Anisotropic universe with power law $f(R)$ gravity

M. Farasat Shamir*

Department of Humanities and Sciences,
National University of Computer and Emerging Sciences, Lahore Campus, Pakistan
**E-mail: farasat.shamir@nu.edu.pk*

This study highlights the dynamics of anisotropic universe in modified gravity. To meet this aim, locally rotationally symmetric Bianchi type I spacetime is studied in the metric theory of $f(R)$ gravity. Anisotropic fluid is considered to study the exact solutions of modified field equations. In particular, a general solution metric is reported using the well-known power law $f(R)$ gravity model. The graphical analysis of equation of state parameter is given which includes the corresponding values predicted for cosmic expansion. The energy conditions are also discussed for a range of specific model parameter. It is shown that anisotropic universe in modified gravity anticipate some interesting solutions and viable power law $f(R)$ gravity models can be reconstructed using suitable values of model parameter.

Keywords: Modified Gravity; $f(R)$ Gravity; Expansion of Universe.

1. Introduction

Sufficient experimental evidence in the recent decade have unveiled a mysterious picture of accelerated expansion of universe[1]. It is also believed that universe is currently exhibiting a transient phase of cosmic expansion with $\omega = -1$ and ultimately it will collapse sometime in the future[2]. This interesting phenomenon of late time acceleration and dark energy is somehow justified by modification of gravity. The $f(R)$ theory of gravity has been debated seriously with some fruitful results[3]. Interesting reviews[4] may be helpful for a better comprehension of the theory.

The involvement of a general function of Ricci scalar in the theory predicts many solutions of modified field equations as compared to ordinary general relativity. However, one can also recover already known solutions. The simplest example is the existence of de Sitter solution when dealing with the vacuum case and power law $f(R)$ gravity model[5]. Similarly, Einstein static universe does exist in $f(R)$ theory with barotropic perfect fluid[6]. Thus investigating modified gravity, in particular, using power law model with anisotropic background seems interesting. In this study, locally rotationally symmetric (LRS) Bianchi type I spacetime is investigated by considering anisotropic fluid and power law $f(R)$ gravity model.

2. Field Equations in $f(R)$ Gravity

The $f(R)$ gravity field equations are

$$f_R(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R(R) + g_{\mu\nu}\Box f_R(R) = \kappa T_{\mu\nu}, \tag{1}$$

where $f_R(R)$ is the derivative of $f(R)$ with respect to R and the other notations have their usual meanings. Moreover, we consider LRS Bianchi type I spacetime

$$ds^2 = dt^2 - X^2(t)dx^2 - Y^2(t)[dy^2 + dz^2], \tag{2}$$

and assume that the universe is filled with anisotropic fluid

$$T^{\mu}_{\nu} = diag[\rho, -p_x, -p_y, -p_z]. \tag{3}$$

We further characterize the anisotropic fluid as

$$T^{\mu}_{\nu} = diag[1, -\omega, -(\omega+\delta), -(\omega+\delta)]\rho, \tag{4}$$

where ω and δ are equation of state and skewness parameters respectively. Since $f(R)$ gravity field equations are highly non-linear in nature, so we adopt a conventional physical assumption of proportionality of shear scalar and expansion scalar which gives

$$X = Y^s, \qquad s \in \mathbb{R}. \tag{5}$$

Using, Eqs. (2), (4) and (5), the modified field equations simplify to

$$(2s+1)\frac{\dot{Y}^2}{Y^2}f_R - \frac{f}{2} - \tilde{Y}f_R + (s+2)\frac{\dot{Y}}{Y}\dot{f}_R = \kappa\rho, \tag{6}$$

$$-\left(\frac{2\ddot{Y}}{Y} + \frac{\dot{Y}^2}{Y^2}\right)f_R + \frac{f}{2} + \tilde{Y}f_R - \frac{2\dot{Y}}{Y}\dot{f}_R - \ddot{f}_R = \kappa\omega\rho, \tag{7}$$

$$-\left((s+1)\frac{\ddot{Y}}{Y} + s^2\frac{\dot{Y}^2}{Y^2}\right)f_R + \frac{f}{2} + \tilde{Y}f_R - (s+1)\frac{\dot{Y}}{Y}\dot{f}_R - \ddot{f}_R = \kappa(\omega+\delta)\rho, \tag{8}$$

where $\tilde{Y} \equiv (s+2)\frac{\ddot{Y}}{Y} + (s^2+s+1)\frac{\dot{Y}^2}{Y^2}$ and the over dot is to denote time derivative.

3. Dynamics of Anisotropic Universe with Power Law $f(R)$ Model

Now we consider the well known $f(R)$ gravity model as

$$f(R) = \zeta R^{r+1}, \qquad \zeta,\ r \in \mathbb{R} - \{0\}. \tag{9}$$

Differentiating Eq.(9) with respect to R, it follows that

$$f_R(R) = \zeta(r+1)R^r, \tag{10}$$

Without loss of any generality, we may choose $\zeta = \frac{1}{r+1}$. This simplifies the work for further analysis. Since the field equations (6)-(8) are still complicated even with the choice of power law model. Thus, a power law form of metric coefficient is chosen

$$Y(t) = \xi t^{\frac{1}{s+2}}, \quad \xi \in \mathbb{R} - \{0\}. \tag{11}$$

Using these settings, Eqs.(6)-(8) provide the corresponding expressions for energy density and pressure components

$$\rho = \frac{-1}{t^2(r+1)(s+2)^2}\left[\left(2rs^2+2r^2s^2-2s+4rs+8sr^2+8r^2+6r-1\right)\left(\frac{2(2s+1)}{t^2(s+2)^2}\right)^r\right], \tag{12}$$

$$p_x = \frac{-1}{t^2(r+1)(s+2)^2}\left[\left(2rs^2-2s+4r^3s^2+6r^2s^2+4rs+20sr^2+16r^3s+16r^2+16r^3-1\right)\left(\frac{2(2s+1)}{t^2(s+2)^2}\right)^r\right], \tag{13}$$

$$p_y = \frac{-1}{t^2(r+1)(s+2)^2}\left[\left(2rs-2s+4r^2s^2+4r^3s^2+16r^3s+18sr^2+4r+16r^3+20r^2-1\right)\left(\frac{2(2s+1)}{t^2(s+2)^2}\right)^r\right] = p_z. \tag{14}$$

Manipulating Eqs.(12)-(14), EoS and skewness parameters turn out to be

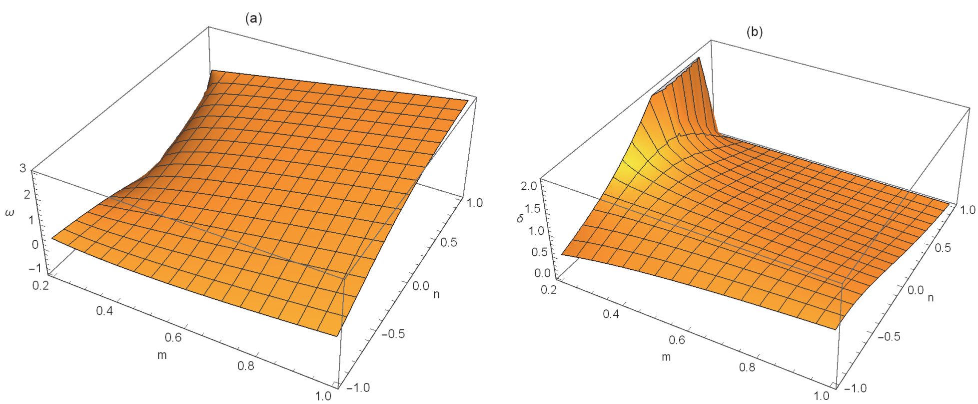

Fig. 1. Behavior of equation of state and skewness parameters

$$\omega = \frac{4r^3s^2+2s^2r+6r^2s^2-2s+20sr^2+16r^3s+4sr+16r^3+16r^2-1}{2r^2s^2+2s^2r+8sr^2+4sr-2s+8r^2+6r-1}, \tag{15}$$

$$\delta = -\frac{2r(r+1)(s+2)(s-1)}{2r^2s^2+2s^2r+8sr^2+4sr-2s+8r^2+6r-1}. \tag{16}$$

The behavior of these parameters is reflected in Fig. $(1a)$ and $(1b)$ respectively. It is worthwhile to notice that many possibilities of equation of state parameter exist. For example, an interesting case arise when $\omega=-1$ describing cosmic expansion[2,7,8].

In this case, explicit solutions of Eq.(15) turn out to be

$$r = \frac{-1 - s \pm \sqrt{3 + 6s + s^2}}{2(s+2)}, \tag{17}$$

and

$$s = \frac{-4r^2 - 3r + 1 \pm \sqrt{-5r^2 - 4r + 1}}{2r(r+1)}. \tag{18}$$

So in the light of above constraints, one can choose suitable power law $f(R)$ gravity model and the corresponding solution metric. In this case, a generic solution metric turns out to be

$$ds^2 = dt^2 - \xi^{2s} t^{\frac{2s}{s+2}} dx^2 - \xi^2 t^{\frac{2}{s+2}} (dy^2 + dz^2). \tag{19}$$

It is important to notice here that the anisotropy parameter involved in the above spacetime may also be used to reconstruct some well known solutions for physical relevance. For example, $s = -1/2$ gives the well known Kasner's universe and obviously this value corresponds to vacuum case (see Eqs.(12)-(14)).

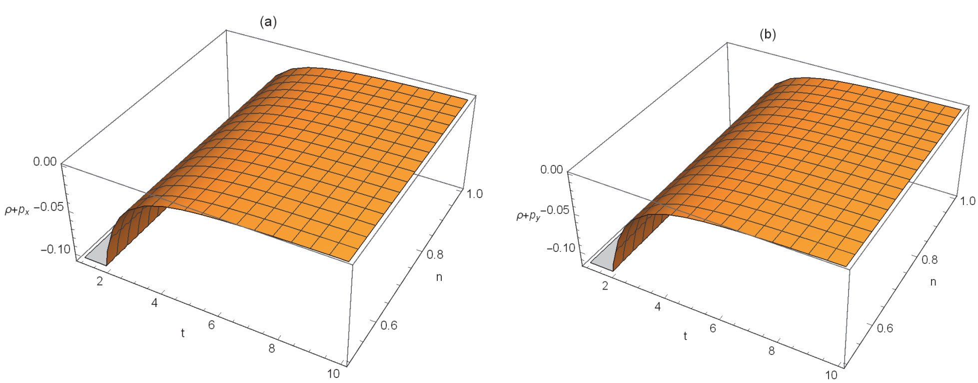

Fig. 2. Plots of NEC and WEC

3.1. *Power Law model and Energy Bounds*

The energy bounds have gained popularity in the modern day cosmology. It is important to mentation here that viability of cosmological models and some important theorems about black holes are associated with the energy bounds. The usual energy conditions namely null energy conditions (NEC), weak energy conditions (WEC), strong energy conditions (SEC) and dominant energy conditions (DEC) are defined as

$$\begin{aligned}
&\text{NEC:} \quad \rho + p_x \geq 0, \quad \rho + p_y \geq 0 \\
&\text{WEC:} \quad \rho \geq 0, \quad \rho + p_x \geq 0, \quad \rho + p_y \geq 0, \\
&\text{SEC:} \quad \rho + 3p_x \geq 0, \quad \rho + 3p_y \geq 0, \quad \rho + p_x \geq 0, \quad \rho + p_y \geq 0, \\
&\text{DEC:} \quad \rho \geq 0, \quad \rho \pm p_x \geq 0, \quad \rho \pm p_y \geq 0. \qquad (20)
\end{aligned}$$

The graphical behavior of these conditions is reflected in Figs. (2)-(4). It is mentioned here that we have fixed $\kappa = 1$ and the model parameter $r = 1/3$. Moreover, we choose the same range of anisotropy parameter s as in the case of EoS parameter plot (see Fig. (1)). Since these values corresponds to negative ω, so it is already anticipated that most of the energy bounds will not be satisfied. In particular, Fig. (3) depicts that SEC is violated. It is interesting as this violation seems to support an accelerating universe.

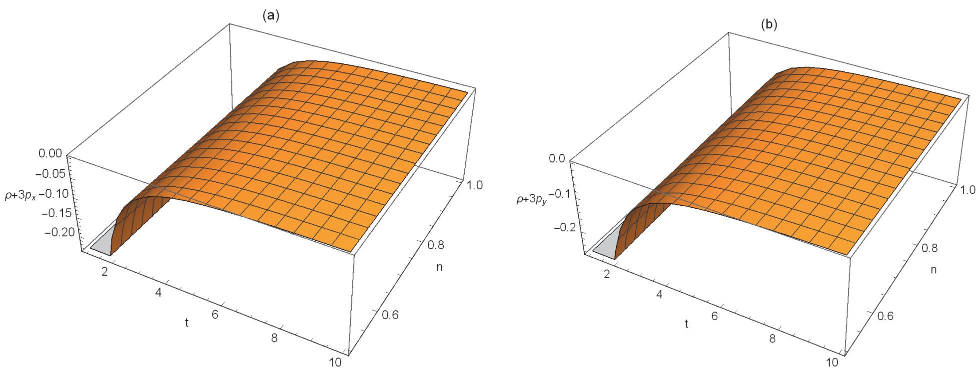

Fig. 3. Plots of SEC

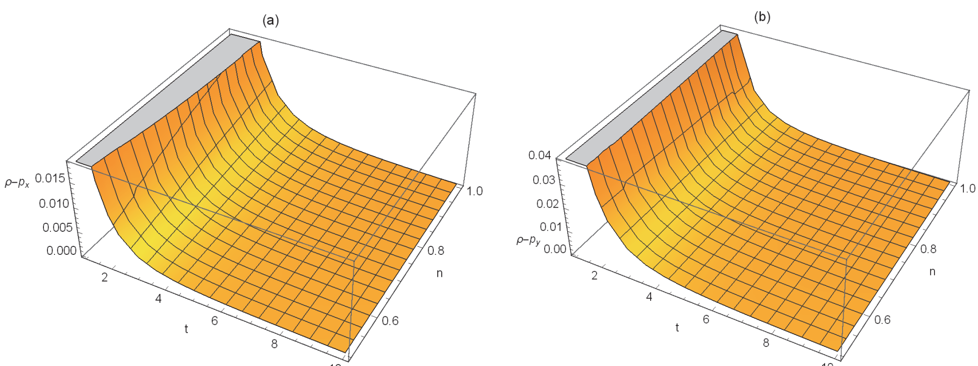

Fig. 4. Plots of DEC

4. Outlook

In the present work, we study the dynamics of anisotropic universe in modified gravity. LRS Bianchi type I spacetime is investigated in the metric theory of $f(R)$ gravity to meet this aim. Moreover, an anisotropic fluid is considered with deviation parameter to discuss the dynamics of modified field equations. A general solution metric is reported using the well-known power law $f(R)$ gravity model. In particular, explicit equation for EoS parameter is reported in terms of anisotropy and $f(R)$ model parameters. The graphical analysis of equation of state parameter is also given which includes $\omega = -1$ predicted for cosmic expansion. The energy conditions are also discussed for a range of specific model parameter. We have only discussed some limited choices and many other possibilities can also be explored. It is shown that anisotropic universe in modified gravity anticipate some interesting solutions and viable power law $f(R)$ gravity models can be reconstructed using suitable values of model parameter.

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