ISBN: 978-3-95450-191-5 doi:10.18429/JACoW-SRF2017-THPB054

CALCULATING THE FIELD DEPENDENT SURFACE RESISTANCE FROM QUALITY FACTOR DATA

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Abstract

The quality factor of an RF cavity and the surface resistance are typically related with a constant geometry factor. The implicit assumption made is that the surface resistance is field independent, which is however not observed experimentally in superconducting cavities. The approximation error due to this assumption becomes larger the less homogeneous the magnetic field distribution along the cavity walls is. In this paper we calculate the surface resistance error for different cavity types. Correction factors as well as a numerical method to correct for this error are presented.

INTRODUCTION

The quality factor Q_0 of an RF cavity relates the stored energy U with the energy dissipated per RF cycle. It is calculated by:

$$Q_0 = \frac{\omega U}{P_{\mathrm{Dis}}} = \frac{\omega \int_{\mathrm{V}} |B|^2 \, \mathrm{d}v}{\mu_0 \int_{\mathrm{S}} R_{\mathrm{S}} \cdot |B|^2 \, \mathrm{d}s} \approx \frac{G}{R_{\mathrm{S}}} \qquad (1)$$

where P_{Dis} is the dissipated power and R_{S} is the surface resistance. In the last term, the geometry factor G is introduced which directly links the quality factor with the surface resistance. This factor is independent of the material and of the size of the cavity and is calculated with:

$$G = \frac{\omega \int_V |B|^2 \cdot \mathrm{d}v}{\mu_0 \int_{\mathrm{S}} |B|^2 \cdot \mathrm{d}s} \qquad (2)$$

Calculating the surface resistance from a quality factor measurement using $R_{\mathrm{S}}^{\mathrm{meas}} = G/Q_0$ will return a mean surface resistance which is only identical to the local material surface resistance $R_{\mathrm{S}}(B)$ if it is field independent or if the field distribution on the cavity surface is uniform. The less homogenous the surface magnetic field is distributed, the larger the approximation error becomes.

The effect of this is shown in Figures 1 and 2. In these plots, the hypothetically measured surface resistance $R_{\mathrm{S}}^{\mathrm{meas}}$ is shown for various different cavity types:

- Two elliptical cavities, a stadard TESLA geometry and low-loss ERL cavity
- An idealized half-wave resonator (HWR), modelled as a coaxial transmission line shorted at both ends [1].
- Two cavities used for sample testing - a TE_{011} host cavity [2, 3] and a Quadrupole Resonator (QPR) [4, 5]

In Figure 1, the assumed 'true' surface resistance is monotonically increasing and has a quadratic and an exponential component. In Figure 1, a linear term with a negative sign is added, giving a shape similiar to those produced with N-doped cavities [6]. As expected, the ERL cavity which has the most homogeneous field distribution produces the smallest error. The cavities in our study with a very inhomogenous surface magnetic field, the HWR and the QPR, have errors as large as 30% at high fields. For the N-doped case one also observes that the surface resistance minium gets shifted significantly.

For calculating these results, Equation (1) was used together with an explicit calculation of P_{Dis}. For the eliptical cavities which have cylindrical symmetry one can use the wall profile $r(z)$ and the surface field $B(z)$ to reduce the calculation to a line integral:

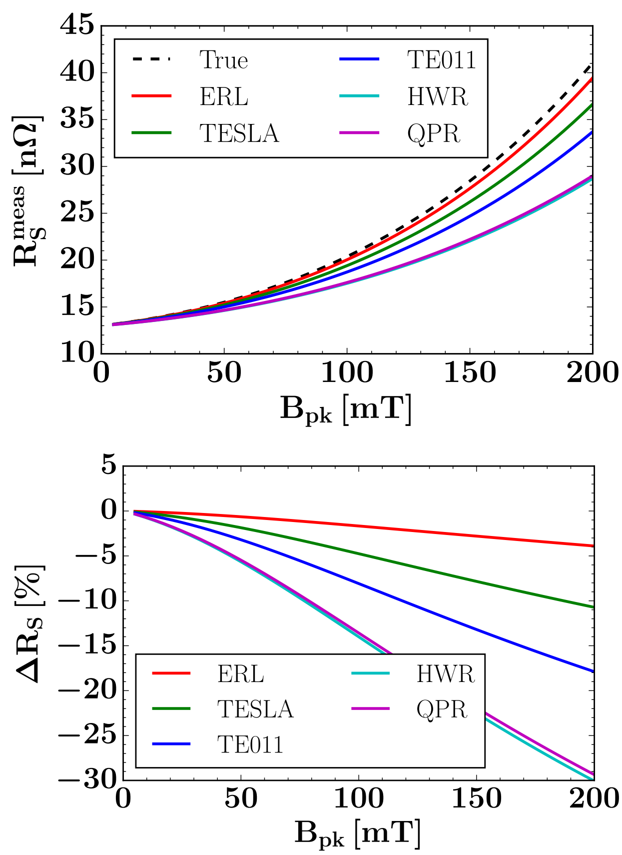

Figure 1: Hypothetical measurement of the same material with different cavities. Shown in the dotted black line is the assumed surface resistance which has a quadratic and an exponential contribution. For cavities types with very inhomogenous surface magnetic fields, the error when calculating the surface resistance as $R_{\mathrm{S}}^{\mathrm{meas}} = G/Q_0$ can be as large as 30%.

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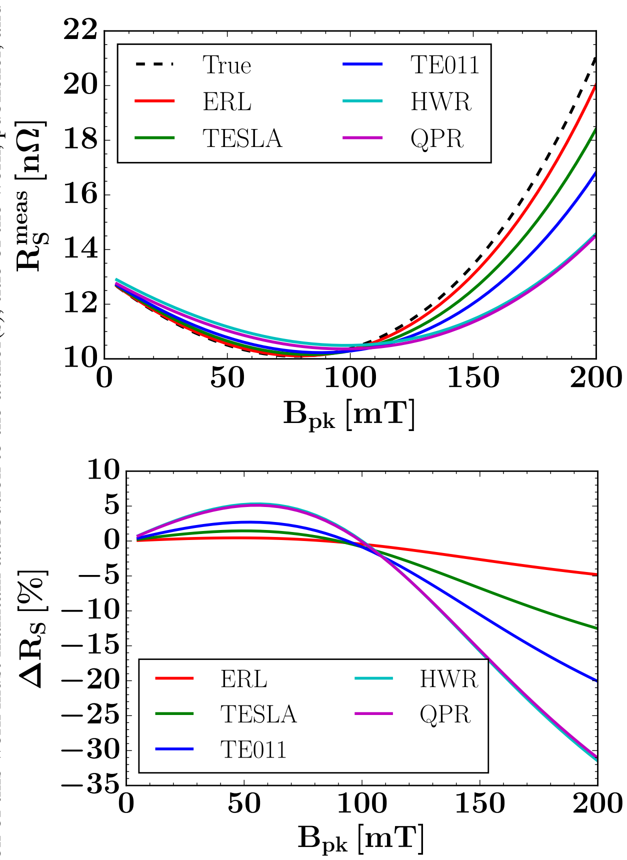

Figure 2: Approximation error for a N-doped like surface resistance. Again large errors are observed, furthermore the surface resistance minimum is shifted significantly for HWR and QPR geometries.

$$P_{Dis} = \frac{1}{2\mu_0^2}\int_S R_S(B)\cdot|B|^2\,\mathrm{d}s = \frac{1}{2\mu_0^2}\int_0^L 2\pi r(z)R_S(B(z))\cdot|B(z)|^2\cdot\mathrm{d}z \tag{3}$$

CORRECTION

So how does one correct for this problem? If one assumes a polynomial dependence of the surface resistance one can explicitly calculate the correction factors for each coefficient [7]. For a quadratic dependency ($R_S(B) = R_0 + \alpha_2 B^2$) one has:

$$R_S^{meas} = \frac{G}{Q_0} = \frac{\int_S (R_0+\alpha_2 B^2)|B|^2\,\mathrm{d}s}{\int_S |B|^2\,\mathrm{d}s} = R_0 + \alpha_2\cdot\frac{\int_S (|B|/B_{pk})^4\,\mathrm{d}s}{\int_S (|B|/B_{pk})^2\,\mathrm{d}s}\cdot B_{pk}^2 = R_0+\alpha_2\cdot\beta_2\cdot B_{pk}^2 \tag{4}$$

Table 1: Correction Factors β_i, Calculated with Equation (5) for Various Different Cavities

	β_1	β_2	β_3
TESLA	0.91	0.85	0.80
ERL	0.97	0.95	0.93
HWR	0.74	0.58	0.48
TE_{011}	0.84	0.74	0.67
QPR	0.72	0.58	0.48

For a general polynomial form ($R_S(B) = R_0 + \sum_{i=1}^{\infty}\alpha_i B^i$), the correction coefficients can be calculated as

$$\beta_i = \frac{\int_S (|B|/B_{pk})^{i+2}\,\mathrm{d}s}{\int_S (|B|/B_{pk})^2\,\mathrm{d}s} \tag{5}$$

The correction factors for the cavities considered here are shown in Table 1. Calculating a correction factor does not work however if the surface resistance is exponential or of a other, non-polynomial form. Here a numerical calcuation, following a perturbative approach is required.

Starting with the naive calculation of the surface resistance ($R_{S,0} = G/Q_0$), an expected quality factor is calculated, using Equations (1) and (3). A field dependent geometry factor is then computed and the surface resistance results are updated. The updated results are used to compute a new geometry factor, and so on.

$$R_{S,0}(B) = \frac{G_0}{Q_0(B)}$$

$$G_1(B) = Q_{calc}(R_{S,0})\cdot R_{S,0} \quad , \quad R_{S,1}(B) = \frac{G_1(B)}{Q_0(B)}$$
$$G_2(B) = Q_{calc}(R_{S,1})\cdot R_{S,1} \quad , \quad R_{S,2}(B) = \frac{G_2(B)}{Q_0(B)} \tag{6}$$
$$\vdots$$

Note that as the measurement data $Q_0(B)$ is discrete, one has to interpolate the intermedeate surface resistance results $R_{S,i}$ to be able to calculate the expected quality factor at each iteration. The application of these update rules are shown in Figure 3, using as an example the half wave resonator and the same surface resistance functions as assumed previously. One can see that in both cases the algorithm converges towards the correct result within a few iterations.

ACKNOWLEDGEMENT

The research leading to these results has received funding from the European Commission under the FP7 Research Infrastructures project EuCARD-2, grant agreement No. 312453.

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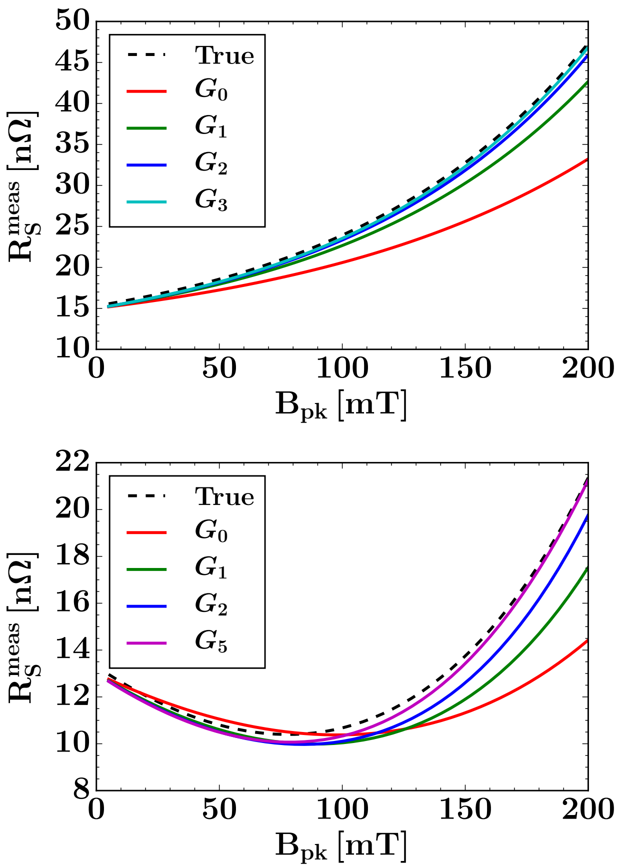

Figure 3: Correction calculation for a Half Wave Resonator assuming a monotonically increasing surface resistance as used in Figure 1. The black dotted line indicates the 'true' surface resistance, the result obtained using a constant geometry factor is shown in red . After only a few iterations, the calculation converges towards the correct result.

CONCLUSION

We have shown that the approximation error caused by calculating the surface resistance directly from the geometry factor can be very significant for realistic scenarios. If the surface resistance follows a polynomial function, one can pre-compute correction factors. Furthermore a simple method was introduced that correctly calculates the surface resistance from Q_0-data without making assumptions about the underlying loss model.

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