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On the mechanisms of heavy-quarkonium hadroproduction

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Abstract We discuss the various mechanisms potentially at work in hadroproduction of heavy quarkonia in the light of computations of higher-order QCD corrections both in the colour-singlet (CS) and colour-octet (CO) channels and the inclusion of the contribution arising from the *s*-channel cut in the CS channel. We also discuss new observables meant to better discriminate between these different mechanisms.

1 Introduction

Heavy quarkonia are among the most studied bound-quark systems. It is however fair to say that, for the time being, there is no consensus concerning which mechanisms are effectively at work in their production in *pp* collisions, that is, at the Tevatron and RHIC. By extension, available theoretical predictions for the production rates at the LHC are rather uncertain. For recent reviews, the reader is referred to [1–3] and for some perspectives for the LHC to [4].

To what concerns the Υ states, the latest NLO predictions [5] in the QCD-based approach of the colour-singlet model (CSM) [6–8], including for the first time some of the important NNLO α_S^5 corrections, show a satisfactory agreement with the data coming from the Tevatron [9, 10].

On the other hand, to what concerns the charmonium family, it is well known that the first measurements by the CDF Collaboration of the *direct* production of J/ψ and ψ' at $\sqrt{s} = 1.8$ TeV [11, 12] brought to light a striking puzzle. They indeed found much larger rates than the prediction of the CSM.

Since then, other approaches were proposed (e.g. the colour-octet mechanism (COM) from NRQCD [13]) or revived (e.g. the colour-evaporation model (CEM) [14–17]). Those are unfortunately not able to reproduce in a consistent way experimental studies of both cross-section and polarisation measurements for charmonia at the Tevatron [18,

19] along with the cross sections measured by PHENIX at RHIC [20]. For instance, the seemingly solid prediction of the COM for a transverse polarisation of ψ 's produced at high transverse momentum is clearly challenged by the experimental measurements. The most natural interpretation of such a failure of NRQCD is that the charmonium system is too light for relativistic effects to be neglected and that the quark-velocity expansion (v) of NRQCD [13] may not be applicable for the rather "light" $c\bar{c}$ system. This might indeed be the case in view of the aforementioned agreement between theory and the available experimental data on the production in pp (and inclusive decays) of the significantly heavier Υ . In this case, relativistic corrections are expected to be less important and the leading state in the Fock expansion, i.e. the heavy-quark pair in a colour singlet 3S_1 , to be dominant. This would in turn explain why a computation incorporating the sole CS channel (LO contribution in the v-expansion of NRQCD) is sufficient—when P_T^{-4} contributions are considered though.

Recently, many new theoretical results became available. Some completed our knowledge of quarkonium production like the up-to-date proof [21-23] of NRQCD factorisation holding true at any order in v in the gluon-fragmentation channel—therefore relevant at large P_T , where this channel should dominate—and the QCD corrections which we shall discuss in the next section. Others mainly questioned our understanding of the mechanisms at work in heavyquarkonium production. Firstly, a complete survey of fixedtarget measurements [24] showed that the universality of the long distance matrix elements (LDMEs) of NRQCD¹ cannot certainly be extended to the description of low- P_T data. Secondly, NRQCD factorisation was shown to require modifications in fragmentation regions where three heavy quarks have similar momenta [25, 26]. Thirdly, the c-quark fragmentation approximation was shown [27] to be only

¹Let us remember that the universality of the LDMEs is clearly challenged by HERA data on photoproduction of J/ψ [3].



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valid at much higher P_T than expected from the pioneering works [28, 29].

Parallel to that, we investigated in [30, 31] the cut contributions due to off-shell and non-static quarks. In particular, we questioned the assumption of the CSM² that takes the heavy quarks forming the quarkonium (Q) as being on shell [6–8]. If they are not, the usual s-channel cut contributes to the imaginary part of the amplitude and needs to be considered on the same footing as the CSM cut. A first evaluation [30] of the latter incorporating constraints for the low- and large- P_T (the scaling limit) region gives rates significantly larger than the usual CSM cut. Moreover, low- P_T data from RHIC are very well described without need of re-summing initial-gluon contributions. However, as expected [30], this approach underestimates the cross section at large values of P_T , and other mechanisms have to be considered in this region.

In Sect. 2, we present the latest results available on QCD corrections to hadroproduction of J/ψ , ψ' and $\Upsilon(nS)$. In Sect. 3, we discuss how the s-channel cut contribution to the CS channel can be evaluated, and we present a comparison with data. In Sect. 4, we briefly review other recent theoretical results. In Sect. 5, we show how the study of the production of quarkonia in association with a heavy-quark pair of the same flavour may be used to disentangle the different mechanisms proposed to explain quarkonium production. Finally, we present our conclusions and outlook.

2 QCD corrections

More than ten years ago now, the very first NLO calculation on quarkonium production became available. It was centred on unpolarised photoproduction of ψ [32] via a colour-singlet (CS) transition. Later on, NLO corrections were computed for direct $\gamma\gamma$ collisions [33, 34], for which it had been previously shown [35] that the LO CS contribution alone was not able to correctly reproduce the measured rates by DELPHI [36]. NLO corrections have also recently been computed for the integrated cross section of two J/ψ -production observables at the *B*-factories: $J/\psi + c\bar{c}$ [37] and $J/\psi + \eta_c$ [38]. As of today, only the full colour-octet (CO) contributions to direct $\gamma\gamma$ collisions have been evaluated at NLO for $P_T > 0$ [33, 34].

At the LHC and the Tevatron, ψ and Υ production proceeds most uniquely via gluon-fusion processes. The corresponding cross section at NLO (α_S^4 for hadroproduction processes) are significantly more complicated to compute than the former ones and became only available one year ago [27, 39]. We shall discuss them in the next section.

²Note by the way that this assumption is also present in NRQCD.



The common feature of all these calculations is the significant size of the NLO corrections, in particular for large transverse momenta, P_T , of the quarkonia for the computations of differential cross sections in P_T . In γp and pp collisions, QCD corrections to the CS production indeed open new channels with a different behaviour in P_T , which substantially raise the cross section in the large- P_T region.

Let us discuss this briefly for the gluon-fusion processes, which dominate the yield in pp. If we only take into account the CS transition to ${}^{3}S_{1}$ quarkonia, it is well known that the differential cross section at LO as a function of P_T scales like P_T^{-8} [6–8]. This is expected from contributions coming from the typical "box" graphs of Fig. 2.1(a). At NLO [27, 39], we can distinguish three noticeable classes of contributions. First, we have the loop contributions as shown on Fig. 2.1(b), which are UV divergent³ but as far their P_T scaling is concerned, they would still scale like P_T^{-8} . Then we have the t-channel gluon-exchange graphs like in Fig. 2.1(c). They scale like P_T^{-6} . For sufficiently large P_T , their smoother P_T behaviour can easily compensate for their α_S suppression compared to the LO (α_S^3) contributions. They are therefore expected to dominate over the whole set of diagrams up to α_s^4 . To be complete, we should not forget the α_s^4 contributions from $Q + Q\bar{Q}$ (where Q is of the same flavour as the quarks in Q). Indeed, one subset of graphs for $Q + Q\bar{Q}$ is fragmentation-like (see Fig. 2.1(d)) and scales like P_T^{-4} . Such contributions are therefore expected to dominate at large P_T , where the smoother decrease in P_T is enough to compensate the suppression in α_S and the one due to the production of four heavy quarks. As mentioned above, in practice [27], this happens at larger P_T than as expected before [28, 29]. We shall come back to this channel later. In the next sections, we shall discuss the impact of the NLO corrections to the CS channels and then a first computation including the a priori dominant α_s^5 contributions, i.e. topologies illustrated by Fig. 2.1(e) and (f).

As regards the CO contributions, the effects of NLO (here α_S^4) contributions are expected to be milder. Indeed, the contributions from CO transitions are a priori suppressed by v^4 , and the reason why they can still appear to be significant lies in a lower power in α_S for similar topologies (and thus similar P_T scaling). For instance, compare Fig. 2.1(g) to Fig. 2.1(e) and Fig. 2.1(h) to Fig. 2.1(c). At LO, P_T^{-6} and P_T^{-4} scaling are therefore already present. As a result, including α_S^4 contributions will not open any new channels and NLO corrections are expected to be described by a roughly constant K factor. As we shall see, this is indeed the trend seen in the results of [40], in which the NLO corrections to ${}^1S_0^{[8]}$ and ${}^3S_1^{[8]}$ colour octets going to J/ψ were considered.

³These divergences can be treated as usual using dimensional regularisation; see e.g. [39].

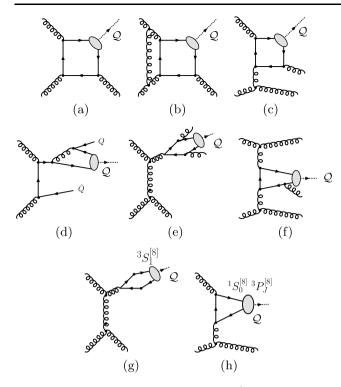


Fig. 2.1 Representative diagrams contributing to 3S_1 hadroproduction via colour-singlet channels at orders α_S^3 (**a**), α_S^4 (**b**, **c**, **d**), α_S^5 (**e**, **f**) and via colour-octet channels at orders α_S^3 (**g**, **h**). The quark and antiquark attached to the ellipsis are taken as on shell and their relative velocity v is set to zero

2.1 NLO corrections for colour-singlet channels

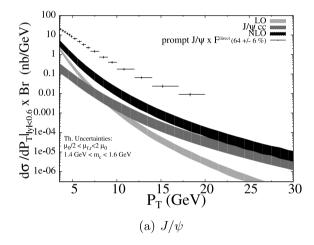
Let us first present a comparison between the measurements by the CDF Collaboration and the result for the J/ψ obtained following the procedure explained in [27, 39]. It is worth noting that the χ_c cross sections are not available at NLO accuracy. This would be necessary if we wanted to predict at this accuracy the prompt- J/ψ production cross section, in order to compare with the most recent measurements of RUN II [41]. These focused only on the prompt yield. As a makeshift, we have multiplied those data by the averaged fraction of direct J/ψ measured during RUN I [12] for the rather similar beam energy of 1.8 TeV and a similar range in P_T and rapidity: $\langle F^{\rm direct} \rangle = 64 \pm 6\%$.

In our calculation, we set $m_c = 1.5 \pm 0.1$ GeV and $m_b = 4.75 \pm 0.25$ GeV. We used the PDF set CTEQ6L1 (respectively CTEQ6_M) [42] for LO (respectively NLO) cross sections, and we always kept the factorisation scale equal to the renormalisation scale: $\mu_f = \mu_r$. Except for the associated production channel, where we took $\mu_0 = \sqrt{(2m_Q)^2 + P_T^2}$, the central scale is fixed at $\mu_0 = \sqrt{m_Q^2 + P_T^2}$ and then was varied by a factor of 2. To what concerns the non-perturbative inputs, we used the values related to the BT potential [43]: $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.16 \text{ GeV}^3$

and $\langle \mathcal{O}^{\Upsilon}(^3S_1^{[1]})\rangle = 9.28 \text{ GeV}^3$. For $\Upsilon(1S)$ production, we considered the prompt measurement at $\sqrt{s} = 1.8 \text{ TeV}$ in [9], multiplied by the averaged direct fraction obtained in [44]: $\langle F^{\text{direct}} \rangle = 50 \pm 12\%$.

In both cases, we have an illustration (Fig. 2.2) of the previous discussion. The differential cross section for the LO contribution, i.e. $gg \to J/\psi g$, has the steepest slope and is already an order of magnitude smaller than the NLO contribution at $P_T \simeq 10$ GeV. The differential cross section for $Q + Q\bar{Q}$ has the smoothest slope. In the case of J/ψ , it starts to be significant for $P_T > 20$ GeV. For the Υ , the suppression due to the production of four b quarks is stronger and this yield remains negligible in the accessible value of P_T . The bands denoted NLO refer to all the contributions up to order α_s^4 .

Those results were recently confirmed in [45, 46]. In these papers, the polarisation information was kept and the observable α was also computed. However, it is important to stress that for ψ and Υ production, the CS yields predicted



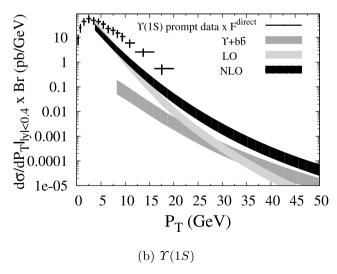


Fig. 2.2 Differential cross sections at NLO accuracy as a function of the quarkonium transverse momentum P_T at the Tevatron $(\sqrt{s}=1.96~{\rm TeV})$



at the NLO accuracy are still clearly below the experimental data, especially at large P_T . In this respect, the predictions for the polarisation at this order cannot be usefully compared to the data.

The conclusion is that, in general, the inclusion of NLO contributions bring the CS predictions considerably closer to the data, although agreement is only reached at NLO in the photoproduction case [32].

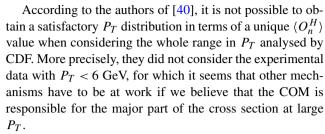
2.2 NLO corrections for colour-octet channels

As aforementioned, NRQCD has reached a certain success by explaining the main features of charmonium and bottomonium hadroproduction via the introduction of the colour-octet (CO) mechanism. It indeed provides a good description of the P_T -differential cross section for the direct J/ψ and ψ' cases for $P_T\gtrsim 5$ GeV as measured by CDF in $p\bar{p}$ [11, 12]. A reasonable agreement was also obtained with the first PHENIX measurements in pp at $\sqrt{s}=200$ GeV [47, 48]. In both cases, the cross section is dominated by the gluon fragmentation into a colour-octet S-wave state. Following the heavy-quark spin symmetry [13] of NRQCD, the latter mechanism leads to a transversally polarised J/ψ and ψ' , the parent fragmentating gluon being mostly on shell and thus transversally polarised at high P_T .

However, J/ψ and ψ' are not seen to be transverse by the CDF experiment [18]. It measured a slight longitudinal polarisation for both the prompt J/ψ and direct ψ' yield. It is worth noting here that the feed-down from χ_c can influence significantly the polarisation of the prompt J/ψ yield—this was taken into account in the NRQCD-based predictions [49]. Moreover, the recent preliminary result from PHENIX [50] indicates a polarisation compatible with zero for the total J/ψ production at forward rapidity (1.2 < |y| < 2.2), but with large uncertainties.

Very recently, CO contributions from S waves (${}^1S_0^{[8]}$) and ${}^3S_1^{[8]}$) have become available [40] for hadroproduction. A complete phenomenological study is not yet available though. Anyhow this confirms that NLO corrections do not significantly affect the P_T dependence as expected from the introductory discussion of this section.

Let us define K factors as the ratios of NLO to LO cross sections for a given CO channel. For the Tevatron, they are about 1.2 for the $^1S_0^{[8]}$ state and 1.1 for the $^3S_1^{[8]}$ (at the LHC, they are both about 0.8). Consequently, the value of the CO long distance matrix elements (LDMEs) fit to the Tevatron data at LO $\langle O^{J/\psi}(^3S_1^{[8]})\rangle \simeq 0.0012~{\rm GeV}^3$ and $\langle O^{J/\psi}(^1S_0^{[8]})\rangle \simeq 0.0045~{\rm GeV}^3$ [3] would be at most reduced by 15%. In this respect, the NLO corrections to the octets do not improve the universality of the matrix elements when the idea of the dominance of the CO transitions is confronted with the data on photoproduction from HERA.



This in any case emphasises the need for more work dedicated to the description of the low- P_T region and maybe the relevance of the study of s-channel cut contribution, which we discuss later. Last but not least, the polarisation from CO transitions appears not to be modified at NLO with respect to LO results. Overall, this recent first study of CO contributions at NLO in hadroproduction at $P_T > 0$ sounds like a confirmation of the flagrant discrepancy between the NRQCD predictions for the polarisation of the J/ψ and the experimental measurements from the CDF Collaboration [18].

2.3 QCD corrections up to α_s^5

As noted above, the discrepancy between the NLO computations for the CSM and the experimental data, both for ψ and Υ , still grows with P_T . If we consider this in parallel that to the existence of a new P_T^{-4} channel at order α_S^5 , it is reasonable to wonder what their size are effectively.

In fact, their contributions can be evaluated in a relatively "simple" 4 and reliable way by computing the α_S^5 contributions consisting of the production of a $\mathcal Q$ with three light partons (denoted j hereafter). Among them are the topologies of Fig. 2.1(d) (gluon fragmentation) and Fig. 2.1(e) ("high-energy enhanced"); these close the list of kinematical enhancements from higher-order QCD corrections. This α_S^5 subset being the LO for a physical process $(pp \to \mathcal Q + jjj)$, its contribution is finite except for soft and collinear divergences.

To avoid such divergences, we impose a lower bound on the invariant mass of any light parton pairs (s_{ij}) . For the new channels opening up at α_S^5 , which specifically interest us, the dependence on this cut is to get smaller for large P_T since no collinear or soft divergences can appear there. For other channels, whose LO contribution is at α_S^3 or α_S^4 , the cut would produce logarithms of s_{ij}/s_{ij}^{\min} . Those can be large. Nevertheless, they can be factorised over their corresponding LO contribution, which scales at most as P_T^{-6} . The sensitivity on s_{ij}^{\min} is thus expected to come to nothing at large P_T .

Thanks to the exact NLO computation of [39], such a procedure can be tested for the process $pp \rightarrow Q + jj$. For



⁴We mean "simple" compared to a full—out-of-reach—NNLO computation and thanks to the automated generator of matrix elements MadOnia [51].

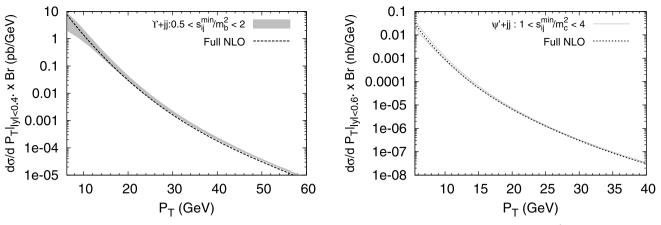
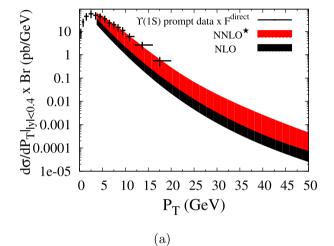


Fig. 2.3 Full computation at NLO for (*left*) $\Upsilon(1S) + X$ (*dashed line*) versus $\Upsilon(1S) + 2$ light partons with a cut on s_{ij}^{\min} (*grey band*), (*right*) $\psi(2S) + X$ (*dashed line*) versus $\psi(2S) + 2$ light partons with a cut on s_{ij}^{\min} (*grey curves*)

instance, the differential cross section for the real α_s^4 corrections, $\Upsilon(1S)+jj$ production, is displayed in Fig. 2.3 (left). The grey band illustrates the sensitivity to the invariant-mass cut s_{ij}^{\min} between any pairs of light partons when it is varied from $0.5m_b^2$ to $2m_b^2$. The yield becomes insensitive to the value of s_{ij}^{\min} as P_T increases, and it reproduces very accurately the differential cross section at NLO accuracy. In the charmonium case, the similar contributions from $pp \to \psi' + jj$ match even better, for lower P_T and with a smaller dependence of s_{ij}^{\min} , the full NLO computation, as seen on Fig. 2.3 (right).

We now turn to the results concerning the real contribution at α_s^5 , which we refer to as NNLO*. We used the approach described in Ref. [51], which allows for the automatic generation of both the subprocesses and the corresponding scattering amplitudes. The differential cross sections for $\Upsilon(1S)$ and $\psi(2S)$ are shown in Fig. 2.4. The red band (referred to as NNLO*) corresponds to the sum of the NLO yield and the Q + jjj contributions. In the Υ case, the contribution from Υ with three light partons fills the gap between the data and the NLO calculation, while for the $\psi(2S)$ there seems to remain a small gap between the NNLO* band and the preliminary CDF data [52]. In both cases, the α_S^5 contribution is very sensitive to the choice of the renormalisation scale, μ_r . This is expected: for moderate values of the P_T , the missing virtual part might be important, whereas at large P_T , the yield is dominated by Bornlevel α_S^5 -channels, from which we expect a large dependence on μ_r . Even though the uncertainty on the normalisation is rather large, the prediction of the P_T shape is quite stable and agrees well with the behaviour found in the data [9, 10, 52].

Concerning the polarisation, the *direct* yield is predicted to be mostly longitudinal; see Fig. 2.5(a). However, the existing experimental data for Υ are centred on the *prompt* yield [9, 53]. In order to draw further conclusions, we would



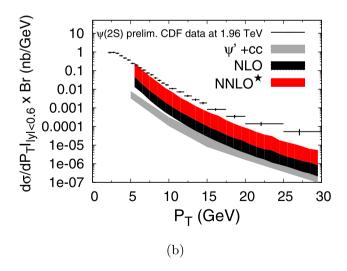
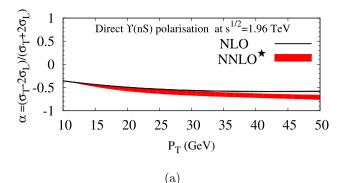


Fig. 2.4 Comparison between differential cross sections at NLO and NNLO* accuracy as a function of the $\mathcal Q$ transverse momentum P_T at the Tevatron ($\sqrt{s}=1.96$ TeV) and the data for (a) $\Upsilon(1S)$ [9] and (b) direct $\psi(2S)$ [52]





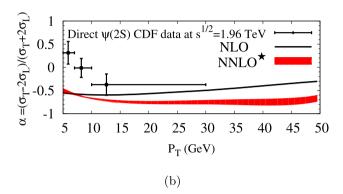


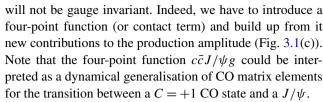
Fig. 2.5 Polarisation of (a) $\Upsilon(nS)$ ((b) $\psi(2S)$) directly produced as a function of its transverse momentum P_T at the Tevatron

need first to gain some insights in the NLO corrections to P-wave production at $P_T > 0$. Yet, since the yield from P-wave feed-down is likely to give transversely polarised Υ , the trend is more than encouraging. As regards $\psi(2S)$, one should be very careful with any comparisons with experimental measurements, since the yield is not exactly reproduced. Having this in mind, one sees in Fig. 2.5(b) that the trend for longitudinally polarised $\psi(2S)$ is reproduced but more marked. At very large P_T where the contribution from $\psi(2S) + c\bar{c}$ becomes more and more significant, the polarisation gets slightly less negative. In any case, further investigations are needed to draw any conclusions.

3 s-Channel cut contribution

In this section, we briefly review our first evaluation of the s-channel cut contribution in hadroproduction of J/ψ [30, 31] and present an outlook for necessary future investigations in this direction.

If the quarks which constitute the \mathcal{Q} are not on their mass shell, it is not possible to factorise in a gauge-invariant way the amplitude for the production of those quarks and the one responsible for their binding into the \mathcal{Q} . In other words, if the latter amplitude is given by a (three-point) Bethe–Salpeter vertex function, the set of Feynman diagrams shown in Fig. 3.1(a, b) constructed from this vertex



Gauge invariance relates the four-point function to the three-point one, but not univocally, since it does not constrain its transverse contribution with respect to the emitted gluon. However there is an elegant way to parametrise this freedom by an auxiliary function F [54–57]. To see this, let us define the three-point function

$$\Gamma_{\mu}^{(3)}(p,P) = \Gamma(p,P)\gamma_{\mu}$$
, (3.1)

where $P \equiv p_1 - p_2$ and $p \equiv (p_1 + p_2)/2$ are the total and relative momenta, respectively, of the two quarks bound as a quarkonium state, with p_1 and p_2 being their individual four-momenta and the four-point one by

$$\Gamma^{(4)} = -ig_s T_{ik}^a M_c^{\nu} \gamma^{\mu} , \qquad (3.2)$$

where g_s is the strong coupling constant, T_{ik}^a the colour matrix, and μ and ν are the Lorentz indices of the outgoing J/ψ and gluon, respectively. For simplicity, we have suppressed all indices on the left-hand side. The $c\bar{c}\,J/\psi$ vertex function $\Gamma^{(3)}$ with the kinematics of the direct graph is denoted here by Γ_1 and for the crossed graph by Γ_2 , i.e., $\Gamma_1 = \Gamma(c_1 - \frac{P}{2}, P)$ and $\Gamma_2 = \Gamma(c_2 + \frac{P}{2}, P)$, as shown in Figs. 3.1(a) and (b).

One easily verifies that any contact current defined by ⁵

$$M_c^{\nu} = \frac{(2c_2 + q)^{\nu} (\Gamma_1 - F)}{(c_2 + q)^2 - m^2} + \frac{(2c_1 - q)^{\nu} (\Gamma_2 - F)}{(c_1 - q)^2 - m^2}$$
(3.3)

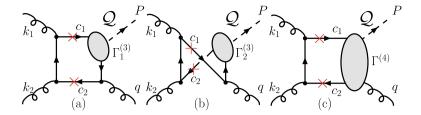
satisfies the gauge-invariance requirement [30] for any value of the function $F = F(c_1, c_2, q)$.

Nonetheless, the function $F(c_1, c_2, q)$ must be chosen so that the current (3.3) satisfies crossing symmetry (i.e., symmetry under the exchange $c_1 \leftrightarrow -c_2$) and is free of singularities. The latter constraint implies $F = \Gamma_0$ at either pole position, i.e., when $(c_2+q)^2 = m^2$ or $(c_1-q)^2 = m^2$, where the constant Γ_0 is the (unphysical) value of the momentum distribution $\Gamma(p,P)$ when all three legs of the vertex are on their respective mass shells. In principle, employing gauge invariance as the only constraint, we may take $F = \Gamma_0$ everywhere. This corresponds to the minimal substitution discussed by Drell and Lee [58] (for a complete derivation, see [59]), who pointed out, however, that this does not provide the correct scaling properties at large energies, which means within the present context that $F = \Gamma_0$ would not lead



⁵We have taken $c_1^2=c_2^2=m^2$ and $P^2=M^2$ with m and M being the masses of the quark and the J/ψ .

Fig. 3.1 (a) and (b) Leading-order (LO) s-channel cut diagrams contributing to $gg \rightarrow Qg$ with direct and crossed box diagrams employing the three-point $c\bar{c}$ Q vertex. The crosses indicate that the quarks are on shell. (c) Box diagram with the (four-point) $c\bar{c}Qg$ contact term mandated by gauge invariance



to the expected P_T scaling of the amplitude. See [60] for a numerical comparison with the data.

In order to obtain a correct scaling at large P_T and a behaviour close to the minimal substitution at low P_T , we have chosen [30, 60]

$$F(c_1, c_2, q) = \Gamma_0 - h(c_1 \cdot c_2) \frac{(\Gamma_0 - \Gamma_1)(\Gamma_0 - \Gamma_2)}{\Gamma_0} , \quad (3.4)$$

where the (crossing-symmetric) function $h(c_1 \cdot c_2)$ rises to become unity for large relative momentum. The phenomenological choice for the interpolating function h used in our calculations is

$$h(c_1 \cdot c_2) = 1 - a \frac{\kappa^2}{\kappa^2 - (c_1 \cdot c_2 + m^2)}, \qquad (3.5)$$

with two parameters, a and κ .

Figure 3.2(a) shows our results for $\sqrt{s}=1.8$ TeV in the pseudorapidity range $|\eta|<0.6$ with parameter values a=4 and $\kappa=4.5$ GeV fixed to reproduce, up to $P_T\simeq 10$ GeV, the cross-section measurement of direct J/ψ by CDF [12], the usual LO CSM from $gg\to J/\psi$ g [6–8] and LO CSM from $gg\to J/\psi$ $c\bar{c}$ [27]. Our results fit well the CDF data up to about $P_T=10$ GeV. At higher P_T , our curve falls below the

data as expected from the genuine $1/P_T^8$ scaling of a LO box diagram (see the discussions of the previous section).

It is interesting to note the different P_T behaviours of σ_T and σ_L leading to a dominance of the latter at large P_T and a negative value for the polarisation α [30] at mid and large P_T . Figure 3.2(b) shows our results at $\sqrt{s} = 200$ GeV, still with a = 4 and $\kappa = 4.5$ GeV, compared with the PHENIX data [20].

Through this first evaluation of the s-channel cut contribution to the imaginary part of the production amplitude, incorporating low- and large-energy constraints as well as gauge invariance, we have shown that this cut can be significant. It is even possible to obtain a very good fit of the data from CDF at mid P_T by proper choices of the parameters of our four-point function. With the same parameters, we obtained an excellent description of the data taken at RHIC and down to very low P_T without re-summing initial-state gluon contributions. The s-channel cut indeed has a threshold at low \hat{s} (thus low P_T), which corresponds to the energy needed to put the two c-quarks on shell.

Now that we have seen that the s-channel cut matters at low and mid P_T , it is necessary to have in the future a first evaluation of the contribution of the real part itself. On the other hand, we can start testing our parametrisation of the

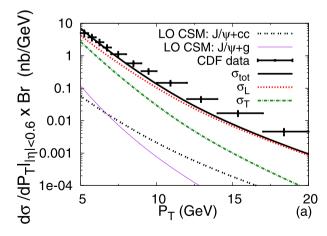
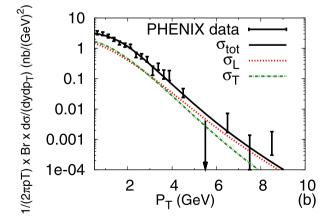


Fig. 3.2 (a) Comparison between polarised (σ_T and σ_L) and unpolarised (σ_{tot}) cross sections [with parameters a=4 and $\kappa=4.5$ GeV in Eq. (3.5)], LO CSM contributions, and CDF experimental data [12] at



the Tevatron ($\sqrt{s} = 1.8$ TeV, pseudorapidity $|\eta| < 0.6$); (**b**) comparison between σ_T , σ_L , σ_{tot} and PHENIX data [20] at RHIC ($\sqrt{s} = 200$ GeV and rapidity |y| < 0.35); taken from [30]



four-point function, in photoproduction for instance, or in any other process involving a final-state gluon.

4 Other theoretical advances

Beside the theoretical advances concerning QCD corrections and the inclusion of the *s*-channel cut contribution discussed in the previous sections, several interesting theoretical results have been obtained in recent years. Let us review some of the most significant ones briefly.

On the side of NRQCD, Nayak, Qiu and Sterman provided an up-to-date proof [21–23] of NRQCD factorisation holding true at any order in v in the gluon-fragmentation channel. They showed that new definitions of NRQCD matrix elements incorporating QCD Wilson lines were to be used, but that this was not to affect the existing phenomenological studies.

Last year, Collins and Qiu [61] showed that in general the k_T -factorisation theorem does not hold in production of high-transverse-momentum particles in hadron-collision processes, and therefore also for ψ and Υ . This is unfortunate, since many studies [62–71], predicting mostly longitudinal yields and smaller CO LDMEs, in better agreement with the idea of LDME universality, were based on the hypothesis of such a factorisation in hadroproduction.

Besides, the c- and b-fragmentation approximation was shown to fail for the P_T ranges accessible in experiments for quarkonium hadroproduction. By studying the entire set of diagrams contributing to ψ and Υ production in association with a heavy-quark pair of the same flavour, we have shown [27] that the full contribution was significantly above (typically of a factor of 3) that obtained in the fragmentation approximation. A precision of 10% accuracy, say, can only be obtained at very large P_T : $P_T \gtrsim 60$ GeV for ψ and $P_T \gtrsim 100$ GeV for Υ . Note that the same observation was previously made for the process $\gamma \gamma \to J/\psi c\bar{c}$ [72] and also for the B_c^* hadroproduction, for which it was noticed that the fragmentation approximation was not reliable at the Tevatron [73, 74].

Moreover, still in double-heavy-quark-pair production, the notion of colour-transfer enhancement was introduced by Nayak, Qiu and Sterman [25, 26]. If three out of the four heavy quarks are produced with similar velocities, then there is the possibility that colour exchanges within this three-quark system could turn CO configurations into CS ones and thus could effectively increase the rate of production of CS pairs. They finally discussed the introduction of specific new three-quark operators—beyond the usual ones of NRQCD—necessary to deal with such an issue. A study of the colour-transfer effects in hadroproduction is still awaited for.



5 Associated-production channels

As previously discussed, the results of QCD corrections for γ production seem to indicate that the CS transitions are dominant. Eventually, this should put an end to the controversy related to Υ production. Contrariwise, the situation remains unclear for the charmonium case. NLO corrections complemented by some dominant α_S^5 contributions are large and seem to bring the prediction for the CS transitions very close to the data in the ψ' case for instance (see Fig. 2.4(b)). Yet, theoretical uncertainties remain large, and there seems to be some space left for CO transitions. Careful comparisons are still therefore due with polarisation observables. In this case, the theoretical uncertainties would certainly be competitive with experimental ones, for instance on prompt J/ψ yield [75]. This requires however some knowledge on the QCD corrections to the P-wave CS yield. For the time being, nothing is known on this side.

It is therefore vital in order to make progress in the understanding of the mechanisms responsible for heavy-quarkonium production to introduce, compute and measure new observables. One of those is the hadronic activity around the quarkonium [76]. Historically, UA1 compared their charged-track distributions with Monte Carlo simulations for a J/ψ coming from a B and a J/ψ coming from a χ_c [77, 78]. At that time χ_c feed-down was still expected to be the major source of prompt J/ψ . Following either the idea of CO transitions or of CS transitions at higher orders, we however expect now more complex distributions even for the prompt yield. It is therefore not clear if such methods are suitable to size up the B-feed-down otherwise than with the measurements of a displaced vertex typical of a B decay.

We therefore urgently need observables rather easy to predict and likely to test the many production models available [1, 2]. We argue here that the study of associated-production channels, first in pp collisions, then in pA and AA, fulfils both these requirements. By associated-production channels, we mean $\psi + c\bar{c}$ and $\Upsilon + b\bar{b}$.

A further motivation for such studies is that similar studies carried out at *B*-factories showed an amazingly large fraction of J/ψ produced in association with another $c\bar{c}$ pair. Indeed, the Belle Collaboration first found [79] $\frac{\sigma}{\sigma}\frac{(e^+e^-\to J/\psi+c\bar{c})}{\sigma(e^+e^-\to J/\psi+X)}$ to be $0.59^{-0.13}_{+0.15}\pm0.12$. Thereafter, the analysis was improved and they obtained [80]

$$\frac{\sigma (e^+e^- \to J/\psi + c\bar{c})}{\sigma (e^+e^- \to J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14$$

$$> 0.48 \text{ at } 95\% \text{ CL.}$$
(5.1)

Whether or not such a high fraction holds for hadroproduction as well is a question which remains unanswered. Analyses at the Tevatron (CDF and $D\emptyset$) and at RHIC (PHENIX and STAR) are already possible. As computed

in [27] for the RUN2 at the Tevatron at $\sqrt{s} = 1.96$ TeV, the integrated cross sections are significant:

$$\sigma(J/\psi + c\bar{c}) \times \mathcal{B}(\ell^+\ell^-) \simeq 1 \text{ nb},$$

$$\sigma(\Upsilon + b\bar{b}) \times \mathcal{B}(\ell^+\ell^-) \simeq 1 \text{ pb}$$
 (5.2)

As an illustration of the potentialities at RHIC, we chose to display in Fig. 5.1 the differential cross section for $pp \rightarrow J/\psi + c\bar{c}$ computed for the STAR kinematics. Such studies could for instance be carried out by STAR in the next run with integrated luminosities of around 50 pb⁻¹, if dedicated triggers are available [81].

Without taking into account the likely reduction of the CO LDMEs induced by the QCD corrections mentioned in the previous sections, the integrated cross sections were found in [82] to be dominated by the CS part, similarly to the differential cross section in P_T up to at least 5 GeV for ψ and 10 GeV for Υ . In other words, such observables can be thought of as a test of the CS contribution, for the first time since the introduction of the idea that CO transitions would be the dominant mechanism responsible for quarkonium production at high transverse momentum. If the effect of CO transitions is confirmed to be negligible for the Υ , the Υ produced in association with a $b\bar{b}$ pair are predicted to be strictly unpolarised, for any P_T (see Fig. 5.2) for the LHC.

Beside the property of discriminating between the CO and the CS transitions, the yield of ψ in association with $c\bar{c}$ should show an a priori completely different sensitivity to the χ_c feed-down than the inclusive yield. The same holds for Υ with $b\bar{b}$ with the χ_b feed-down. As far as concerns the CS transitions, the P-wave yield is expected to be smaller than the S-wave one, since they are being suppressed by powers of the relative velocity v and here there is no extra gluon needed to be attached to the heavy-quark loop to produce ψ (or Υ) compared to P-waves as is the case in the inclusive case.

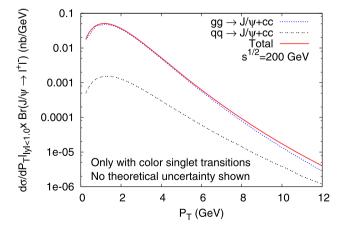


Fig. 5.1 Differential cross section for $pp \to J/\psi + c\bar{c}$ as a function of the J/ψ transverse momentum P_T for the STAR kinematics $(\sqrt{s}=200~{\rm GeV},|y|\le 1.0)$

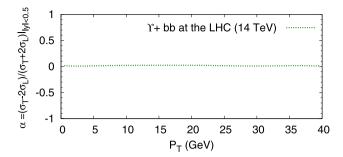


Fig. 5.2 Polarisation of an Υ produced in association with a $b\bar{b}$ pair at the LHC for $\sqrt{s}=14$ TeV for $|y|\leq 0.5$

For the CO transitions, associated production $\chi_c + c\bar{c}$ can occur via the process $gg \to gg$, for which the two final-state gluons split into a $c\bar{c}$ pair, one of them hadronising into a χ_c via the CO mechanism. This contribution is certainly suppressed up to $P_T \simeq 20$ GeV. For larger P_T , a dedicated calculation is needed. However, this mechanism would be very easily disentangled from the CS contributions, since both c quarks are necessarily emitted back to back to the χ_c and thus to the J/ψ .

Concerning the non-prompt signal, it would originate as usual from $gg \to b\bar{b}$, where one b quark hadronises in ψ . Usually, this hadronisation of the b produces the ψ with light quarks only. This means that we have one single c quark in the event. It is produced from the decay of the recoiling b quark and is therefore back to back to the ψ . The non-prompt signal would then be simply cut down by searching for a D meson near the ψ . Now it can happen that the hadronisation of the b produces the ψ and a D meson. In this case, kinematical cuts would not help to suppress the non-prompt yield. Fortunately, this is a priori suppressed compared to the first case and even more than the direct yield, since there is here no gain in the P_T dependence since both the $gg \to b\bar{b}$ and $gg \to \psi + c\bar{c}$ cross sections scale like P_T^{-4} . A cross check by sizing up the non-prompt yield with a displaced vertex measurement would be surely instructive, anyhow.

Let us also mention that associated production has also been studied in direct $\gamma\gamma$ collisions in ultra-peripheral collisions (UPC) [83]. At least for direct $\gamma\gamma$ collisions, associated production is the dominant contribution to the inclusive rate for $P_T \ge 2$ GeV/c.

To conclude, studies can be carried on by detecting either the "near" or "away" heavy quark with respect to the quarkonia. There are of course different ways to detect the D, B, or a b-jet, ranging from the use of a displaced vertex to the detection of their decay in e or μ . As discussed above, this has to be considered by also taking into account the different backgrounds. The forthcoming quarkonium-event generator Madonia 2 [84] will surely be of a great help to achieve this task. In any case, we hope that such measure-



ments would provide us with clear information on the mechanisms at work in quarkonium production⁶.

6 Conclusion

Recently, significant progresses have been made in the evaluation of the QCD corrections to quarkonium production. The situation sounds now rather clear for the *bottomonia*, where an agreement has eventually been obtained using only CS channels when dominant α_S^5 contributions are incorporated. The polarisation predictions for the latter cases seem also quite encouraging considering CDF [9] and $D\emptyset$ [53] measurements. Yet, confirmations are awaited for from the LHC.

On the other hand, those α_S^5 contributions could still be unable to bring agreement with the measured P_T -differential cross section of the direct *charmonia*. Dedicated further studies are however needed especially for the feed-down from P-waves, which is not known at NLO accuracy. In the charmonium case, we have also seen that the s-channel cut can bring a significant contribution to the cross section at low P_T and hence a first evaluation of the real part of the production amplitude is needed.

Additional tests are now undoubtedly needed beyond the *mere* measurements of inclusive cross sections and polarisation at the LHC. For instance, the hadroproduction of J/ψ or Υ with a heavy-quark pair [27, 82] appears to be a new valuable tool to separately probe the CS contribution, at least dominant at low P_T (below 15 GeV), as well as the study of the hadronic activity around the quarkonium.

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⁶Note also that NLO QCD corrections have recently been computed for the production of a J/ψ and Υ in association with a photon [85]. An experimental study of such process could be interesting as well.

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