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COMPARISONS BETWEEN TRANSPORT AND HYDRODYNAMIC CALCULATIONS*

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We study the energy dissipation in an one-dimensional expansion of gluon matter with Bjorken boost invariance and the formation of shock waves in viscous gluon matter by employing a microscopic parton cascade as well as by solving the Israel–Stewart hydrodynamic equations. Comparisons between the present results from the two approaches are reported.

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1. Introduction

Recently, a great interest is taken in estimating the shear viscosity of the quark gluon plasma (QGP) created in heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC). Attempts are made by employing perturbation QCD based parton cascade calculations [1] as well as by solving the second order Israel–Stewart (IS) hydrodynamic equations [2]. In both approaches the ratio of the shear viscosity to the entropy density η/s of the QGP is extracted by matching the experimental data on the elliptic flow v_2 [3] measured at RHIC. A rough agreement is achieved with the conclusion of $0.08 < \eta/s < 0.4$, although details on initial conditions, hadronization, dissipation in hadronic cascade, and freeze-out conditions should be carefully examined to make fair comparisons.

Transport models provide an excellent tool to test the applicability limit of the IS hydrodynamic equations [4,5]. In the rest of the paper, we present further comparisons between the transport and hydrodynamic calculations

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considering two examples: The one is the energy dissipation in an one-dimensional expansion, and another concerns the formation of relativistic shock waves.

The transport model that we use is the parton cascade BAMPS (Boltzmann Approach of MultiParton Scatterings), which has been developed by two of us [6]. BAMPS solves the Boltzmann equations

$$p^\mu \partial_\mu f(x,p) = C(x,p) \tag{1}$$

for on-shell quarks and gluons with full detailed balance, especially for multiple processes. Calculations using BAMPS showed that the pQCD gluon bremsstrahlung and its backreaction $gg \leftrightarrow ggg$ are essential for fast thermalization [6], small η/s ratio [7], large elliptic flow buildup [1], and a reasonable energy loss of high energy gluons [8] in a consistent manner.

2. Dissipation in an one-dimensional expansion

The early stage in ultrarelativistic heavy ion collisions can most probably be described by a longitudinal expansion along the beam axis. We mimic such an expansion of gluon matter in a 3-dimensional tube with a fixed radius and an infinite length. The details of the numerical setup can be found in Ref. [9]. The initial gluon system is assumed to possess Bjorken boost invariance and is further assumed to be in thermal equilibrium with an energy density of $e_0 = 0.3$ GeV^4 at $\tau_0 = 0.4$ fm$/c$.

The shear viscosity is extracted by means of $f(x,p)$ via

$$\eta = 4n \frac{-T^2 \bar{\pi}}{P^{33} - P^{00}/3}\,, \tag{2}$$

where n is the local gluon density, $T = e/(3n)$ is the temperature, $\bar{\pi} = T^{33} - e/3$ denotes the shear component, and $T^{\mu\nu} = \int dw p^\mu p^\nu f(x,p)$ and $P^{\mu\nu} = \int dw p^\mu p^\nu C[f(x,p)]$ with $dw = d^3p/p^0/(2\pi)^3$. $f(x,p)$ is obtained by solving the Boltzmann equation (1) with BAMPS. Eq. (2) is derived using the Grad's method to the second order [5, 10]. The symbols in Fig. 1 show the shear viscosity scaled by T^3.

Because η/T^3 is approximately constant in time (stronger effect is seen when using small QCD coupling $\alpha_s = 0.1$), we set η/T^3 to be time independent (solid lines) as a simplified input to solve the IS hydrodynamic equations. Approximately one obtains $\eta/s \approx \eta/(4n_{eq}) \approx (\pi^2/64)(\eta/T^3) = 0.83$, 0.18, and $1/4\pi$ for $\alpha_s = 0.1$, 0.3, and 0.6, respectively. n_{eq} denotes the gluon number density at thermal equilibrium.

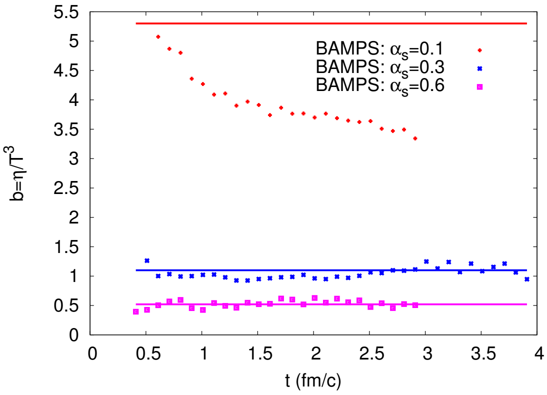

Fig. 1. Shear viscosity scaled by T^3 at the central slice ($z = 0$).

For the one-dimensional expansion with Bjorken boost invariance the IS equations are reduced to

$$\frac{dn}{d\tau} = -\frac{n}{\tau}, \tag{3}$$

$$\frac{de}{d\tau} = -\frac{4}{3}\frac{e}{\tau} + \frac{\bar{\pi}}{\tau}, \tag{4}$$

$$\frac{d\bar{\pi}}{d\tau} = -\frac{\bar{\pi}}{\tau_\pi} - \frac{1}{2}\bar{\pi}\left(\frac{1}{\tau} + \frac{1}{\beta_2}T\frac{\partial}{\partial\tau}\left(\frac{\beta_2}{T}\right)\right) + \frac{2}{3}\frac{1}{\beta_2\tau}, \tag{5}$$

where $\beta_2 = 9/(4e)$ and $\tau_\pi = 2\beta_2\eta$ denotes the relaxation time. The solutions of $e(\tau)$ with the η/s inputs are shown in Fig. 2 and compared with those obtained from BAMPS calculations. Energy density decreases with time

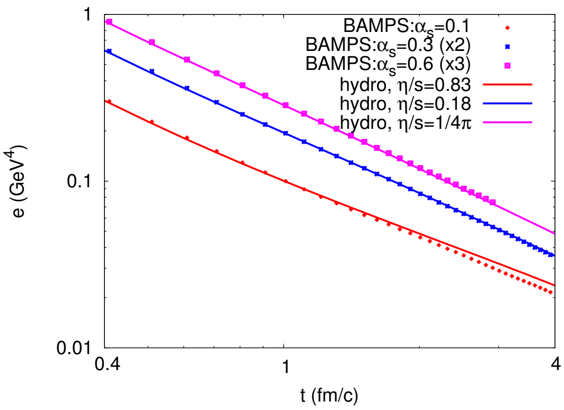

Fig. 2. Local energy density at the central slice. The results for $\alpha_s = 0.3(0.6)$ are amplified by a factor of 2(3) to make clear comparisons.

due to the work done by pressure during the longitudinal expansion [11]. Perfect agreements between the transport and hydrodynamical calculations are seen for small η/s values corresponding to $\alpha_{\rm s} = 0.3$ and 0.6, whereas moderate difference is observed for $\alpha_{\rm s} = 0.1$ at late times. The difference might indicate the break down of the IS hydrodynamics, as expected to occur for larger η/s. However, we must note that the shear viscosity in both calculations is not exactly the same (compare the red symbols with the red line in Fig. 1). Further comparisons using the same value of η are under way.

In general, an expanding viscous medium cannot maintain chemical equilibrium [5]. Fig. 3 shows the gluon fugacity $\lambda = n/n_{\rm eq}$, which quantifies the extent of out of chemical equilibrium. The larger the η/s, the faster is the deviation from the initial chemical equilibrium. Moreover, large difference is visible between the results from the transport and hydrodynamic calculations. This is mainly due to the reason that $gg \leftrightarrow ggg$ processes, which are included in BAMPS, drive the system toward chemical equilibrium again, whereas particle number conservation is assumed in the IS equation (3). To make more detailed comparisons, the present IS equations should in principle be improved by adding terms to describe chemical equilibration.

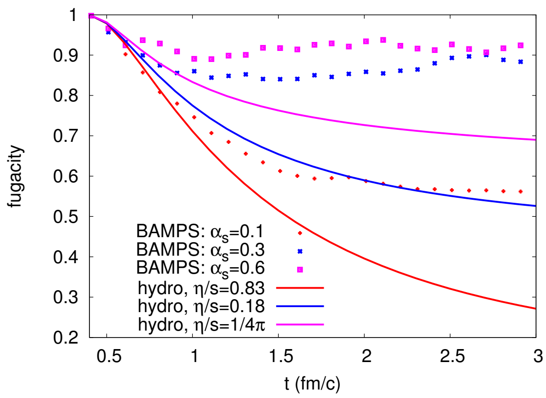

Fig. 3. Gluon fugacity in the central slice.

3. Shock waves in viscous matter

At RHIC, significant and exciting structures in the two-particle and three-particle correlations of associated particles of a high energy jet have been observed, which might indicate the conical emission of propagating Mach cones created by a jet crossing the expanding medium [12]. Important and relevant questions are whether relativistic shock waves can be observed in parton cascade simulations and how finite (shear) viscosity will alter the shock formation.

To answer these questions, we consider the relativistic Riemann problem: Matter initially possesses a discontinuity in pressure at a particular plan (*e.g.* at $z = 0$). The ideal hydrodynamic solution of the Riemann problem represents a propagating relativistic shock, as shown by the solid curve in Fig. 4. Assuming only isotropic binary collisions the shear viscosity is given by $\eta = 0.4en\sigma$ [13], where σ denotes the cross section. The IS equations, with shear viscosity only, are given by

$$\partial_t T^{00} + \partial_z \left(vT^{00}\right) = -\partial_z(vP + v\bar{\pi}) \,, \tag{6}$$

$$\partial_t T^{0z} + \partial_z \left(vT^{0z}\right) = -\partial_z(P + \bar{\pi}) \,, \tag{7}$$

$$\gamma\partial_t\bar{\pi} + \gamma v\partial_z\bar{\pi} = \frac{1}{\tau_\pi}(\pi_{\rm NS} - \bar{\pi}) - \frac{\bar{\pi}}{2}\left(\theta + D\ln\frac{\beta_2}{T}\right) \,, \tag{8}$$

where $\theta \equiv \partial_\mu u^\mu$ and $D \equiv u^\mu \partial_\mu$ with $u^\mu = \gamma(1, 0, 0, v)$. v denotes the collective velocity in z-direction and $\pi_{\rm NS} = -(4/3)\eta\theta$ is the Navier–Stokes value of the viscous pressure.

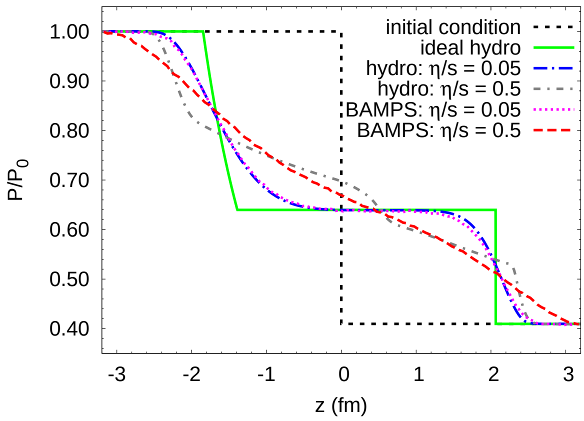

Fig. 4. Pressure normalized by the initial value as a function of the position at a time of 3.2 fm/c.

Results obtained from the BAMPS and the IS hydrodynamic calculations [14] for two η/s values are shown in Fig. 4. Viscous effect changes the shock to a smooth and less abrupt shape. For small η/s a characteristic plateau behind the shock front is still visible, whereas for larger η/s the structure resembles that of diffusion process.

We see a nice agreement between the results from the transport and hydrodynamic calculations for small η/s, while large difference is present for large η/s. Viscous hydrodynamics can be well described by the present IS equations, only if the microscopic scales like the mean free path

$\lambda_{\rm mfp} = 1/(n\sigma)$ are much smaller than the macroscopic dimensions. This condition can be expressed by $Kn \ll 1$, where Kn is the Knudsen number defined as $Kn \equiv \lambda_{\rm mfp}\partial_\mu u^\mu$. Once a shock is built up, $\partial_\mu u^\mu$ becomes large at the shock front. For large η/s values, *i.e.*, for large $\lambda_{\rm mfp}$, the Knudsen number is also large at the shock front. Thus, the application of the IS hydrodynamics there is questionable. This drawback is not present in the microscopic transport calculations.

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