Model of wormholes under Krori-Barua spacetime in the presence of matter

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Abstract.

We intend to investigate particular models of wormholes with ordinary matter source. The exact wormhole solutions are deduced by numerically solving the Einstein's field equations with the stress-energy-tensor components. A particular form of feasible wormhole shape function is utilized. We study the solutions considering the Krori-Barua (KB) ansatz [K.D. Krori and J. Barua, J. Phys. A: Math. Gen. 8, 508 (1975)] in presence of perfect fluid matter. The obtained solutions are free from central singularity. The wormhole model thus obtained under Krori-Barua spacetime satisfies the flare-out conditions. Some physical features are briefly discussed.

1. Introduction

Lorentzian wormholes are believed to exist at the planck scale of 10^{-35} metres [1] though their existence are much speculative. Wormholes are characterised as connecting tunnels between two distinct universes or regions of the same universe. Their actual physical significance was first interpreted by Morris and Thorne [2] with further possible locations in the galactic halo and central regions of galaxies investigated by Rahaman F. et al., [3],[4]. Wormhole models with extra fields such as scalar field and static electric charge have been discussed by Kim S.W et al. [5] and deduced self-consistent solutions. Though wormhole spacetimes are predictions of the GTR, they need to be supported by observations. The modified extended metric for the charged wormhole model of Kim and Lee has been extended by Kuhfittig P.K.F. [6], to represent a charged wormhole that is compatible with quantum field theory. The field equations devoid of the charge Q have been studied by Buchdahl [7] and observed physically meaningful solutions.

Kuhfittig P. K. F. et al. [8] discussed the modified wormhole model in conjunction with a noncommutative-geometry background to avert the traversability problems. In view of quantum field theory, the modified metric in [6] is investigated as a solution of the Einstein fields equations representing a charged wormhole. Stable phantom-energy wormholes admitting conformal motions is studied in [9]. The author have shown that the wormhole is stable to linearized radial perturbations whenever $1.5 < \omega < 1$. Macroscopic traversable wormholes with zero tidal

forces inspired by noncommutative geometry [10]. The author has obverved that whenever the energy density describes a classical wormhole, the resulting solution is found to be incompatible with quantum-field theory.

The Friedmann-Robertson-Walker model with a wormhole has been discussed by Kim S. W. [11]. It is found that the total matter is nonexotic, while it is exotic in the static wormhole or inflating wormhole models. The flare-out condition for the wormhole with the Einstein field equations and the finiteness of the pressure is studied in [12]. The possibility that inflation might provide a natural boost for the enlargement of Lorentzian wormholes of the Morris-Thorne type to macroscopic laws size has been investigated by Roman T. A. [13]. Goulart P.[14] has studied an analytical electrically charged traversable wormhole solution for the EinsteinMaxwell-anti-dilaton theory as well as the deflection angle of a light-ray passing close to this wormhole.

It is discussed in [15], the traversability of asymptotically flat wormholes in Rastall gravity with phantom sources. Cosmic acceleration with traversable wormhole is possible without presence of exotic matter like dark and phantom energy unless the scale factor of the universe obeys a power law dominated by a negative fractional parameter is observed [16]. The existence of a static traversable wormhole solutions in the form of the Lovelock gravity in 7-dimensional universe is studied by El-Nabulsi R. A. [17]. The four-dimensional cosmological implications dominated by quintessence field with a static traversable wormhole by means of an additional bimetric tensor in Einsteins field equations is discussed in [18]. Traversable Wormhole with Extra-Dimensional cosmology has been studied in [19]. Many features of a higher-dimensional cosmology are discussed alongwith a static traversable wormhole dominated by a variable effective cosmological constant which is dependent on the scale factor $a(t)$.

Hererra L. [20] has discussed the conditions under which general relativistic polytropes for anisotropic matter exhibit cracking and/or overturning. The dynamics pertaining to a test particles in stable circular orbits around static and spherically symmetric wormholes in conformally symmetric spacetimes is investigated in [21].

Strange stars in Krori-Barua space-time have been studied in [22]. The authors have explored the possibility of applying the Krori and Barua model to describe ultra-compact objects like strange stars. In this paper we thrive to describe a wormhole model under Krori-Barua spacetime in the presence of matter.

The stress in this paper is on the isotropic matter distribution where we have considered a combined model of quintessence matter and ordinary matter along with charge distribution. A valid wormhole also exists under the isotropic condition where $p_t = p_r$. We iterate that such a combination could in fact support a wormhole in Einstein-Maxwell gravity. One could also like to invite the prospect of theoretical construction of such wormhole with the assumption of zero tidal forces.

The aim of our paper is envisaged as follows:

In section 2. we have solved the Einstein field equations and deduced the wormhole solutions. Sections 3. and 4. deal with a particular class of solution and solutions in Krori-Barua spacetime respectively. The study ends with a concluding remark.

2. Einstein-Maxwell field equations

The self consistent Einstein-Maxwell equations for a charged fluid distribution, where the matter source consists of perfect fluid is,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu},\tag{1}$$

We consider a general wormhole spacetime metric of the spherically symmetric form [23]

(using $G = c = 1$) to be represented by the line element,

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{2}$$

Here the functions $A(r)$ and $B(r)$ are dependent on the radial coordinate r and determined by the Einstein-Maxwell equations (where $'$ denotes d/dr),

$$\frac{e^{A(r)}B'(r)}{re^{B(r)}} + \frac{e^{A(r)}}{r^2} - \frac{e^{A(r)}}{r^2e^{B(r)}} = 8\pi T_{00}$$

$$\frac{A'(r)}{r} - \frac{e^{B(r)}}{r^2} + \frac{1}{r^2} = 8\pi T_{11}$$

$$-\frac{1}{2}\frac{rB'(r)}{e^{B(r)}} + \frac{1}{2}\frac{rA'(r)}{e^{B(r)}} + \frac{1}{2}\frac{r^2A''(r)}{e^{B(r)}} + \frac{1}{4}\frac{r^2(A'(r))^2}{e^{B(r)}} - \frac{1}{4}\frac{r^2(A'(r))B'(r)}{e^{B(r)}}$$

$$= 8\pi T_{22} = 8\pi T_{33}/sin^2(\theta). \tag{3}$$

The Hilbert action coupled to electromagnetism is given by

$$I = \int dx^3\sqrt{-g}\left(\frac{R}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right), \tag{4}$$

The energy momentum tensor (EMT) components are given as,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \tag{5}$$

We take an arbitrary function,

$$h(r) = \frac{h_0 e^{-A(r)/2}}{r^2} \tag{6}$$

where h_0 is a constant.

We consider a local orthonormal coordinate basis $(\hat{t}, \hat{r}, \hat{\theta}, \hat{\phi})$ as,

$$e_{\hat{t}} = e^{-A(r)/2}e_t, \quad e_{\hat{r}} = e^{-B(r)/2}e_r$$
$$e_{\hat{\theta}} = r^{-1}e_\theta, \quad e_{\hat{\phi}} = (rsin(\theta))^{-1}e_\phi \tag{7}$$

Hence eqn.(3) reduce as,

$$T_{00} = e^{A(r)}\left((E(r))^2/2 + (h(r))^2/2\right) = e^{A(r)}T_{\hat{0}\hat{0}}$$
$$T_{11} = e^{B(r)}\left(-(E(r))^2/2 + (h(r))^2/2\right) = e^{B(r)}T_{\hat{1}\hat{1}}$$
$$T_{22} = r^2\left((E(r))^2/2 - (h(r))^2/2\right) = r^2T_{\hat{2}\hat{2}} = r^2T_{\hat{3}\hat{3}}. \tag{8}$$

Comparing eqn.(3) with,

$$ds^2 = -e^{2\phi}dt^2 + (1 - \frac{b(r)}{r})^{-1}dr^2 + r^2d\Omega^2, \tag{9}$$

we get,

$$e^{A(r)} = e^{2\phi(r)}, \quad e^{B(r)} = \left(1 - \frac{b(r)}{r}\right)^{-1} \tag{10}$$

The redshift function $\phi(r)$ and the wormhole shape shape function $b(r)$ should be such that it does not violate the wormhole flare-out conditions as defined below. Also the spacetime is asymptotically flat, i.e $\frac{b(r)}{r} \to 0$ as $|r| \to \infty$. We further show that for particular choice of the shape function and redshift function, the wormhole metric in the context does not violate the energy condition at or near the wormhole throat.

We consider the geometric units $G = c = 1$, in the EM field equations.

The flare-out conditions need to be studied which prevent wormholes to be physical and making it open, thereby paving a way for traversibility. These are as : (1) There should be no event horizon, hence the redshift function, $\phi(r)$ should be finite. (2) The wormhole shape function should obey the conditions, (i) $b(r) \le r$ for $r_0 \le r$, r_0 being the throat radius (ii) $b(r_0) = r_0$ (iii) $b'(r_0) < 1$ (iv) $b'(r) < \frac{b(r)}{r}$ and (v) As $r \to r_0$, $b(r)$ approaches $2M$, the Schwarzschildt mass [1, 24], which is the mass function of the wormhole. Hence $b(r)$ should be a positive function.

Using eqns.(3) and (9) we get,

$$B'(r) = \frac{rb'(r) - b(r)}{r(r - b(r))} \tag{11}$$

$$8\pi(E(r))^2 = \frac{b'(r)}{r^2} + \frac{b(r)}{r^3} - \frac{2\phi'(r)(r - b(r))}{r^2} \tag{12}$$

$$8\pi(h(r))^2 = \frac{b'(r)}{r^2} - \frac{b(r)}{r^3} + \frac{2\phi'(r)(r - b(r))}{r^2} \tag{13}$$

Eqn.(7) and (14) are equivalent to,

$$b(r) = 8\pi h_0^2 r e^{2\phi(r)} \int r^{-3} e^{-4\phi(r)} dr - k_1 r e^{2\phi(r)} + r \tag{14}$$

and

$$\phi(r) = \frac{1}{2} ln \left[8\pi h_0^2 \left(1 - \frac{b(r)}{r}\right)^{-1} \int r^{-3} \left(1 - \frac{b(r)}{r}\right)^{-2} dr + k_2 \left(1 - \frac{b(r)}{r}\right)^{-1} \right] \tag{15}$$

where k_1 and k_2 are arbitrary constants.

3. A particular class of solutions

The wormhole shape function is taken as,

$$b(r) = b_0 r^n \tag{16}$$

where b_0 is the constant wormhole shape finction and n is any positive constant. Hence we get a finite redshift function from eqn.(15) as,

$$2\phi(r) = ln \left[8\pi h_0^2 \left(1 - b_0 r^{n-1}\right)^{-1} \left(-\frac{1}{r^2} + 2b_0 \frac{r^{n-3}}{n-3} + 3b_0^2 \frac{r^{2n-4}}{2n-4} + ...\right) + k_2 \left(1 - b_0 r^{n-1}\right)^{-1} \right] \tag{17}$$

4. Solutions in Krori-Barua spacetime

We consider the KB ansatz (Krori and Barua 1975) [25],

$$B(r) = c_1 r^2 = -ln\left(1 - \frac{b(r)}{r}\right) \tag{18}$$

$$A(r) = c_2 r^2 + c_3 = 2\phi(r) \tag{19}$$

where c_1, c_2 and c_3 are some arbitrary constants which need to be defined using some physical conditions.

Case(i) We observe that for the presence of perfect fluid, eqns.(3),(5) and (9) further reduce as,

$$
\begin{aligned}
(E(r))^2/2 + (h(r))^2/2 &= \rho \\
-(E(r))^2/2 + (h(r))^2/2 &= p \\
(E(r))^2/2 - (h(r))^2/2 &= p.
\end{aligned}
\tag{20}
$$

The following relations are also obvious,

$$\frac{b'(r)}{r^2} = 8\pi\left[(E(r))^2/2 + (h(r))^2/2\right]$$

$$\frac{2\phi'(r)}{r^2}(r - b(r)) - \frac{b(r)}{r^3} = 8\pi\left[-(E(r))^2/2 + (h(r))^2/2\right]$$

$$r(r - b(r))\left[\phi''(r) + (\phi'(r))^2\right] + \frac{\phi'(r)}{2}\left[2r - rb'(r) - b(r)\right] + \frac{b(r) - rb'(r)}{2r}$$
$$= 8\pi r^2\left[(E(r))^2/2 - (h(r))^2/2\right]. \tag{21}$$

which further reduce as,

$$\phi'(r)\left[3r - \frac{5}{2}b(r) - \frac{r}{2}b'(r)\right] - \frac{b(r)}{2r} - \frac{b'(r)}{2} + r(r - b(r))\left[\phi''(r) + (\phi'(r))^2\right] = 0 \tag{22}$$

On solving the above eqn.(23) we find the wormhole shape function as,

$$b(r) = \frac{ke^{-2\phi(r)}}{r(1 + r\phi'(r))^2} + \frac{r^2\phi'(r)(2 + r\phi'(r))}{(1 + r\phi'(r))^2} \tag{23}$$

where k is an integration constant.

This wormhole shape function $b(r)$ as well as the plot $b(r)$-r to deduce the value of r_0, where r_0 is the wormhole throat are shown in figures 1. and 2. respectively. The values of the arbitrary constant k is taken as 1, and the redshift function as is eqn.(19), under Krori-Barua spacetime.
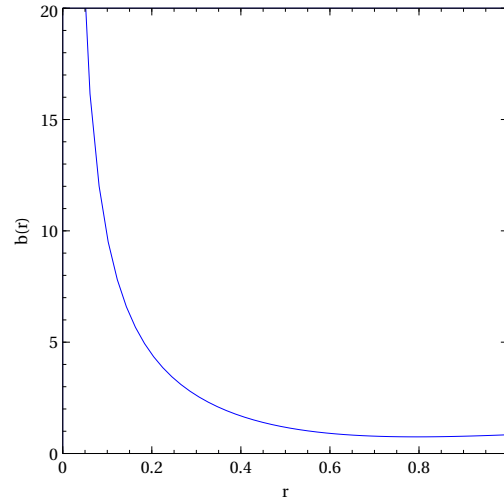
Figure 1. The shape function of the wormhole against $r(k.p.c)$.
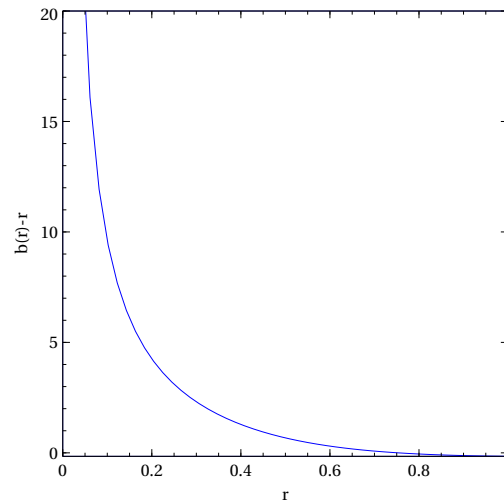
Figure 2. b(r)-r is plotted against $r(k.p.c)$.

We find from the above figure 2. that $b(r) - r$ cuts the r axis at $r_0 = 0.753$. Hence it is obvious that the throat of the wormhole occurs at $r = 0.753$ k.p.c from the center of the wormhole. Also $b(r)/r \rightarrow 0$ as $r \rightarrow \infty$ is confirmed from figure 3. Numerically we also confirm that $b(0.753) = 0.753$ as well as $b'(0.753) = -0.2769 < 1$. Hnece the wormhole throat occurs at a finite distance from it's center.
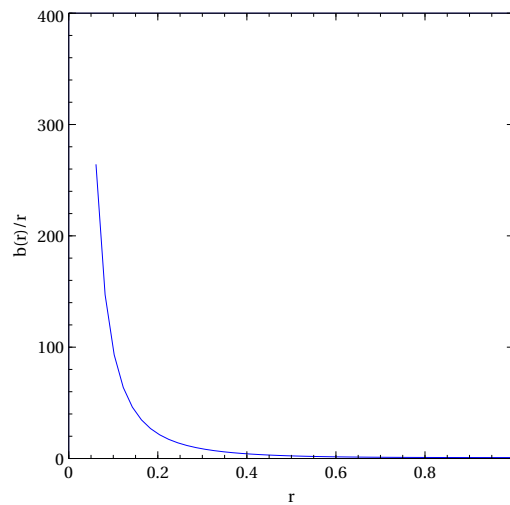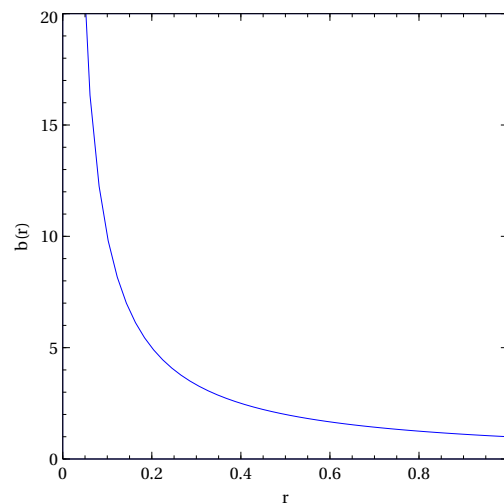
Figure 3. b(r)/r is shown against $r(k.p.c)$.

Case(ii) Constant redshift function

As a matter of fact the constant redshift function would reduce the above eqn.(24) as,

$$b(r) = \frac{p}{r} \tag{24}$$

where p is an arbitrary constant. It is considered as p=1, for brevity and plotting easeness.

Figure 4. The shape function of the wormhole against $r(k.p.c)$.

Figure 5. gives the wormhole throat $r = r_0$ under redshift condition. Obviously the wormhole throat occurs at $r_0 = 1$ when $b(1) = 1$ and $b'(1) = -1 < 1$ and the flare-out conditions are satisfied. The wormhole throat is at a distance 1 k.p.c from it's center under such circumstances.
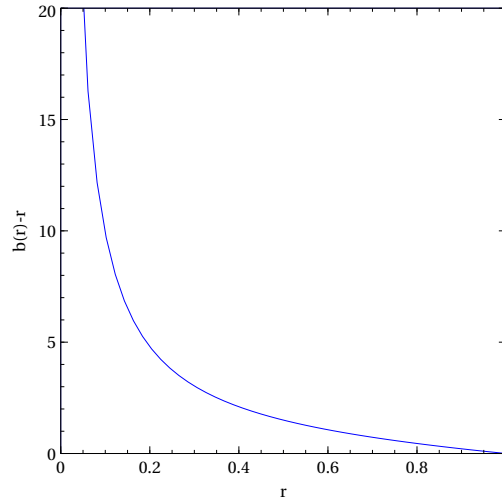
Figure 5. b(r)-r is plotted against $r(k.p.c)$.
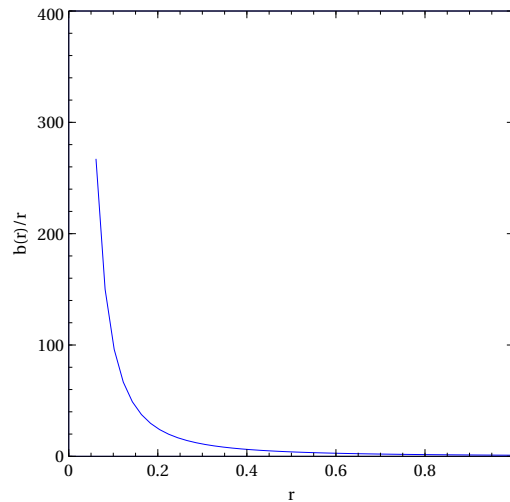
Figure 6. b(r)/r is shown against $r(k.p.c)$.

5. Final Remarks

We have obtained a valid wormhole model under Krori-Barua spacetime in the presence of matter. The energy conditions in this case are being studied to arrive at a decision whether such wormholes could be traversable. Null energy conditions need to be violated under such circumstances. The tidal forces between two parts of a traveller should be less than the gravitational forces, so that he does not get ripped apart. For a wormhole to be physically present, the effective mass should be positive, so that an observer cannot distinguish between the gravitational nature of a wormhole or a compact mass. Such study is in progress for the wormhole model obtained. Hence we are successful in our attempt to derive a wormhole model under Krori-Barua spacetime in the presence of matter, which satisfies the wormhole flare-out conditions.

Acknowledgments

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