Proc. 24th International Spin Symposium (SPIN2021)
JPS Conf. Proc. **37**, 020302 (2022)
https://doi.org/10.7566/JPSCP.37.020302

Collinear Twist-3 Approach to Hyperon Polarization in SIDIS

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(Received February 7, 2022)

We derive the twist-3 cross section formula for the production of transversely polarized hyperons in semi-inclusive deep inelastic scattering in the framework of the collinear factorization. We present the cross section from the twist-3 distribution function and the twist-3 quark fragmentation functions in the leading order (LO) with respect to the QCD coupling constant for all five structure functions which have different dependences on the azimuthal angles. The present result is relevant to large-P_T hyperon production in the future Electron-Ion-Collider experiment.

KEYWORDS: hyperon polarization, twist-3, semi-inclusive deep inelastic scattering

1. Introduction

The description of transverse polarization of hyperons produced in unpolarized proton-proton collisions has been a big challenge in QCD spin physics, since a naive parton model or perturbative QCD at twist-2 gives almost zero polarization. In the collinear factorization, it occurs as a twist-3 observable reflecting multi-parton correlations in the nucleon or in the fragmenting processes. The twist-3 cross section for $pp \to \Lambda^{\uparrow} X$ has been completed recently in the leading order (LO) with respect to the QCD coupling constant [1–5]. This twist-3 mechanism also leads to hyperon polarizaton in semi-inclusive deep inelastic scattering (SIDIS), $ep \to e\Lambda^{\uparrow} X$, which we discuss in this talk. This process is of great interest in the future Electron-Ion-Collider (EIC) experiment.

There are two sources for the polarization: (i) Twist-3 distribution function (DF) in the initial nucleon combined with the twist-2 transversity fragmentation function (FF) for $\Lambda^{\uparrow}$, both being chiral-odd, and (ii) Twist-3 FF for $\Lambda^{\uparrow}$ combined with the usual twist-2 unpolarized DFs, both being chiral-even. In (ii), two types of twist-3 FFs contribute, i.e., (a) quark FFs and (b) purely gluon FFs. In this talk we present the twist-3 cross section from (i) and (ii)(a) [6]. The contribution from (ii)(b) is reported in [7].

2. Twist-3 DFs and FFs for $ep \to e\Lambda^\uparrow X$

Here we give a list of DFs and FFs relevant to our study. For the twist-3 DFs in the unpolarized nucleon, we need only one function $E_F(x_1, x_2)$ defined by

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p|\bar{\psi}_j(0)[0, \mu n] g F^{\alpha n}(\mu n)[\mu n, \lambda n]\psi_i(\lambda n)|p\rangle$$
$$= -\frac{M_N}{4} \epsilon^{\alpha\beta n p}(\gamma_5 \gamma_\beta \not{p})_{ij} E_F(x_1, x_2) + \cdots , \tag{1}$$

where p is the nucleon momentum which can be regarded as lightlike and n is another lightlike vector satisfying $p \cdot n = 1$. M_N is the nucleon mass and $[\mu n, \lambda n]$ is the gauge link operator which makes the correlation function gauge-invariant. We use the convention for the ϵ-tensor as $\epsilon^{0123} = 1$.

The twist-2 transversity FF $H_1(z)$ for $\Lambda^\uparrow$ and the twist-3 intrinsic FFs are defined as [8]

$$\frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[\infty w, 0]\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle$$
$$= \left(\gamma_5 \not{S}_\perp \frac{\not{P}_h}{z}\right)_{ij} H_1(z) + M_h \epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij} \frac{D_T(z)}{z} + M_h(\gamma_5 \not{S}_\perp)_{ij} \frac{G_T(z)}{z} + \cdots , \tag{2}$$

where $N = 3$ is the number of the color SU(N), $|h(P_h, S_\perp)\rangle$ denotes the hyperon state with mas M_h, momentum P_h and the transverse spin vector $S_\perp$ normalized as $S_\perp^2 = -1$, and w is another lightlike vector satisfying $P_h \cdot w = 1$. The twist-3 kinematical FFs are defined as

$$\frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[\infty w, 0]\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle \overleftarrow{\partial}^\alpha$$
$$= -iM_h \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij} \frac{D_{1T}^{\perp(1)}(z)}{z} + iM_h S_\perp^\alpha (\gamma_5 \not{P}_h)_{ij} \frac{G_{1T}^{\perp(1)}(z)}{z} + \cdots , \tag{3}$$

and the dynamical ones are defined as

$$\frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z} - \frac{1}{z_1})} \langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w) g F^{\alpha w}(\mu w)|0\rangle$$
$$= M_h \epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij} \frac{\widehat{D}_{FT}^*(z, z_1)}{z} - iM_h S_\perp^\alpha (\gamma_5 \not{P}_h)_{ij} \frac{\widehat{G}_{FT}^*(z, z_1)}{z} + \cdots , \tag{4}$$

where the gauge link is suppressed for simplicity. The dynamical FFs $\widehat{D}_{FT}(z, z_1)$ and $\widehat{G}_{FT}(z, z_1)$ are complex functions and their complex conjugates are defined in (4). Although these three types of twist-3 FFs shown in (2), (3) and (4) are necessary to derive the cross section, they are not independent but are related by the QCD equation-of-motion relation (EOM relation) and the Lorentz invariance relation (LIR). The EOM relation involving the "T-odd" FFs is given by

$$\int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{1/z - 1/z_1} \left(\Im \widehat{D}_{FT}(z, z_1) - \Im \widehat{G}_{FT}(z, z_1)\right) = \frac{D_T(z)}{z} + D_{1T}^{\perp(1)}(z), \tag{5}$$

and the LIR reads

$$-\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\Im \widehat{D}_{FT}(z, z_1)}{(1/z_1 - 1/z)^2} = \frac{D_T(z)}{z} + \frac{d\left(D_{1T}^{\perp(1)}(z)/z\right)}{d(1/z)}. \tag{6}$$

These relations play crucial roles to guarantee the gauge invariance and the frame independence of various twist-3 cross sections (See, for example, [8]).

3. Twist-3 cross section for $ep \to e\Lambda^{\uparrow}X$

3.1 Kinematics

We consider the process

$$e(\ell) + p(p) \to e(\ell') + \Lambda^{\uparrow}(P_h, S_\perp) + X, \tag{7}$$

which has five Lorentz invariants: $S_{ep} = (p+\ell)^2$, $x_{bj} = \frac{Q^2}{2p\cdot q}$, $Q^2 = -q^2 = -(\ell-\ell')^2$, $z_f = \frac{p\cdot P_h}{p\cdot q}$, $q_T = \sqrt{-q_t^2}$, with the space-like four momentum $q_t^\mu = q^\mu - \frac{P_h\cdot q}{p\cdot P_h}p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu$ satisfying $q_t\cdot p = q_t\cdot P_h = 0$. To derive the cross section for (7), we work in the hadron frame where the virtual photon momentum $\vec{q}$ and $\vec{p}$ are collinear as shown in Fig. 1. In this frame q and p can be

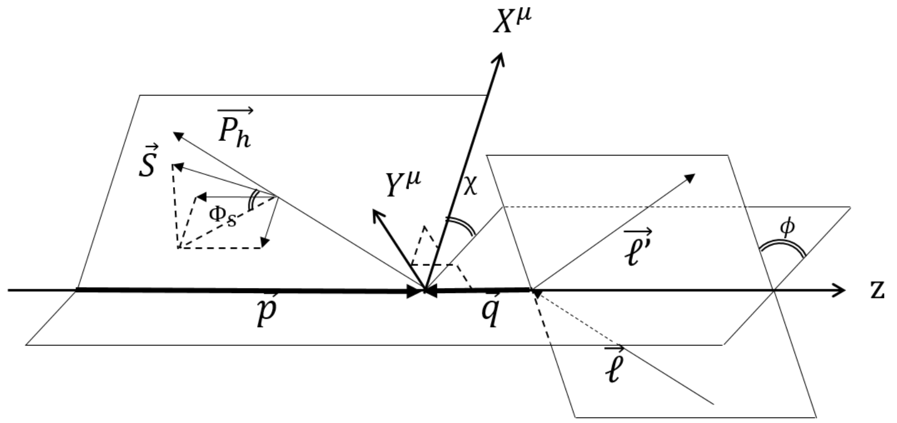

Fig. 1. Hadron frame in which $\vec{q}$ and $\vec{p}$ are collinear. Azimuthal angles for the lepton plane and the hadron plane are, respectively, ϕ and χ, and the azimuthal angle of the transverse spin vector of $\Lambda^{\uparrow}$ measured from the hadron plane is Φ_S.

written as $q^\mu = (q^0, \vec{q}) = (0,0,0,-Q)$ and $p^\mu = \left(\frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}}\right)$. We define the azimuthal angles of the hadron plane and the lepton plane as χ and ϕ, respectively. Then P_h^μ can be written as $P_h^\mu = \frac{z_f Q}{2}\left(1+\frac{q_T^2}{Q^2}, \frac{2q_T}{Q}\cos\chi, \frac{2q_T}{Q}\sin\chi, -1+\frac{q_T^2}{Q^2}\right)$. The lepton momenta ℓ and ℓ' are, respectively, $\ell^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1)$ and $\ell'^\mu = \ell^\mu - q^\mu$ with $\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1$. For convenience, we introduce 4 vectors as $T^\mu = \frac{1}{Q}(q^\mu + 2x_{bj}p^\mu) = (1,0,0,0)$, $X^\mu = \frac{1}{q_T}\left\{\frac{P_h^\mu}{z_f} - q^\mu - (1+\frac{q_T^2}{Q^2})x_{bj}p^\mu\right\} = (0, \cos\chi, \sin\chi, 0)$, $Z^\mu = -\frac{q^\mu}{Q} = (0,0,0,1)$, $Y^\mu = \epsilon^{\mu\nu\rho\sigma}T_\nu X_\rho Z_\sigma = (0, -\sin\chi, \cos\chi, 0)$ which are orthogonal to each other. Then the XZ-plane becomes the hadron plane and the transverse spin vector $S_\perp^\mu$ can be written as

$$S_\perp^\mu = \cos\theta\cos\Phi_S\, X^\mu + \sin\Phi_S\, Y^\mu - \sin\theta\cos\Phi_S\, Z^\mu, \tag{8}$$

where θ is the polar angle of $\vec{P}_h$ measured from the z-axis which can be written as $\tan\theta = 2q_T Q/(q_T^2 - Q^2)$, and Φ_S is the azimuthal angle of $\vec{S}_\perp$ around $\vec{P}_h$ measured from the hadron plane (See Fig. 1). With these kinematic variables, the differential cross section for (7) can be written as

$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2 z_f}{128\pi^4 S_{ep}^2 x_{bj}^2 Q^2} L^{\mu\nu}(\ell,\ell')W_{\mu\nu}(p,q,P_h), \tag{9}$$

where $\alpha_{em} = e^2/(4\pi)$ is the fine structure constant in QED, $W_{\mu\nu}$ is the hadronic tensor and $L^{\mu\nu}$ is the unpolarized leptonic tensor defined by $L^{\mu\nu}(\ell,\ell') = 2(\ell^\mu \ell'^\nu + \ell^\nu \ell'^\mu) - Q^2 g^{\mu\nu}$.

3.2 *Twist-3 DF contribution*

The hyperon polarization for (7) can occur from the twist-3 DF $E_F(x_1, x_2)$ in (1) and the transversity FF $H_1(z)$ for $\Lambda^\uparrow$ in (2). The corresponding partonic cross section occurs from a pole of an internal propagator in the hard part, which can be classified into the hard pole (HP) and the soft-gluon-pole (SGP). The method of the calculation is explained in [9] in great detail. We thus present the final result below.

The HP contribution is obtained as

$$\frac{d^6\Delta\sigma^{\rm HP}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s}{16\pi^2 S_{ep}^2 x_{bj}^2 Q^2}\left(\frac{-\pi M_N}{4}\right)\int\frac{dz}{z}H_1(z)$$
$$\times\int\frac{dx}{x}\left[\frac{2}{1-\hat{x}}E_F\left(x_{bj},x\right)\left(\frac{4}{Nq_T}-\frac{4NQ^2(\hat{x}-1)}{q_T^3\hat{x}}\right)\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}\right.$$
$$+E_F(x_{bj},x_{bj}-x)\left(\frac{-1}{N}\right)\left\{\frac{8\hat{x}}{\hat{z}q_T}(1+\cosh^2\psi)\sin\Phi_S+\frac{8(1-\hat{x})Q}{\hat{z}q_T^2}\sinh 2\psi\sin(\Phi_s+\phi-\chi)\right.$$
$$\left.\left.+\frac{8(1-\hat{x})^2Q^2}{\hat{x}\hat{z}q_T^3}\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}\right\}\right]\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right), \tag{10}$$

where $\hat{x}\equiv x_{bj}/x$ and $\hat{z}\equiv z_f/z$, $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, and the summation over quark flavors and the factor for their electric charges is omitted. The SGP contribution is given by

$$\frac{d^6\Delta\sigma^{\rm SGP}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi}$$
$$=\frac{\alpha_{em}^2\alpha_S}{16\pi^2 S_{ep}^2 x_{bj}^2 Q^2}\left(\frac{-\pi M_N}{4}\right)\frac{q_T}{Q^2}\int\frac{dz}{z}H_1(z)\int\frac{dx}{x}\left(\frac{-1}{2N}\right)$$
$$\times\left[-8(1+\cosh^2\psi)\sin\Phi_S\left\{\frac{2\hat{x}}{1-\hat{z}}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)+\left(\frac{(1+\hat{z})Q^2}{\hat{z}q_T^2}+\frac{2\hat{x}}{1-\hat{x}}\right)E_F(x,x)\right\}\right.$$
$$+8\sinh 2\psi\sin(\Phi_S+\phi-\chi)\left\{\frac{2\hat{x}Q}{(1-\hat{z})q_T}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)-\frac{Q}{2q_T}\left(\frac{Q^2}{q_T^2}+1\right)E_F(x,x)\right\}$$
$$-8\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}\left\{\frac{2\hat{x}Q^2}{(1-\hat{z})q_T^2}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)\right.$$
$$\left.\left.-\left(\frac{3Q^4}{\hat{z}q_T^4}+\frac{Q^2}{q_T^2}\right)E_F(x,x)\right\}\right]\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right). \tag{11}$$

The twist-3 DF contribution is given by the sum of (10) and (11).

3.3 *Twist-3 FF contribution*

In this subsection we present the cross section from the twist-3 FFs given in Sec. 2, combined with the twist-2 unpolarized quark or gluon DFs $f_1(x)$. As opposed to the twist-3 DF case, the twist-3 FF contribution occurs as a nonpole one. Following the formalism developed in [10], we have obtained the cross section in the following form:

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi}$$

$$= \frac{\alpha_{em}^2 \alpha_s(-M_h)}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2} \sum_{k=1}^{9} \mathcal{A}_k(\phi - \chi) \mathcal{S}_k \int \frac{dx}{x} f_1(x) \int \frac{dz}{z} \delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right)\right)$$
$$\times \left[\frac{D_T(z)}{z} \hat{\sigma}_T^k - \left\{ \frac{d}{d(1/z)} \frac{D_{1T}^{\perp(1)}(z)}{z} \right\} \hat{\sigma}_{\perp D}^k - D_{1T}^{\perp(1)}(z) \hat{\sigma}_{\perp}^k \right.$$
$$+ \int \frac{dz'}{z'^2} \mathrm{P}\left(\frac{1}{1/z - 1/z'}\right) \left\{ \Im \widehat{D}_{FT}(z, z') \left[\frac{z'}{z} \hat{\sigma}_{DF1}^k + \frac{1}{z} \left(\frac{1}{z'} - \frac{1}{\left(1 - q_T^2/Q^2\right) z_f} \right)^{-1} \hat{\sigma}_{DF2}^k \right] \right.$$
$$\left. \left. + \Im \widehat{G}_{FT}(z, z') \left[\frac{z'}{z} \hat{\sigma}_{GF1}^k + \frac{1}{z} \left(\frac{1}{z'} - \frac{1}{\left(1 - q_T^2/Q^2\right) z_f} \right)^{-1} \hat{\sigma}_{GF2}^k \right] \right\} \right]. \tag{12}$$

where the sum over k runs $k = 1, \cdots, 4, 8, 9$, $\mathcal{A}_k$ is given by $\mathcal{A}_1(\varphi) = 1 + \cosh^2\psi$, $\mathcal{A}_2(\varphi) = -2$, $\mathcal{A}_3(\varphi) = -\cos\varphi \sinh 2\psi$, $\mathcal{A}_4(\varphi) = \cos 2\varphi \sinh^2\psi$, $\mathcal{A}_8(\varphi) = -\sin\varphi \sinh 2\psi$, $\mathcal{A}_9(\varphi) = \sin 2\varphi \sinh^2\psi$, and $\mathcal{S}_{1,2,3,4} = \sin\Phi_s$, $\mathcal{S}_{8,9} = \cos\Phi_S$. The partonic hard cross sections $\hat{\sigma}_T^k$, $\hat{\sigma}_{\perp D}^k$, $\hat{\sigma}_{\perp}^k$, $\hat{\sigma}_{DF1}^k$, $\hat{\sigma}_{DF2}^k$, $\hat{\sigma}_{GF1}^k$, $\hat{\sigma}_{GF2}^k$ are functions of $\hat{x}$, $\hat{z}$, q_T, Q and color factors, which are listed in [6]. To reach (12), we have used the EOM relation (5) and the LIR (6).

4. Summary

In this talk, we have presented the LO twist-3 cross section for the transversely polarized hyperon production in SIDIS, $ep \to e\Lambda^{\uparrow}X$, in the collinear twist-3 factorization. Contribution from the twist-3 DF and the twist-3 quark FFs are taken into account. The cross section can be decomposed into five components which have different dependence on the azimuthal angles of the lepton plane, hadron plane and the spin vector. This study is relevant to large-P_T hyperon production in the future EIC experiment. The detail of the calculation will be reported elsewhere [6].

Acknowledgments

This work has been supported by the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract Nos. 19K03843 (Y.K.) and 18J11148 (K.Y.), National Natural Science Foundation in China under grant No. 11950410495, Guangdong Natural Science Foundation under No. 2020A1515010794 and research startup funding at South China Normal University.

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