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Plasma Suppression of Beam-Beam Interaction in Circular Colliders*

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Abstract

A possibility to suppress beam-beam interaction in a circular collider by means of introducing a plasma at the interaction point of the colliding beams is considered. It is shown that for TeV proton and muon colliders, the overdense plasma can easily suppress the beam-beam tune-shift parameter several times without degrading the beam lifetimes.

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1. INTRODUCTION

In this paper we study a possibility to overcome a major obstacle in future colliders – limitation of the luminosity caused by the beam-beam interaction [1]. Electromagnetic interaction of the intense colliding beams can result in strong perturbation of particle motion which, in extreme situation, makes the motion of the beam particles unstable. The conventional measure of the beam-beam interaction is given by the so called beam-beam tune shift parameter ξ [2], which, for round beams, is equal to $\xi = N r_{\text{class}} / (2\pi\gamma\varepsilon_n)$, where N is the number of particles in the bunch, γ is the relativistic factor, r_{class} refers to the classical radius of the particles comprising the beam ($r_{\text{class}} = 1.6 \times 10^{-16}$ for protons, and $r_{\text{class}} = 1.4 \times 10^{-15}$ for muons), and ε_n is the normalized emittance of the beam. In the design and operation of modern colliders, the parameter ξ is usually set below 0.05 for electron machines and less then 0.005 for hadron colliders in order to avoid the diminishing of the dynamic aperture. Relatively small values of ξ result in the limitation of the luminosity of the collider because, for a given number of particles in the bunch and its dimensions at the interaction point, the luminosity is proportional to ξ.

The general tendency in high energy physics looks for dramatic increase in collider luminosity, which inevitably push ξ to higher values. Recently, it was found [3] that in the muon collider currently under study [4], the beam emittance increases substantially even after a single beam crossing. It is therefore highly desirable to find a means to ameliorate the long standing problem of beam-beam interaction.

In order to suppress the effect of beam-beam collisions on particle dynamics, we propose to intercept the colliding beams at the interaction region with a plasma. If the plasma density is larger than the particle density of the colliding bunches, the electric field of the beams will be suppressed by repelling (in case of negatively charged bunches) or attracting (in case of bunches of positive charges) plasma electrons. However, the suppression of electric field only is not sufficient; it eliminates electric force of the incident beam, but, at the same time, it releases the effect of the magnetic field of the beam, which in vacuum is canceled by the own electric field within a factor of γ^{-2}. This results in a so-called "self-focusing" effect and has been proposed as a means to strongly focus high energy beams (plasma lens) [5].

In the far overdense regime, however, both the electric and magnetic fields could be canceled. This regime of beam-plasma interaction has been invoked to suppress disruption and beamstrahlung in the beam-beam interaction in linear colliders [6,7]. The issue at stake in that case is the extremely high plasma density required and the associated concern on the induced detector backgrounds. For the case of storage rings, as we will see in what follows, the issues are different. The required plasma density is much smaller, however, of primary importance becomes the degradation of the beam lifetime due to the collisions with plasma particles. This effect practically eliminates a possibility of using our scheme in electron and positron colliders but does not preclude plasma at the interaction point of a heavier particle machine such as muon and proton circular collider. Suppression of the beam-beam interaction with the plasma, if successful, allows, in principle, to increase the number of particles in the bunch and boost the luminosity of the collider.

2. BEAM-PLASMA INTERACTION

In order to suppress the magnetic field, the beam should generate a return current in the plasma in the direction opposite to the beam current. To calculate the magnetic and electric fields in the bunch traveling through the plasma we will assume that the beam density is much smaller than the plasma density and neglect variation of the plasma density in space. In this case, a general expressions can be derived for the fields generated by arbitrary external current density $\mathbf{j}(\mathbf{r},t)$ in a cold plasma [8]. We will consider first the magnetic field $\mathbf{B}(\mathbf{r},t)$,

$$\mathbf{B}(\mathbf{r},t)=\frac{4\pi i}{c}\int\frac{\mathbf{k}\times\tilde{\mathbf{j}}(\mathbf{k},\omega)}{k^2-\omega^2\varepsilon(\omega)/c^2}e^{-i\omega t+i\mathbf{kr}}d^3kd\omega. \tag{1}$$

In Eq. (1), $\varepsilon(\omega)=1-\omega_p^2/\omega^2$ is the dielectric function of the cold plasma, ω_p is the plasma frequency, $\omega_p^2=4\pi n_p e^2/m_e$ (n_p is the plasma density and m_e is the electron mass), and $\tilde{\mathbf{j}}(\mathbf{k},\omega)$ is the Fourier transform of the bunch current,

$$\tilde{\mathbf{j}}(\mathbf{k},\omega)=\frac{1}{(2\pi)^4}\int\mathbf{j}(\mathbf{r},t)e^{i\omega t-i\mathbf{kr}}d^3rdt. \tag{2}$$

For what follows, we will also need a wavelength k_p associated with the plasma frequency, $k_p=\omega_p/c$. For a round beam moving along the z-axis, the beam current has only one component, $\mathbf{j}=(0,0,j_z)$, where

$$j_z(r,t)=Nec\lambda(z-ct)\rho(r), \tag{3}$$

N is the number of particles in the bunch, $\lambda(z)$ is the longitudinal, and $\rho(r)$ – radial distribution functions in the bunch normalized so that $\int_{-\infty}^{\infty}\lambda(\xi)d\xi=1$ and $2\pi\int_0^{\infty}\rho(r)rdr=1$. We assume a long-thin bunch, $\sigma_z\gg\sigma_r$, where σ_z is the rms length of the bunch, and σ_r is its rms radius. For Fourier components of the current we have

$$\tilde{j}_z(\mathbf{k},\omega)=Nec\tilde{\lambda}(k_z)\tilde{\rho}(k_\perp)\delta(\omega-k_zc), \tag{4}$$

where $\mathbf{k}_\perp=(k_x,k_y,0)$ and $k_\perp=|\mathbf{k}_\perp|$,

$$\tilde{\rho}(k_\perp)=\frac{1}{(2\pi)^2}\int\rho(r)e^{-i\mathbf{k}_\perp\mathbf{r}}d^2k_\perp,\quad \tilde{\lambda}(k_z)=\frac{1}{(2\pi)}\int\lambda(\xi)e^{-ik_z\xi}d\xi. \tag{5}$$

Putting Eqs. (4) and (5) into Eq. (1) and carrying out the integration over the frequency ω, we find for the azimuthal component of the magnetic field B_φ,

$$B_\varphi=-4\pi iNe\int\frac{k_\perp\tilde{\rho}(k_\perp)\tilde{\lambda}(k_z)}{k^2-k_z^2\varepsilon(ck_z)}e^{i\mathbf{kr}}d^3k. \tag{6}$$

A typical value of k_z in Eq. (6) will be of the order of σ_z^{-1}. As we will see from the result, the most interesting regime from the point of view of suppression of the magnetic field is when $k_p\sigma_r \geq 1$. Since we assume that $\sigma_z \gg \sigma_r$, $k_p\sigma_z$ will be much greater than one, and the dielectric function $\varepsilon(ck_z)$ can be approximated by its low-frequency limit, $\varepsilon(ck_z) = 1 - k_p^2/k_z^2 \approx -k_p^2/k_z^2$. We can also neglect k_z in comparison with $k_\perp$, $k^2 \approx k_\perp^2$. As a result, Eq. (6) becomes,

$$B_\varphi = -4\pi i N e \lambda(z-ct) \int \frac{k_\perp \tilde{\rho}(k_\perp)}{k_\perp^2 + k_p^2} e^{i\mathbf{k}_\perp \mathbf{r}} d^2k. \tag{7}$$

For a Gaussian beam, $\tilde{\rho}(k_\perp) = (2\pi)^{-2} \exp(-k_\perp^2 \sigma_r^2/4)$, and Eq. (7) yields

$$B_\varphi(r,z,t) = 2Ne\lambda(z-ct) \int \frac{k_\perp^2 dk_\perp}{k_\perp^2 + k_p^2} J_1(k_\perp r) \exp(-k_\perp^2 \sigma_r^2/4). \tag{8}$$

Paralleling the derivation of the expression (8), and using the same approximations, one can find the following equation for the electric field,

$$\mathbf{E} = -4\pi i N e \int \frac{k_z \tilde{\rho}(k_\perp) \tilde{\lambda}(k_z)}{k_\perp^2 + k_p^2} \left(\hat{\mathbf{z}} + \frac{k_z \mathbf{k}}{k_p^2} \right) e^{i\mathbf{k}\mathbf{r}} d^3k, \tag{9}$$

where $\hat{\mathbf{z}}$ is the unit vector along the z-axes. We will not try to simplify further Eq. (9) and note only, that by the order of magnitude

$$E_z \sim \frac{\sigma_r}{\sigma_z} B_\varphi, \quad E_r \sim \left(\frac{\sigma_r}{\sigma_z} \right)^2 B_\varphi, \tag{10}$$

which means that, under the specified conditions, the electric field generated by the beam is always small compared to the magnetic one.

Having found the magnetic field in the plasma, we can calculate the tune shift ξ due to the beam-beam interaction. Note that the tune shift for small-amplitude oscillations is proportional to the derivative of the interaction force at $r = 0$, which in our case gives $\xi \propto \int_{-\infty}^{\infty} \left[\partial B_\varphi(r,z,t)/\partial r \right] \Big|_{r=0} dt$ and, with Eq. (8), reduces to

$$\xi = \mathrm{const} \int \frac{\zeta^3 d\zeta}{\zeta^2 + \sigma_r^2 k_p^2} \exp(-\zeta^2/4), \tag{11}$$

Fig. 1 shows the dependence of the parameter ξ as a function of the product $k_p\sigma_r$ normalized by its value at vacuum ξ_0.

3. BEAM AND LUMINOSITY LIFETIMES

Introduction of the plasma in the interaction region gives rise to parasitic collisions of the beam particles with the plasma ions and electrons which cause a growth of the beam emittance and particle losses. The plasma parameters should be chosen so that these deleterious effects would not overcome beneficial contribution to the suppression of the beam-beam interaction. Below we consider several processes of beam-plasma interaction, following Ref. [9]. Note that in Ref. [9], the cross sections are given for electron beams only; we have modified them to apply to species of arbitrary mass *m*.

1. *Emittance growth of the beam due to small-angle elastic scattering on nuclei.*

The rate of the emittance growth $\dot{\varepsilon}_n$ is given by the following formula,

$$\dot{\varepsilon}_n = \frac{2\pi r_{\text{class}}^2 \sigma_r^2 Z^2 n_p l f}{\varepsilon_n} \Lambda_1, \tag{12}$$

where f is the revolution frequency, and $\Lambda_1 = \ln(\lambda_D/\lambda_B)$ is a logarithmic factor with $\lambda_D = \left(kT/4\pi n_p e^2\right)^{1/2}$ denoting the Debye length in the plasma (T is the plasma temperature), and $\lambda_B = 137\, r_{\text{class}}/\gamma$ denoting the de Broglie wavelength for the beam particles.

In addition to small-angle collisions, the beam particles can be scattered on a nuclei at relatively large angle. Such collisions excite betatron oscillations in the beam, and if the induced amplitude of the oscillations exceeds the vacuum chamber aperture, the particle gets lost. However, for high-energy beams, this process is usually negligible compared to other processes considered below.

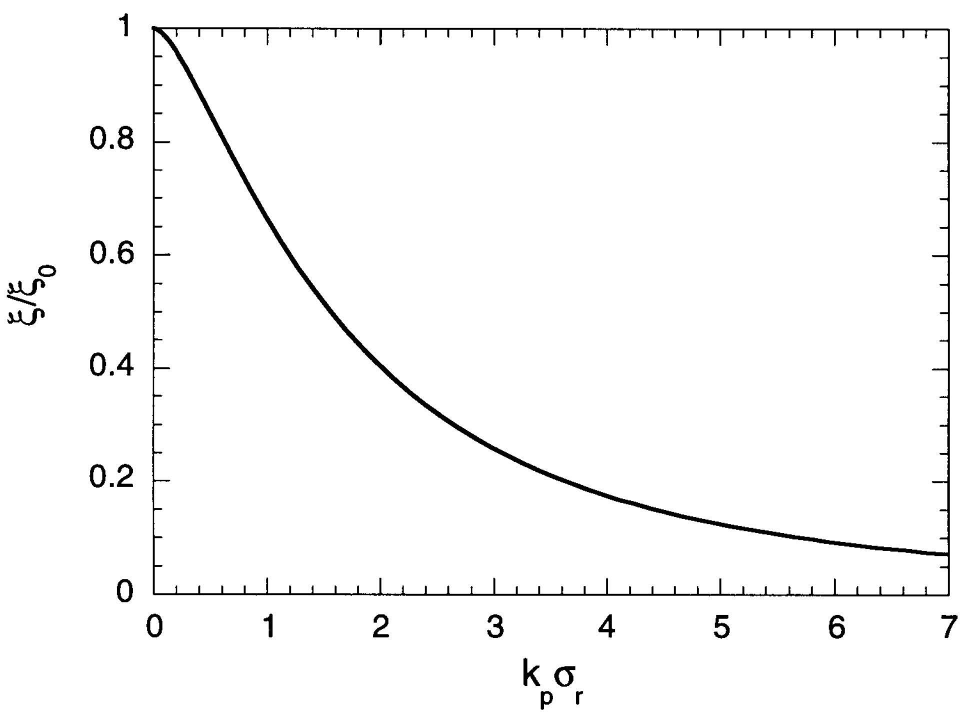

Fig. 1. Beam-beam interaction parameter as a function of the $k_p\sigma_r$.

2. *The bremsstrahlung on nuclei.*

Bremsstrahlung on nuclei of the plasma causes the energy losses of the beam particle due to radiation in collisions with the nuclei. If the relative energy loss $\delta E/E$ exceeds the RF acceptance, ε_{RF}, the particle gets lost. The cross section for this process is

$$\sigma_2 = \frac{16}{3}\frac{r_{\text{class}}^2 Z^2}{137}\Lambda_2\left(\ln\frac{1}{\varepsilon_{RF}} - \frac{5}{8}\right), \tag{13}$$

where $\Lambda_2 = \ln(2\lambda_D/\lambda_C)$ is a logarithmic factor with $\lambda_C = 137 r_{\text{class}}$ denoting the Compton wavelength for the beam particles.

3. *The elastic scattering on electrons.*

In this process, the incident particle collides with the plasma electrons and transfers to them part of its energy. The losses occur if the transfer is larger than ε_{RF}. The cross section for this process is,

$$\sigma_3 = \frac{2\pi r_e r_{\text{class}}}{\gamma \varepsilon_{RF}}. \tag{14}$$

Knowing the cross section for each process, we can evaluate the lifetimes for the beam associated with each loss channel, τ_1 and τ_2, using the following formula, $\tau_i = \left(n_p \sigma_i l f\right)^{-1}$, where l is the length of the plasma layer, and f is the repetition rate (revolution period) for the collisions. For the emittance growth we define the emittance growth time $\tau_1 = \varepsilon_n / \dot{\varepsilon}_n$, on which the initial emittance would increase by a factor of *e*. We will also use the notation τ_0 for the design luminosity lifetime.

4. POSSIBLE APPLICATIONS

For numerical example, which illustrates a possibility of using plasma in interaction region of a circular collider, we chose the plasma parameters such that $k_p \sigma_r = 4$; this guarantees about six-fold decrease in the parameter ξ. The corresponding plasma density is then calculated using the nominal radius of the beam in the interaction region. From the point of view of beam-plasma interaction, the most advantageous plasma species would be hydrogen, so we set $Z = 1$. The length of the plasma l is assumed to be equal to twice the length of the bunch, $l = 2\sigma_z$, in order to ensure that the bunches are overlapped with the plasma throughout the collision event. For the RF exceptance, ε_{RF}, the value of 0.001 was chosen.Table. 1 shows the relevant parameters of the beams, plasma, and calculated lifetimes for two colliders: proton-proton collider LHC [10] and the μ collider currently under study [4]. The required plasma density is in the range of 2×10^{18} cm^{-3} for LHC and 6.2×10^{19} cm^{-3} for the μ-collider. A possible approach to the generation of plasma of such density, with a minimum impact on the vacuum system of the collider, may use a technique based on a supersonic gas jet, currently under development for the plasma lens experiment at the Final Focus Test Beam at SLAC [11].

As is seen from the table, for the μ-collider, due to the extremely short muon's lifetime, the degradation of the beam quality caused by the beam-plasma interaction is negligible. For the LHC, the lifetimes are also larger than the nominal luminosity lifetime τ_0.

Table 1. Beam-plasma parameters and lifetimes

Accelerator	μ-collider	LHC
Beam parameters		
Particle species	μ	p
E [TeV]	2	7
γ	1.9×10^{4}	7.5×10^{3}
σ_z [cm]	0.3	7.5
σ_r [μm]	2.7	15
f [Hz]	2.3×10^{4}	1.1×10^{4}
N	2×10^{12}	10^{11}
ε_n [m rad]	5×10^{-3}	3.75×10^{-6}
ξ_0	0.05	0.003
Plasma parameters		
$k_p\sigma_r$	4	4
ξ/ξ_0	1/6	1/6
n_p [cm^{-3}]	6.2×10^{19}	2×10^{18}
l [cm]	0.6	15
Beam lifetimes		
τ_0 [hour]	1×10^{-5}	10
τ_1 [hour]	4.1×10^{6}	13
τ_2 [hour]	50	8×10^{3}
τ_3 [hour]	2.5	23

5. SUMMARY

We have shown that introduction of the plasma into the interaction region of TeV-range muon and proton colliders can substantially suppress the beam-beam interaction. This allows to overcome the limit set by the beam-beam tune shift parameter and advance into the region of higher luminosity. The proposed method looks especially attractive for muon colliders, where a small lifetime of muons makes effects of the beam degradation caused by the beam-plasma interaction negligible.

6. ACKNOWLEDGMENTS

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