Eur. Phys. J. C (2023) 83:478
https://doi.org/10.1140/epjc/s10052-023-11670-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Dynamical equivalence of the varying G and Λ cosmology with the Brans–Dicke theory

A. Paliathanasis[1,2,a]

[1] *Present address:* Institute of Systems Science, Durban University of Technology, Durban 4000, South Africa
[2] *Present address:* Departamento de Matemáticas, Universidad Católica del Norte, Avda. Angamos 0610, Casilla, 1280 Antofagasta, Chile

Received: 2 March 2023 / Accepted: 29 May 2023

Abstract We investigate the dynamical equivalence of the varying G and Λ cosmology with the Brans–Dicke theory. We show that there exist a point transformation which connects the field equations for a spatially flat FLRW geometry. We conclude that the two theories share the same integrability properties and solution trajectories.

1 Introduction

Although General Relativity is a well tested theory of gravity, it cannot explain the cosmological observations. Nowadays, the universe is going through an accelerated period attributed to an exotic matter source known as dark energy. The latter matter source has a negative pressure component such that the strong energy condition is violated and to drive the dynamics in a way that the universe is accelerated and gravity appears as if it is a repulsive force. Moreover, it is considered that during that the universe has undergone a rapid expansion from its very early stages. That second acceleration phase is called inflation and it is explained by a similar mechanism as that of dark energy. The cosmological constant is the simplest candidate for dark energy, however it suffers from various problems [1,2]. In order to surpass these problems cosmologists have proposed gravitational theories which allow Λ to vary with cosmic time [3–7].

In [8] there has been proposed a gravitational theory in which the gravitational parameters G and Λ are varying and provide new dynamical degrees of freedom in order to explain the observations [9–11]. On the other hand, the Brans–Dicke theory provides a mechanism for a varying parameter G [12]. The varying G and Λ theory [8] and the Brans–Dicke theory have the same degrees of freedom and similar dynamical properties. In this work we show that in cosmological studies there exists a relation between these two theories. Specifically we show that the two theories are dynamical equivalent and the theories share the same trajectory solutions.

2 Brans–Dicke cosmology

For a spatially flat FLRW cosmology with scale factor $a(t)$ with metric tensor $g_{\mu\nu} = diag\left(-1, a^2(t), a^2(t), a^2(t)\right)$, the gravitational field equations of the Brans–Dicke theory are

$$3H^2 = \frac{\omega_{BD}}{2}\left(\frac{\dot\phi}{\phi}\right)^2 + \frac{V(\phi)}{\phi} - 3H\frac{\dot\phi}{\phi}, \tag{1}$$

$$\dot H = -\frac{\omega_{BD}}{2}\left(\frac{\dot\phi}{\phi}\right)^2 + 2H\left(\frac{\dot\phi}{\phi}\right) + \frac{\left(\phi\frac{dV}{d\phi} - 2V\right)}{(2\omega_{BD}+3)\phi}, \tag{2}$$

$$\ddot\phi + 3H\dot\phi = \frac{2\left(-\phi\frac{dV}{d\phi} + 2V\right)}{2\omega_{BD}+3}. \tag{3}$$

where $H = \frac{\dot a}{a}$ is the Hubble function, ϕ is the Brans–Dicke field, ω_{BD} is the Brans–Dicke parameter. The field equations of the Brans–Dicke cosmology admit a point-like Lagrangian description. Indeed, the field equations follow from the variation of the point-like Lagrangian

$$\mathcal{L}_\psi = e^\psi\left(-3a\dot a^2 - 3a^2\dot a\dot\psi + \frac{\omega_{BD}}{2}a^3\dot\psi^2\right) - a^3V(\psi), \tag{4}$$

where $\phi = e^\psi$, ψ is the dilaton field. We remark that the constraint equation (1) is a conservation law for the Euler-Lagrange equations.

[a] e-mail: anpaliat@phys.uoa.gr (corresponding author)

Published online: 08 June 2023

3 Varying G and Λ cosmology

In varying G and Λ theory the field equations follow from the application of the renormalization group in the ADM formalism [8]. For a spatially flat FLRW geometry the cosmological field equations are [8]

$$-3H^2 + \frac{\mu}{2}\left(\frac{\dot{G}}{G}\right)^2 + GV(G) = 0, \tag{5}$$

$$\dot{H} + \frac{3}{2}H^2 - H\frac{\dot{G}}{G} + \frac{\mu}{4}\left(\frac{\dot{G}}{G}\right)^2 - \frac{1}{2}GV(G) = 0, \tag{6}$$

$$\ddot{G} - \frac{3}{\mu}H^2 G + 3H\dot{G} - \frac{3}{2}\frac{\dot{G}^2}{G} + \frac{1}{\mu}G^3 V_G = 0, \tag{7}$$

These gravitational field equations have the property to follow from the variation of the Lagrangian function [8]

$$\mathcal{L}_G = \left(-\frac{3}{G}a\dot{a}^2 + \frac{\mu}{2G}a^3\left(\frac{\dot{G}}{G}\right)^2\right) - a^3 V(G). \tag{8}$$

4 Discussion

Consider the point transformation $(a, G) \to \left(Ae^{\frac{\Psi}{2}}, e^{\frac{1}{2}\Psi}\right)$, then, the point-like Lagrangian (8) becomes

$$\mathcal{L}_\Psi = e^\Psi\left(-3A\dot{A}^2 - 3A^2\dot{A}\dot{\Psi} + \frac{\omega_G}{2}A^3\dot{\Psi}^2\right) - A^3\hat{V}(\Psi), \tag{9}$$

which is of the form of the Lagrangian of the dilaton field (4) where $\hat{V}(\Psi) = e^{\frac{3\Psi}{2}}V(\Psi)$ and ω_G is the effective Brans–Dicke parameter related with μ as $\mu = 2(2\omega_G + 3)$. As a result we conclude that the Brans–Dicke and the varying G and Λ cosmology are dynamically equivalent and solution trajectories of one model can be transferred to solution trajectories for the other model through the aforementioned point transformation. This should not be confused with the conformal transformation which relates the Brans–Dicke action with the minimally coupled scalar field [13].

From the solution trajectories of the varying G and Λ cosmological model for the background geometry $g_{\mu\nu} = diag\left(-1, a^2(t), a^2(t), a^2(t)\right)$, with Hubble function $H = \frac{\dot{a}}{a}$, we can determine the solution trajectories for the Brans–Dicke theory with scalar field Ψ and line element $g_{\mu\nu} = diag\left(-1, A^2e^\Psi, A^2e^\Psi, A^2e^\Psi\right)$ and Hubble function $\hat{H} = H + \dot{\Psi}$. The two theories share not only the same solution trajectories but also all the integrability properties. Indeed, the conservation laws and the admitted symmetries for the one model hold for the other model. Recently, in [14] the Noether symmetries were investigated for the varying G and Λ cosmological theory. However, the above discussion shows that in the case of the vacuum the results of [15]. Furthermore, in the case in which matter is included, the Noether conditions in varying G and Λ constrain the symmetry analysis for the vacuum case. Which means that the symmetry analysis in the case of matter is included inside the family of the symmetry classification in [15]. Furthermore, the above transformation which relates the two theories was presented before [15] in order to simplify the nonlinearity terms in Brans–Dicke theory.

The above transformation is not the unique transformation which relates the two theories. Indeed, under the change of variables $\left\{G \to \left(\alpha e^{\frac{\Theta}{2}}\right)^{-3}, a \to e^{-\frac{\Theta}{6}}\right\}$ the point-like Lagrangian (8) with potential $\hat{V}(G) = V_0 G^{-1}$ is written in the equivalent form of the Brans–Dicke theory (4). This discussion shows that it is of important interest to study the relation of a given gravitational theory with other theories.

Acknowledgements This work was partially financially supported in part by the National Research Foundation of South Africa (Grant Numbers 131604). The author thanks the support of Vicerrectoría de Investigación y Desarrollo Tecnológico (Vridt) at Universidad Católica del Norte through Núcleo de Investigación Geometría Diferencial y Aplicaciones, Resolución Vridt No - 098/2022.

Data Availability Statement Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

References

1. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
2. T. Padmanabhan, Phys. Rep. **380**, 235 (2003)
3. S. Basilakos, Astron. Astrophys. **508**, 575 (2009)
4. E.L.D. Perico, J.A.S. Lima, S. Basilakos, J. Sola, Phys. Rev. D **88**, 063531 (2013)
5. J.V. Cunha, J.A.S. Lima, N. Pires, Astron. Astrophys. **390**, 809 (2002)
6. J. Salim, I. Waga, Class. Quantum Gravity **10**, 1767 (1993)
7. S. Pan, MPLA **33**, 1850003 (2018)
8. A. Bonanno, G. Esposito, C. Rubano, Class. Quantum Gravity **21**, 5005 (2004)
9. A. Bonanno, G. Esposito, C. Rubano, P. Scudellaro, Class. Quantum Gravity **26**, 1443 (2007)

10. A. Bonanno, G. Esposito, C. Rubano, P. Scudellaro, Class. Quantum Gravity **39**, 189 (2007)
11. E. Piedipalumbo, P. Scudellaro, G. Esposito, C. Rubano, Gen. Relativ. Gravit. **44**, 2477 (2012)
12. C.H. Brans, R.H. Dicke, Phys. Rev. **124**, 925 (1965)
13. M. Tsamparlis, A. Paliathanasis, S. Basilakos, S. Capozziello, Gen. Relativ. Gravit. **45**, 2003 (2013)
14. R. Bhaumik, S. Dutta, S. Chakraborty, EPJC **82**, 1124 (2022)
15. A. Paliathanasis, M. Tsamparlis, S. Basilakos, S. Capozziello, Phys. Rev. D **89**, 063532 (2014)