbb{x}, \mathbb{p}$ etc.). Scalar products of both three and four vectors are denoted by a dot between the two vectors.

Choice of Metric:

The space time component $\mathbb{x}^{\mu}$ of a four vector $\mathbb{x}$ are taken as:

$$\mathbb{x}^0 = ct \quad , \mathbb{x}^1 = x$$
$$\mathbb{x}^2 = y \quad , \mathbb{x}^3 = z$$

The metric $g_{\mu\nu}$ is taken to have the following components:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The Lorentz invariant scalar product of two four vectors is given by:

$$\mathbb{P}\cdot\mathbb{X} = g_{\mu\nu}\mathbb{P}^{\mu}\mathbb{X}^{\nu} = \mathbb{X}_0\mathbb{P}_0 - \vec{\mathbb{X}}\cdot\vec{\mathbb{P}}$$

The usual summation convention for repeated indices is always understood.

Units:

We use the units in which,

$$\hbar = c = 1$$

In these units we have

$$m = \text{mass} = \text{energy} = \frac{1}{\text{length}} = \frac{1}{\text{time}}$$

$$= m = mc^2 = \frac{1}{\hbar/mc} = \frac{1}{\hbar/mc^2}$$

For electron,

$$m_e \approx 9.1\times10^{-28}\,\text{gm.} \approx 0.511\,\text{mev} \approx \frac{1}{3.86\times10^{-11}\,\text{cm.}}$$

$$\approx \frac{1}{1.288\times10^{-21}\,\text{sec.}} \qquad \text{(A-1)}$$

Hence in the above units the decay rates should be proportional to mass.

Dirac - Matrices etc.

The Dirac equation, the Dirac - matrices, and the Feynman propagators etc., used in this work are the same as those used by Schweber, Bethe and De Hoffman - Mesons and Fields - Vol. I.

Field operators and spinors

The field operators for spin - $\frac{1}{2}$ particles are usually denoted by the symbol of the corresponding field. For instance, the four-fermion interaction for β^--decay is denoted by:

$$f^2(\bar{p}\gamma_\mu(1+i\gamma_5)n)(\bar{e}\gamma_\mu(1+i\gamma_5)\nu)$$

where each symbol stands for the operator of the corresponding field.

However, we sometimes denote the spinors for corresponding fields occuring in the matrix-elements also by the same symbols. For instance, the matrix element of $K^+ \rightarrow \mu^+ + \nu$ -decay is written as:

$$(2\pi)^4\delta^4(\mathbb{P}_{K^+}-\mathbb{P}_{\mu^+}-\mathbb{P}_\nu)(g_K f f')m_p I, \mathbb{P}_{K\alpha}\bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu$$

This distinction between spinors and field-operators should, however, be clear from the context of the discussion.

APPENDIX II

Angular Distribution of Pions and Longitudinal Polarisation of Nucleon in the Decay of Λ -Hyperon

Let us consider the decay of Λ -hyperon in its rest frame. Let χ_Y denote the initial spinnor of the hyperon. By rotational invariance the transition - operator for $\Lambda \rightarrow N+\pi$ -decay has the form :

$$t = A + B\vec{\sigma}.\hat{P}_\pi \tag{A-2}$$

where A and B are complex quantities, denoting S and P wave amplitudes respectively; $\vec{\sigma}$ is the pauli spin operator and $\hat{P}_\pi$ is a unit vector along $\vec{P}_\pi$.

The final state of (π-N) system is therefore given by:

$$\chi_f = (A + B\vec{\sigma}.\hat{P}_\pi)\chi_Y \tag{A-3}$$

The angular distribution of the decay pions is proportional to the norm of the out going spinor. Denoting the cosine of the angle Θ between the pion-momentum and the hyperon polarisation axis by ξ , the angular distribution of pions is proportional to

$$\begin{aligned} W(\xi) &\propto \chi_f^\dagger \chi_f \\ &= \chi_Y^\dagger (A^* + B^* \vec{\sigma}.\hat{P}_\pi)(A + B\vec{\sigma}.\hat{P}_\pi)\chi_Y \\ &= \left\{|A|^2 + |B|^2\right\}\left\{1 + \frac{A^*B + AB^*}{|A|^2+|B|^2}\, \chi_Y^\dagger\vec{\sigma}\chi_Y.\hat{P}_\pi\right\} \end{aligned}$$

(A-4)

The average of $\chi_Y^+ \vec{\sigma} \chi_Y$ over the initial hyperon spin states denotes the polarisation $\vec{P}$ of the hyperon. Thus the angular distribution of pions is proportional to:

$$W(\xi) \propto 1 + \alpha P \cos\Theta \tag{A-5}$$

where

$$\alpha = \frac{2 \operatorname{Re}(A^* B)}{|A|^2 + |B|^2} \tag{A-6}$$

The total decay rate is obtained from (A-4) by integrating over angles, which yields:

$$W(\Lambda \rightarrow N + \pi) \propto |A|^2 + |B|^2 \tag{A-7}$$

Finally we want to show that the longitudinal polarisation of protons coming from the decay of unpolarised hyperons at rest is $-\alpha$.

Let us choose the Z-axis, the axis of quantisation along $\vec{P}_p \; (= -\vec{P}_\pi)$. Since the parent hyperon is unpolarised, we may represent its spin-wave function by

$$\chi_Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{A-8}$$

Hence the spin wave function of the outgoing proton is given by:

$$\chi_p = t\,\chi_Y$$
$$= \frac{1}{\sqrt{2}}\left(A + B\vec{\sigma}\cdot\hat{p}_\pi\right)\begin{pmatrix}1\\1\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\left(A - B\vec{\sigma}\cdot\hat{p}_p\right)\begin{pmatrix}1\\1\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\left(A - B\sigma_z\right)\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}A-B\\A+B\end{pmatrix} \qquad \text{(A-9)}$$

Hence longitudinal polarisation of the proton is given by:

$$P_{long} = \frac{|A-B|^2 - |A+B|^2}{|A-B|^2 + |A+B|^2}$$
$$= -\frac{2\,\mathrm{Re}(A^* B)}{|A|^2 + |B|^2}$$
$$= -\alpha \qquad \text{(A-10)}$$

__Note:__ For a general consideration of the angular distribution of the decay products of a hyperon into a pion and a nucleon for arbitrary values of hyperon spin, see T.D. Lee and C.N. Yang, Phys. Rev. __109__, 1755 (1958).

APPENDIX III

Matrix Elements of Various K-Meson Decay Modes

A. Convention

We derive in this appendix the matrix elements of the various K-meson decay modes in the lowest order perturbation theory. Thus we assume that the K-meson decays take place only through virtual $(N\bar{\Lambda})$ - system. For the evaluation of the integrals encountered in the matrix elements we neglect $(\Lambda - N)$ - mass difference in the denominators compared to baryon-masses. However we maintain their mass-difference in the numerators. We, furthermore, adopt the convention (as an approximation) that expressions of the form $m_p^2 x$ or $m_\Lambda^2 x$ in the denominator are to be replaced by,

$$\bar{M}^2 x \equiv \frac{m_\Lambda^2 + m_p^2}{2} x \qquad \text{(A-11)}$$

where x is a Feynman variable. We retain only the leading terms in powers of baryon mass in the matrix elements, i.e. we drop terms of the order of m_π^2/m_p^2 compared to unity. It is estimated that these approximations are not expected to lead to more than 10% error in the various decay rates.

As a convention, we do not write the normalisation factors for the incoming and outgoing particles in the matrix elements. We, however, insert them in the calculation of decay rates (Appendix IV).

We assume the parity of K-meson relative to $(\Lambda \cdot N)$ - system to be odd. The results for even - parity case, can in some cases be obtained with simple substitutions. For example, the matrix elements of $K_{\mu 2}$ and $K_{\mu 3}$ -decays for scalar K-meson can be obtained from those for pseudo scalar K-meson by replacing m nucleon by $-m$ nucleon.

We assume the usual Yukawa type pseudo scalar meson - baryon - interactions. The various matrix elements frequently involve certain integrals at the final stage of integration which we list below for convenience.

$$I_0 = \int_0^1 \frac{x}{\Lambda^2 - Bx}\,dx \qquad I_6 = \int_0^1 \frac{x^5}{(\Lambda^2 - Bx)^2}\,dx$$

$$I_1 = \int_0^1 \frac{x^2}{\Lambda^2 - Bx}\,dx \qquad I_7 = \int_0^1 \frac{x^3}{(\Lambda^2 - Bx)^3}\,dx$$

$$I_2 = \int_0^1 \frac{x^3}{\Lambda^2 - Bx}\,dx \qquad I_8 = \int_0^1 \frac{x^4}{(\Lambda^2 - Bx)^3}\,dx$$

$$I_3 = \int_0^1 \frac{x^2}{(\Lambda^2 - Bx)^2}\,dx \qquad I_9 = \int_0^1 \frac{x^5}{(\Lambda^2 - Bx)^3}\,dx$$

$$I_4 = \int_0^1 \frac{x^3}{(\Lambda^2 - Bx)^2}\,dx \qquad I_{10} = \int_0^1 \frac{x^6}{(\Lambda^2 - Bx)^3}\,dx$$

$$I_5 = \int_0^1 \frac{x^4}{(\Lambda^2 - Bx)^2}\,dx \qquad I_{11} = \int_0^1 \frac{x^7}{(\Lambda^2 - Bx)^3}\,dx \tag{A-12}$$

where Λ denotes the Feynman cut off and

$$B \equiv \Lambda^2 - \overline{M}^2 = \Lambda^2 - \frac{m_\Lambda^2 + m_p^2}{2}$$

Note:

Some of the results (Matrix elements of diagrams involving fig. 5 and of the leptonic modes of K-meson decay) derived in this appendix have been derived by others.(188)

188
A. Fujii and M Kawaguchi - Phys. Rev. 113, 1156 (1959)
S. Oneda - Nuclear Physics 9, 476 (1959)
V.S. Mathur - Nuovo Cimento 14, 1322 (1959).

B. Matrix Element of $K^+ \rightarrow \mu^+ + \nu$ -Decay

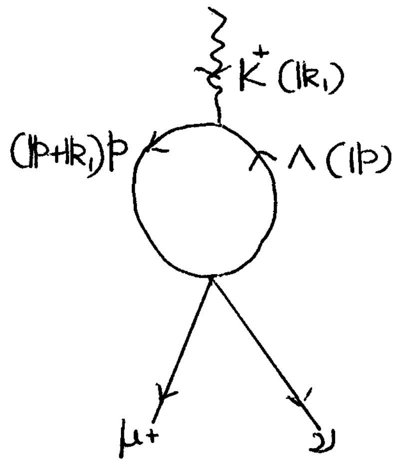


Fig. 15

The matrix element of fig. 15 is given by

$$M(K^+ \rightarrow \mu^+ + \nu)_{\text{fig-15}} = (g_K f f')(-1)\,\delta^4(k_1 - p_{\mu^+} - p_\nu) \int d^4p \,\mathrm{Tr}\Big[i\gamma_5 \left\{(\not{p} + \not{k}_1) - m_p\right\}^{-1} \gamma_\alpha (1+i\gamma_5)\,(\not{p} - m_\Lambda)^{-1}\Big]\,\bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu$$

$$= (2\pi)^4 \delta^4(k_1 - p_{\mu^+} - p_\nu)(g_K f f')\,\mathcal{G}\,\bar{\nu}\,\gamma_\alpha(1+i\gamma_5)\mu \qquad \text{(A-13)}$$

where

$$\mathcal{G} = \frac{(-1)}{(2\pi)^4} \int d^4p \,\mathrm{Tr}\Big[i\gamma_5 \left\{(\not{p} + \not{k}_1) + m_p\right\} \gamma_\alpha (1+i\gamma_5)\,(\not{p} + m_\Lambda)\Big] \left\{(p+k)^2 - m_p^2\right\}^{-1} \left\{p^2 - m_\Lambda^2\right\}^{-1} \qquad \text{(A-14)}$$

Introducing the Feynman cut off factor $(-\Lambda^2/\mathbb{P}^2-\Lambda^2)$ for the evaluation of (A-14) and evaluating the trace by the standard rules, we obtain:

$$\mathcal{G} = \frac{\Lambda^2}{(2\pi)^4}\int d^4\mathbb{P}\ \frac{4\left[m_\Lambda(\mathbb{P}+\mathbb{K}_1)_\alpha - m_p\mathbb{P}_\alpha\right]}{\left\{(\mathbb{P}+\mathbb{K}_1)^2-m_p^2\right\}(\mathbb{P}^2-m_\Lambda^2)(\mathbb{P}^2-\Lambda^2)} \tag{A-15}$$

Putting,

$$\mathbb{P}^2-\Lambda^2 = a \quad , \quad \mathbb{P}^2-m_\Lambda^2 = b\ , \text{ and } \ (\mathbb{P}+\mathbb{K}_1)^2-m_p^2 = c$$

and using

$$\frac{1}{abc} = 2!\int_0^1 dx\int_0^x dy\ \frac{1}{[(c-b)y+(b-a)x+a]^3}$$

we get,

$$\mathcal{G} = \frac{(\Lambda^2)(4)2!}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int d^4\mathbb{P}\ \frac{\left[m_\Lambda(\mathbb{P}+\mathbb{K}_1)_\alpha - m_p\mathbb{P}_\alpha\right]}{D^3} \tag{A-16}$$

where

$$D = (\mathbb{P}+\mathbb{K}_1 y)^2 + \mathbb{K}_1^2 y(1-y) + (m_\Lambda^2-m_p^2)y + (\Lambda^2-m_\Lambda^2)x - \Lambda^2 \tag{A-17}$$

clearly in the range of integration of y

$$k_1^2 y(1-y) \leq \frac{k_1^2}{4} = \frac{m_K^2}{4}$$

Since we are going to choose Λ to be at least of the order of nucleon-mass and since $\frac{1}{4}(m_K/m_p)^2 \sim 6\%$, we shall drop the term $k_1^2 y(1-y)$ Compared to Λ^2. As mentioned in sec. A, we also drop terms of the type $(m_\Lambda^2 - m_p^2)y$ in the denominator. Furthermore, we replace terms of the type $m_\Lambda^2 x$ in the denominator by $\overline{M}^2 x$ $(\overline{M}^2 \equiv \frac{m_\Lambda^2 + m_p^2}{2})$. The last approximation somewhat <u>compensates</u> for the first two. At any rate these approximations are not expected to lead to more than 10% error in the decay rate. Thus we have,

$$D \approx (p + k_1 y)^2 - (\Lambda^2 - Bx) \tag{A-18}$$

where,

$$B = \Lambda^2 - \overline{M}^2 \tag{A-19}$$

Shifting the origin of the variable p to that of p', where

$$p' = p + k_1 y$$

and dropping odd powers of p' from the numerator of (A-15); [since $\int d^4p \, f(p^2)$(odd number of p_μ factors) $= 0$], we have

$$\mathcal{J} = \frac{8\Lambda^2}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int d^4 p' \frac{[m_\Lambda - (m_\Lambda - m_p) y] R_{1\alpha}}{\{p^2 - (\Lambda^2 - Bx)\}^3}$$

$$= \frac{8\Lambda^2}{(2\pi)^4} \left(\frac{-i\pi^2}{2}\right) \int_0^1 dx \int_0^x dy \frac{[m_\Lambda - (m_\Lambda - m_p) y] R_{1\alpha}}{(\Lambda^2 - Bx)}$$

$$= \frac{(-i)\Lambda^2}{(2\pi)^2} \left[m_\Lambda I_0 - \frac{(m_\Lambda - m_p) I_1}{2} \right] R_{1\alpha} \tag{A-20}$$

I_0 and I_1, have been defined before in eq. (A-12). By eqs. (A-13) and (A-20) we have

$$\underset{\text{fig-15}}{M(K^+ \to \mu^+ + \nu)} = (2\pi)^4 \delta^4(k_1 - p_{\mu^+} - p_\nu)(g_K f f') m_p I R_{1\alpha} \bar{\nu} \gamma_\alpha (1 + i\gamma_5) \mu \tag{A-21}$$

where

$$I = \frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right) \left\{ m_\Lambda m_p I_0 - \frac{m_p(m_\Lambda - m_p)}{2} I_1 \right\} \tag{A-22}$$

C. Matrix Elements of $K \to 2\pi$ – Decays

1. $K^+ \to \pi^+ + \pi^0$ – Decay, Fig. 18

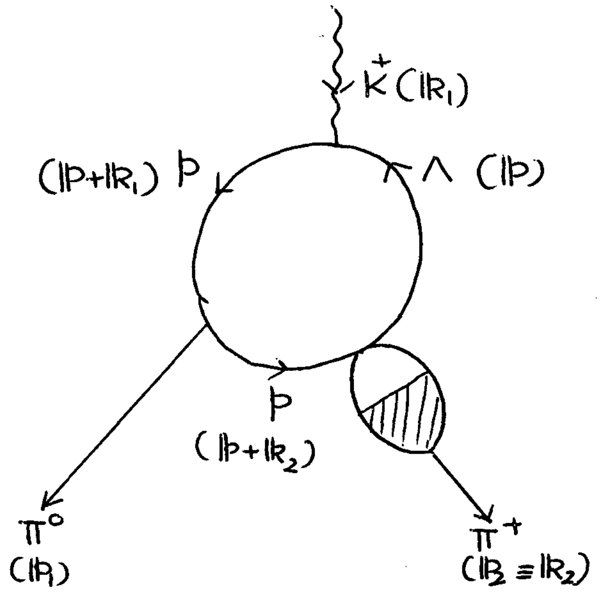


Fig. 18

The matrix element of fig. 18 is given by

$$M(K^+ \to \pi^+ + \pi^0)_{\text{fig.-18}} = \left(\frac{g_K g_\pi G_\Lambda}{m_\pi \sqrt{2}}\right)(-1)\,\delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2) \int d^4\mathbb{p}\, \mathrm{Tr}\,[\,i\gamma_5$$

$$\{(\not{\mathbb{p}} + \not{\mathbb{k}}_1) - m_p\}^{-1}\, i\gamma_5\, \{(\not{\mathbb{p}} + \not{\mathbb{k}}_2) - m_p\}^{-1} \not{\mathbb{k}}_2 (1 + i\gamma_5)$$

$$(\not{\mathbb{p}} - m_\Lambda)^{-1}\,]$$

$$= (2\pi)^4\, \delta^4(\mathbb{k}_1 - \mathbb{p}_1 - \mathbb{p}_2)\left(g_K g_\pi G_\Lambda / m_\pi \sqrt{2}\right) \mathcal{J} \qquad \text{(A-23)}$$

where,

$$\mathcal{J} = \frac{(-1)}{(2\pi)^4}\int d^4 p \, \mathrm{Tr}\left[i\gamma_5 \{(\not p + \not k_1) + m_p\} i\gamma_5 \{(\not p + \not k_2) + m_p\} (1 + i\gamma_5)(\not p + m_\Lambda)\right]\{(p+k_1)^2 - m_p^2\}^{-1}\{(p+k_2)^2 - m_p^2\}^{-1}(p^2 - m_\Lambda^2)^{-1}$$

(A-24)

Introducing the Feynman cut off ($-\Lambda^2/p^2 - \Lambda^2$) for the evaluation of $\mathcal{J}$ and using the standard rules for the evaluation of the trace, we obtain:

$$\mathcal{J} = \frac{4\Lambda^2}{(2\pi)^4}\int \frac{N}{[(p+k_1)^2 - m_p^2][(p+k_2)^2 - m_p^2](p^2 - m_\Lambda^2)(p^2 - \Lambda^2)} d^4 p$$

(A-25)

where,

$$N = -(2\, p.k_2 + k_2^2)(p^2 + p.k_1) + p^2(p.k_2 + k_1 \cdot k_2) + m_p^2 (p.k_2) + m_p m_\Lambda (k_2^2 - k_1 \cdot k_2) \qquad \text{(A-26)}$$

Putting:

$$(p+k_1)^2 - m_p^2 = a \quad ; \quad (p+k_2)^2 - m_p^2 = b$$

$$p^2 - m_\Lambda^2 = c, \quad \text{and,} \quad p^2 - \Lambda^2 = d$$

(A-27)

and using,

$$\frac{1}{a\,b\,c\,d} = 3!\int_0^1 dx\int_0^x dy\int_0^y dz\,\frac{1}{[(a-b)z+(b-c)y+(c-d)x+d]^4}$$

we have;

$$\mathcal{G} = \frac{(4\Lambda^2)\,3!}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int_0^y dz\int d^4\mathbb{P}\,\frac{N}{(D)^4}$$

(A-28)

where,

$$D = \left[\mathbb{P}+\{\mathbb{K}_1 z+\mathbb{K}_2(y-z)\}\right]^2+\mathbb{K}_1^2 z(1-z)+\mathbb{K}_2^2(y-z)(1-y+z)$$
$$-2\,\mathbb{K}_1\cdot\mathbb{K}_2\,z(y-z)+(m_\Lambda^2-m_p^2)y+(\Lambda^2-m_\Lambda^2)x-\Lambda^2$$

(A-29)

Clearly in the range of integration of y and z

$$\mathbb{K}_1^2 z(1-z) \leq \frac{m_K^2}{4}$$

$$\mathbb{K}_2^2(y-z)(1-y+z) \leq m_\pi^2/4$$

$$2\mathbb{K}_1\cdot\mathbb{K}_2\,z(y-z) \leq m_K^2/4$$

We therefore drop such terms in $\mathbb{D}$ compared to Λ^2. Furthermore we neglect (N-Λ) mass difference in $\mathbb{D}$ and replace $m_\Lambda^2 x$ by $\overline{M}^2 x$ ($\overline{M}^2 = \frac{m_\Lambda^2 + m_p^2}{2}$). [see sec. A]. We then obtain:

$$\mathbb{D} \approx \left[\mathbb{P} + \{ \mathbb{k}_1 z + \mathbb{k}_2 (y-z) \} \right]^2 - (\Lambda^2 - Bx) \tag{A-30}$$

where

$$B = \Lambda^2 - \overline{M}^2 \tag{A-31}$$

Shifting the origin of the variable $\mathbb{P}$ to that of $\mathbb{P}'$, where,

$$\mathbb{P}' = \mathbb{P} + e$$

$$e = \mathbb{k}_1 z + \mathbb{k}_2 (y-z) \tag{A-32}$$

and dropping odd powers of $\mathbb{P}'$ from N we have,

$$g = \frac{(4\Lambda^2)(3!)}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int_0^y dz \int d^4\mathbb{P}' \frac{\alpha \mathbb{P}'^2 + \beta}{\left[\mathbb{P}'^2 - (\Lambda^2 - Bx) \right]^4} \tag{A-33}$$

where,

$$\alpha = \frac{\mathbb{k}_1 \cdot \mathbb{k}_2}{2} + \frac{3}{2} e \cdot \mathbb{k}_2 - \mathbb{k}_2^2$$

$$\mathcal{B} = (2\,e\cdot k_2 - k_2^2)(e^2 - k_1\cdot e) - (e^2 + m_p^2)(e\cdot k_2)$$

$$+ e^2(k_1\cdot k_2) + m_p m_\Lambda (k_2^2 - k_1\cdot k_2) \qquad \text{(A-34)}$$

Carrying out the integration straight forwardly, we obtain:

$$\mathcal{G} = \frac{(4\Lambda^2)\,3!}{(2\pi)^4}\left(\frac{i\pi^2}{6}\right)\Big[\Big\{-\frac{(I_1+I_2)}{2} - \frac{m_p m_\Lambda I_3}{2} + \frac{(k_2^2 - m_p^2)}{6} I_4$$

$$+ \frac{(k_2^2 - k_1^2)}{12} I_5 + \left(\frac{k_1^2}{20} + \frac{k_2^2}{60} + \frac{k_1\cdot k_2}{30}\right) I_6\Big\}\, k_1$$

$$+ \Big\{(I_1 - I_2/2) + \frac{m_p m_\Lambda I_3}{2} + \frac{(k_1^2 - m_p^2)}{6} I_4 - \left(\frac{k_1^2}{6} + \frac{k_2^2}{12}\right.$$

$$\left. + \frac{k_1\cdot k_2}{4}\right) I_5 + \left(\frac{k_1^2}{60} + k_2^2/20 + \frac{k_1\cdot k_2}{30}\right) I_6\Big\}\, k_2\Big]\cdot k_2$$

(A-35)

where,

$$I_1 = \int_0^1 \frac{x^2}{\Lambda^2 - Bx}\,dx \qquad I_4 = \int_0^1 \frac{x^3}{(\Lambda^2 - Bx)^2}\,dx$$

$$I_2 = \int_0^1 \frac{x^3}{\Lambda^2 - Bx}\,dx \qquad I_5 = \int_0^1 \frac{x^4}{(\Lambda^2 - Bx)^2}\,dx$$

$$I_3 = \int_0^1 \frac{x^2}{\Lambda^2 - Bx}\,dx \qquad I_6 = \int_0^1 \frac{x^5}{(\Lambda^2 - Bx)^2}\,dx$$

(A-36)

By eqs. (A-23) and (A-35) we have

$$M(K^+ \to \pi^+ + \pi^0)_{\text{fig-18}} = (2\pi)^4 \delta^4(\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2)(g_K g_\pi G_\Lambda / m_\pi \sqrt{2})$$

$$[L_1 \mathbb{K}_1 + L_2 \mathbb{K}_2] \cdot \mathbb{K}_2 \qquad \text{(A-37)}$$

where,

$$L_1 = \frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right)\left[\frac{m_p^2(I_1+I_2)}{2} + \frac{m_p^3(m_\Lambda I_3 + m_p I_4/3)}{2}\right.$$

$$\left. + m_p^2\left\{-\frac{\mathbb{K}_2^2}{6} I_4 + \frac{(\mathbb{K}_1^2 - \mathbb{K}_2^2)}{12} I_5 - \left(\frac{\mathbb{K}_1^2}{20} + \frac{\mathbb{K}_2^2}{60} + \frac{\mathbb{K}_1 \cdot \mathbb{K}_2}{30}\right) I_6\right\}\right]$$

(A-38)

$$L_2 = -\frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right)\left[m_p^2(I_1 - I_2/2) + \frac{m_p^3(m_\Lambda I_3 - m_p I_4/3)}{2}\right.$$

$$\left. + m_p^2\left\{\frac{\mathbb{K}_1^2}{6} I_4 - \left(\frac{\mathbb{K}_1^2}{6} + \frac{\mathbb{K}_2^2}{12} + \frac{\mathbb{K}_1 \cdot \mathbb{K}_2}{4}\right) I_5 + \left(\frac{\mathbb{K}_1^2}{60} + \frac{\mathbb{K}_2^2}{20} + \frac{\mathbb{K}_1 \cdot \mathbb{K}_2}{30}\right) \frac{I}{6}\right\}\right]$$

(A-39)

If we want to keep only the leading terms in L_1 and L_2 in powers of baryon-mass; we have;

$$L_1 \approx \frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right)\left[\frac{m_p^2(I_1+I_2)}{2} + \frac{m_p^3(m_\Lambda I_3 + m_p I_4/3)}{2}\right] \qquad \text{(A-40)}$$

$$L_2 \approx \frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right)\left[m_p^2(I_1 - I_2/2) + \frac{m_p^3(m_\Lambda I_3 - m_p I_4/3)}{2}\right] \qquad \text{(A-41)}$$

2. $K^0 \rightarrow \pi^- + \pi^+$ – Decay, Fig. 21

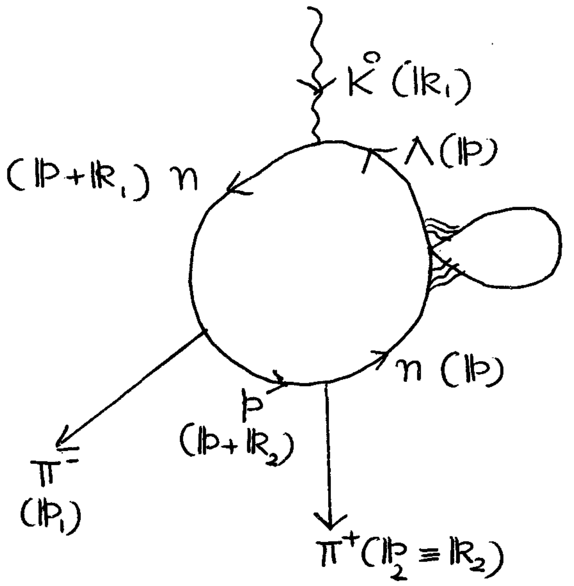


Fig. - 21

The matrix element of fig. 21 is given by;

$$N(K^0 \rightarrow \pi^- + \pi^+) = (g_K(\sqrt{2}g_\pi)^2 \rho m_p)(-1)\,\delta^4(k_1 - p_1 - p_2)\int d^4p\, \mathrm{Tr}[$$

$$i\gamma_5 \{(\not p + \not k_1) - m_p\}^{-1} i\gamma_5 \{(\not p + \not k_2) - m_p\}^{-1} i\gamma_5$$

$$(\not p - m_n)^{-1}(1 + i\gamma_5)(\not p - m_\Lambda)^{-1}]$$

$$= (2\pi)^4\, \delta^4(k_1 - p_1 - p_2)[g_K(\sqrt{2}g_\pi)^2 \rho m_p]\, \mathcal{G} \tag{A-42}$$

where,

$$\mathcal{G} = \frac{(-1)}{(2\pi)^4}\int d^4p\, \mathrm{Tr}[i\gamma_5\{(\not p + \not k_1) - m_p\}^{-1} i\gamma_5 \{(\not p + \not k_2) - m_p\}^{-1}$$

$$i\gamma_5 \{\not p - m_n\}^{-1}(1 + i\gamma_5)(\not p - m_\Lambda)^{-1}] \tag{A-43}$$

Introducing the Feynman cut off ($-\Lambda^2/p^2-\Lambda^2$) for the evaluation of $\mathcal{J}$ and using the standard rules for the evaluation of the trace, we obtain:

$$\mathcal{J} = \frac{4\Lambda^2}{(2\pi)^4}\int d^4 p \frac{N}{[(p+k_1)^2-m_p^2][(p+k_2)^2-m_p^2](p^2-m_n^2)(p^2-m_\Lambda^2)(p^2-\Lambda^2)} \tag{A-45}$$

where,

$$N = (m_n m_\Lambda - p^2)\left\{m_p^2-(p+k_1)^2+(p+k_1).(k_1-k_2)\right\}$$

$$+(m_n m_\Lambda - m_n^2)\left\{p.(k_1-k_2)\right\} \tag{A-46}$$

As mentioned in sec. A, we neglect ($N-\Lambda$) - mass difference in the denominator and put

$$p^2-m_\Lambda^2 = p^2-m_n^2 \tag{A-47}$$

Putting,

$$p^2-m_n^2 = a \qquad ; \qquad (p+k_1)^2-m_p^2 = b$$

$$(\mathbb{P}+\mathbb{K}_2)^2 - m_p^2 = c \quad , \quad \text{and} \quad \mathbb{P}^2 - \Lambda^2 = d \tag{A-48}$$

and using;

$$\frac{1}{a^2\, b\, c\, d} = 4! \int_0^1 dx \int_0^x dy \int_0^y z\, dz \frac{1}{[(a-b)z + (b-c)y + (c-d)x + d]^4}$$

we obtain,

$$\mathcal{G} = \frac{(4\Lambda^2)(4!)}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int_0^y z\, dz \int d^4\mathbb{P} \,\frac{N}{(D)^5} \tag{A-49}$$

where

$$\begin{aligned} D = {} & [\, \mathbb{P} + \{ \mathbb{K}_1 (y-z) + \mathbb{K}_2 (x-y) \}]^2 + \mathbb{K}_1^2 (y-z)(1-y+z) \\ & + \mathbb{K}_2^2 (x-y)(1-x+y) - 2\, \mathbb{K}_1 \cdot \mathbb{K}_2 (y-z)(x-y) \\ & + (\Lambda^2 - m_p^2)\, x - \Lambda^2 \end{aligned} \tag{A-50}$$

Clearly, in the range of integration of x, y and z

$$\mathbb{k}_1^2 (y-z)(1-y+z) \leq \frac{m_K^2}{4}$$

$$\mathbb{k}_2^2 (x-y)(1-x+y) \leq m_\pi^2/4$$

$$2\,\mathbb{k}_1 \cdot \mathbb{k}_2 (y-z)(x-y) \leq m_K^2/4$$

We therefore drop such terms in $\mathcal{D}$ compared to Λ^2. Furthermore we replace $m_p^2 x$ by $\bar{M}^2 x$, where $\bar{M}^2 = \frac{m_\Lambda^2 + m_p^2}{2}$ (see sec. A.). We then obtain,

$$\mathcal{D} \approx (\mathbb{p} + \mathbb{e})^2 - (\Lambda^2 - Bx) \tag{A-51}$$

where

$$\mathbb{e} = \mathbb{k}_1 (y-z) + \mathbb{k}_2 (x-y)$$

$$B = \Lambda^2 - \bar{M}^2 \tag{A-52}$$

Shifting the origin of $\mathbb{p}$ to that of $\mathbb{p}'$, where

$$\mathbb{p}' = \mathbb{p} + \mathbb{e} \tag{A-53}$$

and dropping odd powers of $\mathbb{p}'$ from N, we have,

$$g = \frac{(4\Lambda^2)4!}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int_0^y z\,dz\int d^4 p' \,\frac{(\alpha'\beta+\gamma)-(\alpha'+\beta+\delta)p'^2+p'^4}{[\,p'^2-(\Lambda^2-Bx)]^5}$$

(A-54)

where,

$$\alpha' = m_n m_\Lambda - e^2$$

$$\beta = m_p^2 - (k_1-e)^2 + (k_1-e)\cdot(k_1-k_2)$$

$$\gamma = (m_n m_\Lambda - m_n^2)\{e\cdot(k_2-k_1)\}$$

$$\delta = \frac{e}{2}\cdot(k_1+k_2-2e) \qquad \text{(A-55)}$$

Carrying out the integration (A-54) straight forwardly and substituting in (A-42), we obtain,

$$N(K^0\to\pi^- + \pi^+) = (2\pi)^4\,\delta^4(k_1-p_1-p_2)\,[g_K(\sqrt{2}g_\pi)^2\rho\, m_p]\,\mathfrak{M}$$

(A-56)

where,

$$\mathfrak{M} = \frac{(4\Lambda^2)\,4!\,(-i\pi^2/12)}{(2\pi)^4}\Bigg[\frac{m_n m_\Lambda(m_n^2-k_1\cdot k_2)I_7}{6} + \{m_n m_\Lambda(k_1+k_2)^2$$

$$+\, m_n(m_\Lambda-m_n)(k_2^2-k_1^2)\}\,\frac{I_8}{24} - \{(m_p^2+m_n m_\Lambda - k_1\cdot k_2)(k_1^2+k_2^2+k_1\cdot k_2)\}$$

$$\times\, I_9/60 - \{k_1^4 + 2k_1^2(k_1\cdot k_2) + 2/3\, k_1^2 k_2^2 + 4/3\,(k_1\cdot k_2)^2$$

$$+ 2\mathbb{k}_2^2(\mathbb{k}_1\cdot\mathbb{k}_2) + \mathbb{k}_2^4\}\, I_{10}/120 + \{\mathbb{k}_1^4 + \mathbb{k}_1^2(\mathbb{k}_1\cdot\mathbb{k}_2)$$

$$+ \mathbb{k}_2^2(\mathbb{k}_1\cdot\mathbb{k}_2) + \frac{1}{3}\mathbb{k}_1^2\mathbb{k}_2^2 + 2/3\,(\mathbb{k}_1\cdot\mathbb{k}_2)^2 + \mathbb{k}_2^4\}\, I_{11}/210$$

$$+ \{m_n m_\Lambda + m_p^2 - \mathbb{k}_1\cdot\mathbb{k}_2\}\, I_4/6 + (\mathbb{k}_1+\mathbb{k}_2)^2 I_5/16$$

$$- (\mathbb{k}_1^2 + \mathbb{k}_2^2 + \mathbb{k}_1\cdot\mathbb{k}_2) I_6/20 + I_2/2\,\Big]$$

(A-57)

The quantities I_i's have been defined before by eq. (A-12). If we wnt to keep only the leading terms in $\mathfrak{M}$ in powers of baryon mass, we have,

$$\mathfrak{M} \approx \frac{1}{4\pi^2 i}\left(\frac{\Lambda^2}{m_p^2}\right)\left[\frac{m_p^5 m_\Lambda I_7}{3} + \frac{m_p^3(m_p+m_\Lambda)I_4}{3} + m_p^2 I_2\right]$$

(A-58)

D. Matrix Elements of $K \to 3\pi$ -Decays

1. The $K^+ \to \pi^+ + \pi^- + \pi^+$ -Decay, Fig. 22

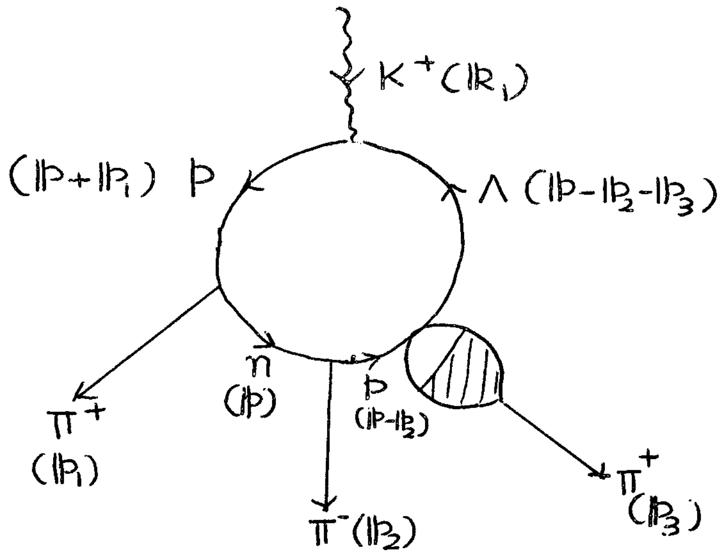


Fig. - 22

The matrix element of fig. 22 is given by

$$M(K^+ \to \pi^+ + \pi^- + \pi^+)_{\text{fig.-22}} = (2\pi)^4\, \delta^4(k_1 - p_1 - p_2 - p_3)\left[g_K (\sqrt{2}\, g_\pi)^2 \frac{G_\Lambda}{m_\pi \sqrt{2}}\right] \mathcal{G} \tag{A-59}$$

where,

$$\mathcal{G} = \frac{-1}{(2\pi)^4} \int d^4 p \,\mathrm{Tr}\left[i\gamma_5 \{(\not p + \not p_1) - m_p\}^{-1} i\gamma_5 (\not p - m_n)^{-1} i\gamma_5 \right.$$
$$\left. \{(\not p - \not p_2) - m_p\}^{-1} \not p_3 (1 + i\gamma_5) \{(\not p - \not p_2 - \not p_3) - m_\Lambda\}^{-1} \right] \tag{A-60}$$

The integral (A-60) is convergent and hence does not need any cut off factor. In the evaluation of the trace in (A-60), We drop contribution of terms $Tr [\gamma_5 \not{p}_A \not{p}_B \not{p}_C \not{p}_D]$; since they contribute only in the form $\epsilon_{\alpha\beta\gamma\delta}\, p_{A\alpha}\, p_{B\beta}\, p_{C\gamma}\, p_{D\delta}$ (where the four momenta p_A , p_B , p_C and p_D have to be all different from each other to yield non zero trace, which is the case for the present diagram; we have 3 independent external momenta and one internal momentum) and hence are smaller by a factor $p_i^2/m_p^2 \approx m_\pi^2/m_p^2$ compared to the leading term in the matrix element due to dimensional considerations. We then obtain,

$$g = \frac{4}{(2\pi)^4}\int d^4p \frac{N}{[(p+p_1)^2-m_p^2][p^2-m_n^2][(p-p_2)^2-m_p^2][(p-p_2-p_3)^2-m_\Lambda^2]} \tag{A-61}$$

where,

$$\begin{aligned}N = {} & \{m_p^3 (p-p_2-p_3) - m_p^2 m_\Lambda (p_1+p-p_2)\}\cdot p_3 + m_p\, p_3 \{ \\ & p_1 [p_2 \cdot (p-p_2-p_3)] + p_2 [p^2 + 2 p_1\cdot p_2 - p_1\cdot(p-p_3)] \\ & + p_3 (p^2 + p_1\cdot p_2) + p(-p^2 - p_1\cdot p_2)\} + m_\Lambda p_3 \cdot \{p_1 [p\cdot \\ & (p-p_2)] - p_2 [p\cdot(p+p_1) + p(p^2 + p_1\cdot p_2)\}\end{aligned} \tag{A-62}$$

We put $m_\Lambda = m_p$ in the denominator. Substituting

$$(\mathbb{p}+\mathbb{p}_1)^2 - m_p^2 = a \quad ; \quad \mathbb{p}^2 - m_n^2 = b$$

$$(\mathbb{p}-\mathbb{p}_2)^2 - m_p^2 = c \quad , \quad \text{and} \quad (\mathbb{p}-\mathbb{p}_2-\mathbb{p}_3)^2 - m_\Lambda^2 = d \tag{A-63}$$

and using,

$$\frac{1}{abcd} = 3! \int_0^1 dx \int_0^x dy \int_0^y dz \frac{1}{[(a-b)z + (b-c)y + (c-d)x + d]^4}$$

we obtain,

$$\mathcal{G} = \frac{(4)3!}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int_0^y dz \int d^4\mathbb{p} \frac{N}{(D)^4} \tag{A-64}$$

where,

$$\begin{aligned} D = {} & [\mathbb{p} + \{\mathbb{p}_1 z - \mathbb{p}_2(1-y) - \mathbb{p}_3(1-x)\}]^2 + \mathbb{p}_1^2 z(1-z) + \mathbb{p}_2^2 y(1-y) \\ & + \mathbb{p}_3^2 x(1-x) - \mathbb{p}_3^2 + 2\,\mathbb{p}_1\cdot\mathbb{p}_2\, z(1-y) + 2\,\mathbb{p}_1\cdot\mathbb{p}_3\, z(1-x) \\ & - 2\,\mathbb{p}_2\cdot\mathbb{p}_3\,(1-y+xy) - m_p^2 \end{aligned} \tag{A-65}$$

We neglect terms of the order of m_π^2 and $m_K^2/4$, as mentioned before (see sec. A), compared to m_p^2. Then we have,

$$D \approx (p + e)^2 - m_p^2 \tag{A-66}$$

where,

$$e = p_1 z - p_2(1-y) - p_3(1-x) \tag{A-67}$$

Shifting the origin of p to that of p', where

$$p' = p + e \tag{A-68}$$

and dropping odd powers of p' from N, we have,

$$\mathcal{J} = \frac{4(3!)}{(2\pi)^4} \int dx \int dy \int dz \int d^4 p' \; \frac{p_3 \cdot [\alpha p'^2 + \beta]}{(p'^2 - m_p^2)^4} \tag{A-69}$$

where,

$$\alpha = m_p (p_2 + p_3 + \tfrac{3}{2} e) + m_\Lambda (p_1 - p_2 - \tfrac{3}{2} e)$$

$$\mathcal{B} = -m_p^3\,(e + p_2 + p_3) - m_p^2 m_\Lambda (p_1 - p_2 - e) + m_p\left[-p_1 \{p_2 \cdot (e + p_2)\} + p_2\,(e^2 + 2\,p_1 \cdot p_2 + e \cdot p_1) + (p_3 \cdot + e)(e^2 + p_1 \cdot p_2)\right] + m_\Lambda\left[p_1 \{e \cdot (e - p_2)\} + p_2 \{e \cdot (p_1 - e)\} - e\,(e^2 + p_1 \cdot p_2)\right] \tag{A-70}$$

Carrying out the integration (A-69) straight forwardly and keeping only the leading terms in powers of baryon mass, we obtain from eqs. (A-59) and (A-69);

$$\underset{\text{fig.-22}}{M(K^+ \to \pi^+ + \pi^- + \pi^+)} = (2\pi)^4\,\delta^4(k_1 - p_1 - p_2 - p_3)\left[g_K(\sqrt{2}\,g_\pi)^2 G_\Lambda / m_\pi \sqrt{2}\right] \left(\frac{1}{m_p}\right)\left[N_1\,p_1 + N_2\,p_2 + N_3\,p_3\right] \cdot p_3 \tag{A-71}$$

where,

$$N_1 \approx -\frac{1}{4\pi^2 i}\quad \frac{m_p + 2m_\Lambda}{6\,m_p}$$

$$N_2 \approx +\frac{1}{4\pi^2 i}\quad \frac{m_\Lambda - m_p}{6\,m_p}$$

$$N_3 \approx -\frac{1}{4\pi^2 i}\quad \frac{2m_p + m_\Lambda}{6\,m_p} \tag{A-72}$$

2. $K^+ \to \pi^+ + \pi^- + \pi^+$ -Decay, Fig. 23

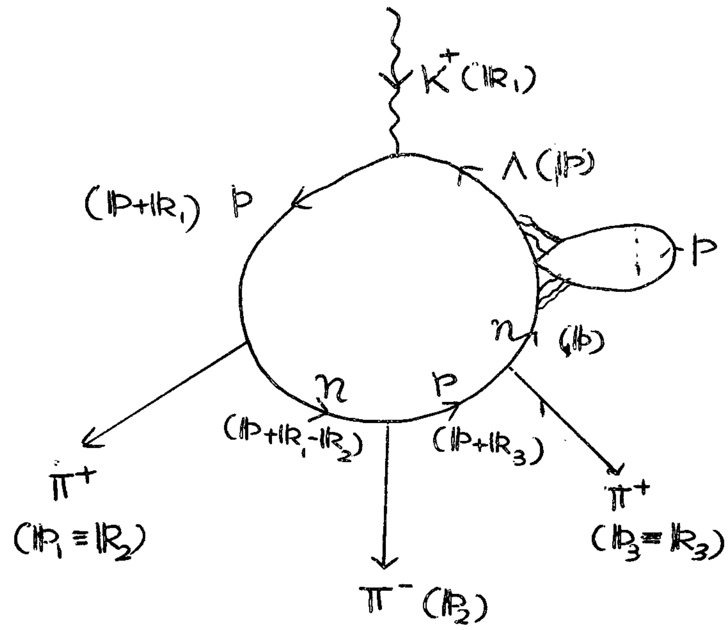


Fig.-23

The matrix element of fig. 23 is given by;

$$N\underset{\text{fig.}-23}{(K^+ \to \pi^+ + \pi^- + \pi^+)} = (2\pi)^4 \delta^4(k_1 - p_1 - p_2 - p_3)\,[g_K(\sqrt{2}\, g_\pi)^3 \rho m_p]\, g \tag{A-73}$$

where,

$$g = \frac{(-1)}{(2\pi)^4} \int d^4p \, \mathrm{Tr}\Big[i\gamma_5 \{(\not p + \not k_1) - m_p\}^{-1} i\gamma_5 \{(\not p + \not k_1 - \not k_2) - m_n\}^{-1}$$
$$i\gamma_5 \{(\not p + \not k_3) - m_p\}^{-1} i\gamma_5 (\not p - m_n)^{-1} (1 + i\gamma_5)(\not p - m_\Lambda)^{-1}\Big] \tag{A-74}$$

Evaluating the trace by the standard rules[189] we obtain,

$$\mathcal{G} = \frac{4}{(2\pi)^4} \int \frac{N}{\{(p+k_1)^2 - m_p^2\}\{(p+k_1-k_2)^2 - m_n^2\}\{(p+k_3)^2 - m_p^2\}(p^2 - m_n^2)} \times (p^2 - m_\Lambda^2)^{-1}\, d^4 p \qquad \text{(A-75)}$$

where,

$$\begin{aligned} N = {} & (m_n + m_\Lambda)\,[-m_p^2\,(p.k_2 + p^2 + p.k_3) + (p+k_1)^2 (p+k_3).p \\ & - \{(p+k_1).k_2\}\{(p+k_3).p\} - \{(p+k_1).p\}\{(p+k_3).k_2\} \\ & + \{(p+k_1).(p+k_3)\}(p.k_2)] + m_p\,(p^2 + m_n m_\Lambda)\,\{-(p+k_1)^2 \\ & + m_p^2 + k_1.k_2 + k_2.k_3 + 2\,p.k_2\} \end{aligned} \qquad \text{(A-76)}$$

we put $m_\Lambda = m_p$ in the denominator, as before and substitute;

$$p^2 - m_n^2 = a \quad ; \quad (p+k_1)^2 - m_p^2 = b$$

$$(p+k_3)^2 - m_p^2 = c \;, \quad \text{and}, \quad (p+k_1-k_2)^2 - m_n^2 = d \qquad \text{(A-77)}$$

Now using ;

---

189 Here also we neglect erms of the form $Tr\,[\gamma_5 \not{p}_A \not{p}_B \not{p}_C \not{p}_D]$ as for matrix element of fig. 22.

$$\frac{1}{a^2bcd} = 4!\int_0^1 dx\int_0^x dy\int_0^y dz \frac{z}{[(a-b)z+(b-c)y+(c-d)x+d]^5}$$

(A-78)

we obtain,

$$\mathcal{G} = \frac{(4)(4!)}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int_0^y z\,dz\int d^4\mathbb{p}\;\frac{N}{(D)^4} \quad \text{(A-79)}$$

where, (with the omission of terms of the order of m_π^2 and $m_K^2/4$ compared to m_p^2, as made before);

$$D \approx (\mathbb{p}+e)^2 - m_n^2 \quad \text{(A-80)}$$

$$e = \mathbb{k}_1(1-x+y-z)+\mathbb{k}_3(x-y)-\mathbb{k}_2'(1-x) \quad \text{(A-81)}$$

Shifting the origin of $\mathbb{p}$ to that of $\mathbb{p}'$, where

$$\mathbb{p}' = \mathbb{p}+e$$

dropping odd powers of $\not{p}'$ from N, carrying out the integrations over $\not{p}'$, z, y and x straight forwardly and maintaining only the leading terms in powers of baryon mass, we obtain

$$N\underset{\text{fig. -23}}{(K^+ \to \pi^+ + \pi^- + \pi^+)} = (2\pi)^4\, \delta^4(\not{k}_1 - \not{p}_1 - \not{p}_2 - \not{p}_3)\, [g_K (\sqrt{2} g_\pi)^3 \rho m_p] \\ (1/m_p)\, \mathcal{O} \qquad \text{(A-82)}$$

where,

$$\mathcal{O} \approx -\frac{1}{4\pi^2 i}\left(\frac{m_\Lambda}{2m_p}\right)$$

E. Matrix Elements of K_{e3}, $K_{\mu 3}$, K_{e4} and $K_{\mu 4}$ Decays

The matrix elements of $K_{\mu 3}$ (K_{e3}) and $K_{\mu 4}$ (K_{e4})-decays are simply obtained from those of $K^+ \to \pi^+ + \pi^0$ and $K^+ \to \pi^+ + \pi^- + \pi^+$-decays respectively. In fact, in the lowest order perturbation theory, by comparing figs. 16 and 17 with figs. 18 and 22, respectively, it is clear that

$$L_1 = J_1 \qquad ; \quad L_2 \pm J_2$$

$$N_1 = K_1 \quad , \quad N_2 = K_2 \text{, and} \quad N_3 = K_3 \tag{A-83}$$

The for factors J_1 , J_2 , K_1 , K_2 , K_3 , L_1 , L_2 , N_1 , N_2 and N_3 have been defined by eqs. (96), (97), (115) and (123). The values of L_1 , L_2 , N_1 , N_2 and N_3 have been derived in the lowest order perturbation theory in Appendix III, secs. C and D.

F. Summary of Results

The following are the expressions obtained for the various form - factors of K-meson decays in the lowest order perturbation theory. (only leading terms in powers of baryon mass are given).

$$K^+ \to \mu^+ + \nu : \quad I \approx \frac{1}{4\pi^2 i} \quad \frac{\Lambda^2}{m_p^2} \left\{ m_\Lambda m_p I_0 - \frac{m_p(m_\Lambda - m_p)}{2} I_1 \right\}$$

$$K^+ \to \pi^0 + \mu^+ + \nu : \quad J_1 \approx \frac{1}{4\pi^2 i} \left(\frac{\Lambda^2}{m_p^2}\right) \left\{ \frac{m_p^2 (I_1 + I_2)}{2} + \frac{m_p^3 (m_\Lambda I_3 + m_p I_4/3)}{2} \right\}$$

$$J_2 \approx -\frac{1}{4\pi^2 i} \left(\frac{\Lambda^2}{m_p^2}\right) \left\{ m_p^2 (I_1 - I_2/2) + \frac{m_p^3 (m_\Lambda I_3 - m_p I_4/3)}{2} \right\}$$

$$K^+ \to \pi^+ + \pi^- + \mu^+ + \nu : \quad K_1 \approx -\frac{1}{4\pi^2 i} \quad \frac{m_p + 2 m_\Lambda}{6 m_p}$$

$$K_2 \approx \frac{1}{4\pi^2 i} \quad \frac{m_\Lambda - m_p}{6 m_p}$$

$$K_3 \approx -\frac{1}{4\pi^2 i} \quad \frac{m_\Lambda + 2 m_p}{6 m_p}$$

$$K^+ \to \pi^+ + \pi^0 \quad : \quad L_1 = J_1 \quad ; \quad L_2 = J_2$$

$$K_1^0 \to \pi^+ + \pi^- \quad : \quad \mathfrak{M} = \frac{1}{4\pi^2 i} \left(\frac{\lambda^2}{m_p^2}\right) \left[\frac{m_p^5 m_\Lambda I_7}{3} + \frac{m_p^3 (m_\Lambda + m_p) I_4}{3} + m_p^2 I_2 \right]$$

$$K^+ \to \pi^+ + \pi^- + \pi^+ : \quad N_1 = K_1 , \quad N_2 = K_2 , \quad N_3 = K_3$$

$$O \approx -\frac{1}{4\pi^2 i} \left(\frac{m_\Lambda}{2 m_p}\right) \qquad \text{(A-84)}$$

The integrals I_0, I_1, and I_2 etc. have been defined by eq. (A-12). The values of the above form factors for cut off Λ = 1.8 mp are listed below in Table X.

Table X

I	$1.02a$
J_1	$0.91a$
J_2	$-0.58a$
K_1	$-0.56a$
K_2	$0.03a$
K_3	$0.53a$
$\mathfrak{M}$	$0.86a$
$\mathcal{O}$	$-0.59a$

$$a = \frac{1}{4\pi^2 i}$$

The values of the form factors in the lowest order perturbation theory. The values of I , J_1, J_2, and $\mathfrak{M}$ are cut off dependent (logarithmically). The above values are for cut off Λ = 1.8 mp.

APPENDIX IV

Decay Rates of Various K-Meson Modes

A. The Rate of $K^+ \to \mu^+ + \nu$ -Decay

The matrix element of $K^+ \to \mu^+ + \nu$ -decay is given by;

$$M = (2\pi)^4 \delta^4 (\mathbb{k}_1 - \mathbb{p}_{\mu^+} - \mathbb{p}_\nu)(g_K f f') m_p I \, \mathbb{k}_{1\alpha} \bar{\nu} \gamma_\alpha (1 + i\gamma_5) \mu \tag{A-85}$$

Hence the decay rate in the rest frame of the K-meson is given by:

$$W(K^+ \to \mu^+ + \nu) = \int \left[\frac{1}{(2\pi)^3} \sqrt{m_\mu / E_{\mu^+}} \sqrt{m_\nu / E_\nu} \sqrt{1 / 2E_K} \right]^2 \sum_{\substack{\mu\nu \\ \text{spins}}} |M|^2 \, d^3\vec{p}_{\mu^+} \, d^3\vec{p}_\nu \tag{A-86}$$

Since we are interested in transition probability per unit space time volume (VT = 1), we make the following replacement[190]:

---

190 See J.M Jauch and F. Rohrlich - Theory of Photons and Electrons, Adison Wesley Publishing Company, 1st. Edition, P. 163.

$$[\delta^4(\mathbb{k}_1-\mathbb{p}_{\mu+}-\mathbb{p}_\nu)]^2 = \delta^4(0)\,\delta^4(\mathbb{k}_1-\mathbb{p}_{\mu+}-\mathbb{p}_\nu)$$

$$\rightarrow \frac{1}{(2\pi)^4 VT}\,\delta^4(\mathbb{k}_1-\mathbb{p}_{\mu+}-\mathbb{p}_\nu)$$

$$\rightarrow \frac{1}{(2\pi)^4}\,\delta^4(\mathbb{k}_1-\mathbb{p}_{\mu+}+\mathbb{p}_\nu) \qquad \text{(A-87)}$$

Eq. (A-86) then reduces to,

$$W(K^+\rightarrow\mu^++\nu) = \frac{m_\mu m_\nu}{(2\pi)^6(2m_K)}(g_K f f')^2 m_p^2 I^2 \int (2\pi)^4$$

$$\delta^4(\mathbb{k}_1-\mathbb{p}_{\mu+}-\mathbb{p}_\nu)\sum_{\substack{\mu\nu \\ \text{spins}}} |\bar{\nu}\,\not{k}_1(1+i\gamma_5)\mu|^2 \frac{d^3\vec{p}_{\mu+}}{E_{\mu+}}\frac{d^3\vec{p}_\nu}{E_\nu} \qquad \text{(A-88)}$$

Sum Over Spins

$$\sum_{\text{spins}} |\bar{\nu}\,\not{k}_1(1+i\gamma_5)\mu|^2$$

$$= \sum_{\text{spins}} \{\bar{\nu}\,\not{k}_1(1+i\gamma_5)\mu\}\{\bar{\mu}\,\not{k}_1(1+i\gamma_5)\nu\}$$

$$= \mathrm{Tr}\,\frac{[\not{k}_1(1+i\gamma_5)(-m_\mu+\not{p}_{\mu+})\not{k}_1(1+i\gamma_5)(m_\nu+\not{p}_\nu)}{4m_\mu m_\nu}$$

$$= \frac{2}{m_\mu m_\nu} \left[2 (\mathbb{k}_1 \cdot \mathbb{p}_{\mu^+})(\mathbb{k}_1 \cdot \mathbb{p}_\nu) - \mathbb{k}_1^2 (\mathbb{p}_{\mu^+} \cdot \mathbb{p}_\nu) \right]$$

$$= \frac{m_K^2 m_\mu^2}{m_\mu m_\nu} \left(\frac{m_K^2 - m_\mu^2}{m_K^2} \right) \qquad \text{(A-89)}$$

To obtain the last step of eq. (A-89), we have used $\mathbb{k}_1 = (0, \vec{k}_1)$ and energy-momentum conservation.

Phase Space Integration

We need the following integration over phase space:

$$R = \int (2\pi)^4 \delta^4 (\mathbb{k}_1 - \mathbb{p}_{\mu^+} - \mathbb{p}_\nu) \frac{d^3 \vec{p}_{\mu^+}}{E_{\mu^+}} \frac{d^3 \vec{p}_\nu}{E_\nu} \qquad \text{(A-90)}$$

using,

$$\int \frac{d^3 \vec{p}_\mu}{E_\mu} f(\mathbb{p}_\mu) = 2 \int \delta^0 (p_\mu^{0\,2} - \vec{p}_\mu^{\,2} - m_\mu^2) \, d^4 p_\mu f(\mathbb{p}_\mu)$$

we have,

$$R = 2\cdot(2\pi)^4 \int \delta^4(\mathbb{k}_1 - \mathbb{P}_{\mu^+} - \mathbb{P}_\nu)\, \delta^0(\mathbb{P}_\mu^{0\,2} - \vec{P}_\mu^{\,2} - m_\mu^2)\, d^4\mathbb{P}_\mu \frac{d^3\vec{P}_\nu}{E_\nu}$$

$$= 2\cdot(2\pi)^4 \int \delta^0\left[(m_K - E_\nu)^2 - \vec{P}_\nu^{\,2} - m_\mu^2\right] \frac{d^3\vec{P}_\nu}{E_\nu}$$

$$= 2\cdot(2\pi)^4 \int \frac{4\pi P_\nu^2\, dP_\nu}{E_\nu}\, \delta^0\left[m_K^2 - 2m_K E_\nu - m_\mu^2\right]$$

$$= (2\pi)^4 (8\pi) \quad \frac{m_K^2 - m_\mu^2}{4m_K^2} \qquad \text{(A-91)}$$

Combining eqs. (A-88), (A-89) and (A-91) we have,

$$W(K^+ \to \mu^+ + \nu) = \frac{m_\mu^2 m_K}{4\pi} \left(\frac{m_K^2 - m_\mu^2}{m_K^2}\right)^2 (g_K f f')^2 m_p^2 I^2 \qquad \text{(A-92)}$$

B. The Rates of $K_{\mu 3}$ and K_{e3} -Decays

Let us first consider the rate of $\overset{+}{K}_{\mu 3}$ -decay. The matrix-element for the prociess is given by:

$$M(\overset{+}{K} \to \overset{0}{\pi} + \overset{+}{\mu} + \nu) = (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_\pi - \mathbb{p}_{\mu+} - \mathbb{p}_\nu)[g_K g_\pi f f']$$

$$[J_1 \mathbb{k}_{1\alpha} + J_2 \ell_\alpha] \bar{\nu} \gamma_\alpha (1 + i\gamma_5) \mu \qquad \text{(A-93)}$$

where $\ell_\alpha \equiv \mathbb{p}_{\mu+} + \mathbb{p}_\nu$.

We will assume, as mentioned before (Ch. III), that J_1 and J_2 are constants, independent of the pion energy. This is, of course, the case in the lowest order perturbation theory (Appendix - III F).

The rate of $K^+_{\mu 3}$ -decay in the rest frame of the K-meson is given by:

$$W(\overset{+}{K} \to \overset{0}{\pi} + \overset{+}{\mu} + \nu) = \int \left\{ \frac{1}{(2\pi)^{9/2}} \sqrt{\frac{m_\mu}{E_{\mu+}}} \sqrt{\frac{m_\nu}{E_\nu}} \sqrt{\frac{1}{2m_K}} \sqrt{\frac{1}{2E_\pi}} \right\}^2$$

$$\sum_{\substack{\mu^+\nu \\ \text{spins}}} |M|^2 \frac{d^3\vec{p}_\pi}{1} d^3\vec{p}_{\mu+} d^3\vec{p}_\nu$$

$$= \frac{m_\mu m_\nu}{(2\pi)^9 (4m_K)} (g_K g_\pi f f')^2 \int (2\pi)^4 \delta^4(\mathbb{k}_1 - \mathbb{p}_\pi - \mathbb{p}_{\mu+} - \mathbb{p}_\nu)$$

$$\sum_{\substack{\mu^+\nu \\ \text{spins}}} \left| [J_1 \mathbb{k}_{1\alpha} + J_2 \mathbb{k}_{2\alpha}] \bar{\nu} \gamma_\alpha (1 + i\gamma_5) \mu \right|^2 \frac{d^3\vec{p}_\pi}{E_\pi} \frac{d^3\vec{p}_{\mu+}}{E_{\mu+}} \frac{d^3\vec{p}_\nu}{E_\nu} \qquad \text{(A-94)}$$

Sum Over Spins

Using the Dirac equations for μ and ν we have,

$$(J_1 k_{1\alpha} + J_2 \ell_\alpha)(\bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu) = J_1\,\bar{\nu}\,\not{k}_1(1+i\gamma_5)\mu$$

$$-\,m_\mu J_2\,\bar{\nu}\,(1-i\gamma_5)\mu \qquad \text{(A-95)}$$

summing over spins we have,

$$\sum_{\text{Spins}} \left|(J_1 k_{1\alpha} + J_2\ell_\alpha)(\bar{\nu}\gamma_\alpha(1+i\gamma_5)\mu)\right|^2$$

$$= \sum_{\text{Spins}} \Big[|J_1|^2 \{\bar{\nu}\,\not{k}_1(1+i\gamma_5)\mu\bar{\mu}\not{k}_1(1+i\gamma_5)\nu\} + |J_2|^2 m_\mu^2 \{\bar{\nu}(1-i\gamma_5)$$

$$\mu\bar{\mu}(1+i\gamma_5)\nu\} + \overset{*}{J_1} J_2(-m_\mu)\{\bar{\nu}\not{k}_1(1+i\gamma_5)\mu\bar{\mu}(1+i\gamma_5)\nu\}$$

$$+ J_1\overset{*}{J_2}(-m_\mu)\{\bar{\mu}\not{k}_1(1+i\gamma_5)\nu\bar{\nu}(1-i\gamma_5)\mu\}\Big]$$

$$= \frac{2}{m_\mu m_\nu}\Big\{ |J_1|^2 \left[2(\mathbb{K}_1\cdot\mathbb{P}_{\mu^+})(\mathbb{K}_1\cdot\mathbb{P}_\nu) - \mathbb{K}_1^2(\mathbb{P}_{\mu^+}\cdot\mathbb{P}_\nu)\right]$$

$$+ |J_2|^2 m_\mu^2 (\mathbb{P}_{\mu^+}\cdot\mathbb{P}_\nu)$$

$$+ (\overset{*}{J_1} J_2 + J_1 \overset{*}{J_2})\, m_\mu^2 (\mathbb{K}_1\cdot\mathbb{P}_\nu)\Big\} \qquad \text{(A-96)}$$

Integration Over Momenta

We need the following integration over the momenta $\vec{P}_\pi$, $\vec{P}_{\mu^+}$ and $\vec{P}_\nu$:

$$R \equiv \int (2\pi)^4 \delta^4(\mathbb{K}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu^+} - \mathbb{P}_\nu) \frac{d^3\vec{P}_\pi}{E_\pi} \frac{d^3\vec{P}_\mu}{E_{\mu^+}} \frac{d^3\vec{P}_\nu}{E_\nu}$$

$$\Big\{ |J_1|^2 \left[2(\mathbb{K}_1\cdot\mathbb{P}_{\mu^+})(\mathbb{K}_1\cdot\mathbb{P}_\nu) - \mathbb{K}_1^2(\mathbb{P}_{\mu^+}\cdot\mathbb{P}_\nu)\right] + |J_2|^2 m_\mu^2 \times$$

$$(\mathbb{P}_{\mu^+}\cdot\mathbb{P}_\nu) + (\overset{*}{J_1} J_2 + J_1 \overset{*}{J_2})\, m_\mu^2 (\mathbb{K}_1\cdot\mathbb{P}_\nu)\Big\} \qquad \text{(A-97)}$$

Using,

$$\int \frac{d^3\vec{P}_{\mu^+}}{E_{\mu^+}} f(\mathbb{P}_{\mu^+}) = 2\int \delta^0(\mathbb{P}_{\mu^+}^{o\,2} - \vec{P}_{\mu^+}^{\,2} - m_\mu^2)\, d^4\mathbb{P}_{\mu^+} f(\mathbb{P}_{\mu^+})$$

we have,

$$R = (2\pi)^4 . 2 \int \delta^4 (\mathbb{k}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu^+} - \mathbb{P}_\nu) \frac{d^3\vec{P}_\pi}{E_\pi} \frac{d^3\vec{P}_\nu}{E_\nu} d^4\mathbb{P}_{\mu^+}$$

$$\delta^0 (\mathbb{P}_{\mu^+}^{0\,2} - \vec{P}_{\mu^+}^2 - m_\mu^2) \left\{ |J_1|^2 [2 (\mathbb{k}_1 \cdot \mathbb{P}_{\mu^+})(\mathbb{k}_1 \cdot \mathbb{P}_\nu) - \mathbb{k}_1^2 (\mathbb{P}_{\mu^+} \cdot \mathbb{P}_\nu)] \right.$$

$$\left. + |J_2|^2 (m_\mu^2)(\mathbb{P}_{\mu^+} \cdot \mathbb{P}_\nu) + (J_1^* J_2 + J_1 J_2^*)(m_\mu^2)(\mathbb{k}_1 \cdot \mathbb{P}_\nu) \right\}$$

$$= (2\pi)^4 . 2 \int \frac{d^3\vec{P}_\pi}{E_\pi} \frac{d^3\vec{P}_\nu}{E_\nu} \delta^0 \left\{ (m_K - E_\pi - E_\nu)^2 - (\vec{P}_\pi + \vec{P}_\nu)^2 \right.$$

$$\left. - m_\mu^2 \right\} \left\{ |J_1|^2 [2 (\mathbb{k}_1 \cdot \mathbb{k}_1 - \mathbb{P}_\pi - \mathbb{P}_\nu)(\mathbb{k}_1 \cdot \mathbb{P}_\nu) - \mathbb{k}_1^2 (\mathbb{k}_1 - \mathbb{P}_\pi - \mathbb{P}_\nu \right.$$

$$\cdot \mathbb{P}_\nu)] + |J_2|^2 m_\mu^2 (\mathbb{k}_1 - \mathbb{P}_\pi - \mathbb{P}_\nu \cdot \mathbb{P}_\nu) + (J_1^* J_2 + J_1 J_2^*)(m_\mu^2)$$

$$\left. \times (\mathbb{k}_1 \cdot \mathbb{P}_\nu) \right\} \qquad \text{(A-98)}$$

Keeping the direction of $\vec{P}_\pi$ fixed, we want to first integrate over all possible directions of $\vec{P}_\nu$. Denoting the angle between $\vec{P}_\pi$ and $\vec{P}_\nu$ by Θ, we have:

$$\delta^{\circ}\left[(m_K - E_\pi - E_\nu)^2 - (\vec{P}_\pi + \vec{P}_\nu)^2 - m_\mu^2\right]$$

$$= \delta^{\circ}\left[2 P_\pi P_\nu \{ K - \cos\Theta \}\right]$$

$$\leftrightarrow \frac{1}{2 P_\pi P_\nu} \delta^{\circ}(K - \cos\Theta)$$

where $K = \dfrac{1}{2 P_\pi P_\nu}\left[(m_K - E_\pi)^2 - 2(m_K - E_\pi) E_\nu - \vec{P}_\pi^{\,2} - m_\mu^2\right]$

(A-99)

Replacing $d^3\vec{P}_\nu$ in (A-98) by $2\pi \sin\Theta \, d\Theta \, P_\nu^2 \, dP_\nu$ and integrating over Θ with the help of the function $\delta^{\circ}(K - \cos\Theta)$ we get;

$$R = (2\pi)^4 (2\pi) \int \frac{d^3\vec{P}_\pi}{E_\pi} \frac{dE_\nu}{P_\pi} \Big\{ |J_1|^2 m_K^2 \big[2 E_\nu (m_K - E_\pi - E_\nu)$$

$$- \frac{1}{2}(m_K^2 - 2 m_K E_\pi + m_\pi^2 - m_\mu^2) \big] + |J_2|^2 \frac{m_\mu^2}{2} (m_K^2$$

$$- 2 m_K E_\pi + m_\pi^2 - m_\mu^2) + (J_1^* J_2 + J_1 J_2^*) m_\mu^2 m_K E_\nu \Big\}$$

(A-100)

Using conservation of energy and momenta, it is easy to check that the maximum and minimum limits of E_ν needed for the above integration are given by:

$$(E_\nu)^{max}_{min} = \frac{(m_K^2 - 2m_K E_\pi + m_\pi^2 - m_\mu^2)(m_K - E_\pi \pm \sqrt{E_\pi^2 - m_\pi^2})}{2(m_K^2 - 2m_K E_\pi + m_\pi^2)}$$

(A-101)

Using the above limits of integration for E_ν ; replacing $d^3\vec{P}_\pi$ in (A-100) by $4\pi P_\pi^2 \, dP_\pi$ and equating $P_\pi dP_\pi$ to $E_\pi dE_\pi$, we obtain:

$$R = (2\pi)^6 (2m_K) m_\pi^5 \, 2m'_K \int dE'_\pi \sqrt{E'^2_\pi - 1} \left\{ |J_1|^2 m'^2_K \right.$$

$$\times (2I_2 - \tfrac{2}{3} m'_K I_4 - I_1) + |J_2|^2 m'^2_\mu I_1 + (J_1^* J_2 + J_1 J_2^*)$$

$$\left. \times \, m'^2_\mu m'_K I_3 \right\}$$

(A-102)

where,

$$I_1 \equiv \frac{(\alpha' - E'_\pi)^2}{(m'^2_K - 2m'_K E'_\pi + m'^2_\pi)}$$

$$I_2 \equiv \frac{(\alpha' - E'_\pi)^2 (m'_K - E'_\pi)^2}{(m'^2_K - 2m'_K E'_\pi + m'^2_\pi)^2}$$

$$I_3 \equiv \frac{(\alpha' - E'_\pi)^2 (m'_K - E'_\pi)}{(m'^2_K - 2 m'_K E'_\pi + m'^2_\pi)^2}$$

$$I_4 \equiv \frac{(\alpha' - E'_\pi)^3 \{3(m'_K - E'_\pi)^2 + (E'^2_\pi - m'^2_\pi)\}}{(m'^2_K - 2 m'_K E'_\pi + m'^2_\pi)^3}$$

and ,

$$\alpha = \frac{m'^2_K + m'^2_\pi - m'^2_\mu}{2 m'_K} \qquad \text{(A-103)}$$

The primed quantities are dimensionless quantities, having been divided by suitable powers of m_{π^0} . Thus $E'_\pi \equiv E_\pi / m_{\pi^0}$, $m'_K \equiv m_K / m_{\pi^0}$ etc.

The limits of integration over E'_π in eq. (A-102) varies between the maximum and minimum values of E'_π , which are given by:

$$(E'_{\pi^0})_{max} = \frac{m'^2_K + m'^2_{\pi^0} - m'^2_\mu}{2 m'_K}$$

$$(E'_{\pi_0})_{min} = m'_{\pi_0} = 1$$

Performing the integration over dE'_{π} between the above two limits numerically, we have from eqs. (A-94), (A-97) and (A-102)

$$W(K^{+} \to \pi^{0} + \mu^{+} + \nu) = \frac{m_{\pi^0}^5 (m_{K^+}/m_{\pi^0})}{(2\pi)^3} (g_K g'_{\pi} f f')^2 \left[0.431 |J_1|^2 + 0.089 (J_1^* J_2 + J'_1 J_2^*) + 0.042 |J_2|^2\right] \qquad \text{(A-104)}$$

Taking account of the (π^{+} , π^{0}) and (K^{+}, K^{0}) -mass differences in the phase space calculations and neglecting the mass of the electron, the corresponding expressions for decay rates of other $K_{\mu 3}$ and K_{e3} -modes are found to be:

$$W(K^{+} \to \pi^{0} + e^{+} + \nu) = \frac{m_{\pi^0}^5 (m_{K^+}/m_{\pi^0})}{(2\pi)^3} (g_K g_{\pi} f f')^2 \left[0.541 |J_1|^2\right] \qquad \text{(A-105)}$$

$$W(K^{0} \to \pi^{-} + \mu^{+} + \nu) = \frac{m_{\pi^-}^5 (m_{K^0}/m_{\pi^-})}{(2\pi)^3} (g_K \sqrt{2} g_{\pi} f f')^2 \left[0.559 |J_1|^2 + 0.074 (J_1^* J_2 + J_1 J_2^*) + 0.034 |J_2|^2\right] \qquad \text{(A-106)}$$

$$W(K^{0} \to \pi^{-} + e^{+} + \nu) = \frac{m_{\pi^-}^5 (m_{K^0}/m_{\pi^-})}{(2\pi)^3} (g_K \sqrt{2} g_{\pi} f f')^2 \left[0.456 |J_1|^2\right] \qquad \text{(A-107)}$$

We may mention that the right hand side of eq.(A-102) is in fact the pion-energy spectrum in $K_{\mu 3}$-decay, which will be discussed for both local and non-local four-fermion interactions in Appendix V.

C. The Rate of K_{e4} -Decay

Let us first consider the rate of $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ - decay. The matrix element for the process is given by:

$$M(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = (2\pi)^4 \delta^4 (\mathbb{P}_{K^+} - \mathbb{P}_1 - \mathbb{P}_2 - \ell) \left[g_K (\sqrt{2} g_\pi)^2 f f' \right]$$

$$\left(\frac{1}{m_p}\right) \Big[K_1 \mathbb{P}_{1\alpha} + K_2 \mathbb{P}_{2\alpha} + K_3 \ell_\alpha + K_4$$

$$\times \epsilon_{\alpha\beta\gamma\delta} \frac{\mathbb{P}_{1\beta} \mathbb{P}_{2\gamma} \ell_\delta}{m_p^2} \Big] \bar{\nu} \gamma_\alpha (1 + i\gamma_5) e \qquad \text{(A-108)}$$

where $\mathbb{P}_1 \equiv \mathbb{P}_{\pi^+}$, $\mathbb{P}_2 \equiv \mathbb{P}_{\pi^-}$ and $\ell \equiv \mathbb{P}_{e^+} + \mathbb{P}_\nu$

Neglecting the mass of the electron, we can drop the K_3 - term in eq. (A-108). We shal also drop the K_4 -term, since it is expected to be two orders of magnitude smaller than the other terms in powers of m_p (See Chapter III A).

The decay rate in the rest frame of K-meson is then given by:

$$W(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = \int \left\{ \left[\frac{1}{(2\pi)^{3/2}}\right]^4 \sqrt{\frac{m_e}{E_e}} \sqrt{\frac{m_\nu}{E_\nu}} \sqrt{\frac{1}{2E_1}} \sqrt{\frac{1}{2E_2}} \sqrt{\frac{1}{2m_{K^+}}} \right\}^2 \sum_{\substack{e\,\nu \\ \text{spins}}} |M|^2 \, d^3\vec{P}_1 \, d^3\vec{P}_2 \, d^3\vec{P}_e \, d^3\vec{P}_\nu$$

$$= \frac{m_e m_\nu}{(2\pi)^{12}(8m_K)} \left[g_K(\sqrt{2}\,g_\pi)^2 f f'\right]^2 (1/m_p^2) \int (2\pi)^4$$

$$\delta^4(\mathbb{P}_{K^+} - \mathbb{P}_1 - \mathbb{P}_2 - \ell) \frac{d^3\vec{P}_1}{E_1} \frac{d^3\vec{P}_2}{E_2} \frac{d^3\vec{P}_e}{E_e} \frac{d^3\vec{P}_\nu}{E_\nu}$$

$$\sum_{\text{spins}} \left|(K_1 \mathbb{P}_{1\alpha} + K_2 \mathbb{P}_{2\alpha})(\bar{\nu}\gamma_\alpha(1+i\gamma_5)e)\right|^2 \qquad \text{(A-109)}$$

Sum Over Spins

$$\sum_{\text{spins}} \left|\bar{\nu}(K_1 \not{\mathbb{P}}_1 + K_2 \not{\mathbb{P}}_2)(1+i\gamma_5)e\right|^2$$

$$= \sum_{\text{spins}} \left[\bar{\nu}(K_1 \not{\mathbb{P}}_1 + K_2 \not{\mathbb{P}}_2)(1+i\gamma_5)e\,\bar{e}\,(\overset{*}{K}_1 \not{\mathbb{P}}_1 + \overset{*}{K}_2 \not{\mathbb{P}}_2)(1+i\gamma_5)\nu\right]$$

$$= \mathrm{Tr}\left[(K_1 \not{\mathbb{P}}_1 + K_2 \not{\mathbb{P}}_2)(1+i\gamma_5)\frac{(-m_e + \not{\mathbb{P}}_{e^+})}{2m_e}(\overset{*}{K}_1 \not{\mathbb{P}}_1 + \overset{*}{K}_2 \not{\mathbb{P}}_2)\right.$$

$$\left.(1+i\gamma_5)\frac{(m_\nu + \not{\mathbb{P}}_\nu)}{2m_\nu}\right]$$

$$= \frac{2}{m_e m_\nu}\left[|K_1|^2 \left\{2(\mathbb{P}_1 \cdot \mathbb{P}_{e^+})(\mathbb{P}_1 \cdot \mathbb{P}_\nu) - \mathbb{P}_1^2 (\mathbb{P}_{e^+} \cdot \mathbb{P}_\nu)\right\} + |K_2|^2\right.$$

$$\times\left\{2(\mathbb{P}_2 \cdot \mathbb{P}_{e^+})(\mathbb{P}_2 \cdot \mathbb{P}_\nu) - \mathbb{P}_2^2 (\mathbb{P}_{e^+} \cdot \mathbb{P}_\nu)\right\} + (\overset{*}{K}_1 K_2 + K_1 \overset{*}{K}_2)\left\{(\mathbb{P}_1 \cdot \mathbb{P}_{e^+})\right.$$

$$\left.\left.\times(\mathbb{P}_2 \cdot \mathbb{P}_\nu) + (\mathbb{P}_2 \cdot \mathbb{P}_{e^+})(\mathbb{P}_1 \cdot \mathbb{P}_\nu) - (\mathbb{P}_1 \cdot \mathbb{P}_2)(\mathbb{P}_{e^+} \cdot \mathbb{P}_\nu)\right\}\right]$$

(A-110)

Substituting (A-110) into (A-109), the decay rate is given by

$$W(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = \frac{[g_K(\sqrt{2}\,g_\pi)^2 f f']^2}{(2\pi)^{12}(4m_K)\,m_p^2}\,\mathcal{R} \qquad \text{(A-111)}$$

where $\mathcal{R}$ is the Lorentz-invariant expression defined by

$$\begin{aligned}\mathcal{R} \equiv \int & (2\pi)^4 \delta^4(\mathbb{P}_{K^+} - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_{e^+} - \mathbb{P}_\nu)\,\frac{d^3\vec{P}_1}{E_1}\,\frac{d^3\vec{P}_2}{E_2} \\ & \frac{d^3\vec{P}_{e^+}}{E_{e^+}}\,\frac{d^3\vec{P}_\nu}{E_\nu}\Big\{|K_1|^2\,[2(\mathbb{P}_1\cdot\mathbb{P}_{e^+})(\mathbb{P}_1\cdot\mathbb{P}_\nu) - \mathbb{P}_1^2 \\ & \times(\mathbb{P}_{e^+}\cdot\mathbb{P}_\nu)] + |K_2|^2\,[2(\mathbb{P}_2\cdot\mathbb{P}_{e^+})(\mathbb{P}_2\cdot\mathbb{P}_\nu) - \mathbb{P}_2^2(\mathbb{P}_{e^+}\cdot\mathbb{P}_\nu)] \\ & + (\overset{*}{K}_1 K_2 + K_1 \overset{*}{K}_2)\,[(\mathbb{P}_1\cdot\mathbb{P}_{e^+})(\mathbb{P}_2\cdot\mathbb{P}_\nu) + (\mathbb{P}_2\cdot\mathbb{P}_{e^+})(\mathbb{P}_1\cdot\mathbb{P}_\nu) \\ & - (\mathbb{P}_1\cdot\mathbb{P}_2)(\mathbb{P}_{e^+}\cdot\mathbb{P}_\nu)]\Big\} \end{aligned} \qquad \text{(A-112)}$$

In order to facilitate the evaluation of R, we shall make a Lorentz-transformation to a frame $\mathcal{O}'$, in which the vector sum of the 3-momenta of π^+, π^- and ν is zero, from the initially adopted frame $\mathcal{O}$, in which the decaying K-meson is at rest.

Denoting the respective momenta and energies in $\mathcal{O}'$ by $\vec{q}_i\,'s$ and $\omega_i'\,s$, which are denoted by $\vec{P}_i\,'s$ and $E_i'\,s$ in $\mathcal{O}$, we have the following relations:

Frame $\mathcal{O}$		Frame $\mathcal{O}'$
$(\vec{P}_1, E_1)$	$\longrightarrow$	$(\vec{q}_1, \omega_1)$
$(\vec{P}_2, E_2)$	$\longrightarrow$	$(\vec{q}_2, \omega_2)$
$(\vec{P}_{e^+}, E_{e^+})$	$\longrightarrow$	$(\vec{q}_{e^+}, \omega_{e^+})$
$(\vec{P}_\nu, E_\nu)$	$\longrightarrow$	$(\vec{q}_\nu, \omega_\nu)$
$(0, m_K)$	$\longrightarrow$	$(\vec{q}_K, \omega_K)$

$$
\left.\begin{array}{l}
\vec{P}_{K^+} = \vec{P}_1 + \vec{P}_2 + \vec{P}_{e^+} + \vec{P}_\nu \\
\quad = 0 \\
m_K = E_1 + E_2 + E_{e^+} + E_\nu \\
\\
\delta^4(\mathbb{P}_{K^+} - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_{e^+} - \mathbb{P}_\nu)
\end{array}\right\} \longrightarrow
\left\{\begin{array}{l}
\vec{q}_K = \vec{q}_{e^+} \\
\vec{q}_1 + \vec{q}_2 + \vec{q}_\nu = 0 \\
\omega_K = \omega_1 + \omega_2 + \omega_e + \omega_\nu \\
\delta^3(\vec{q}_1 + \vec{q}_2 + \vec{q}_\nu)\,\delta(\omega_1 + \omega_2 + \omega_{e^+} + \omega_\nu - \omega_K)
\end{array}\right.
$$

Clearly the velocity of $\mathcal{O}$ relative to $\mathcal{O}'$ is:

$$
\vec{v} = -\frac{\vec{P}_1 + \vec{P}_2 + \vec{P}_\nu}{E_1 + E_2 + E_\nu} = \frac{\vec{P}_e}{m_K - E_e}
$$

Neglecting the mass of the electron, the momentum and energy of K^+ in O' are given by:

$$\vec{q}_K = \frac{m_K \vec{v}}{\sqrt{1-v^2}} \approx \sqrt{\frac{m_K}{m_K - 2E_{e+}}}\ \vec{p}_{e+}$$

$$\omega_K \approx (m_K - E_{e+})\sqrt{\frac{m_K}{m_K - 2E_{e+}}} \qquad \text{(A-113)}$$

Hence the momentum and energy of the electron in O' are given by:

$$\left.\begin{aligned} \vec{q}_{e+} = \vec{q}_{K+} &= \sqrt{\frac{m_K}{m_K - 2E_{e+}}}\ \vec{p}_{e+} \\ \omega_{e+} &\approx \sqrt{\frac{m_K}{m_K - 2E_{e+}}}\ p_{e+} \end{aligned}\right\} \qquad \text{(A-114)}$$

Since the ratios $\frac{d^3\vec{p}_i}{E_i}$ are Lorentz-invariant quantities, R is therefore given by:

$$\begin{aligned} R = (2\pi)^4 \int & \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_\nu)\, \delta(\omega_1 + \omega_2 + \omega_\nu - \epsilon)\, \frac{d^3\vec{q}_1}{\omega_1}\, \frac{d^3\vec{q}_2}{\omega_2} \\ & \frac{d^3\vec{q}_\nu}{\omega_\nu}\, \frac{d^3\vec{p}_e}{E_e} \Big[|K_1|^2 \{ 2(q_1 \cdot p_{e+})(q_1 \cdot q_\nu) - q_1^2 (p_{e+} \cdot q_\nu) \} \\ & + |K_2|^2 \{ 2(q_2 \cdot p_{e+})(q_2 \cdot q_\nu) - q_2^2 (p_{e+} \cdot q_\nu) \} + (K_1^* K_2 + K_1 K_2^*) \\ & \times \{ (q_1 \cdot p_{e+})(q_2 \cdot q_\nu) + (q_2 \cdot p_{e+})(q_1 \cdot q_\nu) - (q_1 \cdot q_2)(p_{e+} \cdot q_\nu) \} \Big] \\ & \times \sqrt{m_K / (m_K - 2E_{e+})} \end{aligned} \qquad \text{(A-115)}$$

where $$\epsilon \equiv \omega_K - \omega_{e+} \approx \sqrt{m_K(m_K - 2E_{e+})} \qquad \text{(A-116)}$$

Using the symmetry of the integrand in q_1, and q_2, R is given by:

$$R = (2\pi)^4 \int \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_\nu)\,\delta(\omega_1 + \omega_2 + \omega_\nu - \epsilon)\,\frac{d^3\vec{q}_1}{\omega_1}\,\frac{d^3\vec{q}_2}{\omega_2}$$
$$d^3\vec{p}_\nu/\omega_\nu \; d^3\vec{p}_e/E_e \Big[(|K_1|^2 + |K_2|^2 \{2(q_1 \cdot p_{e+})$$
$$\times (q_1 \cdot q_\nu) - q_1^2 (p_{e+} \cdot q_\nu)\} + (K_1^* K_2 + K_1 K_2^*)\{2(q_1 \cdot p_{e+})$$
$$\times (q_\nu \cdot q_2) - (q_1 \cdot q_2)(p_{e+} \cdot q_\nu)\}\Big] \sqrt{\frac{m_K}{m_K - 2E_{e+}}} \qquad \text{(A-117)}$$

Let us transform the variables q_1 and q_2 to Q and R defined by:

$$Q \equiv q_1 + q_2 \qquad ; \qquad R \equiv q_1 - q_2 \qquad \text{(A-118)}$$

Using;

$$\frac{d^3\vec{q}_1}{\omega_1}\,\frac{d^3\vec{q}_2}{\omega_2} \rightarrow 4\,\delta^0(q_1^2 - m_\pi^2)\,\delta^0(q_2 - m_\pi^2)\,d^4q_1\,d^4q_2$$
$$\rightarrow \delta^0(Q \cdot R)\,\delta^0(Q^2 + R^2 - 4m_\pi^2)\,d^4Q\,d^4R \qquad \text{(A-119)}$$

we have:

$$R = (2\pi)^4 \int \delta^3(\vec{Q} + \vec{q}_\nu)\,\delta^0(Q_0 + \omega_\nu - \epsilon)\,\delta^0(Q \cdot R)\,\delta^0(Q^2 + R^2$$
$$- 4m_\pi^2)\,d^4Q\,d^4R\,\frac{d^3\vec{q}_\nu}{\omega_\nu}\,\frac{d^3\vec{p}_e}{E_e}\,\left(\tfrac{1}{2}\right)\Big[(Q \cdot p_e)(Q \cdot q_\nu)$$
$$\times |K_1 + K_2|^2 + (R \cdot p_e)(R \cdot q_\nu)|K_1 - K_2|^2 - 2m_\pi^2(p_{e+} \cdot q_\nu)$$

$$\times\,(|K_1|^2+|K_2|^2) - (\mathbb{Q}^2-\mathbb{R}^2)(\mathbb{P}_e\cdot q_\nu)\frac{(K_1^* K_2 + K_1 K_2^*)}{2}\Bigg]\sqrt{\frac{m_K}{m_K-2E_{e^+}}}$$

(A-120)

In writing $\mathcal{R}$ in the above form we have used the fact that terms, which are linear in $\mathbb{R}$ do not contribute to the integral by symmetry.

We shall first perform the $\mathbb{Q}$-integration by using the following:

$$\int \delta^3(\vec{Q}+\vec{q}_\nu)\,\delta^0(Q_0+\omega_\nu-\epsilon)\,f(\mathbb{Q})\,d^4\mathbb{Q} = f(\mathbb{Q}') \qquad \text{(A-121)}$$

where $\mathbb{Q}'$ is a four vector such that;

$$\vec{Q}' = -\vec{q}_\nu$$
$$Q_0' = \epsilon-\omega_\nu \qquad \text{(A-122)}$$

$\mathcal{R}$ then reduces to:

$$\mathcal{R} = (2\pi)^4(1/2)\int \delta^0(\mathbb{Q}'\cdot\mathbb{R})\,\delta^0(\mathbb{Q}'^2+\mathbb{R}^2-4m_\pi^2)\,d^4\mathbb{R}\,\frac{d^3\vec{q}_\nu}{\omega_\nu}\,\frac{d^3\vec{P}_e}{E_e}$$

$$\Big[(\mathbb{Q}'\cdot\mathbb{P}_e)(\mathbb{Q}'\cdot q_\nu)\,|K_1+K_2|^2 + (\mathbb{R}\cdot\mathbb{P}_e)(\mathbb{R}\cdot q_\nu)\,|K_1-K_2|^2$$

$$-\,2m_\pi^2\,(\mathbb{P}_e\cdot q_\nu)(|K_1|^2+|K_2|^2) - (\mathbb{Q}'^2-\mathbb{R}^2)(\mathbb{P}_e\cdot q_\nu)$$

$$\times\,\frac{(K_1^* K_2 + K_1 K_2^*)}{2}\Bigg]\sqrt{\frac{m_K}{m_K-2E_{e^+}}} \qquad \text{(A-123)}$$

We now need the following integrations over $\mathbb{R}$:

$$1 = \int \delta(\mathbb{Q}'.\mathbb{R})\,\delta(\mathbb{Q}'^2+\mathbb{R}^2-4m_\pi^2)\,d^4\mathbb{R}$$

$$2 = \int \delta(\mathbb{Q}'.\mathbb{R})\,\delta(\mathbb{Q}'^2+\mathbb{R}^2-4m_\pi^2)(\mathbb{R}.\mathbb{P}_2)(\mathbb{R}.\mathbb{q}_2)\,d^4\mathbb{R}$$

$$3 = \int \delta(\mathbb{Q}'.\mathbb{R})\,\delta(\mathbb{Q}'^2+\mathbb{R}^2-4m_\pi^2)\,\mathbb{R}^2\,d^4\mathbb{R} \tag{A-124}$$

(1), (2) and (3) can readily be evaluated by first transforming to the rest frame $\mathcal{O}''$ of $\overrightarrow{Q'}$ and then transforming the result back to the initial frame $\mathcal{O}'$. Denoting the transformed four-vectors in $\mathcal{O}''$ by the corresponding starred quantities, we have:

$$1 = \int \delta(\overset{*}{Q}{}'_0\overset{*}{\mathbb{R}}_0)\,\delta(\overset{*}{\mathbb{Q}}{}'^2+\mathbb{R}^{*2}-4m_\pi^2)\,d^4\mathbb{R}^*$$

$$= \frac{1}{\overset{*}{Q}{}'_0}\int \frac{1}{2\sqrt{\overset{*}{Q}{}_0'^2-4m_\pi^2}}\left\{\delta\left(\overset{*}{R}+\sqrt{\overset{*}{Q}{}_0'^2-4m_\pi^2}\right)+\delta\left(R-\sqrt{\overset{*}{Q}{}_0'^2-4m_\pi^2}\right)\right\}d^3\overrightarrow{R^*}$$

$$= \frac{2\pi}{Q_0^{*'}} \sqrt{(Q_0^{*'})^2 - 4m_\pi^2}$$

$$= 2\pi \sqrt{\frac{Q'^2 - 4m_\pi^2}{Q'^2}} \qquad \text{(A-125)}$$

Similarly:

$$② = \int \delta(Q_0^{*'} R_0^*)\, \delta(Q^{*'2} + R^{*2} - 4m_\pi^2)(R^* \cdot k_e^*)(R^* \cdot q_\nu^*)\, d^4R^*$$

$$= \frac{2\pi}{3} (\vec{k}_e^* \cdot \vec{q}_\nu^*) \frac{\left(\sqrt{Q^{*2} - 4m_\pi^2}\right)^3}{Q_0^{*'}}$$

$$= -\frac{2\pi}{3} \left[(k_e \cdot q_\nu) - \frac{(k_e \cdot Q')(q_\nu \cdot Q')}{Q'^2} \right] (Q'^2 - 4m_\pi^2)$$

$$\times \sqrt{(Q'^2 - 4m_\pi^2)/Q'^2} \qquad \text{(A-126)}$$

$$③ = \int \delta(Q_0^{*'} R_0^*)\, \delta(Q^{*'} + R^{*2} - 4m_\pi^2)\, R^{*2}\, d^4R^*$$

$$= -(2\pi)(Q'^2 - 4m_\pi^2) \sqrt{\frac{(Q'^2 - 4m_\pi^2)}{Q'^2}} \qquad \text{(A-127)}$$

Collecting the results (A-123), (A-124), (A-125), (A-126), and (A-127), we have:

$$R = (2\pi)^4 \frac{(2\pi)}{2} \int \sqrt{\frac{Q'^2 - 4m_\pi^2}{Q'^2}} \sqrt{\frac{m_K}{m_K - 2E_{e^+}}} \frac{d^3\vec{q}_\nu}{\omega_\nu} \frac{d^3\vec{p}_e}{E_e}$$

$$\Big[(Q'.p_e)(Q'.q_\nu)|K_1+K_2|^2 - 2m_\pi^2 (p_e\cdot q_\nu)(|K_1|^2+|K_2|^2)$$

$$- Q'^2 (p_e\cdot q_\nu)(K_1^* K_2 + K_1 K_2^*)/2 + (Q'^2 - 4m_\pi^2)\Big\{-\frac{1}{3}\Big[(p_e\cdot q_\nu)$$

$$- \frac{(p_e\cdot Q')(q_\nu\cdot Q')}{Q'^2}\Big]|K_1-K_2|^2 - (p_e\cdot q_\nu)(K_1^* K_2 + K_1 K_2^*)/2\Big\}\Big]$$

(A-128)

Substituting the value of Q', given by eq. (A-122) and integrating over the directions of $\vec{p}_e$ and $\vec{q}_\nu$, we have:

$$R = (2\pi)^4 (4\pi)^2 \pi \int p_e\, dp_e\, q_\nu\, dq_\nu \sqrt{\frac{\epsilon^2 - 2q_\nu\epsilon - 4m_\pi^2}{\epsilon^2 - 2q_\nu\epsilon}}$$

$$\sqrt{\frac{m_K}{m_K - 2p_e}}\ (p_e q_\nu)\Big[(\epsilon^2 - \epsilon q_\nu)|K_1+K_2|^2 - 2m_\pi^2$$

$$\times(|K_1|^2 + |K_2|^2) - (\epsilon^2 - 2\epsilon q_\nu)(K_1^* K_2 + K_1 K_2^*)/2$$

$$+ (\epsilon^2 - 2\epsilon q_\nu - 4m_\pi^2)\Big\{q_\nu/3(\epsilon - 2q_\nu)|K_1-K_2|^2$$

$$- (K_1^* K_2 + K_1 K_2^*)/2\Big\}\Big] \qquad \text{(A-129)}$$

In writing R in the above form, we have neglected the mass of the electron. From the kinematics involved in the decay, it is easy to check that,

$$(p_e)_{max} = \frac{m_K^2 - 4m_\pi^2}{2m_K}$$

$$(q_\nu)_{max} = \frac{\epsilon^2 - 4m_\pi^2}{2\epsilon} \qquad \text{(A-130)}$$

Hence, from eqs. (A-109), (A-112) and (A-129), we have:

$$W(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = \frac{[g_K(\sqrt{2}g_\pi f f')]^2}{64\pi^5 m_p^2} m_\pi^7 P \qquad \text{(A-131)}$$

where,

$$P \equiv \frac{1}{m'_K}\int_0^{\frac{m'^2_K-4}{2m'_K}} p'^2_e \, dp'_e \int_0^{\frac{\epsilon'^2-4}{2\epsilon'}} q'^2_\nu \, dq'_\nu \sqrt{\frac{\epsilon'^2 - 2q'_\nu\epsilon' - 4}{\epsilon'^2 - 2q'_\nu\epsilon'}}$$

$$\sqrt{\frac{m'_K}{m'_K - 2p'_e}} \Big[(\epsilon'^2 - \epsilon' q'_\nu)|K_1 + K_2|^2 - 2(|K_1|^2 + |K_2|^2)$$

$$- (\epsilon'^2 - 2\epsilon' q'_\nu)(K_1^* K_2 + K_1 K_2^*)/2 + (\epsilon'^2 - 2\epsilon' q'_\nu - 4)$$

$$\times \left\{ \frac{1}{3} \frac{q'_\nu}{\epsilon' - 2q'_\nu} |K_1 - K_2|^2 - (K_1^* K_2 + K_1 K_2^*)/2 \right\} \qquad \text{(A-132)}$$

The primed quantities are dimensionless quantities, having been divided by m_π, i.e. $m'_K \equiv m_K/m_\pi$, $\epsilon' = \epsilon/m_\pi$ etc.

By perturbation calculation (See Appendix III F) K_2-term is proportional to ($m_p - m_\Lambda$). Hence for the sake of simplicity we drop the K_2-term in the evaluation of $\mathbb{P}$. By performing the double integral in eq. (A-132) numerically, we then obtain:

$$\mathbb{P} \approx 0.02\,|K_1|^2 \qquad \text{(A-133)}$$

Thus the decay-rate is given by:

$$\boxed{W(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = \frac{[g_K(\sqrt{2}g_\pi)^2 f f']^2}{64\pi^5 m_p^2}\, m_\pi^7\, [0.021|K_1|^2]\cdot}$$

(A-134)

The matrix-element for $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$-decay, after symmetrisation between the two neutral pions, is given by:

$$M(K^+ \to \pi^0 + \pi^0 + e^+ + \nu) = (2\pi)^4 \delta^4(\mathbb{P}_{K^+} - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_{e^+} - \mathbb{P}_\nu)[g_K g_\pi^2 f f']$$

$$(1/m_p)\Big[\frac{(K_1+K_2)(\mathbb{P}_1+\mathbb{P}_2)_\alpha}{\sqrt{2}} + \sqrt{2}\,K_3\,\ell_\alpha$$

$$+\sqrt{2}\,\frac{K_4\,\epsilon_{\alpha\beta\gamma\delta}\,\mathbb{P}_{1\beta}\,\mathbb{P}_{2\gamma}\,\ell_\delta}{m_p^2}\Big]\,\bar{\nu}\,\gamma_\alpha(1+i\gamma_5)e$$

(A-135)

We shall drop the K_3 and K_4 -terms as in case of $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ -decay.

By comparing the matrix elements given by (A-108) and (A-135), the decay rate for $K^+ \to \pi^0 + \pi^0 + e^+ + \nu$ can clearly be obtained from eqs. (A-131) and (A-132) be replacing ($\sqrt{2}\, g_\pi$) by g_π and both K_1 and K_2 by ($\frac{K_1+K_2}{\sqrt{2}}$) . We then obtain:

$$\boxed{W(K^+ \to \pi^0 + \pi^0 + e^+ + \nu) \approx \frac{[g_K g_\pi^2 f f']^2}{64\pi^5 m_p^2}\, m_\pi^7 \left[0.067 \left|\frac{K_1+K_2}{\sqrt{2}}\right|^2\right]} \tag{A-136}$$

<u>Note</u> - For a slightly different method from the one given here to obtain the K_{e4}-rate, see L.B. Okun and P. Shabalin - JETP (U.S.S.R.) - 37, 1755 (December, 1959). For an earlier work, involving the comparison of K_{e4} and K_{e3} -rates from the available phase space, see S. Oneda, Nuclear Physics 4, 21 (1957).

D. The Rate of $K \to 2\pi$ – Decays

Let us write the matrix element of $K \to 2\pi$ -decay in the form

$$M(K \to 2\pi) = (2\pi)^4 \delta^4(\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2)\, t \tag{A-137}$$

where t is a constant, since the present case is a two body decay. (compare with eqs. A-115,116, and 117). $\mathbb{K}_1$, $\mathbb{P}_1$ and $\mathbb{P}_2$ denote the 4-momenta of the K-meson and two pions respectively. Hence the decay-rate, in the rest frame of the K-meson is given by:

$$W(K \to 2\pi) = \int \left[\frac{1}{(2\pi)^3} \sqrt{\frac{1}{2E_1}} \sqrt{\frac{1}{2E_2}} \sqrt{\frac{1}{2E_K}} \right]^2 |M|^2 d^3\vec{P}_1 d^3\vec{P}_2$$

$$= \frac{|t|^2}{(2\pi)^6 (2m_K)} \int (2\pi)^4 \delta^4(\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2) \frac{d^3\vec{P}_1}{2E_1} \frac{d^3\vec{P}_2}{2E_2}$$

$$= \frac{|t|^2}{(2\pi)^2 4m_K} \int \delta^0(\mathbb{P}_1^{o2} - \vec{P}_1^2 - m_{\pi_1}^2)\, \delta^4(\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2)\, d^4\mathbb{P}_1 \frac{d^3\vec{P}_2}{2E_2}$$

$$= \frac{|t|^2}{(2\pi)^2 4m_K} \int \delta^0\left[(m_K - E_2)^2 - \vec{P}_2^2 - m_{\pi_1}^2\right] \frac{d^3\vec{P}_2}{E_2}$$

$$= \frac{4\pi |t|^2}{(4\pi^2)(4m_K)} \int \delta^0(m_K^2 - 2m_K E_2) \sqrt{E_2^2 - m_{\pi_2}^2}\, dE_2$$

$$= \left[\sqrt{1 - 4(m_\pi/m_K)^2} \Big/ 16\pi m_K \right] |t|^2 \tag{A-138}$$

where, we have used the fact that $m_{\pi_1} \approx m_{\pi_2} \approx m_\pi$.

E. The Rate of $K \to 3\pi$ -Decays

Let us write the matrix element of $K \to 3\pi$ -decay in the form:

$$M = (2\pi)^4 \delta^4 (\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_3)\, t \qquad \text{(A-139)}$$

we assume, t is independent of the pion momenta $\mathbb{P}_1$, $\mathbb{P}_2$ and $\mathbb{P}_3$, as is the case for the matrix element of fig. 23. [See eq. (A-82)]. The decay rate, in the rest frame of the K-meson is given by:

$$W(K \to 3\pi) = \int \left[\frac{1}{(2\pi)^{9/2}} \sqrt{\frac{1}{2E_1}} \sqrt{\frac{1}{2E_2}} \sqrt{\frac{1}{2E_3}} \sqrt{\frac{1}{2E_K}} \right]^2 |M|^2 d^3\vec{P}_1 d^3\vec{P}_2 d^3\vec{P}_3$$

$$= \frac{|t|^2}{(2\pi)^9 (16 m_K)} \int (2\pi)^4 \delta^4 (\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_3) \frac{d^3\vec{P}_1}{E_1} \frac{d^3\vec{P}_2}{E_2} \frac{d^3\vec{P}_3}{E_3}$$

$$= \frac{|t|^2}{(2\pi)^5 (8 m_K)} \int \delta^4 (\mathbb{K}_1 - \mathbb{P}_1 - \mathbb{P}_2 - \mathbb{P}_3) \delta^0 (\mathbb{P}_1^2 - \vec{P}_1^2 - m_{\pi_1}^2) d^4\mathbb{P}_1$$

$$\frac{d^3\vec{P}_2}{E_2} \cdot \frac{d^3\vec{P}_3}{E_3}$$

$$= \frac{|t|^2}{(2\pi)^5 (8 m_K)} \int \delta^0 \left[(m_K - E_2 - E_3)^2 - (\vec{P}_2 + \vec{P}_3)^2 - m_{\pi_1}^2 \right] \frac{d^3\vec{P}_2}{E_2} \frac{d^3\vec{P}_3}{E_3}$$

$$= \frac{|t|^2}{(2\pi)^5 8 m_K} \int \frac{d^3\vec{P}_3}{E_2\ E_3} \delta^0 \left\{ 2 P_2 P_3 \left[\cos\theta - \frac{(m_K - E_2 - E_3)^2 - P_2^2 - P_3^2 - m_{\pi_1}^2}{2 P_2 P_3} \right] \right\} (2\pi P_2^2 dP_2 d(\cos\theta))$$

where Θ denotes the angle between $\vec{p}_2$ and $\vec{p}_3$. Using the δ° -function, the above expression reduces to,

$$W(K \to 3\pi) = \frac{|t|^2 \pi}{(2\pi)^5 (8m_K)} \int \frac{d^3\vec{p}_3}{E_3\, p_3} \left[E_2\right]_{min}^{max} \tag{A-140}$$

The above maximum and minimum values of E_2 correspond to given value of E_3. It is easy to check;

$$\left(E_2\right)_{min}^{max} = \frac{1}{2\alpha} \Big[\left\{ \alpha + (m_{\pi_2}^2 - m_{\pi_1}^2) \right\} (m_K - E_3)$$
$$\pm \Big\{ (E_3^2 - m_{\pi_3}^2) \left[\alpha^2 + 2\alpha (m_{\pi_2}^2 - m_{\pi_1}^2) \right.$$
$$\left. + (m_{\pi_2}^2 - m_{\pi_1}^2) - 4\alpha\, m_{\pi_2}^2 \right] \Big\}^{1/2} \Big] \tag{A-141}$$

where,

$$\alpha \equiv m_K^2 - 2m_K E_3 + m_{\pi_3}^2 \tag{A-142}$$

Substituting eq. (A-141) in (A-140), and neglecting the difference between $m_{\pi 1}$, $m_{\pi 2}$ and $m_{\pi 3}$, we have

$$W(K \to 3\pi) = \frac{|t|^2 \pi}{(2\pi)^5 8 m_K} \int_{(E_3)_{min}}^{(E_3)_{max}} 4\pi \sqrt{E_3^2 - m_{\pi_3}^2} \left\{ (m_K^2 - 2m_K E_3 - 3m_{\pi_3}^2) \right.$$

$$\left. (m_K^2 - 2m_K E_3 + m_{\pi_2}^2)^{-1} \right\}^{1/2} dE_3$$

(A-143)

The minimum and maximum values of E_3 are :

$$(E_3)_{min.} = m_\pi$$

$$(E_3)_{max} = \frac{m_K^2 - 3m_\pi^2}{2 m_K} \qquad \text{(A-144)}$$

carrying out the integration over E_3 numerically we obtain,

$$W(K \to 3\pi) \simeq \frac{|t|^2 (m_\pi^2 / m_K)}{128 \pi^3} (0.16)$$

(A-145)

For simplicity we have used <u>only</u> K^+ and π^+ -meson-masses for m_K and m_π .

APPENDIX V

ENERGY SPECTRA IN $K_{\mu 3}$ AND K_{e3}-DECAYS FOR LOCAL AND NON-LOCAL FOUR-FERMION INTERACTION

In this Appendix, we shall derive the energy spectra of the muon and the pion in $K_{\mu 3}$-decay first for the case when the four fermion interaction is mediated by a vector boson. The expressions for the local-interaction are then obtained by taking the limit, in which the mass m_B of the vector boson goes to infinity.

The energy spectra for K_{e3}-decay are obtained from the corresponding expressions for $K_{\mu 3}$-decay by replacing the mass of muon by that of the electron.

We first obtain the energy spectrum of the muon:

Muon Energy Spectrum

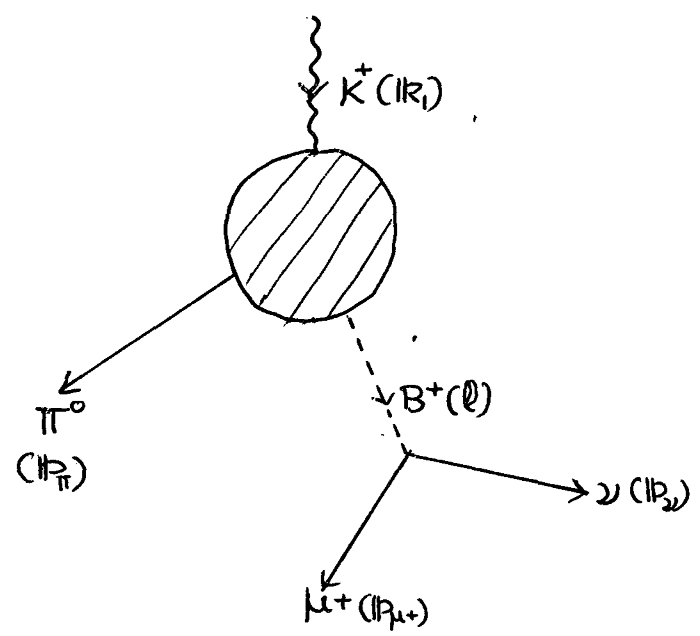


FIG.-37

If we assume that the four-fermion interactions are intermediated by a charged vector boson, the $K_{\mu 3}$ -decay process can be represented by Fig. (37). The matrix-element of Fig. (37) can be written as [See eq. (177)]

$$M\,(\text{fig.-}37) = F\,(2\pi)^4\,\delta^4(\mathbb{k}_1 - \mathbb{p}_\pi - \mathbb{p}_{\mu+} - \mathbb{p}_\nu)\,[J_1\,\mathbb{k}_{1\beta} + J_2'\,\ell_\beta]$$
$$(\ell^2 - m_B^2)^{-1}\,\bar{\nu}\,\gamma_\beta(1 + i\gamma_5)\,\mu \qquad \text{(A-146)}$$

where

$$\ell \equiv \mathbb{p}_{\mu+} + \mathbb{p}_\nu$$

As mentioned before, we neglect the dependence of the form factors J_1 , and J_2' on the pion energy and treat them as constants.

By eq. (A-97) (See Appendix III), the differential transition probability of the K^+ meson at rest decaying to π^0 , μ^+ and ν with four momenta $\mathbb{p}_\pi$, $\mathbb{p}_{\mu+}$ and $\mathbb{p}_\nu$ respectively is given by:

$$W(\mathbb{p}_\mu, \mathbb{p}_\pi, \mathbb{p}_\nu) = \frac{|F|^2}{(2\pi)^5 (2m_K)}\,\delta^4(\mathbb{k}_1 - \mathbb{p}_\pi - \mathbb{p}_{\mu+} - \mathbb{p}_\nu)\,\frac{d^3\vec{p}_\pi}{E_\pi}\,\frac{d^3\vec{p}_{\mu+}}{E_{\mu+}}\,\frac{d^3\vec{p}_\nu}{E_\nu}$$
$$\Big\{ |J_1|^2 \left[2\,(\mathbb{k}_1\cdot\mathbb{p}_{\mu+})(\mathbb{k}_1\cdot\mathbb{p}_\nu) - \mathbb{k}_1^2\,(\mathbb{p}_{\mu+}\cdot\mathbb{p}_\nu) \right]$$
$$+ |J_2'|^2\, m_\mu^2\,(\mathbb{p}_{\mu+}\cdot\mathbb{p}_\nu) + (J_1^* J_2' + J_1 J_2^{*\prime})\, m_\mu^2\,(\mathbb{k}_1\cdot\mathbb{p}_\nu) \Big\}$$
$$\times \left[(m_K^2 + m_\pi^2 - m_B^2)^2 - 2 m_K E_\pi \right]^2$$

$$\text{(A-147)}$$

To obtain the muon-spectrum, we first integrate over $\mathbb{P}_\nu$ and then over $\mathbb{P}_\pi$. Using the relation

$$\int \frac{d^3\vec{P}_\nu}{E_\nu} f(\vec{P}_\nu) = 2\int \delta^0(\mathbb{P}_{\nu 0}^2 - \vec{P}_\nu^2)\, f(\mathbb{P}_\nu)\, d^4\mathbb{P}_\nu$$

and integrating over $d^4\mathbb{P}_\nu$ with the help of the function $\delta^4(\mathbb{R}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu+} - \mathbb{P}_\nu)$, we have from eq. (A-147)

$$W(\mathbb{P}_\mu, \mathbb{P}_\pi) = \frac{|F|^2\, 2}{(2\pi)^5 (2m_K)} \frac{d^3\vec{P}_{\mu+}}{E_{\mu+}} \frac{d^3\vec{P}_\pi}{E_\pi} \delta^0\{(m_K - E_\pi - E_\mu)^2$$

$$- (\vec{P}_\mu + \vec{P}_\pi)^2\} \left[|J_1|^2 \{2(\mathbb{R}_1 \cdot \mathbb{P}_{\mu+})(\mathbb{R}_1 \cdot \mathbb{R}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu+})\right.$$

$$- (\mathbb{R}_1^2)(\mathbb{P}_\mu^+ \cdot \mathbb{R}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu+})\} + |J_2'|^2 m_\mu^2 (\mathbb{P}_{\mu+} \cdot \mathbb{R}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu+})$$

$$\left. + (J_1^* J_2' + J_1 J_2^{*\prime})\, m_\mu^2 (\mathbb{R}_1 \cdot \mathbb{R}_1 - \mathbb{P}_\pi - \mathbb{P}_{\mu+})\}\right]$$

$$\times \{(m_K^2 + m_\pi^2 - m_B^2) - 2 m_K E_\pi\}^{-2}$$

(A-148)

We will first integrate over all possible directions of $\vec{P}_\pi$ by using the δ^0 -function. Denoting the angle between $\vec{P}_\mu$ and $\vec{P}_\pi$ by Θ, we have:

$$\delta^0\{(m_K - E_\pi - E_\mu)^2 - (\vec{P}_\mu + \vec{P}_\pi)^2\} = \frac{1}{2 P_\pi P_\mu}[K - \cos\Theta]$$

(A-149)

where

$$K = \frac{1}{2P_\pi P_\mu}\left[(m_K-E_\pi)^2-2(m_K-E_\pi)E_\mu+m_\mu^2-\vec{P}_\pi^{\,2}\right] \quad \text{(A-150)}$$

Replacing $d^3\vec{P}_\pi$ by $2\pi P_\pi^2\, dP_\pi \sin\Theta\, d\Theta$ and integrating over Θ with the help of the function $\delta^0(K-\cos\Theta)$, we have from eq. (A-148):

$$\begin{aligned} W(P_\mu, P_\pi) = \frac{|F|^2(\pi/m_K)}{(2\pi)^5}\,\frac{d^3\vec{P}_{\mu^+}}{P_\mu E_{\mu^+}}\, dE_\pi \Big[& |J_1|^2 m_K^2 \Big\{ 2E_\mu \\ & \times (m_K-E_\pi-E_\mu) - \tfrac{1}{2}(m_K^2+m_\pi^2-m_\mu^2-2m_K E_\pi)\Big\} \\ & + |J_2'|^2 (m_\mu^2/2)(m_K^2+m_\pi^2-m_\mu^2-2m_K E_\pi) + (J_1^* J_2' + J_1 J_2'^*) \\ & \times (m_\mu^2 m_K)(m_K-E_\pi-E_\mu)\Big]\left[(m_K^2+m_\pi^2-m_B^2)-2m_K E_\pi\right]^{-2} \end{aligned}$$

(A-151)

To get the muon energy spectrum, we finally integrate over E_π. For a given value of E_μ, the maximum and minimum limits of integration for E_π are easily shown to be:

$$(E_\pi)^{max}_{min.} = \frac{1}{2\alpha}\left[\pm(\alpha-m_\pi^2)P_\mu+(\alpha+m_\pi^2)(m_K-E_\mu)\right]$$

(A-152)

where,

$$\alpha \equiv m_K^2 + m_\mu^2 - 2m_K E_\mu \qquad \text{(A-153)}$$

Integrating over E_π between the above two limits and replacing $d^3\vec{P}_\mu$ by $4\pi P_\mu^2 dP_\mu$ in eq. (A-151), we finally obtain the following expression for the muon-energy spectrum with non-local V-A four fermion interaction:

$$W^{\text{NONLOCAL}}(E_\mu) = \frac{|F|^2}{(2\pi)^3 m_K} \Big\{ |J_1|^2 \Big[\{ m_K^2 (2m_K E_\mu - 2E_\mu^2 - a/2)$$

$$+ (b/2) m_K (m_K - 2E_\mu) \} \chi_1 + (m_K - 2E_\mu) m_K^2 \chi_2 \Big]$$

$$+ |J_2'|^2 (m_\mu^2/2) \big[(a-b)\chi_1 - 2m_K \chi_2 \big] + (J_1^* J_2' + J_1 J_2'^*)$$

$$\times (m_\mu^2) \Big[\{ m_K (m_K - E_\mu) - b/2 \} \chi_1 - m_K \chi_2 \Big] \Big\}$$

(A-154)

where,

$$\chi_1 \equiv \frac{(\alpha - m_\pi^2)\sqrt{E_\mu^2 - m_\mu^2}}{\alpha \left\{ b - \frac{m_K}{\alpha}(m_K - E_\mu)(m_\pi^2 + \alpha) - \frac{m_K^2 (\alpha - m_\pi^2)(E_\mu^2 - m_\mu^2)}{2} \right.} \qquad \text{(A-155)}$$

$$\chi_2 \equiv \frac{1}{(2m_K)^2} \log_e \frac{b - \frac{m_K}{\alpha}\left\{ (m_K - E_\mu)(m_\pi^2 + \alpha) + \sqrt{E_\mu^2 - m_\mu^2}\,(\alpha - m_\pi^2) \right\}}{b - \frac{m_K}{\alpha}\left\{ (m_K - E_\mu)(m_\pi^2 + \alpha) - \sqrt{E_\mu^2\, m_\mu^2}\,(\alpha - m_\pi^2) \right\}}$$

(A-156)

with

$$\left.\begin{aligned} a &\equiv m_K^2 + m_\pi^2 - m_\mu^2 \\ b &\equiv m_K^2 + m_\pi^2 - m_B^2 \end{aligned}\right\} \tag{A-157}$$

We can obtain the muon-energy spectrum for contact V-A four-fermion interaction by taking the limit $m_B \to \infty$ and replacing gg'/m_B^2 by ff' (See chapter IV).

It is easy to check that;

$$\left.\begin{aligned} &\lim_{m_B \to \infty} (x_1) = \frac{\eta_1}{m_B^4} \\ &\text{and, } \lim_{m_B \to \infty} \left(m_K x_2 + \frac{b}{2} x_1\right) = \frac{m_K \eta_2}{m_B^4} \end{aligned}\right\} \tag{A-158}$$

where

$$\left.\begin{aligned} \eta_1 &\equiv \frac{(\alpha - m_\pi^2)\sqrt{E_\mu^2 - m_\mu^2}}{\alpha} \\ \eta_2 &\equiv \frac{(\alpha^2 - m_\pi^4)\sqrt{E_\mu^2 - m_\mu^2}\,(m_K - E_\mu)}{2\alpha^2} \end{aligned}\right\} \tag{A-159}$$

Therefore, taking the said limit of $W(E_\mu)$ given by eq. (A-154) and replacing gg' (included in F) by $m_B^2 ff'$, we obtain the following expression for the muon energy spectrum with local V-A four-fermion interaction:

$$\overset{\text{LOCAL}}{W}(E_\mu) = \frac{(g_K g_\pi f f')^2}{(2\pi)^3 (m_K)} \Big\{ |J_1|^2 m_K^2 \big[(2 m_K E_\mu - 2E_\mu^2 - a/2)\eta_1$$

$$+ (m_K - 2E_\mu)\eta_2 \big] + |J_2|^2 (m_\mu^2)(a/2\, \eta_1 - m_K \eta_2)$$

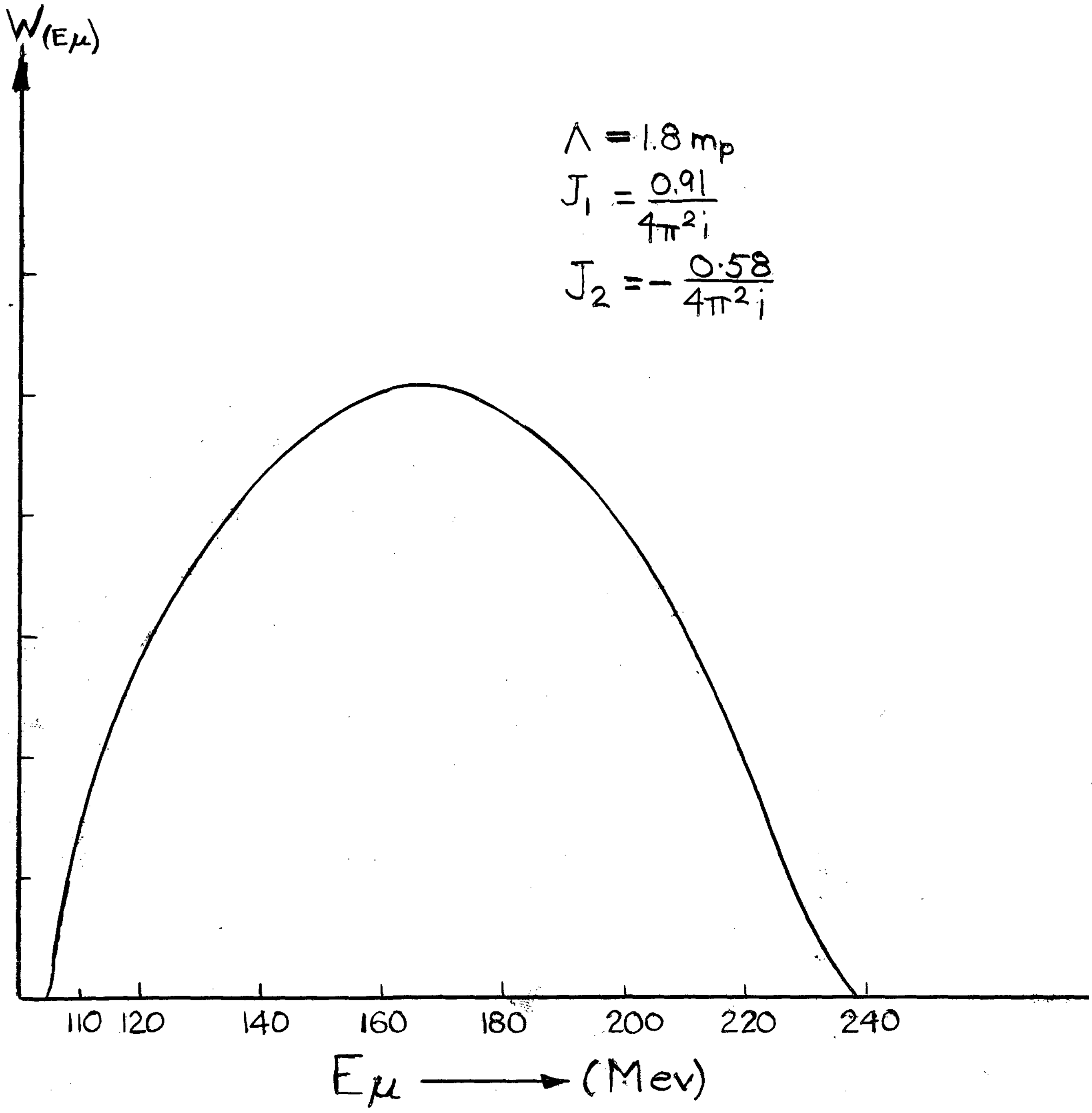
$$+ (\overset{*}{J_1} J_2 + J_1 \overset{*}{J_2})(m_\mu^2)\big[m_K (m_K - E_\mu)\eta_1 - m_K \eta_2 \big] \Big\}$$

(A-160)

The energy spectrum of the muon for local interactions is drawn in fig. 43 with the values of J_1 and J_2 obtained in the lowest order perturbation theory for cut off $\Lambda = 1.8\, m_p$.

Fig. 43

Predicted muon spectrum in $K_{\mu 3}$-decay for local four fermion interaction. J_1 and J_2 have been calculated in the lowest order perturbation theory with cut off $\Lambda = 1.8$ mp.



Pion Energy Spectrum

Comparing eq. (A-97) with eq. (A-147), it is clear that the pion energy spectra for local and non-local four-fermion interactions are related by:

$$\overset{\text{NONLOCAL}}{W}(E_\pi) \propto \frac{\overset{\text{LOCAL}}{W}(E_\pi)}{[(m_K^2+m_\pi^2-m_B^2)-2m_K E_\pi]^2} \qquad \text{(A-161)}$$

The pion spectrum for contact V-A four fermion interaction has been obtained, while deriving the expression for $K_{\mu 3}$ -decay rate (See Appendix IV B). From eqs. (A-94), (A-97) and (A-102) the said spectrum is given by the expression:

$$W(\acute{E}_\pi) = \frac{(g_K g_\pi f f')^2 m_\pi^5 (2\acute{m}_K)}{(2\pi)^3} \sqrt{\acute{E}_\pi^2 - 1}\, \Big[|J_1|^2 \times \acute{m}_K^2 (2I_2 - \tfrac{2}{3}\acute{m}_K I_4 - I_1) + |J_2|^2 \acute{m}_\mu^2 I_1 + (J_1^* J_2 + J_1 J_2^*)\acute{m}_\mu^2 \acute{m}_K I_3 \Big]$$

(A-162)

The corresponding expression for the case of non-local four fermion interaction is therefore given by:

$$\overset{\text{NON LOCAL}}{W(E_\pi)} = \frac{|F|^2 m_\pi (2m'_K)}{(2\pi)^3} \frac{\sqrt{E'^2_\pi - 1}}{(m'^2_K + m'^2_\pi - m'^2_B - 2m'_K E'_\pi)^2}$$

$$\left[|J_1|^2 m'^2_K (2I_2 - \tfrac{2}{3} m'_K I_4 - I_1) + |J_2|^2 m'^2_\mu I_1 \right.$$

$$\left. + (\overset{*}{J_1} J_2 + J_1 \overset{*}{J_2}) m'^2_\mu m'_K I_3 \right]$$

(A-163)

The quantities I_1 , I_2 , I_3 and I_4 have been defined by eq. (A-103).

APPENDIX VI

MATRIX ELEMENT OF THE SINGLE-NEUTRON INTERMEDIATE STATE DIAGRAM FOR $\Lambda \rightarrow N + \pi$ -DECAY WITH INTERMEDIATE VECTOR BOSON

We want to evaluate the matrix-element of fig. (42) with the same modification at the ($p\Lambda$) and (np) -vertices as in case of the local four-fermion interaction (See chapter II).

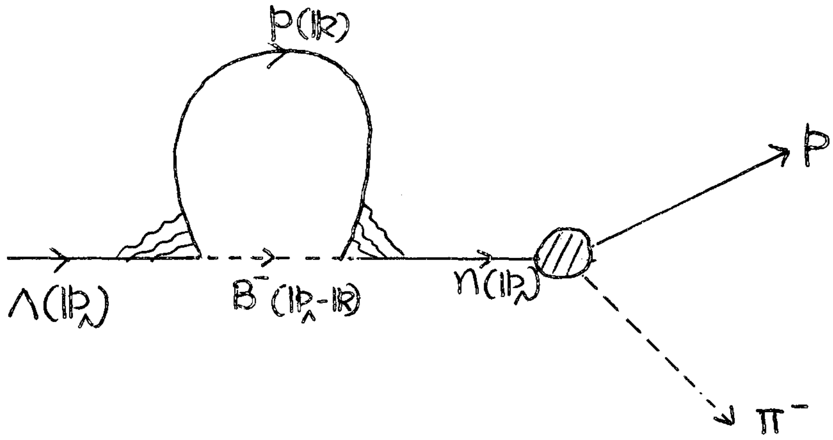


Fig.-42

Choosing the interaction [eq. (142)] adopted in chapter IV the matrix-element for fig. (42) is given by:

$$N(\Lambda \rightarrow p+\pi^-) = (gg')(m_P^2/m_B^2)(i)^6 \delta^4(p_\Lambda - p_p - p_{\pi^-})(\sqrt{2}g_\pi)$$

$$\bar{u}_p i\gamma_5 (\not{p}_\Lambda - m_n)^{-1} I u_\Lambda \qquad \text{(A-164)}$$

where,

$$I = \left(\frac{m_B^2}{m_p^2}\right)\int d^4k\, \gamma_\beta (1+iB\gamma_5)(\not{k}-m_p)^{-1}\gamma_\alpha(1+iA\gamma_5)$$

$$\left[g_{\alpha\beta} - (p_\Lambda - k)_\alpha (p_\Lambda - k)_\beta / m_B^2\right] \Big/ \left[(p_\Lambda - k)^2 - m_B^2\right]$$

(A-165)

As in case of the local interaction, the above integral diverges quadratically, so we introduce a Feynman cut off factor $(-\lambda^2/k^2-\lambda^2)^2$ for the evaluation of the integral. I then reduces to:

$$I = (-\lambda^2)^2\left(\frac{m_B^2}{m_p^2}\right)\int d^4k \frac{[N]}{(k^2-m_p^2)\{(p_\Lambda - k)^2 - m_B^2\}(k^2-\lambda^2)^2}$$

(A-166)

where

$$N = (4m_p\xi - 2\eta\not{k}) - \frac{1}{m_B^2}\Big\{(\not{p}_\Lambda\not{k} - k^2)\eta(\not{p}_\Lambda - \not{k})$$

$$+ (p_\Lambda - k)^2 m_p\xi\Big\} \qquad \text{(A-167)}$$

and,

$$\left.\begin{aligned} \xi &\equiv (A-B) + i(A-B)\gamma_5 \\ \eta &\equiv (1+AB) - i(A+B)\gamma_5 \end{aligned}\right\} \qquad \text{(A-168)}$$

Putting,

$$a = (\mathbb{P}_\Lambda - \mathbb{k})^2 - m_B^2$$

$$b = \mathbb{k}^2 - m_p^2$$

$$c = \mathbb{k}^2 - \lambda^2 \qquad \text{(A-169)}$$

and using the identity

$$\frac{1}{abc^2} = 3! \int_0^1 dx \int_0^x dy \frac{y}{[(c-b)y + (b-a)x + a]^4}$$

we have,

$$I = 3!\,(-\lambda^2)^2 \left(\frac{m_B^2}{m_p^2}\right) \int_0^1 dx \int_0^x y\,dy \int d^4\mathbb{k}\, \frac{[N]}{(D)^4} \qquad \text{(A-170)}$$

where,

$$D = \{\mathbb{k} - \mathbb{P}_\Lambda(1-x)\}^2 - G$$

$$G = \{m_\Lambda^2 x^2 + (m_p^2 - m_B^2)x + m_B^2\} + (\lambda^2 - m_p^2)y \qquad \text{(A-171)}$$

Shifting the origin of $\mathbb{k}$ to $\mathbb{k}'$, where

$$\mathbb{k}' = \mathbb{k} - \mathbb{p}_\Lambda (1-x) \tag{A-172}$$

and dropping odd powers of $\mathbb{k}'$ in N, we have

$$I = 3!\ \lambda^4 (m_B^2/m_p^2) \int_0^1 dx \int_0^x y\,dy \int d^4\mathbb{k}' \ \frac{\mathcal{M} + \mathcal{N}\mathbb{k}'^2}{(\mathbb{k}'^2 - C)^4}$$

$$= \lambda^4 (m_B^2/m_p^2)(i\pi^2) \int_0^1 dx \int_0^x y\,dy \left[\frac{\mathcal{M}}{C^2} - \frac{2\mathcal{N}}{C} \right] \tag{A-173}$$

where,

$$\mathcal{M} = 4 m_p \xi - 2\eta \not{\mathbb{p}}_\Lambda (1-x) - \frac{m_\Lambda^2}{m_B^2} \left\{ \eta \not{\mathbb{p}}_\Lambda x^2 (1-x) + x^2 m_p \xi \right\}$$

$$\mathcal{N} = - \frac{1}{m_B^2} \left[m_p \xi - \frac{(1+3x)}{2} \eta \not{\mathbb{p}}_\Lambda \right] \tag{A-174}$$

The x and y integration in eq. (A-173) can be carried out straight forwardly. The result, after suitable arrangement of the terms is;

$$I = \frac{\lambda^4 (i\pi^2)}{\kappa^2} \Big[m_p (1-AB)\Theta_1 + m_p (A-B)\Theta_1 (i\gamma_5)$$
$$+ \mathbb{p}_\Lambda^\mu (A+B)\Theta_3 \gamma_\mu + \mathbb{p}_\Lambda^\mu (A+B)\Theta_3 (\gamma_\mu i\gamma_5) \Big] \tag{A-175}$$

where,

$$K \equiv \lambda^2 - m_p^2$$

$$\Theta_1 \equiv \Big[2\, I_1 + 2\,(1 - m_p^2/m_B^2)\, I_2 - 3\, m_\Lambda^2/m_B^2\, I_3$$

$$- K\,(4\, I_5 - m_\Lambda^2/m_B^2\; I_7 - 1/m_B^2) \Big] (m_B^2/m_p^2)$$

$$\Theta_3 \equiv \Big[- I_1 + (4 + m_p^2/m_B^2) I_2 + 3\,(m_p^2/m_B^2 - 1)\, I_3$$

$$+ 4\,(m_\Lambda^2/m_B^2)\, I_4 - K\,(- 2 I_5 + 2 I_6 - m_\Lambda^2/m_B^2\, I_7$$

$$+ m_\Lambda^2/m_B^2\; I_8 + 3/2 m_B^2) \Big] (m_B^2/m_p^2)$$

(A-176)

The quantities $I_1, \ldots, I_8$ involve the final x- integration and are defined by:

$$I_1 = \int_0^1 \log_e \frac{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2) x + m_B^2}{m_\Lambda^2 x^2 + (m_p^2 - m_B^2) x + m_B^2}\, dx$$

$$I_2 = \int_0^1 x \log_e \frac{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2) x + m_B^2}{m_\Lambda^2 x^2 + (m_p^2 - m_B^2) x + m_B^2}\, dx$$

$$I_3 = \int_0^1 x^2 \log_e \frac{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2) x + m_B^2}{m_\Lambda^2 x^2 + (m_p^2 - m_B^2) x + m_B^2}\, dx$$

$$I_4 \equiv \int_0^1 x^3 \log_e \frac{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2)x + m_B^2}{m_\Lambda^2 x^2 + (m_p^2 - m_B^2)x + m_B^2}\, dx$$

$$I_5 \equiv \int_0^1 \frac{x}{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2)x + m_B^2}\, dx$$

$$I_6 \equiv \int_0^1 \frac{x^2}{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2)x + m_B^2}\, dx$$

$$I_7 \equiv \int_0^1 \frac{x^3}{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2)x + m_B^2}\, dx$$

$$I_8 \equiv \int_0^1 \frac{x^4}{m_\Lambda^2 x^2 + (\lambda^2 - m_B^2)x + m_B^2}\, dx$$

(A-177)

Inserting eq. (175) into eq. (164), the matrix-element of fig. (42) is given by:

$$\boxed{\begin{aligned} N(\Lambda \to p + \pi^-) = {} & (gg')(\sqrt{2}\, g_\pi)(m_p^2/m_B^2)(i\pi^2)(m_n m_p) \\ & (\lambda^2/\lambda^2 - m_p^2)^2\, \bar{u}_p \not{p}_\pi (\alpha + i\beta\gamma_5) u_\Lambda \\ & \delta^4(p_\Lambda - p_p - p_\pi) \end{aligned}}$$

(A-178)

where,

$$\alpha \equiv (A-B)\Theta_1 - (A+B)(m_\Lambda/m_p)\Theta_3 \quad \text{(A-179)}$$

$$\beta \equiv (1-AB)\Theta_1 - (1+AB)(m_\Lambda/m_p)\Theta_3 \quad \text{(A-180)}$$

The values of Θ_1 and Θ_3 are given in Table - XI for a few values of λ and m_B.

Table - XI

m_B in mev	$\lambda = m_\Lambda \approx 1115.2$ (Mev)		$\lambda' = 1.5\, m_p = 1407$ (Mev)	
	Θ_1	Θ_3	Θ_1	Θ_3
500	+0.0429	−0.0571	+0.3112	−0.4210
700	+0.0488	−0.0549		
938	+0.0555	−0.0520	+0.5479	−0.5360
1100	+0.0597	−0.0499		
1876	+0.0743	−0.0405	+0.5500	−0.3170
4690	+0.1740	−0.0109		

SELECTED BIBLIOGRAPHY

M. Gell-Mann and A.H. Rosenfeld, "Hyperons and Heavy Mesons" - Ann. Rev. Nuclear Science, 7, 407, 1957.

C. Frazinetti and G. Morpurgo - "An Introduction to the Physics of the new particles" - Suppl. Nuovo Cimento, No.2, 2° Semestre 1957.

R.H. Dalitz, "K-Mesons and Hyperons" Reports on Prog. in Phys. 20, 163, 1957.

E.O. Okonov, "Decay Properties of Heavy Mesons and Hyperons" - Soviet Physics, Uspekhi vol. 67(2) Number 1, 119 (January - February, 1959).

T.D. Lee and C.N. Yang, "Elementary Particles and Weak Interactions" Brookhaven National Laboratory (1957).

Proceedings of the 6th Rochester Conference on High Energy Physics, 1956 (Interscience, New York).

Proceedings of the 7th Rochester Conference on High Energy Physics, 1957 (Interscience, New York).

Proceedings of the 1958 Annual International Conference on High Energy Physics at CERN, 1958.

Proceedings of the 1959 Annual International Conference on High Energy Physics at Kiev, 1959.

Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester, 1960.

E. Corinaldesi, "Particles and Symmetries", Nuclear Physics 7, 305, 1958.

N. Kemmer, J.C. Polkinghorne and D.L. Pursey, "Invariance in Elementary Particle Physics". Reports on progress in Physics, 22, 368, 1959.

G.C. Wick "Invariance Principles of Nuclear Physics" Annual Rev. Nuclear Sci., 8, 1 (1958).

W. Pauli, "Exclusion Principle, Lorentz Group and Reflexion of Space-time and charge" in Niels Bohr and the Development of Physics, Pergamon Press, London, 1955.

G. Luders, "Proof of the TCP-Theorem", Annals of Phys. 2, 1, 1957.

E.J. Konopinski, "The experimental Clarification of the Laws of β-Radio Activity". Ann. Rev. Nuclear Sci., 9, 99 (1959).

C.S. Wu, "Parity Experiments in β-Decay". Rev. Mod Phys. 31, 783 (1959).

Ya Smorodinskii, "The Present State of the Theory of β-Decay". Soviet Physics, Uspekhi - 67, 1 (1959).

P. Roman, Theory of Elementary Paricles, North Holland Publishing Company, 1960.

Lectures given at CERN in 1958 and 1959 on "Strange Particle Physics" CERN 59-35.

L. Okun, "Strange Particle Decays", Ann. Rev. Nuclear Sci., 9, 61 (1959).

R.H. Dalitz, "Strange Particle Decay Processes and the Fermi Interaction". Rev. Mod. Phys. 31, 823 (1959).

R. Gatto, "Present Problems in the Theory of Weak Interactions", Suppl. Nuovo Cimento, No. 2, 4° Trimestre 340, 1959.

M. Gell-Mann, "Status of Weak Interactions", Rev. Mod. Phys. 31, 834 (1959).

R.P. Feynman and M. Gell-Mann, "Theory of Fermi Interaction", Phys. Rev. 109, 193 (1958).

E.C.G. Sudarshan and R.E. Marshak, "Chirality Invariance and the Universal Fermi Interaction", Phys. Rev. 109, 1860 (1958).

J.J. Sakurai, "Mass Reversal and Weak Interactions", Nuovo Cimento 7, 649 (1958).

S. Okubo, R.E. Marshak and E.C.G. Sudarshan, "V-A Theory and the Decay of the Λ-Hyperon", Phys. Rev. 113, 944 (1959).

S. Oneda, "Strange Particle Decays and the Universal V-A Four Fermion Interaction", Nuclear Physics, 9, 476 (1958/59).

S.B. Trieman, "Weak Global Symmetry", Nuovo Cimento 15, 916 (1960).

F. Gursey, "On the Symmetries of the Strong and Weak Interactions" Nuovo Cimento 16, 230 (1960).

T.D. Lee and C.N. Yang, "Implications of the Intermediate Boson Basis of the Weak Interactions", (Preprint), Physical Rev. (to be published).

B. Pontecorvo, "Electron and Muon Neutrinos" JETP (U.S.S.R.) 37, 1751 (December, 1959).