

Investigation of polarized proton spin coherence time at storage rings

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Abstract. The idea of the Electric Dipole Moment (EDM) search using the storage ring with polarized beam demands long Spin Coherence Time (SCT). It is the time during which the RMS spread of the orientation of spins of all particles in the bunch reaches one radian. Long SCT is needed to observe a coherent effect on polarization induced by the EDM. The possibility of getting a 1000 s SCT for deuterons has been shown experimentally at COoler SYnchrotron (COSY), accelerator at FZJ Jülich, Germany. Reaching high values of SCT for protons is more challenging due to a higher anomalous magnetic moment. Obtaining sufficient proton SCT is obligatory for planned EDM search experiments at COSY and the ProtoType EDM Ring (PTR). It has been shown that the second order momentum compaction factor (α_1) has to be optimized along with chromaticities ($\xi_{x,y}$) to get high SCT. Three families of sextupoles have to be used. The optimal values of chromaticities and α_1 are discussed. The racetrack option of PTR is investigated.

1. Principles of SCT optimization

To study the effects of spin decoherence one has to observe spin precession component orthogonal to the invariant axis. The in-plane spin precession is governed by the T-BMT equation and in a storage ring with magnetic bending the spin-tune is $\nu_s = G\gamma$, where G is anomalous magnetic moment. For non-reference particles the spin-tune is determined by the oscillation amplitudes in the phase space and lattice parameters such as chromaticities.

The first step in optimization of SCT, referring to [1], is turning on an RF cavity to suppress the first order spin-decoherence. It helps to increase SCT about three orders of magnitude.

In the presence of an RF cavity longitudinal motion is nonlinear in general case. The solution of nonlinear equations for the principle of synchronous acceleration gives the rise of equilibrium (average) energy level $\Delta\delta_{eq}$ [1]:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1} \left[\frac{\delta_m^2}{2} \left(\alpha_1 + \frac{3\beta_s^2}{2\gamma_s^2} - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4} \right) + \left(\frac{\Delta L}{L} \right)_\beta \right], \quad (1)$$

where betatron orbit lengthening term is:

$$\left(\frac{\Delta L}{L} \right)_\beta = -\frac{\pi}{L_0} [\epsilon_x \xi_x + \epsilon_y \xi_y]. \quad (2)$$



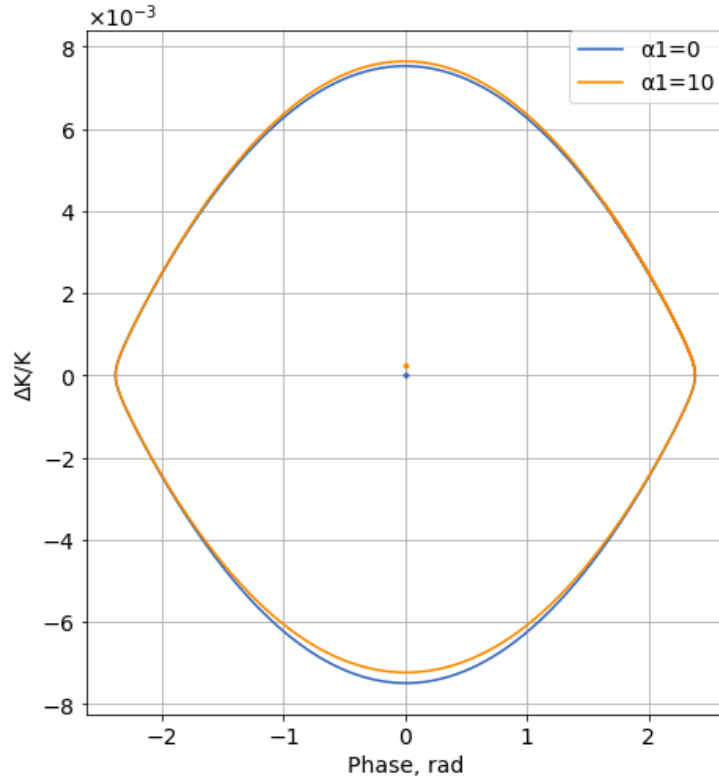


Figure 1. Phase trajectories in longitudinal plane for different values of α_1 ; $\xi_{x,y} = 0$. $\frac{\Delta K}{K}$ is a relative kinetic energy deviation.

Here α_0 and α_1 is a first and second order momentum compaction factor. γ_s is a Lorentz factor and β_s is a relativistic beta factor for the synchronous particle. δ_m is an amplitude of synchrotron oscillations in $\frac{\Delta p}{p}$. $\xi_{x,y}$ are beam chromaticities and $\epsilon_{x,y}$ are the Courant-Snyder invariants.

The shift of equilibrium energy level for different values of α_1 is presented in figure 1. The calculations were done for the case of synchrotron motion without betatron oscillations.

From equations (1) and (2) it can be seen that ξ_x , ξ_y , α_1 have to be optimized to influence nonlinear longitudinal and spin motion. Three sextupole families need to be placed at points with different optical functions and dispersion to optimize these three parameters. Denoting the ansatz in equation (1) at δ_m^2 as κ , one can say that the sextupoles have to be tuned to zero ξ_x , ξ_y , κ to achieve long SCT. This comes from the fact that spin-tune deviation is determined by equilibrium energy level shift:

$$\Delta\nu_s = G\Delta\gamma_{eq}. \quad (3)$$

The investigation of this concept is presented below.

2. SCT optimization at COSY

COSY is a racetrack synchrotron providing unpolarized and polarized protons and deuterons in the momentum range from 300 MeV/c up to 3.7 GeV/c. The typical optical setting with zero dispersion for the experiment is depicted in figure 2. The polarization of the injected beam is flipped from vertical into the ring plane with an rf-solenoid. And SCT is determined as the decay time of the in-plane polarization signal of particle ensemble. Three sextupole families: MXS, MXL and MXG are used for SCT optimization [2].

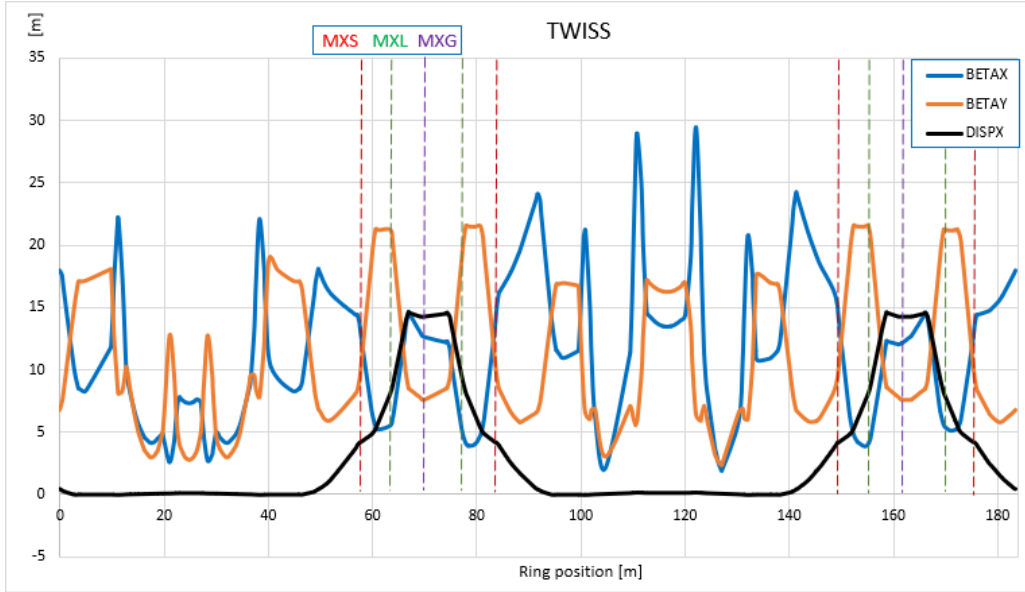


Figure 2. Layout of COSY Twiss functions. Dashed lines indicate the locations of three sextupole families in the arcs.

2.1. Investigation of spin dynamics

In this section spin-tracking simulation results are presented to verify the theoretical predictions for the optimal values of ξ_x, ξ_y and κ coming from equation (3). Calculations are made with COSY Infinity code [3].

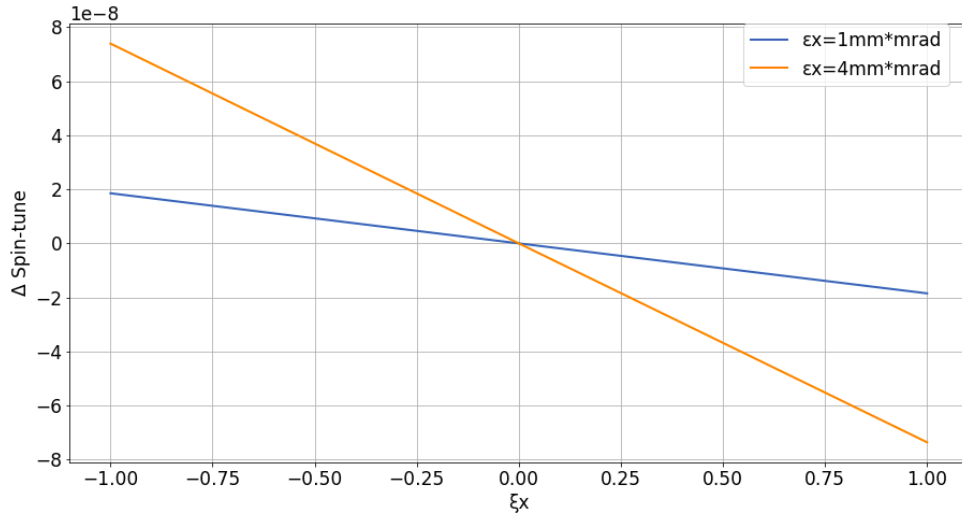


Figure 3. Spin-tune deviation for non-reference particles with different horizontal betatron amplitudes.

First the dependence of spin-tune on ξ_x, κ for non-reference particles was investigated. Here the amplitude dependent spin-tune was computed with the normal form algorithm. The results from figures 3 and 4 confirm that optimal $\xi_x, \kappa = 0$.

However, vertical betatron motion is subject to the influence of spin resonances, hence optimal $\xi_y \neq 0$ [4]. For protons there are several intrinsic resonances in the entire momentum range of

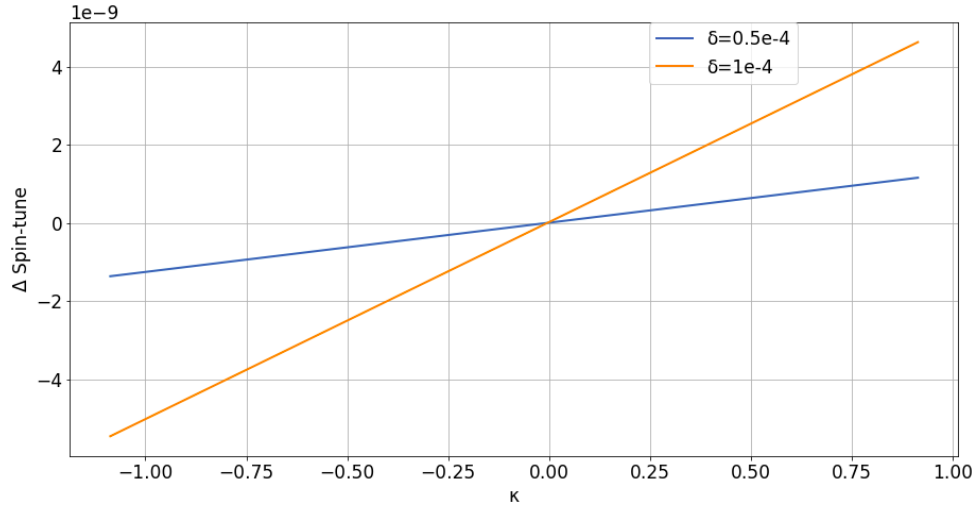


Figure 4. Spin-tune deviation for non-reference particles with different energies.

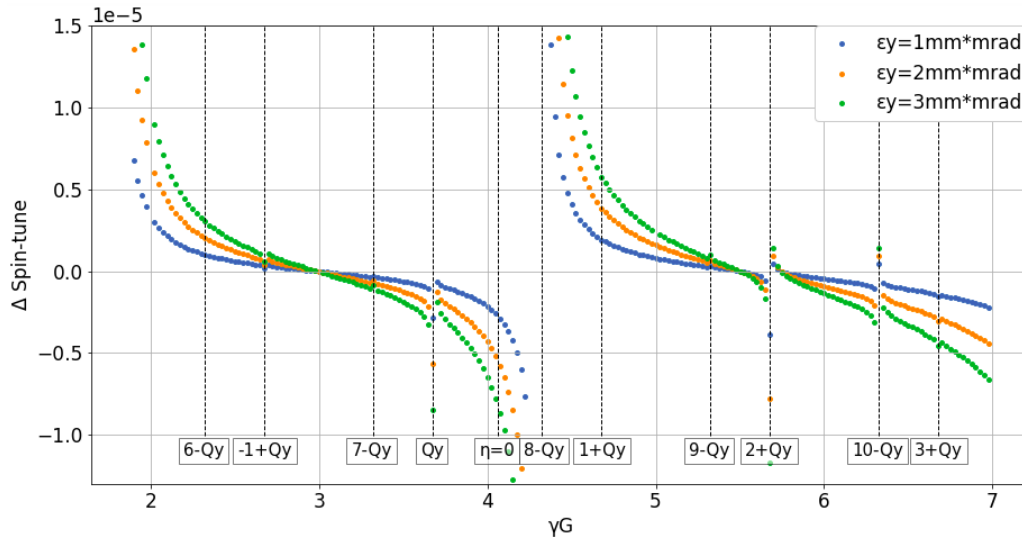


Figure 5. Spin-tune deviation for non-reference particles with different vertical betatron amplitudes; $\xi_y = 0$.

COSY with the strongest one: $\gamma G = 8 - Q_y$ (figure 5). The working point for longest SCT corresponds to the point where $\Delta\nu_s = 0$ for different betatron amplitudes. It can be adjusted by varying ξ_y so that the location of zero crossing corresponds to the energy or γG of the experimental setup. The results from figure 6 show that positive vertical chromaticity shifts the working point to lower energies.

It has to be pointed out that normal form algorithm diverges near spin resonances due to small denominators. And the spin-tune was calculated by tracking particles and averaging one turn spin phase advance during several RF periods. The influence of imperfection resonances was not taken into account in this study.

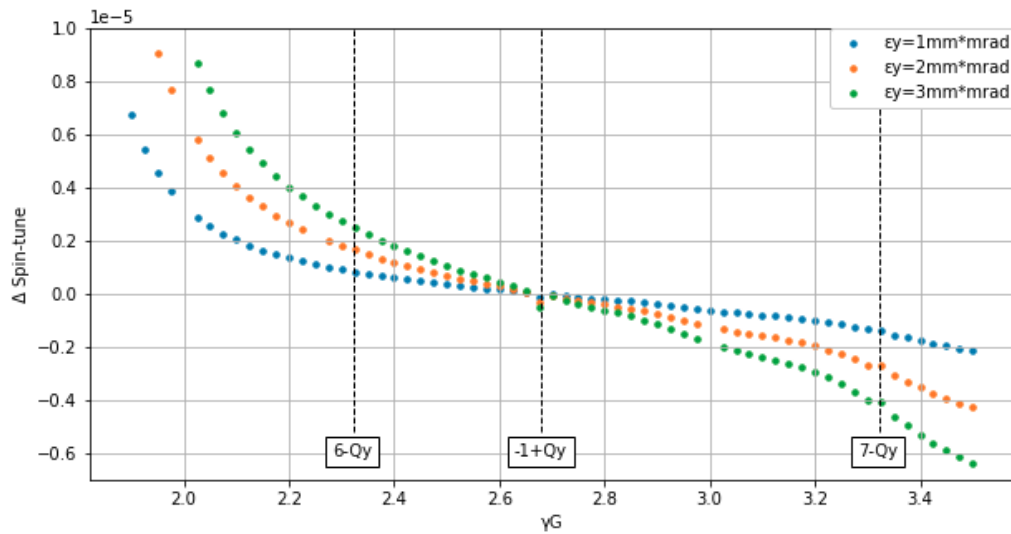


Figure 6. Spin-tune deviation for non-reference particles with different vertical betatron amplitudes; $\xi_y = 6.5$.

3. SCT optimization at prototype EDM storage ring

The PTR is a 45 MeV proton storage ring with combined E+B cylindrical bending elements [5]. Combined elements were chosen to allow for frozen spin operation. In this mode EDM signal is observed as a buildup of vertical polarization for a coherent ensemble of longitudinally polarized particles.

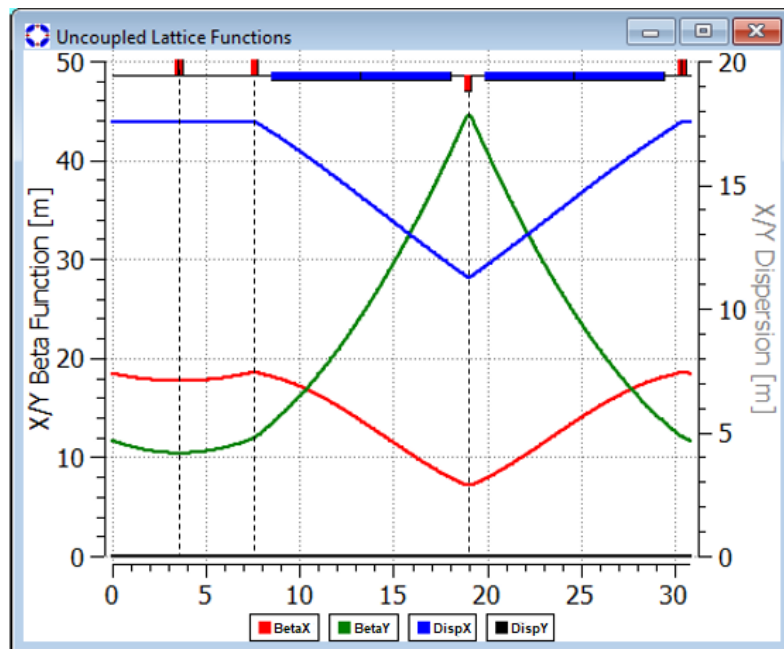


Figure 7. Layout of one superperiod of PTR with $P = 4$ with optical functions. Dashed lines indicate the locations of sextupole families.

The PTR has a superperiodicity $P = 4$ and "weak" focusing in the vertical plane, $Q_y \sim$

$0.1 \div 1.6$ (figure 7), to allow for a substantial spatial separation of the two beams by residual radial magnetic fields. Compensating this separation means mitigating systematic effects, which is crucial for the EDM experiment. For this purpose the PTR supports clockwise and counterclockwise injection of proton beams after the procedure of magnetic field reversal.

3.1. Racetrack option of PTR

At the PTR there are three sextupole families placed on top of the quadrupoles. As it can be seen from figure 7, the phase advance between the first two sextupoles is quite small.

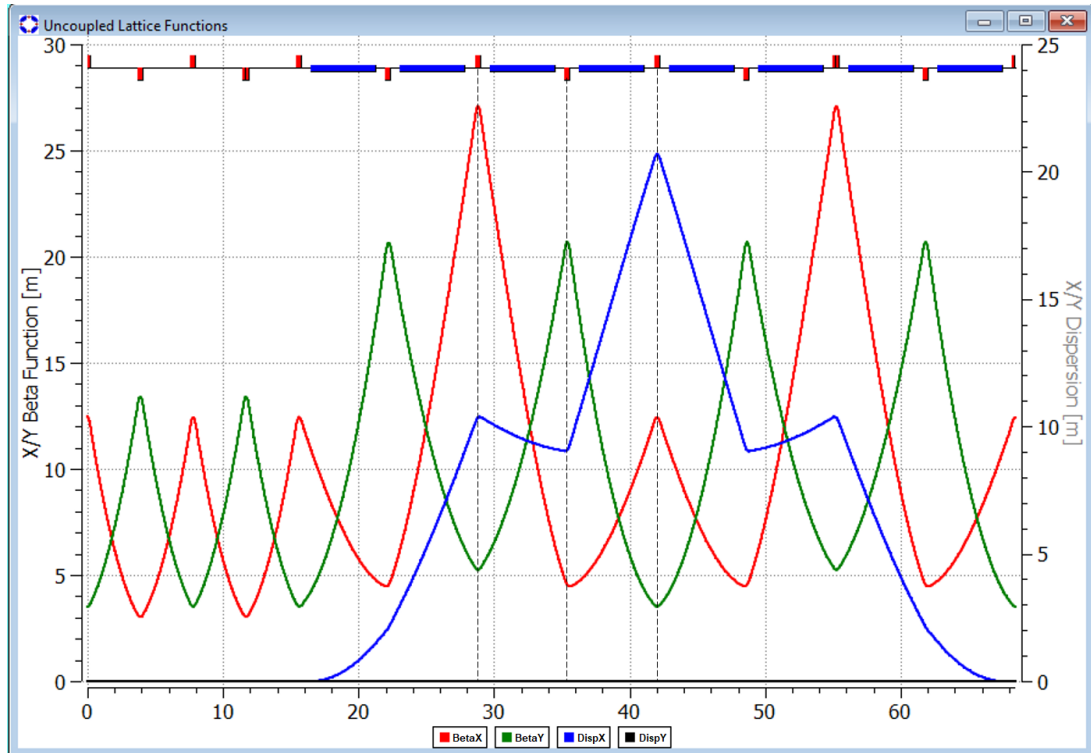


Figure 8. Layout of one superperiod of a racetrack option of PTR with $P = 2$ with optical functions. Dashed lines indicate the locations of sextupole families.

To allow for efficient manipulation of three parameters: ξ_x, ξ_y, κ and to achieve long SCT a racetrack option of PTR is proposed (figure 8). In this lattice the superperiodicity is reduced to $P = 2$ to insert three families of sextupoles at points with different ratios of optical functions and dispersion. Dispersion-free straight sections are also favorable in terms of beam dynamics.

Spin-tracking simulation results for the PTR lattice with $P = 4$ show that with two families of sextupoles it is possible to achieve proton SCT ~ 100 s. While for a racetrack option with three families of sextupoles it comes to ~ 1000 s. That is sufficient for the purpose of the EDM experiment.

4. Conclusion

The numerical calculations prove that SCT is influenced by three parameters: ξ_x, ξ_y, κ . The optimal values for ξ_x, κ are zero, that matches the theoretical predictions. However, for protons optimal ξ_y is not zero due to the influence of intrinsic resonances.

To optimize ξ_x, ξ_y, κ one needs three families of sextupoles with substantial phase advance between them. This concept is realized in the proposed racetrack option of PTR. While the

initial design of the PTR is based on the idea of mitigating systematic effects observing vertical separation of two beams, "weak" vertical focusing and less ability to increase SCT come as a drawback. The racetrack option of PTR is more suitable in achieving high SCT. Detailed study on how both lattices can handle systematic effects is foreseen.

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