Validity of rotational energy formulae for triaxial superdeformed bands in ^{168}Hf

Honey Sharma* and H. M. Mittal ‡
**Department of Physical Sciences,*
Sant Baba Bhag Singh University,
Padhiana, Punjab-144030, India
and
‡Department of Physics,
Dr. B. R. Ambedkar National Institute of Technology,
Jalandhar, Punjab-144008, India

Introduction

Potential energy surface calculations [1, 2] reveal that nuclei with Z$\sim$72 and N$\sim$94 produce a novel region of peculiar shapes coexisting with regular prolate deformation($\varepsilon_2 \sim$ 0.23). These nuclei can occupy stable triaxial superdeformed (TSD) shape at high spins, with a deformation parameter of $\varepsilon_2 \sim 0.4$ and distinct moments of inertia for each of the major axis. Experimentally, such TSD bands have been observed in $^{163-165}$Lu, with two bands in ^{163}Lu [3–5], eight in ^{164}Lu [6], and one in ^{165}Lu [7]. The wobbling mode is a universal phenomena in the A$\sim$160 mass region, as demonstrated by the studies of the first and second phonon wobbling bands in ^{161,163,165,167} Lu and ^{167} Ta. These investigations also provide the strongest indication for the stable triaxial structure. Amro et al.[8] provided the first proof of triaxial superdeformation in ^{168}Hf. In ^{168}Hf, three TSD bands have been identified. The strongest band (TSD1) has a significant quadrupole moment ($Q_t \sim 11.4eb$) according to lifetime measurements. Band head spin of TSD bands in ^{163,164,165}Lu through two-parameter formulae was determined by Sharma and Mittal [9]. The applicability of two parameter formula [9] in TSD bands motivated us to test the validity of rotational energy formulae for TSD bands in ^{168}Hf. In this present work, we have calculated the band head spin (I_0) and band head moment of inertia ($\Im_0$) of TSD bands in ^{168}Hf by using four parameter formula and nuclear softness formula. However, one can check which of the two formulae works better to study the SD spectroscopy of TSD bands in ^{168}Hf.

*Electronic address: honeyshrma777@gmail.com

Formalism

A)Four parameter formula [10]

$$\begin{aligned} E_\gamma(I \to I-2) &= A(I(I+1) - (I-2)(I-1)) \\ &+B((I(I+1))^2 - ((I-2)(I-1))^2) \\ &+C((I(I+1))^3 - ((I-2)(I-1))^3 \\ &+D((I(I+1))^4 - ((I-2)(I-1))^4) \end{aligned} \quad (1)$$

B) Nuclear softness formula [11]

$$E_\gamma(I) = \frac{\hbar^2}{2\Im_0} \times \left[\frac{I(I+1)}{(I+\sigma I)} - \frac{(I-2)(I-1)}{1+\sigma(I-2)}\right] \quad (2)$$

Results and Discussion

In order to achieve the band head spin (I_0) and band head moment of inertia ($\Im_0$), the experimentally noticed transition energies of TSD bands in ^{168}Hf stated in Ref.[12] have been fitted in four parameter formula [10] and nuclear softness formula [11]. The obtained band head spin (I_0) of TSD bands in ^{168}Hf calculated from four parameter formula and nuclear softness formula are presented in Table I. It has been observed from Table I that the four parameter formula is in good agreement with the experimentally observed bandhead spin (I_0). Signature partner pairs are those which have the identical band head moment of inertia ($\Im_0$) values. It has been noticed from Table II that the obtained band head moment of inertia ($\Im_0$) from four parameter formula for TSD bands in ^{168}Hf(2) and ^{168}Hf(3) are same. Hence, ^{168}Hf(2) and ^{168}Hf(3) are the signature partner pairs.

TABLE I: The band head spin (I_0) obtained from four parameter formula and nuclear softness formula for TSD bands in ^{168}Hf. Here 1, 2 and 3 in parenthesis represent band 1, band 2 and band 3 respectively. Here F.P denotes four parameter formula, N.S denotes nuclear softness formula.

TSD Bands	$E\gamma$ (keV)	F.P (I_0)	N.P (I_0)	Ref.[12] (I_0)
$^{168}Hf(1)$	677	23	49	21
$^{168}Hf(2)$	771	26	16	24
$^{168}Hf(3)$	811	24	46	28

TABLE II: The band head moment of inertia ($\Im_0$) obtained from four parameter formula and nuclear softness formula for TSD bands in ^{168}Hf. Here 1, 2 and 3 in parenthesis represent band 1, band 2 and band 3 respectively. Here F.P denotes four parameter formula, N.S denotes nuclear softness formula.

TSD Bands	$E\gamma$ (keV)	F.P $\Im_0$ ($\hbar^2 MeV^{-1}$)	N.P $\Im_0$ ($\hbar^2 MeV^{-1}$)
$^{168}Hf(1)$	677	51.0	22.4
$^{168}Hf(2)$	771	62.7	28.4
$^{168}Hf(3)$	811	62.1	16.5

Conclusion

In this present work, we have employed four parameter formula and nuclear softness formula to check their validity by deducing the band head spin (I_0) and band head moment of inertia ($\Im_0$) of TSD bands in ^{168}Hf. It is concluded from present study that the four parameter formula works very well to explain the general nature of TSD bands in ^{168}Hf. Similar value of band head moment of inertia ($\Im_0$) for TSD bands in ^{168}Hf(2,3) obtained by four parameter formula indicates the presence of signature partner pairs in ^{168}Hf .

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