

Dark energy and matter perturbations in scalar-tensor theories of gravity

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Abstract. We solve analytically and numerically the generalized Einstein equations in scalar-tensor cosmologies to obtain the evolution of dark energy and matter linear perturbations. We compare our results with the corresponding results for minimally coupled quintessence perturbations. We find that Scalar-Tensor dark energy density perturbations are amplified by a factor of about 10^4 compared to minimally coupled quintessence perturbations on scales less than about $1000h^{-1}\text{Mpc}$ (sub-Hubble scales). On these scales dark energy perturbations constitute a fraction of about 10% compared to matter density perturbations. Scalar-Tensor dark energy density perturbations are anti-correlated with matter linear perturbations on sub-Hubble scales. This anti-correlation of matter with negative pressure perturbations induces a mild amplification of matter perturbations by about 10% on sub-Hubble scales. The evolution of scalar field perturbations on sub-Hubble scales is scale independent and therefore corresponds to a vanishing effective speed of sound ($c_{s\Phi} = 0$). We briefly discuss the observational implications of our results which may include predictions for galaxy and cluster halo profiles which are modified compared to ΛCDM . The observed properties of these profiles are known to be in some tension with the predictions of ΛCDM .

1. Introduction

A wide range of cosmological observations indicate that the universe has entered a phase of accelerating expansion. These observations include both direct geometric probes of the expanding metric and dynamical probes of the growth rate of matter perturbations. This growth depends on both the expansion rate and the gravitational law on large scales.

Geometric probes of the cosmic expansion include (a) Type Ia supernovae (SnIa) standard candles [1, 2], (b) the angular location of the first peak in the CMB perturbations angular power spectrum, which probes the integrated cosmic expansion rate using the last scattering horizon as a standard ruler [3], (c) baryon acoustic oscillations of the matter density power spectrum. These oscillations also probe the integrated cosmic expansion rate on more recent redshifts using the last scattering horizon as a standard ruler [4], and (d) other less accurate standard candles (Gamma Ray Bursts [5], HII starburst galaxies[6]) and standard rulers (cluster gas mass fraction [7]) as well as probes of the age of the universe [8].

Dynamical probes of the cosmic expansion and the gravitational law on cosmological scales include X-Ray cluster growth data [9], large scale structure power spectrum at various redshift

slices [10, 11], redshift distortion observed through the anisotropic pattern of galactic redshifts on cluster scales [11, 12] and weak lensing surveys [13, 14, 15].

These cosmological observations converge on the fact that the simplest Friedmann-Robertson-Walker (FRW) model describing well the cosmic expansion rate is the one corresponding to a cosmological constant [16] in a flat space, namely $H(z)^2 = H_0^2 [\Omega_{0m}(1+z)^3 + (1 - \Omega_{0m})]$ where $H(z)$ is the Hubble expansion rate at redshift z , $H_0 = H(z=0)$ and Ω_{0m} the present matter density normalized to the present critical density for flatness. This form of $H(z)$ and other similar, more complicated forms of it that are also consistent with cosmological data are predicted by broad classes of models: Dark energy models [17, 18, 19, 20, 21, 22, 23, 24, 25], modified gravity models [26, 27, 28, 29, 30, 31, 32] and local void models [33, 34].

Scalar-Tensor (ST) cosmological models [29, 30] (extended quintessence [35]) constitute a fairly generic representative of modified gravity models. They are based on the promotion of Newton's constant to a non-minimally coupled to curvature scalar field whose dynamics is determined by a potential $U(\Phi)$ and by the functional form of the non-minimal coupling $F(\Phi)$. The deviation of these models from GR is tightly constrained locally by solar system observations and by small scale gravitational experiments [36, 37]. These constraints, however, are significantly less stringent on cosmological scales [38, 39, 40] and may also be evaded by chameleon type arguments [41]. In the following we assume that the model considered passes the solar system tests in the context of such a mechanism.

The main focus of this work is to study the growth of perturbations in ST theories and its quantitative comparison with the corresponding growth in GR. This comparison can lead to the derivation of potential signatures of ST theories on the power spectrum and on other observables related to the growth of density perturbations. In ST theories, the non-minimal coupling of the scalar field to curvature perturbations (which in turn are driven by matter perturbations) leads to an amplification of the scalar field perturbations on sub-Hubble scales. Thus, it may be shown that on sub-Hubble scales the field perturbations $\delta\Phi$ are scale independent [29] and therefore the effective speed of sound $c_{s\Phi}$ for ST field perturbations vanishes. As discussed below, the corresponding scalar field density perturbations are also amplified but they are *anti-correlated* with respect to matter perturbations.

2. Perturbations in Scalar-Tensor Cosmologies

We consider the following ST action in the physical Jordan frame [29, 31]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(F(\Phi)R - Z(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right) + S_m[\psi_m; g_{\mu\nu}], \quad (2.1)$$

where G is the bare gravitational constant, R is the scalar curvature of the metric $g_{\mu\nu}$ and S_m is the action of matter fields. In what follows we use units such that $8\pi G = 1$. The variation of the dimensionless function $F(\Phi)$ describes the variation of the effective gravitational constant. This variation (spatial or temporal) is severely constrained by solar system experiments [36, 37]. The GR limit of ST theories is obtained either by fixing $F(\Phi) = \Phi_0 \simeq 1$ (Φ_0 is a constant) or by freezing the dynamics of Φ using the function $Z(\Phi)$ or the potential $U(\Phi)$.

Considering a flat FRW background where matter is described by a pressureless perfect fluid with density ρ_m we obtain the equations for the evolution of the background

$$3FH^2 = \rho_m + \frac{1}{2}\dot{\Phi}^2 - 3H\dot{F} + U \equiv \rho_{tot} \quad (2.2)$$

$$-2F\dot{H} = \rho_m + \dot{\Phi}^2 + \ddot{F} - H\dot{F} \equiv \rho_{tot} + p_{tot} \quad (2.3)$$

$$\ddot{\Phi} + 3H\dot{\Phi} = 3F_{,\Phi} \left(\dot{H} + 2H^2 \right) - U_{,\Phi} \quad (2.4)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (2.5)$$

where we have rescaled Φ so that $Z = 1$ (assuming that $Z > 0$) and a subscript Φ stands for derivative with respect to Φ . To solve this system numerically we set initial conditions corresponding to the time of recombination ($z \simeq 1000$) and find the evolution of the background homogeneous field $\Phi(t)$, scale factor $a(t)$ and matter density $\rho_m(t) \propto a(t)^{-3}$. This scaling of the matter density would not be true in the Einstein frame due to the direct coupling of matter to the scalar field. However, here we assume that the physical frame is the Jordan frame and do not consider a direct coupling of the scalar field to matter.

In order to obtain the evolution of perturbations we consider the perturbed FRW metric which in the Newtonian gauge takes the form

$$ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)\delta_{ij}dx^i dx^j \quad (2.6)$$

The linear gravitational potentials ϕ and ψ are related by $\phi = \psi - (F_{,\Phi}/F)\delta\Phi$, and along with the scalar field perturbations $\delta\Phi$ are sufficient to fully determine the cosmological perturbations in these theories. It is straightforward to obtain two additional differential equations for the perturbations ϕ , ψ and $\delta\Phi$ [17, 29, 42]

$$\dot{\xi} + 2H\xi + \left(3\dot{H} - \frac{k^2}{a^2}\right)\phi - \frac{1}{2}(\delta\mu + 3\delta q) = 0, \quad \text{with } \xi \equiv 3(H\phi + \dot{\psi}) \quad (2.7)$$

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} + \left(\frac{k^2}{a^2} + U_{,\Phi\Phi} - \frac{R}{2}F_{,\Phi\Phi}\right)\delta\Phi - \dot{\Phi}\dot{\phi} - (2\ddot{\Phi} + 3H\dot{\Phi})\phi - \dot{\Phi}\xi - \frac{F_{,\Phi}}{2}\delta R = 0, \quad (2.8)$$

where R is the Ricci curvature scalar, δR is its perturbation, and $\delta\mu$ and δq are the effective energy density and pressure perturbations respectively. Also, to solve for $\delta\Phi$ and ψ we use of the generalized Poisson equation

$$\delta\mu = -2(H\xi + (k^2/a^2)\psi). \quad (2.9)$$

Assuming specific forms for the field potentials F and U in Eqs. (2.7)-(2.8) we may numerically solve for the perturbations $\psi(k, t)$, $\delta\Phi(k, t)$. We solve the system with initial conditions at recombination ($z \simeq 1000$) corresponding to an initially smooth scalar field Φ and gravitational potential ψ in a background with small initial deviation from GR.

It is straightforward to obtain the density perturbations of both dark energy ($\delta_\Phi \equiv \delta\rho_\Phi/\rho_{tot}$) and matter ($\delta_m \equiv \delta\rho_m/\rho_{tot}$) in terms of the numerically derived perturbations $\phi(k, t)$, $\psi(k, t)$ and $\delta\Phi(k, t)$ and the corresponding background. Alternatively, the matter density perturbation, when normalized with respect to ρ_m , i.e. $\hat{\delta}_m = \delta\rho_m/\rho_m$ is given by [17]

$$\ddot{\hat{\delta}}_m + 2H\dot{\hat{\delta}}_m + \frac{k^2}{a^2}\left(\psi - \frac{F_{,\Phi}}{F}\delta\Phi\right) - 3(\ddot{\psi} + 2H\dot{\psi}) = 0. \quad (2.10)$$

Even though the system (2.7), (2.8) for the evolution of metric and field perturbations on a scale k can only be solved numerically, there are several useful qualitative conclusions that can be obtained by considering appropriate limits of the scale k in these equations. There are four scales involved in Eqs. (2.7), (2.8): The physical scale $\frac{k}{a}$ of the perturbations, the Hubble expansion rate H , the mass scale of the potential $U_{,\Phi\Phi}^{1/2}$, and the shifted ST perturbation scale $F_{,\Phi}^{1/2}\frac{k}{a}$. In addition, in a cosmologically interesting setup, the expansion of the universe is driving the time evolution of every physical quantity f . Therefore, $|\dot{f}| \simeq H|f|$. For scalar fields that can play a role in the present accelerating expansion of the universe we require $U_{,\Phi\Phi}^{1/2} \simeq H$. Thus each term in Eqs. (2.7), (2.8) is determined by one of the three scales: $\frac{k}{a}$, H , $F_{,\Phi}^{1/2}\frac{k}{a}$. By

identifying the terms that dominate in each range of perturbation scales $\frac{k}{a}$ we may simplify the perturbation equations and obtain approximate solutions for the corresponding range of scales. We consider the following ranges of perturbation scales:

- Sub-Hubble ST scales: $\frac{k}{a} \simeq F_{,\Phi}^{1/2} \frac{k}{a} \gg H$, $F_{,\Phi} \gtrsim 1$. In this case the perturbation scale and the shifted ST perturbation scale are of the same order. Ignoring subdominant terms we find that

$$\delta\Phi \simeq -\psi \frac{FF_{,\Phi}}{F + F_{,\Phi}^2}. \quad (2.11)$$

Thus, on sub-Hubble scales the field perturbations $\delta\Phi$ are independent of the scale k and can be a significant fraction of the total energy perturbations as demonstrated in the next section. The corresponding behavior in GR is very different. Setting $F = 1$ and $\psi = \phi$ in (2.8) we obtain

$$\delta\Phi \simeq A \frac{a^2 H^2}{k^2} (\Phi - \Phi_i) \psi \rightarrow 0, \quad (2.12)$$

where $\dot{\Phi} \simeq (\Phi - \Phi_i) H$ and A is a proportionality factor necessary to fit the numerical solution. As expected, the above implies that $\delta\Phi$ is negligible in GR on sub-Hubble scales. The field and matter density perturbations on these scales are respectively given by

$$\delta\rho_\Phi \simeq \frac{k^2}{a^2} \psi \frac{FF_{,\Phi}^2}{F + F_{,\Phi}^2} \quad \text{and} \quad \delta\rho_m \simeq -\frac{k^2}{a^2} \psi F \left(\frac{F_{,\Phi}^2}{F + F_{,\Phi}^2} + 2 \right). \quad (2.13)$$

The sub-Hubble ratio $\delta\rho_\Phi/\delta\rho_m$ is therefore scale independent, indicating that the effective speed of sound for ST dark energy perturbations is $c_{s\Phi} = 0$. Here we refer to the effective (rest frame) speed of sound as defined in Ref. [43]. This quantity determines the sound horizon, i.e. the scale below which perturbations can not grow. The fact that $\delta\rho_\Phi/\delta\rho_m$ is negative indicates an interesting anti-correlation between dark matter and dark energy perturbations, which is also confirmed numerically in the next section.

It is also straightforward to derive the equation for the evolution of matter density perturbations on sub-Hubble scales in ST theories. On such scales, where $\delta\Phi$ is given by Eq. (2.11) and $\delta\rho_m$ from Eq. (2.13), we obtain [44]

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\rho_m \hat{\delta}_m}{2} \frac{1}{F} \frac{2F + 4F_{,\Phi}^2}{2F + 3F_{,\Phi}^2} = 0. \quad (2.14)$$

It may be shown that $\frac{1}{F} \frac{2F + 4F_{,\Phi}^2}{2F + 3F_{,\Phi}^2} = \frac{G_{\text{eff}}}{G}$, where G_{eff} is the effective gravitational constant in Cavendish-like experiments in the context of ST theories [29]. Therefore, Eq. (2.14) has the anticipated scale independent form (as in the case of GR) but Newton's constant G has been replaced by the effective

- Sub-Hubble GR scales: $\frac{k}{a} \gg H \gg F_{,\Phi}^{1/2} \frac{k}{a}$, $F_{,\Phi} \ll 1$. If $F_{,\Phi} \ll 1$ there is a range of sub-Hubble scales corresponding to $F_{,\Phi} \ll \frac{H^2 a^2}{k^2} \ll 1$ for which the terms depending on the non-minimal coupling in Eq. (2.8) are negligible compared to all other terms. For this range of sub-Hubble scales the scalar field perturbations are negligible, scale dependent and behave as in GR (Eq. (2.12)). However, on small enough scales, i.e. when $\frac{H^2 a^2}{k^2} \ll F_{,\Phi} \ll 1$, we recover the ST scale independent behavior of Eq. (2.11).
- Super-Hubble scales: $\frac{k}{a} \ll H$. In this case we may ignore the scale dependent terms in Eq. (2.8) to obtain $\delta\Phi \sim \psi$. Clearly, there is no scale dependence for the perturbations on super-Hubble scales. Similarly, for the matter component we find scale independent perturbations $\delta_m \sim \psi$.

The above qualitative features of the cosmological perturbations are confirmed and extended by the numerical derivation in the next section.

3. Numerical Solution

Firstly, we define specific forms for the potentials $F(\Phi)$ and $U(\Phi)$

$$F(\Phi) = 1 - \lambda_f \Phi^2 \quad , \quad U(\Phi) = 1 + \exp(-\lambda\Phi) . \quad (3.1)$$

This form of $F(\Phi)$ is consistent with solar system tests for $\Phi \simeq 0$. Indeed solar system constraints of ST theories imply that [36]

$$F_{,\Phi}^2/F|_{t=t_0} < 10^{-4} , \quad (3.2)$$

where t_0 refers to the present time.

We next solve the system (2.2),(2.4),(2.5) to determine the background evolution in the context of the above potentials. We set the initial conditions at t_i corresponding to recombination close to GR and check that our results are robust with respect to reasonable changes of the initial conditions. The parameters that need to be fixed for the solution of the background system (2.2),(2.4),(2.5) are λ , λ_f and Ω_{0m} . In most solutions discussed in this section we set $\Omega_{0m} = 0.3$ and $\lambda = 2$, $\lambda_f = 5$ (ST cosmology) or $\lambda_f = 0$ (minimally coupled quintessence). We also check that deviations from the Λ CDM expansion rate are less than 3.5% for any value of the parameter λ with fixed value of the initial condition $\Phi(t_i) \simeq 0.1$. At late times, the constant part of $U(\Phi)$ prohibits a large deviation from the Λ CDM expansion rate despite of the somewhat increased kinetic energy of Φ .

In Fig. 1 we plot the background dynamics for a non-minimally coupled quintessence field Φ setting $\lambda_f = 5$ and $\lambda = 2$. In ST gravity the scalar field is moving under the influence of the effective potential $U_{\text{eff}}(\Phi) = U(\Phi) - \frac{1}{2}RF(\Phi)$. Assuming $F \sim O(1)$ at all times implies that at early times $U_{\text{eff}}(\Phi)$ is dominated by the second term. The effective mass of the field is then $m_{\text{eff}} \sim (-RF_{,\Phi\Phi})^{1/2} \sim \lambda_f^{1/2}H$. As a result, for $\lambda_f \gtrsim 1$ and $\Phi_i \neq 0$ the scalar field is dynamically rapidly driven to its value corresponding to GR ($\Phi = 0$) and remains performing oscillations of decreasing amplitude around the attractor $\Phi = 0$ as the Universe expands. If $\Phi_i = 0$ then the field remains at $\Phi = 0$ ($F \simeq 1$) until $U_{\text{eff}}(\Phi) \simeq U(\Phi)$. Therefore, Φ behaves as minimally coupled quintessence at early times and as non-minimally coupled quintessence at late times when the field is driven away from $\Phi = 0$. Deviations with respect to the Λ CDM expansion rate can be significant at early times if the field begins far from $\Phi = 0$. Therefore, large deviations can be avoided simply by tuning Φ_i so that $F(\Phi_i) \simeq 1$. With such initial conditions, the system approaches GR while the Hubble expansion rate becomes practically identical to Λ CDM at late times. When U starts dictating the field dynamics (late time evolution), we find that such deviations are always below 3% even for large values of λ . In particular, for steep potentials $U(\Phi; \lambda = 2)$ the dynamical evolution of Φ leads to an amplified value $F_{,\Phi}^2/F \simeq 10^{-1}$ at redshifts $z \lesssim O(1)$ violating solar system constraints (Eq. (3.2)) but not cosmological constraints $F_{,\Phi}^2/F|_{t=t_0} \lesssim O(1)$ [39]. Thus, in the context of the chameleon mechanism the increased value of $F_{,\Phi}^2$ has the potential of being consistent with observational constraints. Furthermore, and as discussed in the previous section (Eq. (2.13)), the value of $F_{,\Phi}^2$ determines the magnitude of the dark energy density perturbations compared to those of matter. We therefore anticipate *amplified dark energy perturbations* (compared to GR) when the dynamics of Φ is turned on by increasing the value of λ .

The non-minimal coupling of the field Φ to the curvature scalar results in a very characteristic behaviour of certain background quantities. In the lefthand panel of Fig. 2 we plot the evolution of the density parameter Ω_m as a function of the redshift. Owing to the negative effective energy contribution in Eq. (2.2) sourced by the field's motion, Ω_m features and oscillatory behaviour

around $\Omega_m = 1$ at early times. Notice that it is possible to have $\Omega_\phi < 0$, $\Omega_m > 1$ since ρ_Φ is not positive definite (Eq. (3.3)). This effect is discussed in detail in Ref. [35]. Another remarkable feature distinguishing minimally coupled quintessence from the non-minimally coupled one is that the latter is able to cross the phantom divide line [45, 46, 47] corresponding to an effective dark energy equation of state $w_{\text{eff}} = -1$. In general, the effective equation of state for the scalar field Φ is given by (see Eqs. (2.2), (2.3))

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{1}{2}\dot{\Phi}^2 - U(\Phi) + \ddot{F} + 2H\dot{F}}{\frac{1}{2}\dot{\Phi}^2 + U(\Phi) - 3H\dot{F}}. \quad (3.3)$$

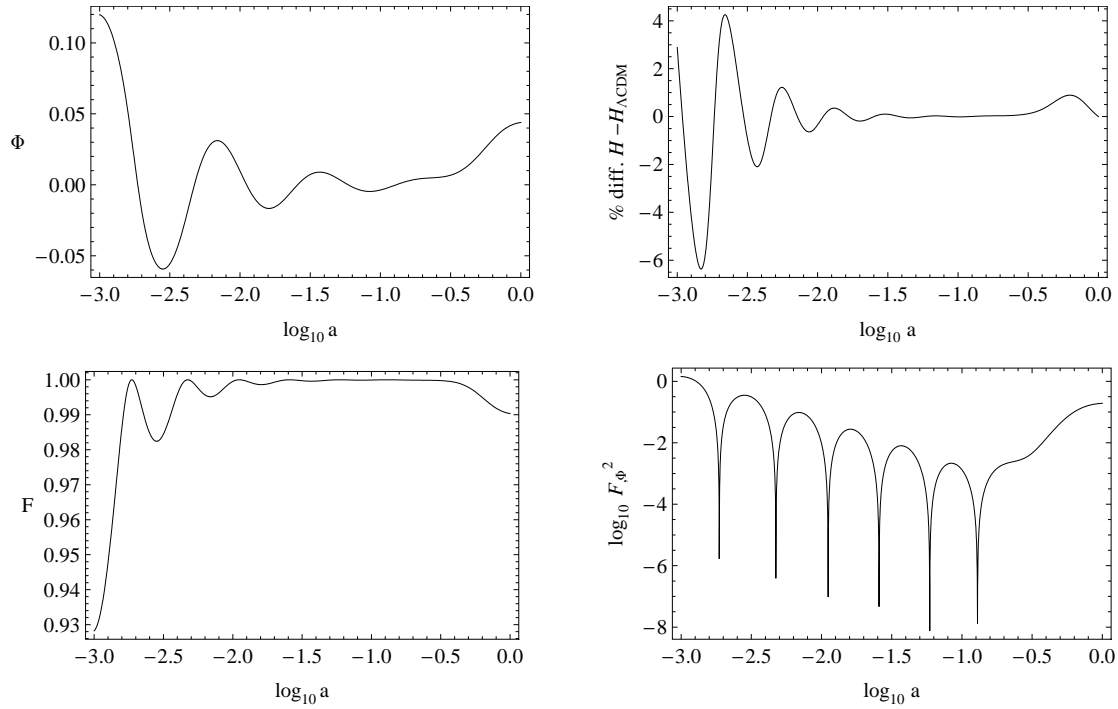


Figure 1. Background dynamics giving rise to the field contrasts depicted in Figs. 3 and 4. From left to right and from top to bottom: 1. Field oscillations about $\Phi = 0$; 2. % difference between H and $H_{\Lambda\text{CDM}}$; 3. Evolution of $F(\Phi)$; 4. F_Φ^2 ($\simeq 10^{-1}$ at present). The parameters are $\lambda_f = 5$ and $\lambda = 2$.

In the righthand panel of Fig. 2 we show the evolution of the equation of state parameter for minimally ($\lambda_f = 0$, solid line) and non-minimally ($\lambda_f = 2$, dashed line) coupled quintessence, corresponding to GR and ST gravity respectively. At early times the background field oscillations give rise to divergences in w_Φ but such divergences do not reflect on the Hubble expansion rate. At late times, the scalar potential $U(\Phi)$ becomes relevant for the field dynamics, the field starts growing and w_Φ oscillates around the phantom divide line. Crossing of the phantom divide line is allowed by all current cosmological observations and is in fact favored by some of them [46]. This behavior is characteristic of ST gravities and cannot be achieved in minimally coupled quintessence [48].

In order to find the evolution of perturbations we solve Eqs. (2.7) and (2.8) using the numerical

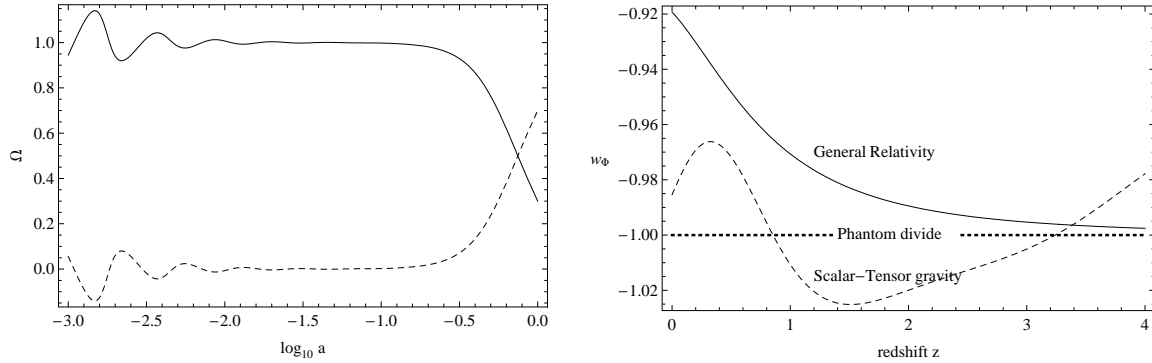


Figure 2. Plot of the density parameters Ω_m and $\Omega_\phi = 1 - \Omega_m$ corresponding to the background dynamics in Fig. 1 (lefthand panel). Equation of state parameter (EOS) w_Φ for the background field Φ as obtained in General Relativity (solid line) with $\lambda = 2$ and Scalar-Tensor gravity (dashed line) with $\lambda_f = 5$ (righthand panel).

solution of the background and impose initial conditions

$$\delta\Phi(k, t_i) = \dot{\delta\Phi}(k, t_i) = 0 \quad (3.4)$$

$$\psi(k, t_i) = 1; \quad \dot{\psi}(k, t_i) = 0. \quad (3.5)$$

The evolution of the field $\delta\Phi$ vs the scale factor a (in logarithmic scale) for a perturbation of wavelength $\lambda_p = 30 \text{ h}^{-1}\text{Mpc}$ is shown in Fig. 3. In the lefthand panel we plot the numerical solution for the evolution of $\delta\Phi$ as obtained in GR (solid line) and as given by the approximation in Eq. (2.12) (dashed line) on subhorizon scales. We set $A \simeq 25$ in Eq. (2.12) in order to match the numerical solution. The righthand panel shows the numerical solution for the evolution of $\delta\Phi$ in ST gravity (solid line) and as given by Eq. (2.11) (dashed line) on subhorizon ST scales.

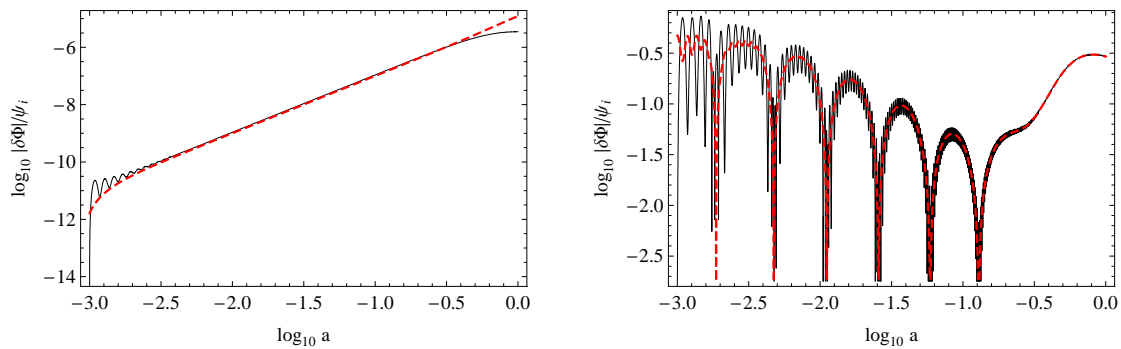


Figure 3. Evolution of the field perturbation $\delta\Phi$ as obtained in General Relativity (lefthand panel) and Scalar-Tensor gravity (righthand panel) for the scale $\lambda_p = 30 \text{ h}^{-1}\text{Mpc}$. The numerical solution is the solid line and the analytical approximation on subhorizon ST and GR scales are the dashed lines. Oscillations in the background field Φ induce oscillations in $\delta\Phi$ through $F_{,\Phi} \propto \Phi$. We use $\lambda_f = 5$ for ST gravity and $\lambda = 2$ in both cases. The spikes in righthand panel correspond to changes of sign of $\delta\Phi$.

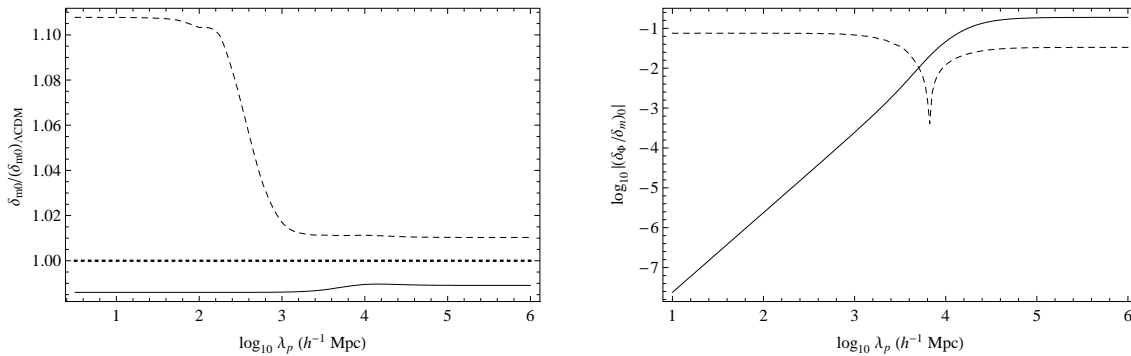


Figure 4. The lefthand panel shows the ratio $\frac{\delta_m(k, t_0)}{\delta_{m\Lambda\text{CDM}}(k, t_0)}$ for ST gravity (dashed line) and GR quintessence (solid line). The righthand panel shows the scale dependence of the ratio δ_Φ/δ_m at present as obtained in GR (solid line) and ST gravity (dashed line). The spike corresponds to a change of sign in δ_Φ/δ_m , thus revealing an anti-correlation between dark energy and matter perturbations. We use $\lambda_f = 5$ and $\lambda = 2$.

The evolution of matter overdensities is also affected by the introduction of a non-minimal coupling. This is demonstrated in the lefthand panel of Fig. 4 where we plot the present value of the ratio of δ_m , for minimally coupled quintessence (solid line) and ST gravity (dashed line), to the matter contrast in the ΛCDM cosmology for a range of scales. Clearly, the 10% amplification of matter perturbations in ST gravity is applicable on sub-Hubble scales while on larger scales the amplification is negligible. This mild amplification of matter perturbations may be attributed to the corresponding amplification of dark energy perturbations in these theories which also affects matter perturbations despite the predicted anti-correlation. Indeed, the dark energy void in a cluster of galaxies reduces the negative pressure inside the cluster and amplifies the gravitational collapse.

In the righthand panel of Fig. 4 we plot the scale dependence of the ratio δ_Φ/δ_m at present in GR (solid line) and in ST gravity (dashed line). Setting $\lambda_f = 5$, $\lambda = 2$, we find that the GR solution leads to negligible dark energy perturbations on sub-Hubble scales $\delta_\Phi \sim k^{-2}$ (Eq. (2.12)). In contrast, ST gravity produces amplified, anticorrelated with matter, dark energy perturbations on sub-Hubble scales. The ratio $\delta_\Phi/\delta_m = \delta\rho_\Phi/\delta\rho_m$ is scale independent on these scales as predicted by Eq. (2.13). In such models, where dark energy perturbations can grow on all scales, it may be shown that the speed of sound c_s vanishes [43]. The anti-correlation is evident by the spike of the dashed line, which corresponds to a change of sign of δ_Φ on sub-Hubble scales.

4. Conclusion

We have investigated in detail, analytically and numerically, the evolution of dark energy and matter linear density perturbations in Scalar-Tensor (ST) cosmologies. We have found that the evolution of dark energy perturbations in ST cosmologies is significantly different from the corresponding evolution in minimally coupled (GR) quintessence.

For natural values ($\mathcal{O}(1)$) of the ST Lagrangian parameters leading to a background expansion similar to ΛCDM (Fig. 1), dark energy density perturbations are amplified by a factor of about 10^6 compared to minimally coupled quintessence perturbations on scales less than about $100h^{-1}\text{Mpc}$ (Fig. 4). On sub-Hubble scales dark energy perturbations constitute a fixed fraction of about 10% compared to matter density perturbations (Fig. 4). The fixed scale independent fraction implies that the effective speed of sound for ST dark energy is

$c_{s\Phi} = 0$. The corresponding fraction for minimally coupled quintessence perturbations scales as k^{-2} and is about $\lesssim 10^{-4}\%$ (Fig. 4) corresponding to $c_{s\Phi} = 1$. Scalar-Tensor dark energy density perturbations are anti-correlated with matter linear perturbations on sub-Hubble scales (Eqs. (2.13) and Fig. 4). Thus clusters of galaxies overlap with voids of dark energy.

The evolution of scalar field perturbations on sub-Hubble scales, is scale independent and involves large oscillations (Fig. 3) induced by the amplified effective mass of the field. This mass amplification is due to the non-minimal coupling of the field to curvature, and therefore to matter (Eqs. (2.3), (2.4)). No such oscillations are present in minimally coupled quintessence perturbations which are suppressed on sub-Hubble scales and vary as k^{-2} (Eq. (2.12)). The evolution of matter density perturbations is affected by the introduction of non-minimal coupling (Fig. 4) and is amplified by about 10% in ST cosmology compared to minimally coupled quintessence and Λ CDM on sub-Hubble scales.

These results have interesting observational consequences. In particular

Dark Matter Halo Profiles: Λ CDM predicts shallow low concentration density dark matter halo profiles for clusters and galaxies in contrast to observations which indicate denser high concentration cluster haloes [49]. The amplified anti-correlated with matter dark energy perturbation profiles can lead to a modification of the predicted by Λ CDM dark matter halo profiles. In particular, the dark energy voids in clusters of galaxies can amplify locally dark matter clustering due to the local reduction of negative pressure in the region of the cluster.

Large Scale Structure Power Spectrum $P_m(k)$: For a non-minimal coupling $F_{,\Phi} = O(1)$ the ratio $(\delta_{\Phi}/\delta_m)^2 \sim P_{\Phi}(k)/P_m(k)$ is scale independent for practically all sub-Hubble scales (see Fig. 4). Thus it would be hard to identify a scale dependent signature of dark energy perturbations on the matter power spectrum for such values of $F_{,\Phi}$. For smaller values of the non-minimal coupling however, there exists a GR regime for large sub-Hubble scales where the dark energy perturbations are predicted to be scale dependent whereas on smaller scales we enter the ST regime where the ratio $P_{\Phi}(k)/P_m(k)$ becomes again scale independent. This transition from the GR regime on large sub-Hubble scales to the ST regime in small sub-Hubble scales may leave a trace (small glitch) on the matter power spectrum on a scale $k \simeq aH F_{,\Phi}^{-1/2}$.

In conclusion, the amplified and anti-correlated with matter, dark energy ST perturbations investigated in the present study provide a new direction of observational signatures for this class of modified gravity models.

Numerical Analysis Files: The mathematica files used for the numerical analysis and the production of the figures may be found at <http://leandros.physics.uoi.gr/deports/deports.htm>.

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