A Better Quantum Extremal Surface and Island Interpretation that Explains the Associated Massive Gravity

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Abstract

In a paper about multi-fold black holes, we describe the evolution of black holes, their evaporation and entropy. With a particle-based model, we recover the correct Page curve, and resolve the black hole information paradox. Doing so we encounter a quantum extremal surface, as proposed by others using the conventional approach of the island and replica trick. In a subsequent paper, we show that the definitions of the quantum extremal surfaces coincide, while our physical interpretation are rather radically different: an horizon for the multi-folds attached to virtual particles in the multi-fold universe, versus an island à la Wheeler's bag of gold for the conventional replica trick. We dispute as a result a widely held view that Wheeler's bag of gold could isolate most of their content from the parent universe.

Others have questioned the replica trick with its use of JT gravity, and shown that it implies massive gravity, which might not correspond to a universe where General Relativity (GR) reigns. We show, in multi-fold scenarios, that the inability for multi-folds to escape the quantum extremal surface implies massive gravity effects, while remaining able to recover GR. Therefore, we can provide microscopic mechanisms to explain how massless gravity becomes massive. We also justify relying on JT gravity, by the 2D massless Higgs bosons dilaton random walks.

On the other hand, in an asymptotic AdS universe, the replica trick requires the introduction of a reservoir for the evaporated particles, beyond the boundary of AdS, otherwise evaporation can't take place. Letting gravitons enter the reservoir implies massive gravity, while preventing them from entering the reservoir leads to a different Page curve, and therefore, no resolution of the black hole information paradox. We provide new microscopic interpretations to these scenarios, explaining away these issues, and dealing with the encounter of quantum extremal surface outside the black hole in the presence of a thermal bath. Doing so, we confirms the better suitability of the multi-fold interpretation of the quantum extrema surface, and island, while supporting the replica trick and island formula approach, and resolving the black hole information paradox once and for all.

Already knowing that multi-folds appear with GR at Planck scales, and with other hints, the multi-fold analysis applies to our real universe, and the paper provides a microscopic interpretation of quantum extremal surfaces, the correct page curve, the resolution of the black hole information paradox, and the apparition of massive gravity associated to quantum extremal surfaces.

From the island formula and replica trick at low masses, we encounter hints of our past proposal for new life cycles of black holes (small extremal black holes splitting into another extremal black hole and an evaporating black hole), which invalidate the strict Weak Gravity Conjecture, and justify the Ultimate Unification (UU) proposal. It is quite a confirmation.

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1. Introduction

In a multi-fold universe [1,8-10,22,*196*], gravity emerges from entanglement through the multi-fold mechanisms. As a result, gravity-like effects appear in between entangled particles [1,24,25,*202,203*], whether they be real or virtual. Long range, massless gravity results from entanglement of massless virtual particles [1,26]. Entanglement of massive virtual particles leads to massive gravity contributions at very small scales [1,27]. It is at the base of the E/G Conjecture [24], and the main characteristics of the multi-fold theory [22]. Multi-folds mechanisms also result in a spacetime that is discrete, with a random walk fractal structure and non-commutative geometry that is Lorentz invariant, and where spacetime nodes and particles can be modeled with microscopic black holes [1-4]]. All these recover General Relativity (GR) at large scales, and semi-classical model remain valid till smaller scales than usually expected. Gravity can therefore be added to the Standard Model (SM) resulting into what we define as SM_G: the SM with gravity effects non-negligible at its scales. This can contribute to resolving several open issues with the Standard Model and with the standard cosmological model, without new Physics other than gravity [1,4-27,40,55-61,76,79,90-131,138-160,*176-214,256*]. These considerations hint at an even stronger relationship between gravity and the Standard Model, as shown in [23,93,99,*181*].

Note added on December 22, 2023: In this paper, references in italic were introduced on December 22, 2023.

Among the multi-fold SM_G discoveries, the apparition of an always-in-flight, and hence non-interacting, right-handed neutrinos, coupled to the Higgs boson is quite notable. It is supposedly always around, due to chirality flips by gravity of the massless Weyl fermions, induced by 7D space time matter induction and scattering models, and hidden behind the Higgs boson or field at the entry points and exit points of the multi-folds [1,57,92,96,97,98,100,101,107,112,*a181,206*]. Massless Higgs bosons, modeled as minimal microscopic black holes mark concretized spacetime locations. Below the energies of the multi-fold gravity electroweak symmetry breaking, they can condensate into Dirac Kerr-Newman soliton Qballs, appearing as neutral charged or rotating microscopic blackholes, often over extremal [1-4], to produce massive and charged particles [1,4], thereby providing a microscopic explanation for a Higgs driven inflation, the electroweak symmetry breaking, the Higgs mechanism, the mass acquisition, the chirality of fermions and the orientation of spacetime; all resulting from the multi-fold gravity electroweak symmetry breaking [99]. Above the energies of the multi-fold gravity electroweak symmetry breaking, massless particles are induced patterns of the 2D random walks of the massless Higgs bosons [1,23,94,*177*]. The multi-fold theory has also concrete implications on New Physics like supersymmetry, superstrings, M-theory and Loop Quantum Gravity (LQG) [1,8-21,131,*199,209*].

Multi-folds are encountered in GR at Planck scales [5,6] and in Quantum Mechanics[2] (QM) if different suitable quantum reference frames (QRFs) are to be equivalent relatively to entangled, coherent or correlated systems [7], as well as in QFT [118,129,152,*191*] and M-theory [40,129]. It hints that GR and QM are different facets of something that they cannot well model: the multi-folds [1,*206*].

We start the paper with a short summary of the black hole information paradox [28,29], resulting from the Hawking radiation [1,59-61], its proposed resolution with the correct Page curve [30-33], and how a path integral resolution of the island formula with the replica trick indeed recovers the correct Page curve under the right circumstances [34-37]. The island formula computes the fined grained entropy, which corresponds to the Von Neuman entropy, i.e., the entanglement entropy, and encounters an island inside the black hole bounded by a second surface called the quantum extremal surface [36,37]. The conventional interpretation is that the black hole evaporates both towards the outside of the black hole and towards the island, where it

[2] Standing in for Quantum Physics in general.

would no longer be accessible to, entangled with, or gravitationally felt by the outside: the island is an "à la Wheeler's bag of gold" [38-40]. This is what would produce the correct page curve [28,37].

The approach has not yet convinced everybody in the Physics community, in part because of the limited situations where it has been developed, and reasonably computed [28,41]: mostly Jackiw–Teitelboim (JT) Gravity [42-44] (e.g., rather than Liouville gravity that uses a different potential to match the limit of Einstein gravity in 2D [45,54,80), 2D spacetime, and mostly in AdS universes that do not match our universe [*209,221*]; and because more recent papers have pointed potential issues like the allegedly problematic apparition of massive gravity due to island, or, possible inconsistencies in AdS. Therefore there are concerns that the island, and its resolution of the black hole information paradox, may relate to a universe with a different gravity than GR, or may violate some tenets of holography that motivates the need to maintain unitarity and not lose any information [46-53]. In discussing these, we reject the view that Wheeler's bag of gold would be able to isolate most of their content from the parent universe or black hole [37,39,167]: only what may disappear behind a cosmological universe apparent to the wormhole implementing the Wheeler's bag of gold can be isolated. It is also why a Wheeler's bag of gold does not violate the covariant entropy bound [40].

Then, the paper discusses black holes in a multi-fold universe [1,40,76], and how a quantum extremal surface, minimizing the same formula as earlier, hence adhering to the same definition, again recovers the correct Page curve, but, this time, with a different physical interpretation, which seems more natural when understood in terms of particles, which we previously argued as the preferred models for studying many multi-fold aspects [1,55].

Considering the multi-fold massive gravity contributions from entangled massive virtual particles [1,25], we show that, indeed, massive gravity must appear in the presence of the quantum extremal surface, as a new phase of spacetime, and that the behavior encountered in AdS with the island formula and replica trick, can be fully microscopically explained, with multi-folds considerations applied to such a negatively curved spacetime, therefore validating the approach of the island formula and replica trick, even if the conventional interpretation is disputed. So far, such microscopic / physical interpretations have been missing in the literature; they are key contributions of this paper. Doing so, our justifications of the definition and our physical interpretation are based on the multi-fold explanation rather than the island as a Wheeler's bag of gold. *Note added on December 22, 2023: We also explain that baby universes created via the wormholes and Wheeler's bags of gold are usually suppressed (resorbed) as soon as formed, yet their plausible formation implies another of our multi-fold result: gravity does not systematically decohere [146,182,187,198,209,222].*

We conclude by explaining how other papers on the island formula and replica trick encountered the new lifecycle for black holes, that we had proposed in [1,56], where small extremal black holes split up into smaller black holes, with violation of the strict weak gravity conjecture (WGC). This validates the first steps of our proposal of the ultimate unification (UU) [1,57,58].

2. The Black Hole Information Paradox

The black hole information paradox [28,29] refers to the challenges resulting from the discovery of the Hawking radiation [59-61] (and references therein), and the model of black hole entropy: the information, captured when matter, fermions, bosons, or composites, are swallowed by a black hole, seems lost through the subsequent processes of Hawking evaporation that generates thermal black body radiations that erase all the information as the black hole fully evaporates.

The challenge is that quantum Physics is expected to be unitary [62]: no information can be lost; a contradiction with the outcome of the Hawking evaporation. In 1993, Page proposed a curve, known as the Page curve, which describes the evolution of the black hole and entanglement entropy that, if satisfied, would resolve the paradox [30-33].

Many have tried to find solutions including, as a non-exhaustive list, in and out entanglement of the horizon [67-70], the horizon as (AMPS) firewall [63,64], holographic models like [65,66], and finally the island formula with replica trick [34-37]. Each leads to an answer to the black hole information paradox, but at the cost of new issues or paradoxes, except for the latter, which resolves the paradox, once we address the challenges as done in the remainder of this paper.

We focus on the latter approach because it is built on path integrals of semi classical spacetime, and it is supposedly the answer to the problems, and handwaving, encountered by the other proposals. It leads to computing the correct Page curve, in simplified cases, e.g., (mostly) 2D, (mostly) AdS, with JT gravity [42-45,54,80]. It led popular articles to claim victory over the black hole information paradox [14], while others remained more skeptical [28,41]. They are rights to be so, with the conventional interpretation of the island, and without the multi-fold arguments presented here.

We know, with some level of confidence, that a solution to the black hole information paradox must exist. Indeed, if the AdS/CFT correspondence conjecture is correct [65] (See [1,14], and references therein, for our derivation, and analysis of it), something that, today, most physicists seem to believe, then unitarity of (quantum) gravity is mandated: indeed according to AdS/CFT correspondence, gravity in AdS is the dual of a quantum CFT (Conformant Field Theory) model on the boundary of AdS (which can be de Sitter (dS)). CFT can be seen as renormalized QFT at high energies. Accordingly, per the properties of quantum physics, CFT is unitary and per the duality, (non-perturbative) (quantum) gravity must be unitary in the AdS (bulk). Of course, we need to add that this says nothing about an asymptotic dS universe, which better characterizes our real universe [209,221]. For asymptotic dS universe, [40,76] and this paper address it: it is indeed unitary as the information paradox is recovered through the recovery of the correct Page curve.

3. Conventional Quantum Extremal Surfaces, Replica Trick, Island and Massive Gravity

3.1 The correct Page curve

[37], and [40] for the conventional approach and associated references therein, discuss how the use of path integrals over an Euclidian spacetime, obtained via Wick rotation, allows approximatively solving the island formula. The island formula is a generalized equation for the entropy of a black hole, and its evaporation. The path integrals are resolved with the replica trick, which relies on taking the limit of n replicas tending towards 1 of the Rényi entropies computed in the presence of n replicas, as discussed in [40,71] and references therein. The extremization of the entropy is achieved over a set of surfaces that includes the black hole horizon, and possibly other surfaces, including internals ones. Doing so, an additional internal surface appears to extremize the entropy. It is called a quantum extremal surface. See [34-37], and references therein.

To do this, computations are in general, with some exceptions (e.g., for dS, see [72-75], more than 2D, see [77,78,85]), done in 2D, with JT gravity, a dilaton based gravity, instead of the Liouville gravity that would be more directly the limit for $\varepsilon \rightarrow 0$ of the 2D+ε version of GR [42-45, 54,80,161,*215*], and/or in AdS spacetime, and with approximations to the dominant order(s). All these considerations and limitations are not exactly convincing everybody [28,41]. Yet, the approach leads to recovering the required correct Page curve to resolve the black hole information paradox as discussed in section 2. A good sign, hopefully appropriately extensible to GR in 4D.

Note that some works on the island may not necessarily use all aspects of the replica trick. We will not discuss these details, and rather consider that all the methods conventionally extracting islands are encompassed in what we mean by island and replica trick. It does not change the overall analysis.

3.2 About JT gravity consistently used with the island formula and replica trick

About JT gravity vs Liouville gravity [161,*215*], let us first say that the advantage of 2D gravity come from the ability to solve mathematically some of the path integrals and models; something still overwhelmingly complex in higher dimensions, or even with Liouville models. Liouville gravity is often associated to world sheets associated to branes and superstrings [161,162], and it is already more complicated. Liouville gravity represents the limit of 2D+ε GR gravity when $\varepsilon \rightarrow 0$ [161], but in the end, the 2D gravity obtained this way has no dynamics (because the action has only surface term, which we can understand physically as: spacetime locations can't be perturbed by a massless graviton (that wouldn't be with longitudinal polarization, the only available option for motion) [*216*].

JT gravity is associated to massless dilatons in 2D, as is Liouville gravity, but with different choices of potentials [161,*215*]. We can switch from one potential to another by transformations and/or field redefinitions. Motivations for the dilaton gravity action is provided for example in [163], where we see that it results from compactifying other dimensions, hence correctly corresponding to a dilaton as in Kaluza Klein approaches [165] (Multi-fold space time matter induction and scattering [23,93,96-98,*a6*] is a particular case of unconstrained Kaluza Klein, where we do not compactify, that aspects comes from the multi-fold dynamics [*177,189,206*]. Dilatons gravity can extend to higher dimensions [163].

When considering the Liouville gravity, one can see that the Teitelboim action varied over the scalar field in conformally flat coordinates leads to the Liouville gravity [164,*218*]. So, for small (constant) curvatures, or on conformal spacetime as encountered with CFTs, i.e., ~ renormalizable QFTs at very small spatial scales, JT is a suitable approximation of the 2D GR gravity, and converges to it [164]. Other ways to see JT gravity as a limit of Liouville gravity exist, typically in the context of string theory, which is typically expressed in a conformal gauge [162], or as a limit on the conformal parameter that eliminates the conformant factor, or Liouville potential exponential in the scalar field [164,*219,220*].

With this analysis, we see that JT gravity seems to be a good candidate, suitable for modeling 2D gravity and spacetime with a constant small cosmological constant, positive or negative. This is why, despite the concerns of others as mentioned earlier, JT gravity works so well to model and compute the path integrals (Euclidian or Lorentzian) encountered with the replica trick and island formula. We are not concerned. It correctly approximates GR in 2D. After all, what matters is 2D physics [147,148,*195*]. With JT gravity, the cosmological constant can be strictly negative or positive (not really zero if the scalar dilaton field is not zero).

Conventionally, in the literature, it is often taken as negative because of the link to conformal theory / CFTs and strings [161,*209*,*217*], and AdS/CFT correspondence conjecture, [1,14,65] and references therein.

Note added on December 22, 2023: yet have showed that the cosmological constant must be strictly positive (small) in [209].

In section 7, we will see that in a multi-fold universe, JT gravity is a good model, supported by microscopic interpretations of multi-fold gravity at 2D.

3.3 Physical interpretation

Figure 1 shows the conventional interpretation of the quantum extremal surface around the island. Radiation due to evaporation takes place in part towards the outside of the black hole, across the horizon, and in part towards the island through the quantum extremal surface, where it does not contribute to the entanglement with the outside anymore. As a result, as the black hole evaporates, its entropy reduces, and the entanglement entropy with the outside also decreases, eventually roughly one unit decrease of each per unit of associated increase of the radiation entropy. This gives us the right evolution of the Page curve as in Figure 4 of [76], with a progressive decrease of the entanglement entropy along with the black hole entropy. This way, the black hole information paradox is resolved: entanglement entropy disappears as the black hole shrinks, and evaporates, and no information is lost. Physics remains unitary, as it had to be for the sake of Quantum Physics, and the AdS/CFT correspondence conjecture.

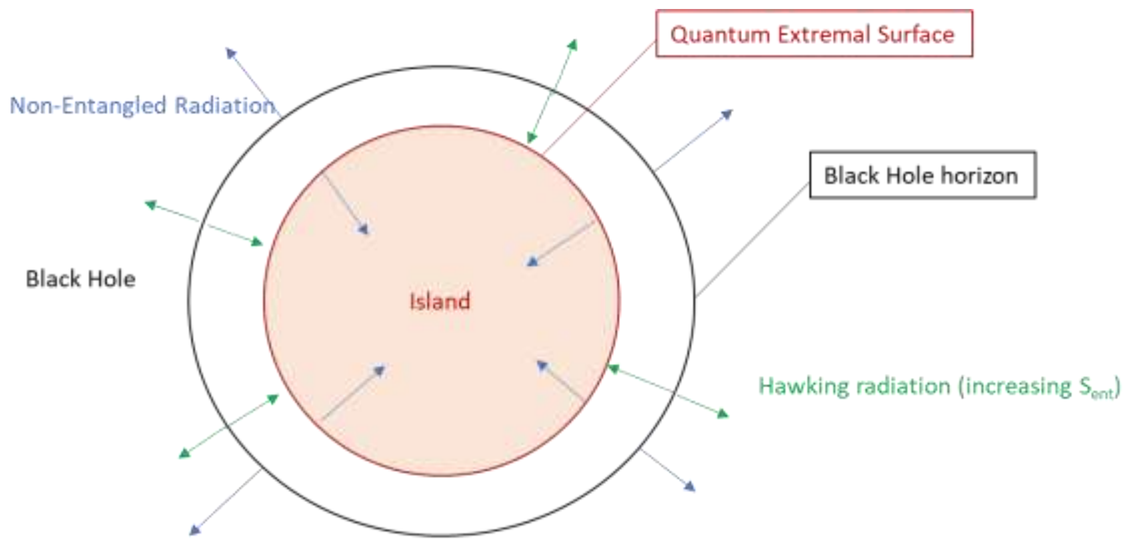

Figure 1. The black hole is shown with its horizon and island, bounded by the quantum extremal surface. In the conventional island formula and replica trick interpretation, Hawking radiation takes place at the horizon and contributes entanglement (green), while additional radiation (blue) takes place towards the outside and the island. The latter (blue) does not entangle the interior of the black hole (outside the island) with the exterior of the black hole.

Figure 2 shows the, or rather a plausible, geometry, to explain the island: a Wheeler's bag of gold [37,38,39]. Accordingly, bosons and fermions can easily enter it but it is hard to exit. It can be modeled as stitching a wormhole to say a FLWR universe, or like a baby expanding universe [39]. As intuitively seen, it is easy to get in, but once there it is hard to get back to the original universe because of the baby universe expansion. [37], and the proponents of the conventional definition for the quantum extremal surface, then argue that the area of throat of the wormhole defines the (entanglement) entropy, no matter how much matter / entropy is in the bag of gold.

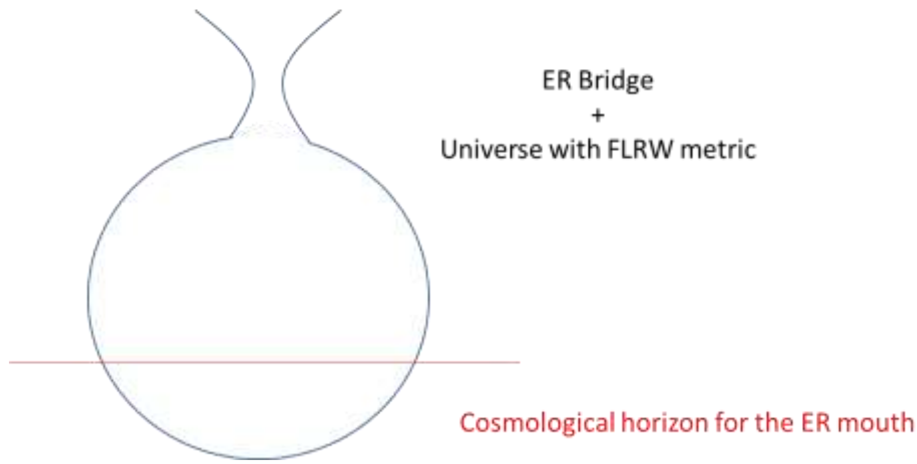

Figure 2: A geometry à la Wheeler's bag of gold per [38,39]. The expanding universe (~FLRW [170]) is often call baby universe. Our analysis further calls for matter, gravity (and entanglement) to be captured behind the apparent cosmological horizon of the FLRW expanding baby universe, with respect to the mouth of the ER wormhole as hinted in red. A conventional Wheeler's bag of gold as in [38,39], and the mechanisms invoked in [37], do not rely, to the best of our understanding, on the apparent cosmological horizon, but they should.

The model is that as entanglement follows area laws (e.g., See [1] and references therein), only a fraction is exposed to the external parent universe (i.e., the black hole interior), and so most of the entropy appears lost in the parent universe [37,167]. We will come back to this in sections 3.4 and 7.

The results seem impressive, with the caveats of above in terms of 2D, AdS, JT gravity, which we have already justified by now in section 3.2, and approximations. All seems solved isn't it?

3.4 Trouble in paradise?

Some physicists remain cautious about the results of the island formula and replica trick, because of all the assumptions or approximations involved: 2D, AdS, JT gravity, and only first order dominant approximations. But there are other important issues with the island method: when in an AdS universe, the model used needs to slightly modify AdS to add a reservoir where the radiation is assumed to accumulate (and be entangled with the island, as entanglement wedge). Otherwise, the black hole evaporation would be reflected by the AdS boundary back into the black hole, and the black hole would be eternal, without a suitable Page curve (and in equilibrium of evaporation and absorption).

Others have shown that there are potentially other problems with the approach used:

- [77] shows that, in AdS with more than 2D, e.g., [78], (as in 2D, with the absence of graviton, there is no dynamics [80,81], and therefore this issue is not apparent), letting evaporated particles reach the reservoir, but blocking gravitons (Here we do not further argue if we are talking of gravitons as carriers or dilatons, from our point of view, gravitons do not exist, they are unphysical and quasi particles, even if gravity is quantum [1,118,130,131,*226*]. The use of graviton here is meant for a carrier of gravity: massless or massive; which in the models discussed here are dilatons.) on the boundary, results in gravitons/gravity becoming massive, and therefore no longer characterizing GR. Note it is not the same situation as the massive gravity encountered in multi-fold universe [1,26]. We will come back to this.
- [82] shows that if gravitons are let into the reservoir, then the Page curve becomes flat (constant), and therefore not resolving the black hole information paradox.

- [83], shows that, even when just the evaporated particles go to the reservoir, all information about the gravity of the island is also on the boundary, or at least as close as we want to be to the boundary, instead of lost in the island disconnected form the boundary. This is also reviving the black hole information paradox: the reasoning of the replica trick would not work. Again the recipe to address this is to suggest that the gravity be massive so that gravity never reaches the AdS boundary while photons do. It also relates to the next bullet.
- Furthermore, [84] then argues that the AdS boundary must see everything within the universe, and that, therefore, the information, and entanglement, within the black hole towards the island is not hidden from it. This is why the previous bullet problem occurs, and therefore the reasoning that it can be excluded to compute the Page curved would be flawed. Again, the entanglement entropy curve becomes constant with an horizontal curve, that is not the Page curve that (linearly) decreases.

The last two bullets, if correct, mean that the method of the island and replica trick in AdS would not be consistent with GR, and would not resolve the black hole information paradox, at least with its current conventional interpretation[3]. It corroborates in 2D the problem identified in the first bullet.

Note that [84], if correct, could also be used to argue that there never was a black hole information paradox: all the information would have always remained conserved and visible to the boundary of the universe[4] (^^).

Note that for the latter bullet, some have argued that an island à la Wheeler's bag of gold is a counter example to the holographic principle [87], and that therefore the original information paradox would still exist. In [40], we argued that a Wheeler's bag of gold is actually not a counter example to the covariant entropy bound, one just need to know what surface (on both sides of the wormhole) to consider, and what contributes to the bound. On the basis of such a reasoning, we can see indeed any virtual exchange involved with conveying gravity / mass information beyond the wormhole (to the parent universe) is fully maintained. It is illustrated in figure 3. So the mass of the black hole, and its gravitational field is not reduced when particles fall into the baby universe.

In multi-fold universes, entanglement is similarly conveyed along the domain support of the entangled particles/systems [1,25*,202,203,206*], and the same reasoning, as explained with figure 3, applies. It is at the basis of the E/G conjecture factual in multi-fold universes [24]. In our real universe, and 2D toy universes with JT gravity and Physics, it is more a conjecture at the level of entanglements. So we can't really definitively say that the arguments of papers like [37] about the loss of entanglement entropy in a Wheeler's bag of gold do not work, but we definitively can see that figure 19 in [37] is confusing, and confused. We can only agree with the doubts expressed by others, and point out however that our reasoning and figure 3, shows at least one inconsistency in the conventional approach: the mass of the black hole is not reduced, by falling in the island as a Wheeler's bag of gold, and its gravity effects remain in the wormhole and parent universe, and so the surrounding black hole entanglement with the outside aren't universe is not changing. We tend to argue that this (no change of mass and gravity and hence of black hole entanglement with its outside, per the area laws linking gravity and entanglement at the black hole horizon ([1,76] and references therein)) alone is sufficient on its own to invalidate the interpretation of the island as a Wheeler's bag of gold, and the conventional interpretation of the quantum extremal surface, as delimiting a region where the black hole would also radiate particles that would disappear the rest of the black hole and the rest of the (parent) universe; except for when matter slips beyond the apparent cosmological horizon of the baby universe for the wormhole. It would only get a portion to disappear.

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[3] As we presented in [40,76], we will see in a later section that the approach works way better once we change the interpretation of the quantum extremal surface, as defined in the multi-fold theory, but directly applicable to the (conventional) real universe, that it be multi-fold or not. It weights towards our definition and interpretation as the winner.

[4] And frankly, in [1], we have already resolved the paradox also simply with the particle interpretation, and the UU proposal that defined a new black hole lifecycle without remnant [1,56-58].

Figure 3. It shows a 3D spacetime image that better explains how a Wheeler's bag of gold works and the suspected mistakes with the interpretation of [37] and alike. Any point in the baby universe (A) can emit virtual particles (or multi-folds) that can reach any reachable point (B) in the parent universe. This fails only if A goes behind the cosmological horizon of the baby universe as seen by the mouth of the wormhole. If the mouth of the wormhole is in a black hole on the parent universe side (unless if again beyond the apparent cosmological horizon of that universe, for the wormhole), then B can only be reached e.g., by virtual particles (and attached multi-folds) when in (some of the) interior of the black hole. So there are not many particles in the baby universe that would be hidden to the parent universe, especially in terms of gravity effect / mass. Black holes do not filter mass effect [78]. In terms of entanglement, in the case of a multi-fold universe, with multi-fold mechanisms [1,25,202,203,206], the multi-fold support domain would be analogous to the red paths of the virtual particles, and entanglement is not suppressed either. The 2D can be seen as a slice of the "balloons" comprising their axes.

Our analysis, as in figure 3, differs from the decoupling argued for in [28,167], which maybe what [37] relies on. [167] argues that an horizon and black hole would form in front of the wormhole mouth on the side of the parent universe, on the sole basis that the energy conditions are violated [*223*]. We disagree with [167]. Violations of the energy conditions solely indicate that the wormhole is traversable [*224,225*], assuming that they can be traversable. If they aren't, then the proposal for the interpretation of island as a Wheeler's bag of gold does not hold anyway, and our multi-fold interpretation would be the only game in town. As discussed in [10,23,40,92,*194,206*,224,225], we have reasons to believe that there are ways to ensure that wormholes can be traversable. Then applying the reasoning of figure 3, we invalidate the view of [28,37,167]. It had to be so, per the covariant entropy bound, proven in [40], and independently of any of our views or multi-fold models, in [171-173]. Indeed, if the interpretation of [167] was correct, the bound would be invalidated. Note that [167] does not address the dynamics ("… instantaneous expansion …", which maybe a different context, explaining the differences).

That being said, if we can add the cosmological horizon (apparent to the wormhole) as an additional requirement to what happens within the Wheeler's bag of gold, any particle falling behind it would be decoupled and disconnected form the parent universe. A priori that would be only a portion of these particles and would take a long time (we can't be saved by hoping that the baby universe expands extremely rapidly, as a FLRW universe is homogeneous and hard to justify if we willing to stray away from FLRW [170]).

Note that the notion of cosmological horizon or apparent horizon for the wormhole is relevant and exists as discussed in [168,169]. It works to the extent that gravity, entanglement and other interactions are no more felt at wormhole. It could still be felt elsewhere in the baby universe, hence the "apparent" qualification. Well characterizing all these effects are complex, but it is not needed to prove our claim that it is only once beyond that horizon, the content decouples totally from the parent universe / black hole interior on the other side of the wormhole.

What is the boundary to consider when stating that the AdS boundary knows everything? A spacetime with baby universes is no more purely an AdS, or an asymptotically AdS universe. Its usual boundary is complemented by the apparent cosmological horizon of the baby universe, if such an horizon exists, and its closed topology, and not by the surface covering the throat of the wormhole when going towards the baby universe. Per the above, the AdS boundary is still fully knowledgeable of everything, that hasn't reached the apparent cosmological horizon, and it is satisfying the holographic principle. As anything beyond the apparent cosmological horizon inside the baby universe, or island, is not accessible to the outside (or the wormhole), the portion of information of the boundary associated to it is lost: so, the information paradox exists a priori in that case. So the note (^^) does not really hold.

So there may be ways to ensure that some information becomes lost with a truly disconnected region; but only if everything aligns the right way to have a lot of particles crossing the apparent cosmological horizon. And for everyone crossing, it is fair to expect that some aren't crossing anytime soon, unless maybe the expansion is at a tremendous rate, something that we also struggle to justify, and does not fully compute with FLRW being homogeneous. Also, what to do of the particles that haven't crossed the cosmological horizon yet? They are not accounted for in the conventional approach, unless if considered as part of a long and slow evaporation process towards the island, i.e., entering the wormhole then waiting that they reach the apparent cosmological horizon.

Anyway, we can't fully reject the conventional model. With the apparent cosmological horizon considerations we may have saved it under some conditions, but if we had to consider all the possible paths in a path integral of topologies, such conventional paths would have very low probabilities.

Note added on December 22, 2023: in [209], we further argue that baby universe of the island type, and many other cases, are (almost) immediately suppressed and resorbed. It is another reason why we can expect that no such type of island forms, in our real / conventional universe black holes, and that therefore the conventional interpretation of the island and its quantum extremal surface is problematic and most certainly wrong. With this, we believe that we can more firmly consider conventional interpretation of the sand and its quantum extremal picture as very unlikely. Note that it is also why we believe that the quantum foam is too wild as also mentioned in [5,21,150].

3.5 Quantum extremal surface piercing outside of the black hole horizon.

To add to the challenges, [85] obtains two islands in AdS(4), in a 5D embedding space, which renders everything hard to compare apple to apple, one quantum extremal surface is internal to the horizon and the other external to it. Such a quantum extremal surface was also obtained in AdS(2), with JT gravity, when coupling the black hole to a thermal bath [86]. Then the quantum extremal surface seems, or is believed, to be inside the horizon when the coupling disappears[5]. Explaining the doubling of the islands with the interpretation of the Wheeler's bags of gold, is conventionally very challenging.

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[5] In our opinion this is a red herring, that does not correspond to a viable physical scenario, even if plausible per the AdS/CFT correspondence conjecture. The coupling implies pumping in, or out, extra energy (depending on if the bath temperature is higher or lower than the black hole (Unruh) temperature [61]), from what is supported just by the Hawking evaporation (and the extra non-entangled evaporation discussed in figure 1). The latter, i.e. pumping out energy, would require conventionally that more energy can be ejected. The only way is that the island, where particles would have also evaporated, would now let them escape while Hawking evaporation takes place. It requires therefore that part of its surface be external to the horizon, even if hard to picture. A microscopic / physical interpretation is that the black hole is in an excited mode where its horizon and quantum extremal surface deform to allow some excited particle from inside to escape. Note also that this note does not imply that

We will revisit these aspects in a multi-fold universe in section 7.6, where there is a very simple and convincing microscopic interpretation.

3.6 Universe-scale replica trick

In [87-89], others have adapted the island formula and replica trick to the entanglement between different universes with, or without, gravity. We discussed it in details in [40]. A big part of the reasons, for considering such use cases in [87-89], relies on the credibility of the algorithm, as it recovers the correct Page curve for black holes; not a small feat. In [40], we give similar credit to the approach, but we question the definition and physical interpretation of the quantum extremal surfaces, and produced more analyses and different interpretation aligned with the multi-fold approach, and interpretation for the quantum extremal surface.

3.7 The State of Conventional Affairs

On one hand, the semi classical gravity path integral, over Euclidian spacetime, with the generalized entropy formula, islands and quantum extrema surfaces seems to recover the correct Page curve, and resolve the black information paradox. On the other hand, it seems that it is at the cost of significant approximations: restricted to 2D JT gravity, which may be argued as not GR at 2D, but something with which we are not personally concerned per section 3.2, or to AdS, with some limited dS exceptions where computations may be more shaky so far, and with feared possibly divergences from GR (massive gravity) for dimensions larger than 2, as well as for JT gravity in 2D (Note it is not a different gravity, as established in this paper).

In addition, if one accepts that the holographic principle applies, or that the AdS/CFT correspondence conjecture is true, it seems that the approach may be on shaky grounds, either if we believe that it implies that the island is isolated, and not tracked by the boundary of the black hole, which does not seem correct as we discussed, or that the black hole information paradox never existed because black hole interiors would be similarly trackable no matter what. Remember also that we argue that the latter is not the case, at least if we consider an apparent cosmological horizon appearing in the baby universe. So [84] holds except for what may be beyond the baby universe apparent cosmological horizon, when it appears[6].

In conclusion, we believe that the bullets argument in section 3.4 may hold. The are issues for the conventional interpretation of the island and its quantum extremal surface, not with the methodology of the approach. We will revisit these topics in section 7 for multi-fold universes.

4. Multi-fold Black Holes

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we agree with this conventional interpretation of the island. We don't per [40,76], and the rest of the present paper. It will be further discussed in section 7.6. Here we just want to reason on how [85,86] could make sense.

[6] As we argue later on that there is no Wheeler's bag of gold, in blacks holes, *(Note added on December 22, 2023: or in general: they immediately resorb, which suppresses any growth of baby universes [209]),* this is however not a satisfactory enough resolution. We will discuss the multi-fold (inspired) alternative that should better work, although it is granted that a multi-fold AdS may not be of any physical interest [1].

In [1,76], we discuss how black holes exist in multi-fold universes, and recover an area law for the entropy [1,90]. It is based on the quantum fluctuations of the horizon that allows entanglement of particles between the inside and the outside of the horizon, which can be fluctuating virtual particles, or Higgs bosons at concretized spacetime locations [1,76,91,92,94] (it's all the same as particles are composed of them anyway [4,23,57,91,99-101,*a2,a3*]). These mechanisms are responsible for the apparent encounter with strings looking behaviors on the horizon [1,69]. They aren't strings, but just the result of the particle fluctuations at or near the horizon, a little bit as this is also how gravity prevents magnetic monopoles per [1,106]. This, and the discreteness of multi-fold spacetime [1,16,93], also ensures satisfaction of the Trans-Planckian censorship conjecture (TCC) [95].

In [1,4,76,91,92,94], we show that particles, and concretized spacetime locations can be seen as microscopic black holes that can be seen as the result of space time matter induction and scattering of 7D solitons [1,23,96-98]. [4,93,99,101] then discuss the gravity electroweak symmetry breaking that occurs to give mass to massive particles via Higgs bosons [*205*]. [57,100,101] expand on what happens at higher energies scales, or smaller spatial scales, than the gravity electroweak scales. At higher energy scale, we end up encountering the Ultimate Unification (UU) [1,56,57]

[59-61,76,138,157,*179,190*] discuss black hole evaporation, and the evolution of their entropy. In particular, based on multi-fold mechanisms, and a particle-based analysis, we show that multi-fold black holes have a correct Page curve for the evolution of the entanglement entropy, and therefore resolves the black hole information paradox.

Because a multi-fold spacetime is discrete (and spin, rotations [1], multi-fold dark energy [27], non-commutativity [1,93,*178,209*] and total collision effects [155]) , there are no gravitational or cosmological singularity, except maybe as starting point of the big bang, in some scenarios [1], yet it remains consistent with the SM [57,100]. It is compatible with black holes or evaporation [1,102].

Note added on December 22, 2023: See also discussions like [138,157,179,190,191,198].

5. Multi-fold Quantum Extremal Surface

[40,76] also encounters a quantum extremal surface defined as follows: the surface, within the black hole, from which no multi-fold, or multi-fold effect can escape: the virtual (or real) particles emitted, (radially) towards the outside, by matter (fermions or bosons) inside the black hole, are frozen in place, and particles external to it again take forever to cross it. As a result, the inside of the quantum extremal surface can't contribute new entanglement, unless and until this surface shrinks to reach and pass these particles, as discussed in [76]. In [40] we showed that this matches the algorithm of the island formula as in [37]. Indeed consider the following:

- If the surface increases from its multi-fold definition, it will contribute less short term reduction of the entanglement entropy, but will have more future reduction impact [76], yet that is countered by the secondary effect of more possible radiations from the quantum extremal surface that can increase short term reduction of the entanglement entropy.
- If the surface decreases from its multi-fold definition, it will contribute more short term reduction of the entanglement entropy but will have less future reduction impact [76], yet that is countered by the secondary effect of less possible radiations from the quantum extremal surface that can decrease short term reduction of the entanglement entropy.
- We have extrema, across these options, and we pick the minimum, to ensure we have the fine grained entropy [37,40], by requiring that everything that could contribute short term to reduction, is outside the surface, which makes physical sense as radiation from within the quantum extremal surface is future.

There was however a fundamental interpretation difference between the interpretation of what happens and matter on the quantum extremal surface here vs. in the conventional cases: instead of looking at a surface that radiates towards the island, we now see it as a region that, at the present moment, can't be contributing to the reduction of the entanglement entropy, by evaporating particles already entangled with the outside. These already entangled particles, when evaporating, decrease the entanglement entropy. For now such contributions only come from the region between the horizon and the quantum extremal surface. It leads to the Page curve evolution of the entanglement entropy.

It is shown in figure 4.

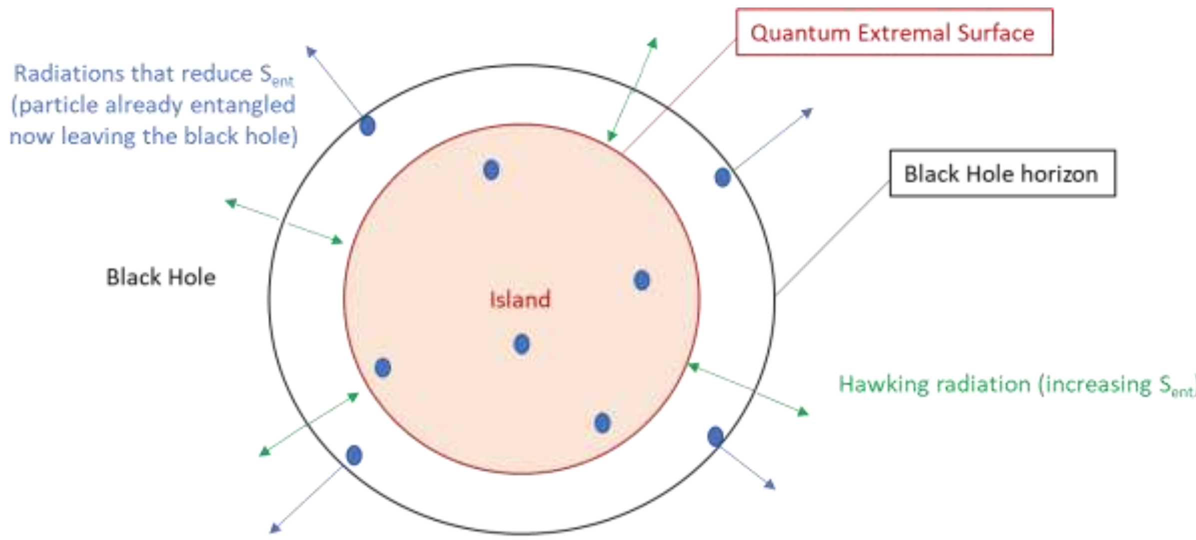

Figure 4: It shows the microscopic interpretation of the multi-fold quantum extremal surface based on figures 2 and 3 in [76]. Instead of reducing the entanglement entropy by losing entanglement though radiations through the quantum extremal surface, towards the singularity, the multi-fold version of it, denotes the part from where no multi-fold can escape for now, till it shrinks further to allow their escape. Some particles in the black holes (blue) who were entangled with the outside (when captured) will now reduce the entanglement entropy when they are caught up by the shrinking horizon and its fluctuations.

Of course, this, and [1,40,76] is for a static black hole. With charged or rotating black holes more considerations are to be added, but they fundamentally amount to adding surfaces (second horizon and quasi-singularities, avoided in multi-fold theory because of multi-fold dark energy effects, spacetime discreteness and non-commutativity, and possibly torsion etc. [1]) [166]. More cases are to be considered, but it does not change the core of the discussion.

Note that quantum fluctuations allows some virtual particles attached to multi-fold from within the quantum extremal surface to reach the in-between horizon and quantum extremal surface region, as can some of the contained particles entangled with the outside. It results into a corresponding progressive contraction of the quantum extremal surface as the life cycle progress. Based on the past history of the black hole (collapse, distribution of past absorbed matter etc.), there may be multiple such quantum extremal surface arranged as Russian dolls, with limited impact on the overall story. If it does not shrink, it is caught up by the shrinking in horizon, and matches its evolution as the particles that formed the island are radiated without creating new entanglement with the outside of the black hole, repeating the contributions of the island to the different entropy terms, as in the conventional definition, but with different microscopic interpretation and effects (e.g., the radiation is external to the black hole not towards the island).

6. Multi-fold Massive Gravity

[1,24,25,*202,203*] show that multi-fold gravity results from the entanglement, and its multi-fold effects, of pairs of virtual particles and anti-particles, created around and by the mass or rather energy of an energy source, particle, or set of particles. In addition, real entangled systems, non-hierarchical, have multi-fold associated to them that create gravity-like effects [1,24,25,*202,203,206*]. It is the base of the E/G conjecture, factual in multi-fold universes [1,24]. The equivalence principle can also be recovered from this analysis [1,103]

[1,6] show that the multi-fold spacetime reconstruction recovers GR from the contribution of entangled massless virtual pairs emitted by mass and energy sources. Except for neutrinos that closely match massless gravity, we also encounter massive gravity with range matching the ranges of these carriers, which corresponds roughly to the scales of the Standard Model (SM) [1,26], contributing to justify the SM_G, which seems to address many open issues with the SM and the standard cosmological model [1,4-27,40,55-61,76,79,90-131,138-160,*176-214,256*]. Such massive gravity looks like GR with possibly different coupling constants at such scales, and at semi classical and classical scales GR is recovered [1,6], especially with the top-down-up-and-upper reconstruction algorithm of [6].

The most important message is that with the multi-fold approach, massive gravity does not modify GR at classical scales, it is compatible with it through its massless contribution [1,6] (and maybe neutrinos as quasi massless contributions).

7. Multi-fold microscopic interpretation of massive gravity with islands and quantum extremal surfaces, including in AdS

Based on the multi-fold theory, and keeping in mind that GR recovers hints of multi-folds at Planck scales [6], let us now provide a microscopic interpretation of the massive gravity encountered with the conventional island formula and replica trick.

7.1 Massive gravity within the black hole

As discussed in sections 4 and 5, the quantum extremal surface encountered in multi-fold universes, is associated to an horizon for multi-folds, that they can't escape to reach the region between it and the black hole horizon, till the quantum extremal surface shrinks past them, as discussed in section 5, to free some multi-folds and particles, in between the horizon and the quantum extremal surface [76]. At some point the quantum extremal surface may coincide and/or follow the horizon as we mentioned also in section 5.

As multi-folds trying to escape slow down to a crawl within the island, below the quantum extremal surface, we identified in [1] that it amounts to stalling the propagation of the gravity effective potential radially towards the outside. In other words, gravity becomes massive, through an effect different from what we discussed in section 6. Although not physical, gravitons are quasi particles actually living outside spacetime, multi-folds are attached to radiated entangled virtual particle pairs some massive, some massless, all stalled at the quantum extremal surface [1,25,26,130,131].

With the quantum extremal surface matching the island formula, at least as minimum of the extrema of the generalized entropy formula for the entanglement entropy of the system, we can see that the island, now defined as the region bounded by the quantum extremal surface (the multi-fold definition), is also by definition the region within the black hole where gravity becomes massive as multi-folds can't escape, per section 5 and [76]. Maybe, it can be useful to envisage it as a phase change, with the quantum extremal surface as domain wall between one phase, the island and the other: the region between the quantum extremal surface and the black hole horizon. Within the island, gravity is massive, a bit like within a superconductor, where in presence of a magnetic field, photons become massive creating the London effect [132,133], although, here, massiveness is not due to Higgs mechanisms of symmetry breaking, but to the stalled virtual particles and their attached multi-folds.

This is different from the massive gravity discussed in section 6. But, similarly, we can see that such massive effects do not affect gravity outside the black hole that remains massless, except for the probably minimal and quasi undistinguishable contribution of neutrinos, at large ranges. For the rest, effects of massive gravity as section 6 occur only as hairs and uncertainties around the black hole. Above the multi-fold quantum extremal surface, multi-fold associated to photons carry the day and gravity has its massless component. The massive components will transition from affected by black hole interior to business as usual outside.

Therefore, we see how the apparition of massive gravity in the island (and elsewhere per the rest of the discussion) does not prevent GR to be the gravity that reigns in the multi-fold spacetime outside the black hole horizon, when considering it at large enough scales. In a multi-fold universe, we are not concerned by the massive gravity encountered in the island by the replica trick and island formula computation, or the multi-fold explanation of its appearance. It does not differ from GR. In an upcoming section we will also show that JT gravity is a good model from a multi-fold point of view, therefore addressing another concern about the gravity involved with the apparition of the island vs. GR.

An important takeaway is that, indeed gravity is massive within the island, delimited by the quantum extremal surface, where also no massless component can move away radially anymore. So the hints of massive gravity appearing in the papers, discussed in section 3.4, are probably facets that the model, based on GR, or rather JT, can't handle better, and describe what happen as well as the multi-fold theory can, and so it is forced to appear to have a (slightly) massive gravity effect, as a hint of what happens.

7.2 Massive gravity with the AdS reservoir

With respect to the reservoirs used by the island formula with the formula trick when computed in AdS, we already understand the need to introduce a thermal reservoir, beyond the boundary of spacetime, where evaporated particles are collected. See [34-37,41], and references therein. Without such reservoir, the particles would be bounced back / reflected at the boundary, and fed back to the black hole.

It shouldn't matter because particles outside, maybe far enough, of the black hole horizon are already evaporated, no need to await them at infinity. Here the need appear because the computed path integrals are inspired by Hawking's original approach, which leads to confusions on where to locate the evaporation and require computing the effects all the way up to infinity. [59-61].

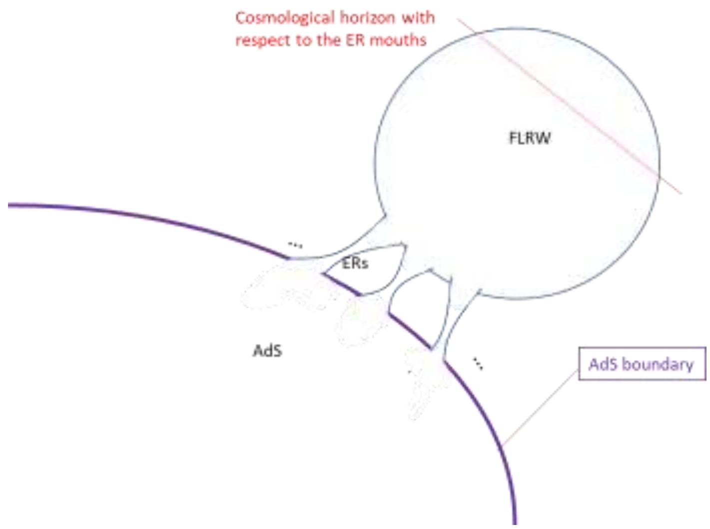

Figure 5: The AdS reservoir as a Wheeler's bag of gold with many ER bridges to the AdS boundary, as a way to address the use case where AdS is fitted with a reservoir that can filter entry of the evaporated particles, but not of gravity, with the problematic interpretation of [37] discussed in section 3.4. Of course it is not exactly correct, but à la [37], one could argues that with this approach, gravity can enter, and only partially exit, which we take as a good image of no subsequent gravitational interaction between the content of the baby universe and the AdS or black hole. And yes with this picture, gravity exists within Wheeler's bag of gold. Such ER wormholes would be covering the whole boundary. Also we have argued in section 3.4, that the interpretation from [37] is flawed expect for content that would be behind the apparent cosmological horizon of the FLRW baby universe. So we should add to the scenarios that we consider particles crossing such an apparent cosmological horizon, so that they have no gravitational (or other entanglement or causal) effect any more on the AdS spacetime or on the blackhole. It is symbolically shown by the red line.

We will now attempt to give physical interpretations to these different use cases. We must start with some caveat. The multi-fold theory assumes a positive cosmological constant unless if built on background spacetime with already a negative cosmological constant. So the analysis that we present may amount to stitching together aspects that shouldn't coexist. However, we think that the analysis still results into interesting interpretations of what happens, and what the AdS(2) JT replica trick with the island formula obtains.

With respect to [77], and the first bullet of the list in section 3.4, with a reservoir, the evaporated particles end-up outside spacetime / AdS, in the baby FLRW expanding baby universe, and then beyond its apparent cosmological horizon, so that no gravity effects, or entanglement occur back in the parent universe. Does it even make sense?

One can imagine a geometry à la Wheeler bag of gold encountered in section 3.3, except that it now needs to cover the whole boundary, maybe with a geometry as shown in figure 5, with an apparent cosmological horizon in the baby universe. Such an example would in fact explain the next consideration: such a bag could let matter in and not out, of the cosmological horizon, and gravity there would not come back to the AdS spacetime.

However, otherwise, in order to absolutely not let gravity enter the reservoir, the only way is that some of the carriers must become all massive so that none can reach the boundary at infinity, even if they come close. Only missing some component suffice. So in the multi-fold theory, the massive gravity contribution of neutrinos would immediately satisfy the requirement, thereby keeping the approach consistent. Gravity can still combine massless gravity, as long as massive gravity exists and therefore introduces a contribution that never reaches the boundary. That is why conventionally, when gravity cannot come in, as in [77], gravity must have a massive component, while when gravity can go into the reservoir, then it does not necessarily need a massive components. Within the reservoir we may or may not have a need for massive gravity, depending on the size/history of the bag(s), or if we can invoke apparent cosmological horizons. If too small, black holes may form, and hence we will encounter massive gravity within the bag(s), just as explained in section 7.1.

In other words, in a multi-fold AdS, whatever that means, as we know it would have to have been constructed from an originally negatively curved background spacetime [1], massive gravity naturally appears in the evaporating black hole (always), in AdS (if it can't enter the reservoir, it must be prevented to reach the AdS boundary) and in the reservoir, when a new black holes may form.

Note that other examples were also proposed in the analysis of the AdS cases. We have for example the case where gravity enters the reservoir. The black hole(s) are now impacting fully the AdS spacetime. If particles and gravity can enter and exit, it's just still AdS, near or at the boundary. The boundary does not exists anymore, it has moved. The region can evaporate (e.g., leave the reservoir or black hole evaporation if the reservoir was such that a black hole forms with the particles evaporated by the initial black hole in AdS), and we will end up with a balanced exchange of radiations between the reservoir and the black hole. This is a variation of the reflection at the boundary. The radiation entropy would become constant (what is emitted by the first black hole is absorbed by the second one then re-immitted towards the first black hole, and so on.), as predicted by the [82]: we would have a different curve than the Page curve. We have in fact in our view recovered, and motivated, a situation analogous to Maldacena eternal black holes as in [134], with the eternal loss of information described in that paper.

With respect to the last two bullets of section 3.4, we know that there exist no gravity shield in a multi-fold universe [79]. From that point of view, the AdS boundary can indeed track almost everything, with the exception of what happens with the Wheeler's bags of gold with whatever reaches the cosmological horizon. So there is indeed a problem with the conventional proposal for the Quantum extremal surface and the island, but the problem is with its physical and microscopic interpretation, not with the mathematical approach that determines it.

In general, in a black hole, even the quantum extremal surface (QES) can't block it entanglement or gravity effects.

We have a chain of potential energy evolution:

$$U_{withinQES} \rightarrow U_{QES} \rightarrow U_{betweenQHS\&BHHorizon} \rightarrow U_{BHHorizon} \tag{1}$$

So a priori, the island is tracked by a multi-fold AdS spacetime boundary, e.g., in terms of its mass, or gravity effects. But, in a multi-fold universe, that is not an issue as we do not rely on "decoupling", or "Hiding" the inside of the QES, to obtain a suitable Page curve. In the conventional pictures this falls apart: you can't hide within the QES, unless partially through the Wheeler's bag of gold geometry, by falling beyond the (apparent) cosmological horizon of the baby universe, as discussed earlier.

The issues of [83,84], mentioned in section 3.3, indeed indicates a problem with the convention QES interpretation, not with the multi-fold interpretation, which can also be extended to the conventional results.

7.3 Wheeler's bag of gold in a multi-fold universe

What about the Wheeler's bag of gold scenario as an island option and interpretation? Could it happen?

Unless and until a cosmological horizon, apparent for the wormhole, forms in the baby universe, nothing has really disappeared from the parent universe, or in this case, the black hole interior. Mass, and its gravitational effects, and entanglement remain. Therefore it is already obvious that the wheeler bag of gold proposal does not decouple all of the island content from the rest of the black hole and parent universe: another horizon would be needed as for example in Reissner Nordström [2], and Kerr Newman [3] black holes [166,*194*]. That region could be considered as decoupled.

In addition, wheeler's bag of gold are suppressed in multi-fold universes as in our real universe. *Note added on December 22, 2023: See [209] for a detailed discussion and proof of this statement.*

As already discussed, [167] concludes differently, but it seems that it is based on an incorrect assumption that the apparition of a violation of the energy condition, associated to a traversable wormhole would imply apparition of an horizon, and black hole behavior That is absolutely not the case. Therefore the entropy exposed to the parent universe side of the worm hole is not bounded by the black hole area law, but by the right covariant entropy bound, which is not violated by Wheeler's bag of gold [171-173], as we argued for, also in a multi-fold universe [40]. Physically, the analysis of figure 3, now also encompasses entanglement per the E/G conjecture factual in a multi-fold universe [1,24,25,*202,203,206*].

With this, we have proven that the conventional microscopic interpretation of the island is not valid in a multi-fold universe. The analysis of this section also applies to non-multi-fold universes.

On the other hand, we have shown that the model of the island once correctly understood, and in a multi-fold universe does not encounter, or can address without problem the other issues raised in section 3.4.

7.4 Massive Gravity and GR

Massive gravity is typically assumed to be at large scales, and, therefore, to modify GR. See references in [26]. As discussed in section 6, it is not necessarily the case, and it is not the case for the small scale massive gravity contributions encountered in multi-fold universes [1,26]. Instead, they rather impact the SM scales and below, with now a standard model with gravity effects not negligible at its scales, SM_G, and the standard cosmology models model [1,4-27,40,55-61,76,79,90-131,138-160,*176-214,256*].

In the present case, the massive gravity effects, within the black hole, result from the constraints on the multi-fold dynamics and kinematics, within the black hole quantum extremal surface. It is a different massive effect but constrained to such a region. It is an effect not modeled by GR, but consistent with GR reigning beyond that region: GR and gravity is not modified beyond the new phase in the black hole, especially beyond its quantum extremal surface.

As we saw in section 3.2, the main advantage of 2D gravity using JT gravity vs Liouville gravity [161,*a41*], come from the ability to solve mathematically some of the path integrals and models.

In a multi-fold universe, spacetime results from 2D (processes of) random walk of massless Higgs bosons [1,16,17,23,93,94,129,*177,189,209*]. When spatial scales are reduced, spacetime always undergoes a dimensional reduction from 4D to 3D to 2D as discussed in [1,94,147,148], with (2D) CFTs also modelling the random walk of massless (free) bosons [175,*177*]. Liouville gravity really represents this limit of 2D+ε gravity when $\varepsilon \rightarrow 0$ [161], but in the end, the 2D gravity obtained this way has no dynamics (because the action has only surface term, or physically because spacetime locations can't move with a massless graviton (that wouldn't be with longitudinal polarization, the only available option) [*216*].

In a multi-fold universe, we understand the massless Higgs boson as the dilaton, associated to the multi-fold mechanisms (compactification and dynamics) [*177,189,206*], stabilizing the theory [174,*184,206*], and the massless bosons in 2D random walks [175,*177*], while the Higgs potential models their many body behavior in QFT [16,27,91,93]. This way, we see that a dilaton gravity theory matches the Hilbert Einstein action expectation with a scalar, and all can be reduced to JT under reasonable expectations (and other variations in potentials will be of the same type of behaviors) [*215*]. See also [17,118,129,152] for more encounters of the dilaton and its relationship to GR, through the double copy duality.

With this analysis, we further confirm that JT gravity is indeed suitable for modeling 2D multi-fold gravity and spacetime. This is why, despite the concerns of others as mentioned earlier, JT works so well to model and compute the path integrals (Euclidian or Lorentzian) encountered with the replica trick and island formula. After all what matters is 2D physics [*195*].

Because the multi-fold reconstruction [1,6] recovers GR, we are not concerned any more that the 2D work be for a different gravity. In a multi-fold universe, we know that JT gravity is 2D GR implemented by massless Higgs bosons, and their 2D random walks. Also, anybody interested in studying multi-fold gravity (at 2D) can start from JT gravity in general.

Our ability to justify the superiority of JT gravity to model 2D gravity in the real universe, not only at 2D scales, but as a substitute for 4D computations, is another encouraging hint in favor of the multi-fold theory.

So, we do not agree with the characterization of [41,77] that the island and replica trick would apply for a different gravity than GR. It is not (necessarily) true, and in the multi-fold universe, it is firmly not the case. It is still GR. Conventionally, we believe that the apparent differences is the reflection of GR not being able to perfectly model what happens, not a change to gravity away from GR. We always knew that GR has limitation within a black hole, with say singularities for example. Now we see another example. But outside the black hole, it still works as plain old GR, and its semi classical equivalent. It's not that different from us also dealing with the Trans-Planckian censorship conjecture [1,95]: it also does not impact GR validity at larger scales.

7.5 About the holographic principle, and information storage, in multi-fold universes

In a multi-fold universes, [1,40,49] shows that the holographic principle exists and that it is satisfied, with the covariant entropy bound. [79] further illustrates how there are no shield or barrier to multi-fold based gravity.

As such, indeed, in an AdS multi-fold universe, a shaky physical concept, we would also find all the information about the content of the universe at the boundary, which contains all the information including from the island [41,83,84]. But then again, our real universe is not AdS and has no boundary, and a multi-fold universe is with a positive cosmological constant [1,*a34*]. So that discussion is moot. But based on what we argued, there are no issues with the concept of island and quantum extremal surface if we take the multi-fold microscopic interpretation. Problems occur when we want to take the conventional interpretation and propose topologies as the Wheeler's bag of gold as the microscopic interpretation. There things fall apart, and are only partially addressed with an apparent cosmological horizon.

7.6 About quantum extremal surface outside of the black hole

About section 3.5, and [86], in a multi-fold universe, we can interpret what happens as follow:

- If the temperature of the bath is high versus the black hole temperature, the flow of particles will increase from the outside to the universe then to the black hole. As a result the horizon and quantum extremal surface will grow, and grow faster than normally, as it absorbs more energy per particle, e.g., in the form of kinetic energy). However the quantum extremal surface should remain within the black hole.
- If the temperature of the bath is lower, the black hole and quantum extremal surface will shrink faster as particles leave horizon at a higher rate, with additional fluctuations of the horizon on top of the quantum fluctuations, accelerating the evaporation and depleting the region between the quantum extremal

surface and the horizon faster, making the two coincide very closely, letting the island to directly contribute to the evaporation along with the horizon.

- If the difference of temperature is even larger, then we can see it as if the horizon has grown, with the extra energy coming from the thermal energy, with the quantum extremal surface coming along: it appears as if it as outside the horizon would be without the thermal bath. Again, an example of facets that GR can't model other than by imperfect hints.

This reasoning works if the radiation is from the island to the outside, not the other way around. Indeed with a Wheeler's bag of gold, for the second bullet, more radiation towards the island does not help, nor does less.

This is in our view another indication that the multi-fold interpretation is the right one of the island and the quantum extremal surface.

7.7 Lessons so far

With the explanation of massive gravity, and the behaviors in AdS using the multi-fold theory, and its particle analysis, we can confirm the suitability of the island formula and replica trick, but using the multi-fold interpretation of the Quantum Extremal Surface instead of the conventional island as a Wheeler's bag of gold. The approach is sound, but it is the conventional interpretation that is problematic.

Conversely, we saw that the notions of using an (asymptotic) AdS space with evaporating black holes are shaky. It is another hint that our real universe is probably not (asymptotic) AdS. *Note added on December 22: See [209] for a proof that the cosmological constant must be small but strictly positive.*

8. Black Hole Evaporation vs. Split

[1,56] proposes that towards the end of evaporation, extremal black holes can split into smaller extremal and non-extremal black holes. The extremal black holes do not evaporate, although, without affecting our discussion, some suggest that, with higher order corrections to GR, they possibly could evaporate [135]. In fact, with the quantum fluctuations, already behind the model of evaporation in [1,76], we would tend to agree that extremality should not prevent evaporation, although, it might slow it down.

It is also that scenario that led us to proposing not following the strict weak gravity conjecture, and eventually introducing the Ultimate Unification (UU) regime [1,56].

The analysis presented here, and in [40,76], does not change our analysis and previous proposals.

Interestingly, papers like [136,137] show that in a AdS universe, the entropy seems to diverge when its mass goes to zero, and spacetime becomes flat [137]. It is understood as the effect of Schwarzschild black hole explosion at low mass. There is no real mystery there, it can be hinted simply from black hole temperature.

[137] describes an approach, which produces with the island formula with replica trick two solutions. One without island an island and one without island . Both configurations are shown to have finite entropy; no black hole explosion anymore.

We interpret this, as a hint of when the black hole split occurs, even if the conditions are a bit different (no charge or rotation in the model): one configuration amounts to the black hole still able to evaporate (with island), while the other would be the extremal one unable to evaporate, and candidate for further splits until we end up with particles as microscopic black holes [1,4,56].

This is an approximate but interesting illustration, and possible a veiled confirmation of our proposal of Ultimate Unification (UU) from [1,56], key to understand high energy behaviors [23,57]. We may publish a more detailed paper in the future on this, but we would be very interested by any results of applying the island / no island analysis in dS spacetime, and with extremally charged and rotating black holes, whose mass also goes towards zero. We should recover the above.

9. Multi-fold universes and the real universe

[6] shows that multi-fold are encountered in GR at Planck scales. Along with other results, like [1,4-27,40,55-61,76,79,90-131,138-160,*176-214,256*], that shows how SM_G seems to contribute to addressing open issues with the SM, and the Standard cosmological model, or even how we encounter superstrings [1,14-20,40,129], it leads us to assert that our real universe is multi-fold.

On that basis, the results and models of sections 4 to 7, apply as suitable candidate interpretations and solutions for sections 2 and 3. We do consider the key following differences and results for the real universe:

- The quantum extremal surface is not at the mouth or throat of a Wheeler's bag of gold as island, where the black hole content would enter and lose information with respect to the rest of the black hole interior, thereby reducing its entropy, as the external black hole entropy grows. Instead, the quantum extremal surface delimits the inside region of the black hole from where particles inside the black hole were previously entangled with the outside, but can't evaporate for now. As the black hole evaporates and its horizon shrinks, so does the quantum extremal surface, and more internal entangled particles become available for evaporation, but their contributions then decrease entanglement, leading to the correct Page curve, resolution of the black hole information paradox. The evolution ends when we reach a scale where the UU effects starts to appear as discussed in [1,56,57], where microscopic black holes will rather break apart instead of evaporating, until everything returns to individual particles.
- In an asymptotic dS spacetime, there is no need for a reservoir. The model stands on it own, and it does so with GR in 4D spacetime. Not that it is an issue as there is no need to worry about 2D JT gravity not characterizing GR in toy models. These models actually represent GR at 2D, with gravity and spacetime implemented by 2D random walks of massless Higgs bosons: the dilatons. The models converge to recover GR at higher scales and dimensions.
- Evaporation is a process as described in [59-61] that can be computed in several ways, contrary to the view of some [59,60].
- All our models and results reinforce the message we gave in [1,55] that particles are as relevant, if not more relevant than just QFT fields, which are great mathematical tool, also sometimes only approximating Physics.

We do not have to comment further on non multi-fold AdS universes, as we do not believe that it is physically relevant in terms of the real universe.

10. Conclusions

The paper reached a few key conclusions. Firstly, we have provided an new physical interpretation for the quantum extremal surface encountered in multi-fold theory and with the island formula + replica trick. We have shown that while the definitions match (the fine grained entropy minimalization), the conventional interpretation of the quantum extremal surface, as a Wheeler's bag of gold, makes way less sense than the multi-fold interpretation. We have also shown that the conventional interpretation is not that physically likely in a black hole, and that, even if it was, it leads to problems by not decoupling the content of the Wheeler's bag of gold from the parent universe, at the difference of what many believe. This can only be partially achieved when we encounter an apparent cosmological horizon for the wormhole; but that does not save the conventional interpretation from its problems. Furthermore, the multi-fold model is derived in a universe that is 4D, multi-fold or (asymptotic) dS, with GR; not limited to 2D,or AdS with JT gravity and first order approximations. With this, the island formula with the replica trick, encounter the correct Page curve, and resolve the black hole information paradox, as doe sthe multi-fold model.

Hereafter, we discussed how massive gravity appears within black holes in multi-fold universes. It is essentially a phase transition, analogous to how photons acquire mass in superconductor in the London effect. It is a different form of massive gravity from the multi-fold massive gravity, associated to commonly massive SM virtual particles: the virtual particles all acquire masses, including the massless ones, and, as a result, gravity is entirely massive, i.e., there are no massless gravity contribution in such a region.

We showed that JT as a dilaton 2D gravity is actually correctly modeling GR in a 2D spacetime constructed by 2D random walks of massless Higgs bosons, the dilatons, and correctly recovers GR at higher scales and dimensions. It confirms that the computations of the island formula with the replica trick, and JT gravity in 2D, are able to model 4D GR black holes. On the other hand, asymptotic AdS is not physical, and that impacts some of the non-multi-fold results, or relevance of the conventional results.

As GR can't model multiple gravity models like massive and massless gravity, we expect that GR may therefore be reduced to assume a massive gravity in its model for these use cases; just as, for example, we argued in [6] that GR can't well model multi-folds, and therefore resorts to Planck scale wormholes to hint at them. We have encountered other examples of these inabilities to model the multi-fold facets that the theories can't conceptualize. See for example [5,7,118], or the black hole and quantum extrema surface configurations discussed in sections 7.6 and 8.

The paper opportunistically explored a black hole in an AdS multi-fold spacetime, in order to see how it would handle the controversies that have arisen with respect to the conventional island formula with the replica trick. Massive gravity appears as linked to the island, the boundary, and the correct Page curve; but now also outside the black hole. We explained how such results are immediately expected in multi-fold AdS universes, i.e., a multi-fold universe with a negative cosmological constant, something that requires starting from a negatively curved manifold. Also, we can explain why the other options for the reservoirs, added to AdS to handle reflections at the AdS boundary, would lead to a constant radiation entropy. No mystery or issues, it all makes sense.

We explained the conventional encounter of quantum extremal surface outside the black hole horizon in the presence of a thermal bath. Doing so, we concluded that only the multi-fold microscopic interpretation for the quantum extremal surface works.

Additional papers on the island formula and replica tricks allowed us to see hints confirming the new black hole lifecycle, at small masses, that we used to invalidate the strict WGC, and inspire then motivate our UU proposal. We see that as an unexpected, and welcomed result.

We admit that we were thinking that we might find a contradiction forbidding AdS, or black holes in AdS. But it hasn't happened: while reservoirs are clearly unphysical, all options for the setup of the reservoir can be made consistent with was observed by those concerned with the island formula with the replica trick. So AdS is not ruled out, other than due to a negatively curved initial conditions as discussed in [1], and all our other works which already made asymptotic AdS / negative cosmological constant spacetime unphysical [1,14,15,*209*].

Finally, it is worth repeating that our approach is particle driven, instead of only QFT driven. This is consistent with our arguments that particles are key to studying entanglement and quantum gravity [1,55].

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