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SIXTY CYCLE MAGNETIC FIELDS

by

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It has been previously noted that unless corrective measures are made, the sixty-cycle magnetic fields of the power-lines and cables will seriously deflect the electron beam. Since it is desirable to limit the requirements placed upon the quadrupole-focusing magnets their consideration will be neglected here. The sources of magnetic induction can be divided into two parts: a) the 220 kv power lines and b) the lower voltage cables. Considering the latter, any effect on the electron beam can be made negligible by the use of multiple-conductor or coax cables. Multiple-conductor cables are made with a uniform spiral and will cost little or no more than the straight cables.

Permanent shielding of the accelerator appears unattractive because of the complications added in the construction. However the advantage as a method of field reduction are undeniable. Using Mu metal, with appropriate care in handling, a shielding factor greater than ten can be achieved with a thickness of four mils at a radius of 4 inches. The cost of the material will be about $15,000 for 3,000 lbs. The fabrication costs should be under $100,000. Using transformer material (e.g., U.S. Steel #72) the same estimate as above would be given since a thickness of about 100 units is required.

The rolling or transposing of the high-voltage lines is not such an inexpensive matter. However, it can be done and probably at a lower cost than the shielding of the accelerator to limit deflections to less than two millimeters.

There are apparently many ramifications depending upon the mode of operation since the power lines do not parallel the full length of the accelerator. Thus, a one Bev beam in the first third would be deflected less than two millimeters by the power line fields.

Approximate Power Line Configurations

The magnetic induction at a position (x, y) created by the currents in a three phase balanced-line system of the two-dimensional geometry shown in figure 1 is

$$\vec{B} = \frac{I_o}{5}\Bigg\{\vec{i}\left[y_1\left(\frac{1}{r_{11}^2}+\frac{1}{r_{12}^2}\right)\cos\omega t + y_2\left(\frac{1}{r_{21}^2}+\frac{1}{r_{22}^2}\right)\cos\left(\omega t+\frac{2\pi}{3}\right)\right.$$

$$\left. + y_3\left(\frac{1}{r_{31}^2}+\frac{1}{r_{32}^2}\right)\cos\left(\omega t+\frac{4\pi}{3}\right)\right]$$

$$+ \vec{j}\left[\left(\frac{x_1}{r_{11}^2}+\frac{x_2}{r_{12}^2}\right)\cos\omega t + \left(\frac{x_1}{r_{21}^2}+\frac{x_2}{r_{22}^2}\right)\cos\left(\omega t+\frac{2\pi}{3}\right)\right.$$

$$\left. + \left(\frac{x_1}{r_{31}^2}+\frac{x_2}{r_{32}^2}\right)\cos\left(\omega t+\frac{4\pi}{3}\right)\right]\Bigg\} ,$$

where the units are gauss, amperes, and centimeters.

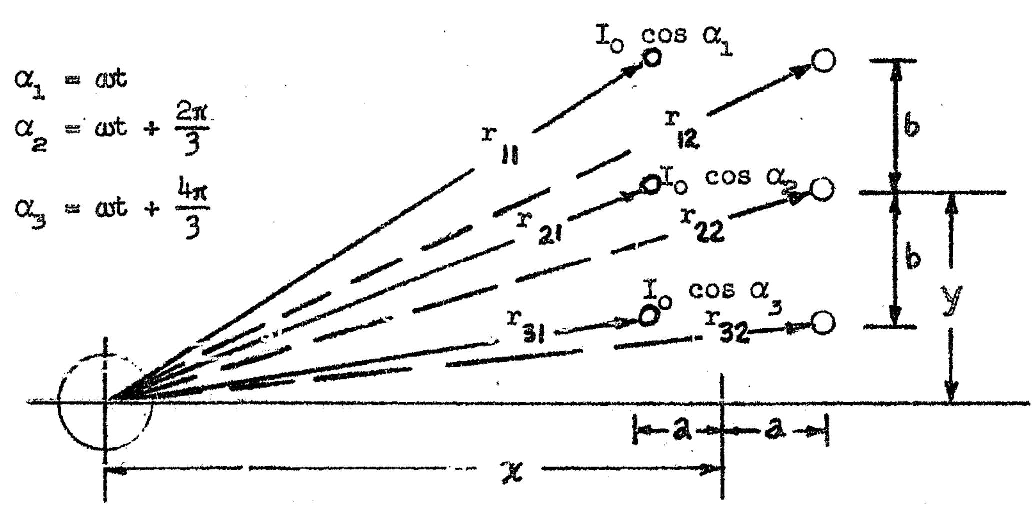

FIG. 1.

$$\frac{1}{r_{11}^2} + \frac{1}{r_{12}^2} = \frac{1}{(x - a)^2 + (y + b)^2} + \frac{1}{(x + a)^2 + (y + b)^2}$$

$$\frac{1}{r_{11}^2} + \frac{1}{r_{12}^2} = \frac{1}{r^2 + a^2 + b^2 + 2(- ax + by)} + \frac{1}{r^2 + a^2 + b^2 + 2(ax + by)}$$

$$\frac{1}{r_{11}^2} + \frac{1}{r_{12}^2} \approx \frac{1}{r^2}\left[1 - \frac{2}{r^2}(- ax + by)\right] + \frac{1}{r^2}\left[1 - \frac{2}{r^2}(ax + by)\right]$$

$$\frac{1}{r_{11}^2} + \frac{1}{r_{12}^2} \approx \frac{2}{r^2}\left(1 - \frac{2\,by}{r^2}\right)$$

$$\frac{1}{r_{21}^2} + \frac{1}{r_{22}^2} = \frac{1}{(x - a)^2 + y^2} + \frac{1}{(x + a)^2 + y^2}$$

$$\frac{1}{r_{21}^2} + \frac{1}{r_{22}^2} \approx \frac{1}{r^2}\left[1 + \frac{2ax}{r^2}\right] + \frac{1}{r^2}\left[1 - \frac{2ax}{r^2}\right] = \frac{2}{r^2}$$

$$\frac{1}{r_{31}^2} + \frac{1}{r_{32}^2} = \frac{1}{(x - a)^2 + (y - b)^2} + \frac{1}{(x + a)^2 + (y - b)^2}$$

$$\frac{1}{r_{31}^2} + \frac{1}{r_{32}^2} \approx \frac{1}{r^2}\left[1 + \frac{2}{r^2}(ax + by)\right] + \frac{1}{r^2}\left[1 - \frac{2}{r^2}(ax - by)\right]$$

$$\frac{1}{r_{31}^2} + \frac{1}{r_{32}^2} \cong \frac{2}{r^2}\left(1 + \frac{2by}{r^2}\right)$$

$$B_x \approx \frac{2I_o}{5r^2}\left[(y + b)\left(1 - \frac{2by}{r^2}\right)\cos \omega t + y \cos\left(\omega t + \frac{2\pi}{3}\right) + (y - b)\left(1 + \frac{2\ by}{r^2}\right)\cos\left(\omega t + \frac{4\pi}{3}\right)\right]$$

$$B_x \approx \frac{2I_o}{5r^2}\left[\left(-\frac{3by^2}{r^2} + \frac{3b}{2} - \frac{b^2y}{r^2}\right)\cos \omega t - \frac{\sqrt{3}}{2}\left(+\frac{2by^2}{r^2} - b - \frac{2b^2y}{r^2}\right)\sin \omega t\right]$$

$$B_x \approx \frac{I_o b}{5r^2}\left(3 \cos \omega t + \sqrt{3}\sin \omega t\right) = \frac{2\sqrt{3}\ I_o b}{5r^2}\cos\left(\omega t - \frac{\pi}{6}\right)$$

$$\frac{x - a}{r_{11}^2} + \frac{x + a}{r_{12}^2} \approx \frac{x - a}{r^2}\left[1 - \frac{2}{r^2}(-\ ax + by)\right] + \frac{x + a}{r^2}\left[1 - \frac{2}{r^2}(ax + by)\right]$$

$$\frac{x - a}{r_{11}^2} + \frac{x + a}{r_{12}^2} \approx \frac{2x}{r^2}\left(1 - \frac{2by}{r^2}\right) - \frac{4a^2x}{r^4}$$

$$\frac{x - a}{r_{21}^2} + \frac{x + a}{r_{22}^2} \approx \frac{x - a}{r^2}\left(1 + \frac{2ax}{r^2}\right) + \frac{x + a}{r^2}\left(1 - \frac{2ax}{r^2}\right)$$

$$\frac{x - a}{r_{21}^2} + \frac{x + a}{r_{22}^2} \approx \frac{2x}{r^2} - \frac{4a^2x}{r^4}$$

$$\frac{x - a}{r_{31}^2} + \frac{x + a}{r_{32}^2} \approx \frac{x - a}{r^2}\left[1 + \frac{2}{r^2}(ax + by)\right] + \frac{x + a}{r^2}\left[1 - \frac{2}{r^2}(ax - by)\right]$$

$$\frac{x - a}{r_{31}^2} + \frac{x + a}{r_{32}^2} \approx \frac{2x}{r^2}\left(1 + \frac{2by}{r^2}\right) - \frac{4a^2x}{r^4}$$

$$B_y \approx \frac{2I_o x}{5r^2}\left[\left(1 - \frac{2by}{r^2} - \frac{2a^2}{r^2}\right)\cos \omega t + \left(1 - \frac{2a^2}{r^2}\right)\cos\left(\omega t + \frac{2\pi}{3}\right)\right.$$

$$\left. + \left(1 + \frac{2by}{r^2} - \frac{2a^2}{r^2}\right)\cos\left(\omega t + \frac{4\pi}{3}\right)\right]$$

$$B_y \approx \frac{2I_o x}{5r^2}\left[-\frac{3by}{r^2}\cos \omega t - \frac{\sqrt{3}by}{r^2}\sin \omega t\right]$$

$$B_y \approx -\frac{4\sqrt{3}\, I_o bxy}{5r^4}\cos\left(\omega t - \frac{\pi}{6}\right)$$

$$\vec{B}(1) \approx \frac{2\sqrt{3}\, I_o b}{5r^2}\left[\vec{i} - \vec{j}\,\frac{2xy}{r^2}\right]\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\left|\vec{B}(1)\right| \approx 5.5 \times 10^{-4} \text{ gauss}$$

for $x = 1.32 \times 10^4$, $y = 2.3 \times 10^3$, $b = 4.5 \times 10^2$, and $I_o = 300$.

The penetration through the copper accelerator wall reduces this to

$$\left|\vec{B}(1)\right| e^{-\frac{t}{s}} = (5.5 \times 10^{-4})\, e^{-\frac{1}{.852}} = 1.7 \times 10^{-4}$$

The resultant magnetic induction for the configuration of Fig. 2 is

$$B_x \approx \frac{4\sqrt{3}\, b(ax - by)I_o}{5r^4} \cos\left(\omega t - \frac{\pi}{6}\right)$$

$$B_y \approx \frac{4\sqrt{3}\, I_o aby}{5r^4} \cos\left(\omega t - \frac{\pi}{6}\right)$$

$$B(2) \approx \frac{4\sqrt{3}\, abI_o}{5r^3} \left[\vec{i} \cos \theta + \vec{j} \sin \theta\right] \cos\left(\omega t - \frac{\pi}{6}\right)$$

$$\frac{|B(2)|}{|B(1)|} \approx \frac{2a}{r} \approx \frac{1}{15}$$

Thus $|\vec{B}(2)| e^{-\frac{t}{s}} \approx 1.1 \times 10^{-5}$.

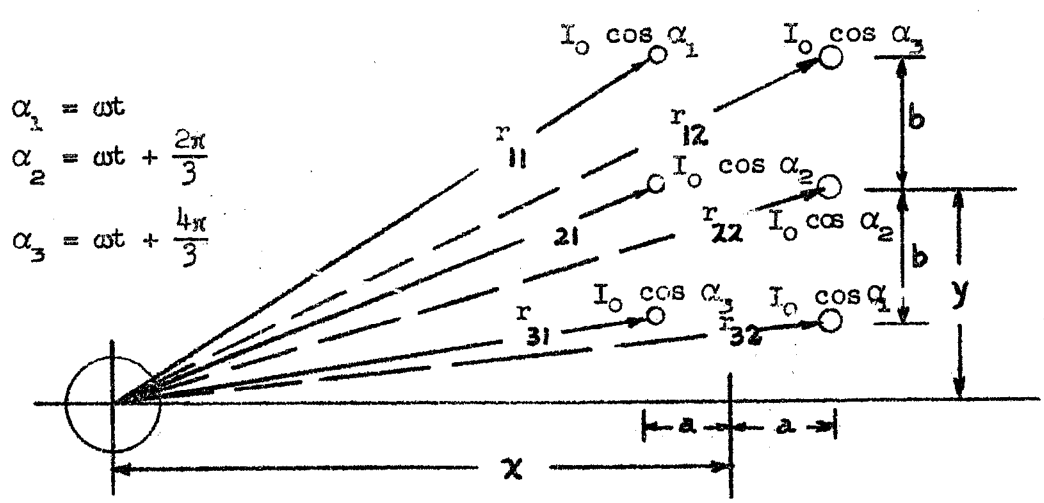

FIG. 2.

Magnetic Shielding

The exact solution of the problem of one or two cylindrical shields has been found previously.[1,2] With the boundary conditions of $B_{n1} = B_{n2}$ and $H_{t1} = H_{t2}$ and the assumptions that $\frac{\mu t}{2r} \gg 1$ and r (radius of the shield) $\gg t$ (thickness of the shield), the shielding of constant magnetic fields for

a) one shield of permeability μ is

$$S_1 = \frac{\mu t}{2r} ,$$

b) two concentric shields

$$S_2 = \frac{\mu^2 t_1 t_2}{4 r_1 r_2}\left[1 - \left(\frac{r_1}{r_2}\right)^2\right] , \text{ and}$$

c) n concentric shields

$$S_n = \left(\frac{\mu}{2}\right)^n \frac{t_1 t_2 \text{---} t_n}{r_1 r_2 \text{---} r_n}\left\{\left[1 - \left(\frac{r_1}{r_2}\right)^2\right]\left[1 - \left(\frac{r_2}{r_3}\right)^2\right] \text{---} \left[1 - \left(\frac{r_{n-1}}{r_n}\right)^2\right]\right\} .$$

If the thicknesses and $r_n/r_{n-1} = c$ are constant, then

$$S_n = \left(\frac{\mu t}{2r}\right)^n \left[\frac{1}{c}\left(1 - \frac{1}{c^2}\right)\right]^{n-1}$$

Although this is a maximum for $c = \sqrt{3}$, it is a slowly varying function of c. For example, for $c = \sqrt{3}$,

$$S_n = 0.385^{n-1} S_1^n$$

and for $c = \sqrt{2}$,

$$S_n = 0.354^{n-1} S_1^n .$$

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[1] Stratton, Electromagnetic Theory, McGraw-Hill, 1941, p. 265.

[2] R. B. Neal, M.L. Report No. 185, February 1953, pp. 203-207.

To obtain a reduction of the earth's field by a factor of 5×10^4 using two concentric shields of $\mu = 2 \times 10^4$ would require $t/r = 3.6 \times 10^{-2}$. If $r = 3.5$ in., then $t = 0.126$ in., yielding a very expensive shield. Three concentric shields yield $t/r = 7 \times 10^{-3}$ or $t = .0245$ in. for the above conditions. The radius of the outer shield in the latter case is 10.5 inches.