OPEN ACCESS
IOP Publishing
Journal of Physics B: Atomic, Molecular and Optical Physics
J. Phys. B: At. Mol. Opt. Phys. **57** (2024) 142001 (9pp)
https://doi.org/10.1088/1361-6455/ad3ff4

Topical Review

Boson bloom

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Received 1 February 2024
Accepted for publication 17 April 2024
Published 17 June 2024

Abstract

The year 2024 marks the 100th anniversary of the first article on Bose statistics. Bose breathed life into the Planck distribution of radiation by a microscopic derivation (Bose 1924 *Z. Phys.* **26** 178), adding a new insight, namely *indistinguishability* into the then evolving quantum theory. Einstein recognized the importance of this article and got it published. Using Bose statistics Einstein wrote an article on the theory (Einstein 1924 *Sutzungsber. Preuss. Akad. Wiss Phys.-Math Kl.* 261) of an ideal Bose gas and Bose–Einstein condensation. The groundbreaking discovery of Bose, an unveiling of a secret of quantum mechanics, continues to reverberate after a century. Bose's paper is considered the fourth important paper in old quantum theory, following Planck's (1900) article (Planck 1900 *Verh. Disch Phys. Ges.* **2** 202), Einstein's (1905) photoelectric effect (Einstein 1905 *Ann. Phys., Lpz.* **17** 132) and Bohr's model (1913) of the atom (Bohr 1913 *London, Edinburgh Dublin Phil. Mag. J. Sci.* **26** 1). Dirac (1926 *Proc. R. Soc.* A **112** 661) coined the name *boson* for one of the two families of indistinguishable particles, the other family being fermion. The edifice of modern quantum field theory, many-body quantum theory, quantum-information and quantum-computing are built on bosons, fermions and anyons. The ever-blooming quantum garden of bosons has photons, gluons, W-bosons, mesons, Higgs-bosons, gravitons, phonons, magnons, excitons, plasmons, polaritons and so on. We present a brief historical account of Bose's life and his discovery, followed by a bird's eye view of the impacts of bosons in modern science and technology: from Bose's distribution of 3-degree background radiation reaching us in the form of cosmic microwave background from the big bang era to *boson sampling*, a novel quantum computing method.

> Bosogenesis before Baryogenesis?: And God said, Let there be light: and there was light (Genesis, 1:4)

Keywords: Bose statistics, history of physics, quantum mechanics

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1. Introduction

Science and understanding of the inner workings of nature have transformed society, its evolution and the way we think. While reminding us of the limitations of human beings as individuals, science opens doors to unlimited potentials, when we work together. *Inspired guesses* of the secrets of quantum mechanics, based on experimental realities, is one of the great creative acts and intuitive leaps of human minds working in collaboration during one of the darkest, war-filled periods in human history. Planck stood on the shoulders of giants Maxwell, Boltzmann, Rayleigh, Wien and others. Einstein stood on the shoulders of Planck and others. Niels Bohr stood on the shoulders of Planck, Einstein and others. Bose stood on the shoulders of these giants and found a new quantum statistics, a remarkable consequence arising from Bose's notion of the indistinguishability of the quanta of particles, use of the uncertainty principle and the possibility of intrinsic *quantized* spin of photons, etc.

The all-pervasive Bose distribution is visible in the Sun, the cosmic microwave background (CMB) from the big bang, black hole radiation with a Hawking temperature and several day-to-day phenomena. Bosons and fermions are two pillars of modern quantum field theory and technology. Bose and Fermi quantum fields interact in the standard model of elementary particles and many-body theories of quantum matter. Lasers and semiconductor electronics dominate modern technologies. Quantum entanglement of two photons exemplifies the Einstein-Podolsky-Rosen (EPR) paradox. Quantum entanglement is a basis of modern quantum information theory and is an excellent resource for quantum computing.

In the first part of this article we recall briefly the life of Bose and his statistics. A wealth of literature is available on the life and science of Bose: a book 'Satyendra Nath Bose (His Life and Times)' [7] 'Selected Works (with Commentaries)' by Wali [8], a popular Science Book, 'Bose and his Statistics' by Venkataraman [9], among other articles [10, 11]. Insightful discussions about Bose statistics and how it impacted the growth of wave mechanics etc are found in the articles of Ghose [12] and Mukunda [13]. Most of the above resources are available at the SN Bose archive [14] of SN Bose Center for Basic Sciences, Kolkata.

In the second part we indicate how the quantum garden of bosons continues to bloom and impact modern science and technology in ways unforeseen. The vast Hilbert space of interacting bosons provides unlimited opportunities for *quantum voyagers* at the theoretical and experimental frontiers.

2. A brief life history of Bose

2.1. Remarkable year for Kolkata

Bose was born in Calcutta (now Kolkata), the eldest of seven children in a Bengali family. He was the only son, with six sisters after him. His ancestral home was in the village of Bara Jagulia, in the district of Nadia during the Bengal Presidency. His father, Surendranath Bose, worked in the Engineering Department of the East Indian Railway Company [15]. In 1914, Satyendra Nath Bose married Ushabati Ghos, and they had nine offspring, two of whom died in early childhood. When Bose died in 1974, he left behind his wife, two sons, and five daughters.

Bose began his studies at Presidency College in Kolkata, where MN Saha, JC Ghose, and JN Mukherjee joined him. This was a remarkable young group [16]. His professors paid particular attention to him because of his abilities across a range of subjects. His talents in languages quickly matched his abilities in mathematics. He later graduated from Kolkata University with a BSc in Physics in 1913 and an MSc in 1915, just as Bohr's theory of atomic structure was being developed [5]. It is noteworthy that within that same yeargroup, other graduands consisted of luminaries such as MN Saha, S Dutta, A Chakraborty, S Ghosh in physics, JC Ghost, JN Mukherjee, PK Bose, PB Sarkar in chemistry and NR Sen of applied mathematics.

In 1917, Bose was appointed lecturer in Physics at Kolkata University. The revolutionary new physics of Max Planck's quantum hypothesis and Einstein's quantization of electromagnetism had already greatly influenced him. Planck's 1901 hypothesis led to a successful derivation of the energy distribution for black body radiation, which was not subject to either the ultraviolet catastrophe of the classical theory at short wavelengths using Rayleigh–Jeans' law or the failure of Wien's law at longer wavelengths [3]. Planck formulated his famous law based on the electromagnetic theory of statistical mechanics.

Within statistical mechanics, Bose contributed two joint papers with MN Saha between 1918 and 1920 on the influence of finite volume molecules on the physical equation of state, and a deduction of the Rydberg Law. The former was later named the Saha-Bose equation of state. Furthermore, in 1919, he published two papers on the solution of stress equations of equilibrium in elasticity. His 1920 deduction of Rydberg's Law from a quantum theory of spectral emission hinted at a pattern of variation in his work.

2.2. Dacca University

In 1921, Bose reluctantly accepted the offer of a readership at the newly established Dacca University with an increased salary and academic freedom. He was particularly interested in using his Bengali mother tongue to teach science [17], and in this new and conducive atmosphere, he studied extensively to become a more effective teacher. Parallel to this, he acquired a deep knowledge of classical thermodynamics and statistical mechanics based on the ideas of Maxwell and Boltzmann. It is true that Gibbs' earlier ideas on the topics of entropy and the paradox in his name greatly assisted Bose [19].

In 1924, he used these ideas to further develop a new quantum statistical physics, deriving Planck's law without any assumptions regarding classical electromagnetic theory nor a wave treatment of photons [1]. Bose submitted this work as a paper to the Philosophical Magazine, but after a six-month wait, it was rejected. Undeterred, he sent a personal copy to Einstein with a cover note. Einstein immediately recognized

Bose [18].

the novelty of this contribution and replied to Bose, assuring him that he would submit it to Zeitschrift für Physik on his behalf [20, p266].

2.3. *In Paris and Berlin*

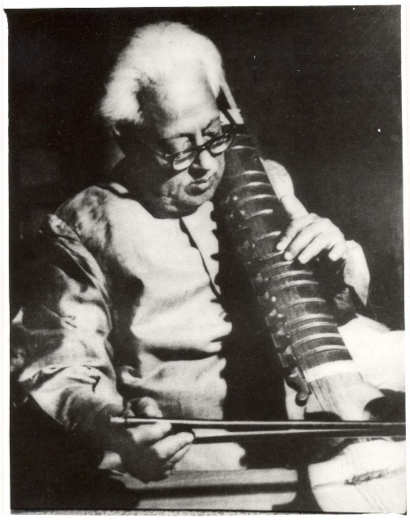
Early in 1924, Bose applied to Dacca University for a two-year leave of absence to study abroad in Paris and Berlin. Requiring a visa to travel, Bose attended the German consulate in Kolkata and was issued one at no cost upon showing Einstein's earlier note [17].

He duly sailed to Paris in the summer of 1924 where, upon arrival, he worked in Madame Curie's laboratory. In this environment, he had the rare opportunity to meet Louis de Broglie, Paul Langevin, and Maurice de Broglie. Here, he became familiar with the experimental techniques of x-ray spectroscopy and crystallography, which motivated KN Krishnan and his other students to undertake significant work in crystal physics back in Dacca [8].

In October 1925, Bose set his sights on Einstein's invaluable help and guidance, and he traveled to Berlin in the hope of meeting with him. As luck would have it, Einstein returned to Berlin only weeks later from an annual visit to Leyden, and they were to finally meet in person [20, p354]. However, by this time, Einstein had lost interest in further research into statistical physics, and they never did work together. But, while in Berlin, he met with many other world-renowned scientists, including Peter Debye, Wolfgang Pauli, Werner Heisenberg, Eugene Wigner, and Max Born. Of particular note was the unique opportunity to discuss the current developments in quantum mechanics, of which attending a lecture by Max Born in Gottingen left its mark.

2.4. *Professorship at Dacca University*

During his time in Berlin, Bose learned through friends that a full professor position was vacant at Dacca University. Although he was reluctant to apply for the post as it required a PhD degree, which he did not have, Einstein was pleased to send a personal recommendation indicating that Bose was held in such high esteem that he was working in Berlin to the benefit of Germany. As a result, in the summer of 1926, he returned to Dacca with this position and that of Head of Physics. He was to stay here for 22 years, working on a series of theoretical and experimental research projects with his students.



Bose as a modest man of many talents, playing the Esraj [14].

2.5. *Triumphant return to his alma mater*

By 1945, Bose had reached the peak of his academic life and was a well-respected scientist and a household name in India. He was then offered the Khaira Professor of Physics position at Kolkata University, returning full circle to his alma mater. Over the following decade until his formal retirement in 1956, he continued to be awarded numerous honours, including election as a Fellow of the Royal Society, London, and following that the national title of Padma Vibhushan in 1958 by the then President of India.

2.6. *A modest man with many talents*

During the 1930s and 1940s, Bose became increasingly passionate about Indian independence from two hundred years of British rule. When this finally occurred in 1947 he was elated but could not accept the partition of Bengal and India. In 1962, Bose went to Moscow, via Sweden, to attend the Peace Conference, organized in memory of the dropping of the atomic bomb on Hiroshima and Nagasaki in August 1945. He then visited Japan and was impressed by Japan's scientific, economic and social progress.

With his deep-held concerns for the illiterate and poor of India, he was particularly taken by the use and power of the mother tongue in Japan in their science education. Bose long held an interest in his native language, Bengali, and a faith in humanity. He had this affinity with the ideas of Pierre Teilhard de Chardin [17]—to overcome all obstacles that impede the building of future civilization, irrespective of religious or national differences.

Bose and Dirac together [14].

Based on these teachings, Bose encouraged the promotion of science to the illiterate and poor in native Bengali in the magazine Jnan O Bijnan. It was Bose's combination of a profound gift of character and intelligence that he was to become such a popular figure.

2.7. Bose–Dirac meeting in Kolkata

Dirac had a great admiration for Bose's work. Dirac coined the name boson to describe the family of indistinguishable particles whose many particle wave functions are symmetric under the interchange of any two particle coordinates. In his article, 'SN Bose—Some Vignettes of a Many-sided Personality' Bose [8, p462] writes, *'Dirac was surprised because he had never met Bose, although Bose was in England during his European visits. He was also surprised that he was not a Fellow of the Royal Society of London. It was shortly after Dirac's visit that S.N. Bose and S.K. Mitra, the latter well-known for his pioneering researches on radio and wireless, were elected belatedly in 1958 as Fellows of the Royal Society.'*

2.8. Bose, Saha and Raman

There has been a special friendship and mutual admiration between Bose and Raman. Two anecdotes illustrate this. Ramaseshan in his article [21] quotes Bose, 'In fact I was so impressed with the discovery that when we were on a walk along the Hoogley, I told him near the Prinseps Ghat "Professor Raman you have made a great discovery. You will surely get the Nobel Prize for it" '. Another one, which we will discuss later, is the appreciation of Raman and Bhagavantam in their article 'Experimental Discovery of Photon Spin', of the introduction of quantized angular momentum for photons in the original version of his paper in 1924.

The friendship of MN Saha (well known for the Saha ionization formula and the emergent spin-$\frac{1}{2}$ orbital angular momentum of electric and magnetic monopole bound states, etc) and SN Bose from their student days is well documented. Bose, Saha and Raman have been called the *pre-independent Quantum Indians*.

2.9. Bose and some of his pupils

Two senior people who were influenced by Bose and whom GB (one of the authors) knows personally are Partho Ghose and Mani Bhaumik. Partho Ghose is a theoretical physicist who is also well known for his science popularization activities. His PhD supervisor was Bose. Partho Ghose works on fundamental issues in quantum mechanics, including the prediction of quantum tunnelling-like phenomena of photon particles with Home and Agarwal [22]. This effect has been experimentally confirmed [23].

GB met Professor Mani Lal Bhaumik last year in Los Angeles. Mani Bhaumik, a billionaire, is about 91 and continues to think and write on issues such as quantum entanglement, the Einstein-Rosen bridge, etc. Mani Bhamik, who comes from a poor family, was encouraged by Bose from his younger days. Bhaumik completed a MSc at the University of Calcutta and was first to get a PhD in physics from IIT Kharagpur. Soon after his post-doctoral work at the University of California at Los Angeles (UCLA), Bhaumik made important discoveries at the Northrop Corporate Research Laboratory. He has made remarkable progress in making excimer lasers. Excimer lasers have become a valuable tool for painless laser surgery.

Mani Bhaumik's fascination for theoretical physics and educating the poor has made him a philanthroper. The Mani Bhaumik Institute of Theoretical Physics at UCLA, among others, stands tall and exemplifies his philanthropy [24]

3. Bose and quantum statistics

3.1. Planck distribution before Bose

Statistical mechanics studies the properties of systems in thermal equilibrium. It uses the properties of constituent atoms and molecules to understand their macroscopic properties. For example, the celebrated equation of state of ideal gas (Boyle's law) and real liquids (van der Waal isotherm) can be derived using classical statistical mechanics. As the name implies, it uses the statistical properties of microstates (velocities and positions) of interacting particles and arrives at a macroscopic description. Atoms and molecules obey Newtonian laws of motion.

In this context the spectrum of black body thermal radiation offered a challenge. Using Maxwell's equations, classical electrodynamics and classical statistical mechanics, one is unable to describe black body radiation. In 1900, Max Planck was at a crossing point. He introduced [3] the concept of Hertzian oscillators, which emit energy packets (quanta) and the famous relation $\epsilon_\nu = h\nu$, connecting energy quanta ϵ_ν to the frequency of the oscillator ν and empirically introduced the famous Planck constant h. The resulting Planck distribution nicely interpolated the low frequency part, Wien's displacement law and the high frequency Rayleigh–Jeans law.

The possibility that light could consist of particles was so outrageous that the referees for Einstein's nomination to the Prussian Academy of Sciences included an apology for his speculative extravagance.

In 1907 Einstein [25] attempted to understand the marked deviation of the specific heat of solids at low temperatures from the classical Dulong–Petit law. He introduced uncoupled quantized harmonic oscillators, which came to be known as *Einstein atom oscillators*. Debye [26] went further and studied coupled Einstein oscillators and found normal modes of acoustic vibrations, described by quantum harmonic oscillators. Einstein and Debye phonons, which are quantized excitations of the lattice of atoms, are not material particles. This was similar to the quanta of normal modes of electromagnetic radiation in a cavity, for example.

However, by 1917, Einstein [27] had pushed his idea further and introduced spontaneous and radiation-induced molecular transition probabilities and recoil momentum upon absorption and emission processes. He associated wave and particle properties of the quanta via A and B coefficients. At this juncture, there were some logical inconsistencies in all derivations of the Planck distribution, including Einstein's. It is here that Bose enters the scene.

3.2. Birth of Bose statistics

In his short and historic article, which he communicated to Einstein, Bose explicitly states his unhappiness about the state of affairs of the derivation of the Planck distribution. Bose is polite but frank and critical:

A remarkably elegant derivation has been presented by Einstein. This author has recognized the logical deficiency of all previous derivations and attempted to deduce the formula without reference to the classical theory. Starting from very elementary assumptions concerning the exchange of energy between molecules and the radiation field, he finds the relation

$$\rho_\nu = \frac{\alpha_{mn}}{e^{\frac{\epsilon_m-\epsilon_n}{kT}}-1}. \tag{1}$$

However, in order to bring this formula into agreement with the one by Planck, he has to make use of Wien's displacement law and Bohr's correspondence principle. Wien's law is based on the classical theory, and the correspondence principle supposes that the quantum theory be in agreement with the classical theory in certain limits.

In all cases it seems to me that the derivations are not justified in a sufficiently logical manner. On the other hand, it seems to me that the hypothesis of light quanta in connection with statistical mechanics (as it has been adapted to the demands of quantum theory by Planck) is sufficient for the derivation of the law without taking recourse to the classical theory. In the following I will briefly sketch the method'.

Bose avoids any reference to electromagnetic modes in his articles. He assumes Einstein's photon quanta are massless particles, moving at the velocity of light c. Light quanta carry an energy $h\nu = pc$ and vector momenta with magnitude $p_s = \frac{h\nu_s}{c}$. Here, ν_s is the frequency of the radiation quanta. Frequency ν_s varies between 0 and ∞, in steps of $\Delta\nu$.

It is the enumeration of the arrangement of these microstates that determines the overall macrostate. If microstates are assumed to be equally probable, the system's entropy is given by the well-known Boltzmann entropy equation. The macrostate can be understood if this entropy is maximized subject to the constraints of constant total energy and number of particles. Bose's unique viewpoint was to generalize coordinates to include the positions and momenta of particles constrained to a spherical volume in quantum phase space, (x, y, z, p_x, p_y, p_z). The total volume of phase space is given by

$$\int \mathrm{d}x\mathrm{d}y\mathrm{d}z\ \mathrm{d}p_x\mathrm{d}p_y\mathrm{d}p_z = V\,4\pi\left(\frac{h\nu}{c}\right)^2\frac{h\mathrm{d}\nu}{c} = 4n\frac{h^3\nu^3}{c^3}V\mathrm{d}\nu.$$

He divides, following Planck, the total phase space volume into cells of magnitude h^3. The frequency interval $\mathrm{d}\nu$ thus corresponds to $4\pi V\frac{\nu^2}{c^3}\mathrm{d}\nu$ cells. Bose also states,

As regards the nature of this division, nothing definite can be said. However, the total number of cells has to be interpreted as the number of possible arrangements of a quantum in the given volume. To account for the fact of polarization it seems mandatory to further multiply this number by 2, so that we obtain $8\pi V\frac{\nu^2}{c^3}\mathrm{d}\nu$ *for the number of cells belonging to* $\mathrm{d}\nu$.

Bose also implicitly assumes a varying total number of photon particles (as the chemical potential is zero). Without explicitly stating it he seems to use indistinguishability in his counting of microstates and arrives at the Planck distributionby the standard Boltzmann entropy maximization approach.

According to Pais [28], *'Bose recalled many years later that he had not been aware of the extent to which his paper defied classical logic. (Such a lack of awareness is not uncommon in times of transition, but it is not the general rule. Einstein's light-quantum paper of 1905 is a brilliant exception.) 'I had no idea that what I had done was really novel... I was not a statistician to the extent of really knowing that I was doing something which was really different from what Boltzmann would have done, from Boltzmann statistics. Instead of thinking of the light-quantum just as a particle, I talked about these states. Somehow this was the same question which Einstein asked when I met him [in October or November 1925]: how had I arrived at this method of deriving Planck's formula ?'.*

3.3. Spin of the photon

Photons carry a quantized angular momentum with an intrinsic spin $S = 1$. In special relativity massless bosons have only two physical components of the spin-1 state. Only two polarization states of light were known to Bose. Bose introduced a factor of 2 to take into account the polarization state of the photon in his counting of the microstates of photon particles without any justification. In the modern interpretation this corresponds to intrinsic quantized angular momentum $\pm\frac{h}{2\pi}$ carried by the photon particle.

Einstein removed the factor of 2 from Bose's article, without realizing that Bose had provided another deep insight into the evolving quantum theory, namely the intrinsic quantized angular momentum of photons. This factor came from an unexpected corner. In their famous paper [29], Raman and Bhagavantam state the following:

In his well-known derivation of the Planck radiation formula from quantum statistics, Prof. S N Bose obtained an expression for the number of cells in phase-space occupied by the radiation, and found himself obliged to multiply it by a numerical factor 2 in order to derive from it the correct number of possible arrangements of the quantum in unit volume. The paper as published did not contain a detailed discussion of the necessity for the introduction of this factor, but we understand from a personal communication by Prof. Bose that he envisaged the possibility of the quantum possessing besides energy $h\nu$ *and linear momentum* $h\frac{h\nu}{c}$ *also an intrinsic spin or angular momentum* $\pm\frac{h}{2\pi}$ *round an axis parallel to the direction of its motion. The weight factor 2 thus arises from the possibility of the spin of the quantum being either right-handed or left-handed, corresponding to the two alternative signs of the angular momentum.*

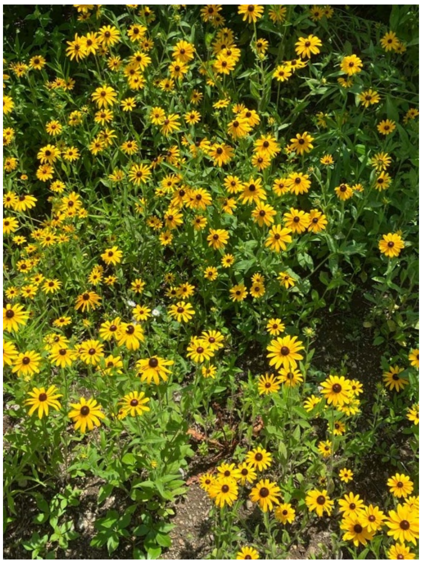
3.4. Einstein and ideal Bose gas

Within a week of submitting Bose's paper, Einstein published [2], using the brand new Bose statistics, his work on the quantum theory of monatomic ideal gases (of a given density determined by the chemical potential). He highlights that there appears to be an implicit paradox, as the Bose gas has less pressure than that of a classical gas. With Bose's indistinguishable counting, the entropy of the gas is much lower than in the classic case with distinguishable counting. It is implied that if two such gases were brought into heat contact, the Bose–Einstein gas would lose energy, leading to a pressure anomaly. Einstein also got excited that at low temperatures entropy respected the Ehrenfest theorem of vanishing entropy. Einstein also discovered a *macroscopic quantum phenomena*, namely Bose–Einstein condensation in an ideal Bose gas of nonrelativistic material particles, below a finite temperature.

Shortly afterwards, Einstein published a second paper [30] where he calculated density fluctuations according to his new theory. He works backwards from entropy fluctuations to determine that entropy consists of two additive parts, one due to the expectation of independent particles and another independent of the density. He also speculates that this can be further interpreted as an interference between particle matter waves. It is noteworthy that at that time, Bose was advised by Langevin of the recent work of one of his students, de Broglie, within his thesis.

4. Blooming bosons in the last 100 years

As mentioned earlier, four key papers caused a paradigm shift through their new insights in the evolution of quantum mechanics. The first one was Planck's hypothesis of energy quanta and phase space analysis. The second was Einstein's elevation of the energy quanta to the level of quantum particles in his explanation of the photoelectric effect. The third is Bohr's model of the atom, invoking special orbits which obeyed certain angular momentum quantization conditions. The fourth one is Bose's idea of the indistinguishability of relativistic photon particles.



A bloom of ∼100 sunflowers (photo by the authors), in tribute to S.N. Bose and Einstein.

The new insight into the form of the indistinguishability of photon particles and liberation from the notion of electromagnetic modes helped Einstein study free Bose gases with a finite density and discover Bose–Einstein condensation. Unexpected developments took place following the lead of Planck, Einstein and Bohr. New participants were de Broglie, Schrödinger, Max Born, W Heisenberg, Pauli, Dirac and Fermi, among others.

These developments in turn lead to developments such as the standard model of elementary particles, superfluidity, superconductivity, quantum magnetism and so on. Lasers were developed. Lasers and quantum materials are revolutionizing the modern electronic industry, including potential applications to quantum information theory and novel qubits for quantum computation.

4.1. Birth of modern quantum mechanics and quantum field theories

In succession, several remarkable developments took place between 1924 and 28. Events unfolding after Bose and Einstein's articles were discussed in depth and detail by several authors. We would recommend the readers to study the well researched articles by Partho Ghosh and N Mukunda [12, 13]. In his article in the journal *Resonance* Mukunda states, "*Soon after the above events, modern quantum mechanics was discovered during 1925–26; and in 1927 Paul Adrien Maurice Dirac completed the task of quantising the classical Maxwell field, something which Einstein had foreseen as early as in 1917. And with that the photon was here to stay.*"

The special theory of relativity joined quantum mechanics and quantum field theory. The Pauli exclusion principle led to Fermi Dirac statistics. The spin statistics theorem, which showed that bosons have integer spins (0, 1, 2, ...) and fermions have half-integer spins ($\frac{1}{2}$, $\frac{3}{2}$, ...). The single particle wave function introduced by Schrödinger resulted in surprising new permutation symmetries for the wave functions of many identical particles, while the boson wave function remained the same when the coordinates of the two particles were interchanged, and the fermion wave functions changed sign. That is, one obtained symmetric and antisymmetric many-body wave functions. The notions of Fock space, Slater determinants, etc, soon followed .

Beginning with the quanta of electromagnetic radiation, the quantum garden of bosons contains elementary particles, photons, gluons, mesons, W-bosons, Higgs bosons, gravitons, axions, etc., and emergent particles in quantum matter: phonons, magnons, excitons, plasmons, polaritons, Cooper pairs, etc. These bosons interact among themselves and with fermions such as electrons, quarks, neutrinos, electrons, neutrons, protons, etc, and create an exciting and incomprehensibly rich Universe that we on earth are part of.

4.2. Big bang and black holes

According to Einstein's theory of general relativity our Universe began in a big bang singularity. After the big bang, inflation, etc, thermalization took place, which is what is believed to be seen as a nearly perfect Planck distribution of CMB at about 3 K. Other remarkable theoretical predictions were black body radiation in the form of Hawking radiation from black holes and the Unruh effect, a thermal Planck radiation seen by an accelerated observer emerging from the electromagnetic vacuum.

Hawking radiation gives rise to the well-known information paradox, still an active field [31] in the context of AdS correspondence, string theory, black hole thermalization, etc.

4.3. Quantum tunnelling of photon particle

If a photon is a quantum particle, will it undergo quantum tunnelling over suitable barriers? This question was raised, and a positive answer was given by Ghosh, Home and Agarwal [22]. This effect, deeply related to the evanescence phenomenon, has been seen in recent experiments [23].

4.4. Laser, a cooperative phenomenon

Laser is a remarkable cooperative phenomenon. At the heart of lasers is the notion of stimulated emission arising from the Bose character of photons. In the theoretical description of the properties of the quantum state of a laser, the *coherent state* of the simple harmonic oscillator (Schrödinger, Sudarshan, Glauber and others) became useful. The nature of quantum fluctuations contained in thermal radiation, laser or non-classical radiation were distinguished and unusual features were observed.

A remarkable development in the field of lasers ere femtosecond lasers and recently attosecond lasers. They have revolutionized spectroscopy in chemistry, physics and biology. Laser tweezers and optical trapping, which use radiation pressure in a fundamental way, began a field of ever growing activities, particularly in biology-related problems.

Optical vortices, arising from the condensation of of photon spin, are important in basic science and applications. The notion of random lasers or Anderson lasers, where mirrorless lasing takes place due to the localized nature of the cavity modes (induced by disorder) is also an exciting front. At the opposite end we have photonic crystals with surprising properties.

The Dicke theory of superradiance was developed and experimentally confirmed. Recent works suggest possible ways to create quantum entanglement among two level atoms using the physics of Dicke superradiance.

4.5. Superfluids, superconductors and macroscopic quantum phenomena

The theory of free Bose gas predicted Bose–Einstein condensation in a collection of identical particles of finite density (determined by a non-zero chemical potential). Back in 1938 superfluidity in liquid He-IV was discovered by Kapitza [32] below about 2 K. London [33] identified it with Bose–Einstein condensation that survives in interacting He^4 atoms in a liquid state and leads to superfluidity. Remarkably, the observed superfluid transition temperature in strongly repelling He^4 atoms is about 2.2 K, which is close to the Bose–Einstein transition temperature $Tc \approx 3.1K$ of free Bose gas (as estimated by London) at the same density. Behind these phenomena is the physics of renormalization of interaction parameters (becoming effectively *irrelevant*), at low energy scales.

Strong interactions generate quantized vortices in superfluid He^4 with a core size of atomic dimensions. It is also remarkable [34] that nano droplets of He^4 atoms containing numbers of atoms as low as 100, exhibit superfluid properties and support quantized vortices!

BEC has been observed experimentally in optically trapped cold atom gas systems, with atom densities several orders of magnitude smaller than that of superfluid He^4. This has led to the creation of quantum matter waves, their interference, etc. [35]

In 1910 Kamerlingh Onnes discovered superconductivity in solid Hg at low temperatures with a $Tc \sim 4$ K. This is in some sense a Bose condensation phenomenon, where charge -2e Cooper pairs form spin singlet pairs (effective bosons) and undergo condensation, leading to the observed dissipationless superconductivity. The Ginzburg-Landau theory and BCS theory provided new insights into this macroscopic condensation phenomenon, resulting in quantized vortices, coherence phenomena Josephson effects and so on.

Stable superconductivity at ambient temperatures and pressures, when discovered, will revolutionize science and technology. Partly encouraged by the discovery of high Tc superconductivity in cuprates by Bednorz and Muller [36] and RVB theory [37], it has been proposed that room temperature superconductivity is possible in principle [38] in real materials. With intense experimental searching and the synthesis of new superconductors, the observation of elusive and unstable superconductivity [39] at ambient temperatures continues to be reported.

Josephson effects, both DC and AC, in a spontaneous supercurrent that is present between two weakly coupled superconductors, have been used in forming superconducting quantum interference devices, etc. Their low energy properties are used to construct qubits for quantum computing. Quantized vortices in 2D spin triplet $p + ip$ superconductors [40] and the ends of a p-wave superconducting wire have been suggested by Kitaev to support Majorana modes [41], which could be used for topological quantum computation [42].

In liquid He^3, where we have strongly repelling spin-half neutral fermions, pairing and superfluidity take place in a spin triplet channel at milli Kelvin scales. This results in a rich phase diagram. In the above consideration, synthetic systems such as neutral cold atoms, Rydberg atoms, ion traps and novel quantum processors are serious participants and competitors.

The notion of the pairing neutral fermions has reached the sky—it has been suggested that a thin crust of neutrons in rapidly rotating neutron stars form a fermionic superfluid, akin to terrestrial He^3. The creep of the vortex lattice (arising from the rotation of neutron stars) and their stick slip behaviour have been used to explain the observed glitches in the rotation period of neutron star pulsars [43]. There have also been suggestions that paired neutrinos in the Universe condense as a possible explanation for dark matter/energy in the Universe.

In heavy ion collisions and other contexts, such as quark matter in collapsing stars, it has been suggested that coloured quarks condense into a state called colour superconductivity [44].

4.6. Quantum entanglement of photons

Quantum entanglement is a non-classical phenomenon present in quantum mechanics, first identified by Schrödinger. It also leads to EPR paradox and an intense historic dialogue between Einstein, Bose and others. Quantum entanglement is illustrated by the well-known entangled state of two photon particles in their polarization degree of freedom: $\frac{1}{\sqrt{2}}(|L,R\rangle - |R,L\rangle)$. Here, L and R refer to the left and right circularly polarized states of a photon particle.

We will not go to details of the EPR paradox. However, entanglement between two identical photons separated by miles, a non-local quantum correlation effect, has been experimentally confirmed. It is also remarkable that even for a gas of identical photons in a pure state, all right circularly polarized (say) particles are also entangled. This arises from the permutation symmetry possessed by the many, many photon wave functions. That is, indistinguishability of even spin zero bosons is a quantum resource, similar to quantum entanglement. [45]

In the field of radio astronomy, the famous Hanbury-Brown-Twiss experiments [46] exemplify the presence of two-photon interference of radio frequency quanta between distant antennas. Similarly, the Hong-Ou-Mandel effect [47] is a two-photon interference phenomenon, leading to non-classical effects and potential applications. A remarkable recent suggestion is that of a *Quantum Radar* that was suggested by Seth Lloyd [48], which use two entangled photons, to get a sharp picture of an object in a hazy background.

4.7. Bosonic quantum correlations in the warm wet biology

Frohlich suggested decades ago [49] that interacting electric dipoles, ubiquitous in biology, will under some conditions support an extended collective mode of quantum electric dipoles. He suggested the possibility of Bose condensation and macroscopic occupancies in these modes at room temperature. Similarly, the presence of quantum entanglement between photon particles and excitons in photosynthetic systems [50] has been invoked.

4.8. Boson sampling route to quantum computation

There seem to be many avenues to achieve quantum computation using bosons. One of the recent avenues is the so-called *boson sampling* method suggested by Scott Aaronson and Arkhipov [51]. In this remarkable linear optical system there are N input ports and N output ports. One prepares and sends photons in a number of eigen states, a set of integers in the N ports. There are only linear elements like reflectors, partial transmitters, interference, etc. They enable quantum interference among traversing multi-photon states.

In general output ports will have different photon numbers than the corresponding input ports with varying probabilities. Given the input photon numbers, finding the probabilities of various outputs has been shown to be a hard problem. It involves the evaluation of *permanents*, which is hard for classical computers.

There was a claim of quantum supremacy using boson sampling experiments with 50–70 ports by Chaoyang Lu and Jianwei Pan at USTC in China [52].

5. Discussion

It is clear that Bose had a remarkable life, all the while enjoying science and touching a variety of subjects including mineralogy. As we saw in this article, his bold venture to derive the Planck distribution in an unconventional fashion, avoiding logical contradictions, produced Bose statistics. We see boson bloom all over, from quantum many body theory to quantum computation. A scientist, however great his achievement, is a human being. Bose's greatness is visible in the anecdote described by Ramaseshan [21] in his article:

… I told Bose that it was the only statement in a scientific journal that he had proposed for the first time that photon had a spin. I asked him whether something should be done so that he be given the credit for proposing the existence of the spin of the photon for the first time.

Bose suddenly showed some annoyance. 'You are descending from the sacred to the profane—as the saying goes. What is important is whether a photon has a spin and it is not important as to who proposed it first and when. The high aims of *science get very debased by such talks of priority etc' I told him that I stand rebuked but my respect for the man went up greatly.*

… Now I have to punish you. You have to listen to my playing the esraj. It was a delightful ending … After half an hour of esraj I rose and said goodbye.

Bose played esraj and Einstein violin. Bose statistics and Bose–Einstein phenomena continue to provide music to our ears.

Data availability statement

No new data were created or analysed in this study.

Acknowledgment

We thank Professor Tanusri Saha Dasgupta for invitation to write this article. GB thanks N Kumar, KP Sinha and PW Anderson, for insightful discussions on interacting bosons, such as plasmons, magnons, supersolid He^4 etc, from 1971; and Rajaram Nityananda for a very recent discussion. AM thanks the IoP History of Physics Group for their assistance in writing this article. Certain images in this publication have been obtained by the author(s) from the Wikimedia website, where they were made available under a Creative Commons licence or stated to be in the public domain. Please see individual figure captions in this publication for details. To the extent that the law allows, IOP Publishing disclaim any liability that any person may suffer as a result of accessing, using or forwarding the image(s). Any reuse rights should be checked and permission should be sought if necessary from Wikimedia and/or the copyright owner (as appropriate) before using or forwarding the image(s).

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