



Life of the homogeneous and isotropic universe in dynamical string tension theories

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Abstract Cosmological solutions are studied in the context of the modified measure formulation of string theory, then the string tension is a dynamical variable and the string tension is an additional dynamical degree of freedom and its value is dynamically generated. These tensions are then not universal, rather each string generates its own tension which can have a different value for each of the string world sheets and in an ensemble of strings. The values of the tensions can have a certain dispersion in the ensemble. We consider a new background field that can couple to these strings, the “tension scalar” which is capable of changing locally along the world sheet and then the value of the tension of the string changes accordingly. When many types of strings probing the same region of space are considered this tension scalar is constrained by the requirement of quantum conformal invariance. For the case of two types of strings probing the same region of space with different dynamically generated tensions, there are two different metrics, associated to the different strings. Each of these metrics have to satisfy vacuum Einstein’s equations and the consistency of these two Einstein’s equations determine the tension scalar. The universal metric, common to both strings generically does not satisfy Einstein’s equation. The two string dependent metrics considered here are flat space in Minkowski space and Minkowski space after a special conformal transformation. The limit where the two string tensions are the same is studied, it leads to a well defined solution. If the string tension difference between the two types of strings is very small but finite, the approximately homogeneous and isotropic cosmological solution lasts for a long time, inversely proportional to the string tension difference and then the homogeneity and isotropy of the cosmological disappears and the solution turns into an expanding braneworld where the strings

are confined between two expanding bubbles separated by a very small distance at large times. The same principle is applied to the static end of the universe wall solution that lasts a time inversely proportional to the dispersion of string tensions. This suggest a scenario where quantum fluctuations of the cosmological or static solutions induce the evolution towards braneworld scenarios and decoherence between the different string tension states.

1 Introduction

String Theories have been considered by many physicists for some time as the leading candidate for the theory everything, including gravity, the explanation of all the known particles that we know and all of their known interactions (and probably more) [1, 2]. According to some, one unpleasant feature of string theory as usually formulated is that it has a dimension full parameter, in fact, its fundamental parameter, which is the tension of the string. This is when formulated the most familiar way. The consideration of the string tension as a dynamical variable, using the modified measures formalism, which was previously used for a certain class of modified gravity theories under the name of Two Measures Theories or Non Riemannian Measures Theories, see for example [3–10]. The modified measure approach has also been used to construct braneworld scenarios [11, 12].

When applying these principles to string theory, this leads to the modified measure approach to string theory, where rather than to put the string tension by hand it appears dynamically.

This approach has been studied in various previous works [13–21]. See also the treatment by Townsend and collaborators for dynamical string tension [22, 23].

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We have also introduced the “tension scalar” [24], which is an additional background fields that can be introduced into the theory for the bosonic case (and expected to be well defined for all types of superstrings as well) that changes the value of the tension of the extended object along its world sheet, we call this the tension scalar for obvious reasons. This formalism is then the basis to construct several cosmological scenarios [21] modified string theory braneworlds scenarios [25]

Before studying issues that are very special of this paper we review some of the material contained in previous papers, first present the string theory with a modified measure and containing also gauge fields that like in the world sheet, the integration of the equation of motion of these gauge fields gives rise to a dynamically generated string tension, this string tension may differ from one string to the other. Then we consider the coupling of gauge fields in the string world sheet to currents in this world sheet. As a consequence, this coupling induces variations of the tension along the world sheet of the string. Then we consider a bulk scalar and how this scalar naturally can induce A world sheet current that couples to the internal gauge fields. The integration of the equation of motion of the internal gauge field lead to the remarkably simple equation that the local value of the tension along the string is given by $T = e\phi + T_i$, where e is a coupling constant that defines the coupling of the bulk scalar to the world sheet gauge fields and T_i is an integration constant which can be different for each string in the universe.

Each string is considered as an independent system that can be quantized. We take into account the string generation by introducing the tension as a function of the scalar field as a factor inside a Polyakov type action with such string tension, then the metric and the factor $g\phi + T_i$ enter together in this effective action, so if there was just one type of string the factor could be incorporated into the metric and the condition of world sheet conformal invariance will not say very much about the scalar ϕ , but if many strings are probing the same regions of space time, then considering a background metric $g_{\mu\nu}$, for each string the “string dependent metric” $(\phi + T_i)g_{\mu\nu}$ appears

Considering the other background fields, like dilaton and antisymmetric tensor fields as trivial implies then that the vacuum Einstein’s equations apply for each of the metrics $(\phi + T_i)g_{\mu\nu}$.

We consider the simplest non trivial multy string case where two types of strings with different tensions are considered. We call $g_{\mu\nu}$ the universal metric, which in fact does not necessarily satisfy Einstein’s equations.

Here we will study a situation where we consider the metrics $(\phi + T_i)g_{\mu\nu}$, considering two types of strings tensions. The two metrics will again satisfy Einstein’s equations and the two metrics will represent Minkowski space

and Minkowski space after a special conformal transformation.

We will study first the non trivial solutions that persist in the limit $T_2 - T_1 \rightarrow 0$, which is possible when the transformation of the two conformally related string is infinitesimal as well. The solutions in this limits are 1)Homogeneous and isotropic cosmological solution, 2)The static “end of space” wall .

Each of these solutions becomes valid only for a finite lifetime Δt proportional to $1/\Delta T$, where $\Delta T = T_2 - T_1$, which resembles a type of time – energy uncertainty principle. This resembles uncertainty principles proposed for the uncertainty of the cosmological constant and the volume of the universe (which contains as a factor the total time of course) [26] in the context of the unimodular theory when formulated in a generally covariant form [27]. As we have pointed out in our review [28], the unimodular theory when formulated in a generally covariant form is a very particular case of a modified measure theory, in this case the cosmological constant becomes a dynamical variable just like in the case being discussed now the string tension is a dynamical variable, the $\Delta T = T_2 - T_1$ may be an effective classical way to describe a quantum fluctuation in the string tension.

For $T_2 - T_1 \neq 0$ but small, the long lived homogeneous and isotropic cosmological solution transforms itself into a braneworld scenario, where there are two locations where the strings acquire an infinite tension which are given by two surfaces. This is when the vector that defines the special conformal transformation is time like, which lead to the cosmological solution in the case $T_2 - T_1 \rightarrow 0$, if the vector is space like the static end of universe solution holds in the limit $T_2 - T_1 \rightarrow 0$, If the vector is space like the two surfaces are spherical and expanding and the distance between them approaches zero at large times (positive or negative). In both cases (the vector field being space like or time like) this represents a genuine brane world scenario.

The basic idea of the brane worlds is that the universe is restricted to a brane inside a higher-dimensional space, called the “bulk”. In this model, at least some of the extra dimensions are extensive (possibly infinite), and other branes may be moving through this bulk. Some of the first braneworld models were developed by Rubakov and Shaposhnikov [29], Visser [30], Randall and Sundrum [31,32], Pavsic [33], Gogberashvili [34–37]. At least some of these models are motivated by string theory. Braneworlds in string theory were discussed in [38], see for a review for example [39], our approach is very different to the present standard approaches to braneworlds in the context of string theories however. In our approach a dynamical string tension is required. Our scenario could be enriched by incorporating aspects of the more traditional braneworlds, but these aspects will be ignored here to simplify the discussion.

2 The modified measure theory string theory

The standard world sheet string sigma-model action using a world sheet metric is [40–42]

$$S_{\text{sigma-model}} = -T \int d^2\sigma \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (1)$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = \det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

Now instead of using the measure $\sqrt{-\gamma}$, on the 2-dimensional world-sheet, in the framework of this theory two additional worldsheet scalar fields $\varphi^i (i = 1, 2)$ are considered. A new measure density is introduced:

$$\Phi(\varphi) = \frac{1}{2} \epsilon_{ij} \epsilon^{ab} \partial_a \varphi^i \partial_b \varphi^j. \quad (2)$$

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. The modified-measure theory is an example of such a theory.

Then the modified bosonic string action is (as formulated first in [13] and latter discussed and generalized also in [14])

$$S = - \int d^2\sigma \Phi(\varphi) \left(\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right), \quad (3)$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a : $F_{ab} = \partial_a A_b - \partial_b A_a$.

It is important to notice that the action (3) is invariant under conformal transformations of the internal metric combined with a diffeomorphism of the measure fields,

$$\gamma_{ab} \rightarrow J \gamma_{ab}, \quad (4)$$

$$\varphi^i \rightarrow \varphi'^i = \varphi^i(\varphi^j) \quad (5)$$

such that

$$\Phi \rightarrow \Phi' = J \Phi \quad (6)$$

Here J is the jacobian of the diffeomorphism in the internal measure fields which can be an arbitrary function of the world sheet space time coordinates, so this can be called indeed a local conformal symmetry.

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (3).

The variation with respect to φ^i leads to the following equations of motion:

$$\epsilon^{ab} \partial_b \varphi^i \partial_a \left(\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right) = 0. \quad (7)$$

since $\det(\epsilon^{ab} \partial_b \varphi^i) = \Phi$, assuming a non degenerate case ($\Phi \neq 0$), we obtain,

$$\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M = \text{const.} \quad (8)$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2} \gamma_{ab} \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0. \quad (9)$$

One can see that these equations are the same as in the sigma-model formulation. Taking the trace of (9) we get that $M = 0$. By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}$ from (8) (with $M = 0$) we obtain the standard string eqs.

The emergence of the string tension is obtained by varying the action with respect to A_a :

$$\epsilon^{ab} \partial_b \left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0. \quad (10)$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \quad (11)$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc. Let us stress that the modified measure string theory action does not have any *ad hoc* fundamental scale parameters associated with it. This can be generalized to incorporate super symmetry, see for example [14–17]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [22, 23]. However the fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before.

Notice that Each String in its own world sheet determines its own tension. Therefore the tension is not universal for all strings.

3 Introducing background fields including a new background field, the tension field

Schwinger [43] had an important insight and understood that all the information concerning a field theory can be studied by understanding how it reacts to sources of different types.

This has been discussed in the text book by Polchinski for example [44–46]. Then the target space metric and other external fields acquire dynamics which is enforced by the requirement of zero beta functions.

However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [24]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings that is induced by such external scalar field.

3.1 Introducing world sheet currents that couple to the internal gauge fields

If to the action of the string we add a coupling to a world-sheet current j^a , i.e. a term

$$S_{\text{current}} = \int d^2\sigma A_a j^a, \quad (12)$$

then the variation of the total action with respect to A_a gives

$$\epsilon^{ab} \partial_a \left(\frac{\Phi}{\sqrt{-\gamma}} \right) = j^b. \quad (13)$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

3.2 How a world sheet current can naturally be induced by a bulk scalar field, the tension field

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^b = e \partial_\mu \phi \frac{\partial X^\mu}{\partial \sigma^a} \epsilon^{ab} \equiv e \partial_a \phi \epsilon^{ab}, \quad (14)$$

where e is some coupling constant. The interaction of this current with the world sheet gauge field is also invariant under

local gauge transformations in the world sheet of the gauge fields $A_a \rightarrow A_a + \partial_a \lambda$.

For this case, (13) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \quad (15)$$

or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \quad (16)$$

The constant of integration T_i may vary from one string to the other. Notice that the interaction is metric independent since the internal gauge field does not transform under the the conformal transformations. This interaction does not therefore spoil the world sheet conformal transformation invariance in the case the field ϕ does not transform under this transformation. One may interpret (16) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i . The consequences of an independent quantization of many strings with different T_i covering the same region of space time will be studied in the next section.

3.3 Consequences from world sheet quantum conformal invariance on the tension field, when several strings share the same region of space

3.3.1 The case of two different string tensions

If we have a scalar field coupled to a string or a brane in the way described in the sub section above, i.e. through the current induced by the scalar field in the extended object, according to Eq. (16), so we have two sources for the variability of the tension when going from one string to the other: one is the integration constant T_i which varies from string to string and the other the local value of the scalar field, which produces also variations of the tension even within the string or brane world sheet.

As we discussed in the previous section, we can incorporate the result of the tension as a function of scalar field ϕ , given as $e\phi + T_i$, for a string with the constant of integration T_i by defining the action that produces the correct equations of motion for such string, adding also other background fields, the anti symmetric two index field $A_{\mu\nu}$ that couples to $\epsilon^{ab} \partial_a X^\mu \partial_b X^\nu$ and the dilaton field φ that couples

to the topological density $\sqrt{-\gamma}R$

$$S_i = - \int d^2\sigma (e\phi + T_i) \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \int d^2\sigma A_{\mu\nu} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu + \int d^2\sigma \sqrt{-\gamma} \varphi R. \quad (17)$$

Notice that if we had just one string, or if all strings will have the same constant of integration $T_i = T_0$.

In any case, it is not our purpose here to do a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields, instead, we will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that all the metrics

$$g_{\mu\nu}^i = (e\phi + T_i) g_{\mu\nu} \quad (18)$$

will satisfy simultaneously the vacuum Einstein's equations.

The interesting case to consider is when there are many strings with different T_i , let us consider the simplest case of two strings, labeled 1 and 2 with $T_1 \neq T_2$, then we will have two Einstein's equations, for $g_{\mu\nu}^1 = (e\phi + T_1) g_{\mu\nu}$ and for $g_{\mu\nu}^2 = (e\phi + T_2) g_{\mu\nu}$,

$$R_{\mu\nu}(g_{\alpha\beta}^1) = 0 \quad (19)$$

and, at the same time,

$$R_{\mu\nu}(g_{\alpha\beta}^2) = 0 \quad (20)$$

These two simultaneous conditions above impose a constraint on the tension field ϕ , because the metrics $g_{\alpha\beta}^1$ and $g_{\alpha\beta}^2$ are conformally related, but Einstein's equations are not conformally invariant, so the condition that Einstein's equations hold for both $g_{\alpha\beta}^1$ and $g_{\alpha\beta}^2$ is highly non trivial.

Then for these situations, we have,

$$e\phi + T_1 = \Omega^2(e\phi + T_2) \quad (21)$$

which leads to a solution for $e\phi$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2} \quad (22)$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} \quad (23)$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} \quad (24)$$

Both tensions can be taken as positive if $T_2 - T_1$ is positive and Ω^2 is also positive and less than 1. It is important that we were forced to consider a multi metric situation. One must also realize that Ω^2 is physical, because both metrics live in the same spacetime, so even if Ω^2 is a constant, we are not allowed to perform a coordinate transformation, consisting for example of a rescaling of coordinates for one of the metrics and not do the same transformation for the other metric.

Other way to see that Ω^2 is physical consist of considering the scalar consisting of the ratio of the two measures $\sqrt{-g^1}$ and $\sqrt{-g^2}$ where $g^1 = \det(g_{\alpha\beta}^1)$ and $g^2 = \det(g_{\alpha\beta}^2)$, and we find that the scalar $\frac{\sqrt{-g^1}}{\sqrt{-g^2}} = \Omega^D$, showing that Ω is a coordinate invariant.

3.3.2 Flat space in Minkowski coordinates and flat space after a special conformal transformation

Let us study now a case where Ω^2 is not a constant. For this we will consider two spaces related by a conformal transformation, which will be flat space in Minkowski coordinates and flat space after a special conformal transformation.

The flat space in Minkowski coordinates is,

$$ds_1^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta \quad (25)$$

where $\eta_{\alpha\beta}$ is the standard Minkowski metric, with $\eta_{00} = 1$, $\eta_{0i} = 0$ and $\eta_{ij} = -\delta_{ij}$. This is of course a solution of the vacuum Einstein's equations.

We now consider the conformally transformed metric

$$ds_2^2 = \Omega(x)^2 \eta_{\alpha\beta} dx^\alpha dx^\beta \quad (26)$$

which we also demand will satisfy the D dimensional vacuum Einstein's equations.

Let us use the known transformation law of the Ricci tensor under a conformal transformation applied to $g_{\alpha\beta}^1 = \eta_{\alpha\beta}$ and $g_{\alpha\beta}^2 = \Omega(x)^2 \eta_{\alpha\beta}$, defining $\Omega(x) = \theta^{-1}$, we obtain

$$\begin{aligned} R_{\alpha\beta}^2 &= R_{\alpha\beta}^1 + (D-2)\nabla_\alpha \nabla_\beta (\ln \theta) + \eta_{\alpha\beta} \eta^{\mu\nu} \nabla_\mu \nabla_\nu (\ln \theta) \\ &\quad + (D-2)\nabla_\alpha (\ln \theta) \nabla_\beta (\ln \theta) \\ &\quad - (D-2)\eta_{\alpha\beta} \eta^{\mu\nu} \nabla_\mu (\ln \theta) \nabla_\nu (\ln \theta) \end{aligned} \quad (27)$$

Since $g_{\alpha\beta}^1 = \eta_{\alpha\beta}$, we obtain that $R_{\alpha\beta}^1 = 0$, also the covariant derivative above are covariant derivatives with respect to the metric $g_{\alpha\beta}^1 = \eta_{\alpha\beta}$, so they are just ordinary derivatives.

Taking this into account, after a bit of algebra we get that,

$$\begin{aligned} R_{\alpha\beta}^2 &= (D-2)\frac{\partial_\alpha\partial_\beta\theta}{\theta} + \eta_{\alpha\beta}\eta^{\mu\nu}\left(\frac{\partial_\mu\partial_\nu\theta}{\theta} - \frac{\partial_\mu\theta\partial_\nu\theta}{\theta^2}\right) \\ &\quad -(D-2)\eta_{\alpha\beta}\eta^{\mu\nu}\frac{\partial_\mu\theta\partial_\nu\theta}{\theta^2} = 0 \end{aligned} \quad (28)$$

by contracting (28) we obtain a relation between $\eta^{\mu\nu}\frac{\partial_\mu\partial_\nu\theta}{\theta}$ and $\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta/\theta^2$

$$2\eta^{\mu\nu}\frac{\partial_\mu\partial_\nu\theta}{\theta} = D\frac{\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta}{\theta^2} \quad (29)$$

using (29) to eliminate the nonlinear term $\eta^{\mu\nu}\frac{\partial_\mu\theta\partial_\nu\theta}{\theta}$ in (28) we obtain the remarkably simple linear relation,

$$\partial_\alpha\partial_\beta\theta - \frac{1}{D}\eta_{\alpha\beta}\eta^{\mu\nu}\partial_\mu\partial_\nu\theta = 0 \quad (30)$$

So we now first find the most general solution of the linear equation (30), which is,

$$\theta = a_1 + a_2 K_\mu x^\mu + a_3 x^\mu x_\mu \quad (31)$$

and then impose the non linear constraint (29), which implies,

$$a_1 = \frac{a_2^2 K_\mu K^\mu}{4a_3} \quad (32)$$

we further demand that $\theta(x^\mu = 0) = 1$, so that,

$$\theta = 1 + a_2 K_\mu x^\mu + \frac{a_2^2 K_\mu K^\mu}{4} x^\mu x_\mu \quad (33)$$

This coincides with the results of Culetu [47] for $D = 4$ and to identify this result with the result of a special conformal transformation, see discussions in [48,49], to connect to standard notation we identify $a_2 K_\mu = 2a_\mu$, so that

$$\theta = 1 + 2a_\mu x^\mu + a^2 x^2 \quad (34)$$

where $a^2 = a^\mu a_\mu$ and $x^2 = x^\mu x_\mu$.

In this case, this conformal factor coincides with that obtained from the special conformal transformation

$$x^\mu = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2 x^2)} \quad (35)$$

As discussed by Zumino [49] the finite special conformal transformation mixes up in a complicated way the topology of space time, so it is not useful to interpret the finite special conformal transformations as mapping of spacetimes.

In summary, we have two solutions for the Einstein's equations, $g_{\alpha\beta}^1 = \eta_{\alpha\beta}$ and

$$g_{\alpha\beta}^2 = \Omega^2 \eta_{\alpha\beta} = \theta^{-2} \eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2} \eta_{\alpha\beta} \quad (36)$$

We can then study the evolution of the tensions using $\Omega^2 = \theta^{-2} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2}$. We will consider the cases where $a^2 \neq 0$.

4 The homogeneous and isotropic universe in dynamical string tension theories

We now consider the case when a^μ is not light like and we will find that for $a^2 \neq 0$, irrespective of sign, i.e. irrespective of whether a^μ is space like or time like, we will have thick braneworlds where strings can be constrained between two concentric spherically symmetric bouncing higher dimensional spheres and where the distance between these two concentric spherically symmetric bouncing higher dimensional spheres approaches zero at large times. The string tensions of the strings one and two are given by

$$\begin{aligned} e\phi + T_1 &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \end{aligned} \quad (37)$$

$$\begin{aligned} e\phi + T_2 &= \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \end{aligned} \quad (38)$$

Let us consider the case where a^μ is time like, then without lossing generality we can take $a^\mu = (A, 0, 0, \dots, 0)$. Now, in order to get homogeneous and isotropic cosmological solutions we must consider the limit $A \rightarrow 0$ and $(T_2 - T_1) \rightarrow 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. In that case the spatial dependence in the tensions (37) and (38) drops out and we get,

$$e\phi + T_1 = e\phi + T_2 = \frac{K}{4t} \quad (39)$$

The embedding metric can now be solved.

$$g_{\mu\nu} = \frac{1}{(e\phi + T_1)} g_{\mu\nu}^1 = \frac{4t}{K} \eta_{\mu\nu} \quad (40)$$

which is not a vacuum metric, as opposed to $\eta_{\mu\nu}$ because of the conformal factor $\frac{4t}{K}$.

4.1 Life of the homogeneous and isotropic universe and emergence of a braneworld at large times

One should notice that the homogeneous and isotropic solution has been obtained only in the limit $A \rightarrow 0$ and $(T_2 - T_1) \rightarrow 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. If A and $T_2 - T_1$ are small but finite, then for large times, of the order of $1/A$. We can formulate this as an uncertainty principle,

$$(T_2 - T_1)\Delta t \approx \text{constant} \quad (41)$$

where we have used that A is of the order of $(T_2 - T_1)$. So a small uncertainty in the tension $(T_2 - T_1)$ leads to a long lived homogeneous and isotropic phase, while a big uncertainty in the tension $(T_2 - T_1)$ leads to short lived homogeneous and isotropic phase.

In fact in these situations, for finite $(T_2 - T_1)$ and A , it is the case that the string tensions can only change sign by going first to infinity and then come back from minus infinity. We can now recognize at those large times the locations where the string tensions go to infinity, which are determined by the conditions

$$2a_\mu x^\mu + a^2 x^2 = 0 \quad (42)$$

or

$$2 + 2a_\mu x^\mu + a^2 x^2 = 0 \quad (43)$$

Let us start by considering the case where a^μ is time like, then without loosing generality we can take $a^\mu = (A, 0, 0, \dots, 0)$. In this case the denominator in (37), (38) is

$$\begin{aligned} & (2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2) \\ &= (2At + A^2(t^2 - x^2))(2 + 2At + A^2(t^2 - x^2)) \end{aligned} \quad (44)$$

The condition (42), if $A \neq 0$ implies then that

$$x_1^2 + x_2^2 + x_3^2 \dots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = -\frac{1}{A^2} \quad (45)$$

if $A \rightarrow 0$, it is more convenient to write this in the form

$$A(x_1^2 + x_2^2 + x_3^2 \dots + x_{D-1}^2) - At^2 - 2t = 0 \quad (46)$$

which for the limit $A \rightarrow 0$ gives us the single singular point $t = 0$, which is the origin of the homogeneous and isotropic cosmological solution.

The other boundary of infinite string tensions is, (43) is given by,

$$x_1^2 + x_2^2 + x_3^2 \dots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = \frac{1}{A^2} \quad (47)$$

This has no limit for $A \rightarrow 0$, all these points disappear from the physical space (they go to infinity).

For $A \neq 0$ we see that (47) represents an exterior boundary which has an bouncing motion with a minimum radius $\frac{1}{A}$ at $t = -\frac{1}{A}$, The denominator (44) is positive between these two bubbles. So for $T_2 - T_1$ positive the tensions are positive and diverge at the boundaries defined above.

The internal boundary (45) exists only for times t smaller than $-\frac{2}{A}$ and bigger than 0, so in the time interval $(-\frac{2}{A}, 0)$ there is no inner surface of infinite tension strings. This inner surface collapses to zero radius at $t = -\frac{2}{A}$ and emerges again from zero radius at $t = 0$.

For large positive or negative times, the difference between the upper radius and the lower radius goes to zero as $t \rightarrow \infty$

$$\sqrt{\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} - \sqrt{-\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} \rightarrow \frac{1}{tA^2} \rightarrow 0 \quad (48)$$

of course the same holds $t \rightarrow -\infty$. This means that for very large early or late times the segment where the strings would be confined (since they will avoid having infinite tension) will be very narrow and the resulting scenario will be that of a brane world for late or early times, while in the bouncing region the inner surface does not exist. We can ignore the part of the solution where $t < -\frac{2}{A}$ and instead take $t = 0$ as the origin of the Universe and only consider positive values of cosmic time because the part of the solution with $t < -\frac{2}{A}$ is disconnected, at least at the classical level from the part of the solution with positive cosmic time.

We see then that for the exact limit of $\Delta T \rightarrow 0$ and $A \rightarrow 0$ we get a perfect homogeneous and isotropic cosmology, but as ΔT and A are deformed to be small but finite, the scenario is modified at large times into a braneworld scenario.

5 The static end of space wall

Let us start by considering the case where a^μ is space like, then without loosing generality we can take $a^\mu = (0, A, 0, \dots, 0)$. Now, in order to get a static, only dependent on x solution again we must consider the limit $A \rightarrow 0$ and $(T_2 - T_1) \rightarrow 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. In that case the temporal dependence in the tensions (37) and (38) drops out now and we get,

$$e\phi + T_1 = e\phi + T_2 = -\frac{K}{4x^1} \quad (49)$$

if we conventionally take $K < 0$, then the strings cannot cross the point $x^1 = 0$ from positive values, since as they approach that point the strings acquire a very large string

tension, so that is why we can call $x^1 = 0$ the end of space wall.

5.1 Lifetime of the static end of space wall and its evolution towards an emergent braneworld scenario

In this case we look at the denominator in the case when a^μ is space like, now for finite A , then without loosing generality we take $a^\mu = (0, A, 0, \dots, 0)$, then for finite values of A in (37), (38), we get

$$\begin{aligned} & (2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2) \\ &= (-2Ax^1 - A^2(t^2 - \vec{x}^2))((2 - 2Ax^1 - A^2(t^2 - \vec{x}^2)) \end{aligned} \quad (50)$$

where $\vec{x} = (x^1, x^2, \dots, x^{D-1})$ represents the spacial part of x^μ , and $\vec{x}^2 = (x^1)^2 + (x^2)^2 + \dots + (x^{D-1})^2$. We then consider the first boundary where the string tensions approach infinity according to (42),

$$-\left(x^1 - \frac{1}{A}\right)^2 - x_2^2 - x_3^2 \dots - x_{D-1}^2 + t^2 = -\frac{1}{A^2} \quad (51)$$

which can be written as .

$$2x^1 - A(-x_1^2 - x_2^2 - x_3^2 \dots - x_{D-1}^2 + t^2) = 0 \quad (52)$$

which in the limit $A \rightarrow 0$ implies $x^1 = 0$, so that then the only singularity is at the wall $x^1 = 0$

The case (43) gives

$$-\left(x_1 - \frac{1}{A}\right)^2 - x_2^2 - x_3^2 \dots - x_{D-1}^2 + t^2 = \frac{1}{A^2} \quad (53)$$

(53) is an internal boundary which exists only for times t smaller than $-\frac{1}{A}$ and bigger than $\frac{1}{A}$. Between $-\frac{1}{A}$ and $\frac{1}{A}$ there is no inner surface of infinite tension strings. For $A = 0$ this solution does not exist at all, since the forbidden interval for the inner solution $(-\frac{1}{A}, \frac{1}{A})$ includes all the real line as $A \rightarrow 0$.

For finite A this inner surface collapses to zero radius at $t = -\frac{1}{A}$ and emerges again from zero radius at $t = \frac{1}{A}$. So the situation is very similar to that of the case where the vector a^μ is time like, just that the roles of the cases $\Omega = 1$ and $\Omega = -1$ get exchanged. Between these two boundaries the two factors in the denominator (50) are positive, while at the boundaries one or the other approach zero and the tensions diverge, so again for $T_2 - T_1$ positive the tensions are positive and diverge at the boundaries.

Once again for large positive or negative times, the difference between the upper radius and the lower radius goes to zero. Implying that the strings will be confined to a very small segment at large early or late times, so then again we get an emergent brane world scenario.

The strings and therefore all matter and gravity will be consequently confined to the very small segment of size $\frac{1}{tA^2}$, very small for large t . At the moment of the bounce there is no brane world, there is only one exterior bubble which represents infinite tension location, the brane is generated dynamically after a period of time by the appearance of the inner bubble which completes the trapping of the strings between two surfaces.

The static end of space wall as initial state that then evolves into the cosmological scenario resembles the scenario called The Emergent Universe where the start of the Universe is assumed to be a static Einstein Universe, which then evolves into a cosmological inflationary Universe [50–59].

6 Restoration of the equivalence principle in the limit $\Delta T \rightarrow 0$

As we have seen, in the case $\Delta T \neq 0$, we have different string metrics (18) for each type of string characterized by a different constant of integration T_i as the dispersion of the tensions vanishes, that is $\Delta T \rightarrow 0$, all these string dependent metric converge into a single one.

7 Discussion: motivations, ΔT from quantum fluctuations, braneworld creation and decoherence

The approach we want to promote in this paper is to formulate first of all the dynamical tension theories where each string can have its own tension. The string interactions are usually formulated for strings of the same tension, in dynamical string tension this may not be an obstacle if the string tensions are close enough, so that quantum fluctuations of the string tension will make possible interactions. This is then a good motivation to consider the dispersion of the tension parametrized by ΔT in the string ensemble to be very small, in fact we consider solutions where this dispersion goes to zero, $\Delta T \rightarrow 0$.

This lead us to consider the life time (1) the homogeneous and isotropic cosmological solution, (2) the static “end of space” wall. If $\Delta T \rightarrow 0$, these solutions remain forever undisturbed, if however $\Delta T \neq 0$, these solutions have a certain lifetime, after which (1) and (2) become braneworld scenarios.

It would be most appealing to have the origin of a $\Delta T \neq 0$ as a quantum fluctuation scenario. The subsequent evolution to braneworld scenarios in (1) and (2) should then lead to decoherence of these quantum fluctuations decohere and become classical, in a very similar way as the primordial quantum fluctuation in inflationary scenarios [60–65].

As we have pointed out the inverse relation between ΔT and Δt resembles the inverse relation between the uncer-

tainty of the cosmological constant and the total volume of the universe, which is also proportional to the age of the universe [26]. The difference is that here Δt refers to the age of a certain phase of the universe, like the homogeneous and isotropic phase, and not the total age of the universe. Here, after this period Δt , still there is a universe, but there is a braneworld universe.

This braneworld is very different to the standard approaches to braneworlds in the context of string theories however. In our approach a dynamical string tension has been used. Our scenario could be enriched by incorporating aspects of the more traditional braneworlds, like introducing D-branes between the surfaces where the string tensions go to infinity, so open strings could end before their tensions approach an infinite value, or the surfaces where the tensions diverge could be themselves be defined as D branes for open strings. These possibilities have been ignored here to simplify the discussion.

In any case, given that for times larger than Δt the tension of the strings diverge at the two boundaries we have defined, all strings are confined between those, the closed strings also, so unlike more traditional braneworlds, gravity does not escape to the bulk, in fact in the framework proposed here a braneworld scenario using just closed strings is perfectly possible. The question of the backreaction of these divergent string tensions, if we were to populate these regions near the boundaries keeping a flat or almost flat space in dynamical tension string theories in a braneworld scenario was discussed in [66, 67].

Finally one may consider in the future generalizations with n types of different types of string configurations instead of just two. This does not seem possible under the simplified assumptions used here where all the other background fields (dilaton, two index antisymmetric potential) are considered to be trivial. The reason that we may need more of these background fields is that each string type implies a new condition through the quantum conformal invariance, so for many strings we may need more background fields to be active to provide solutions, this is to be studied in details in future publications.

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