Exploring the Quasi-Universal Relation in Neutron Star Properties Through Symbolic Regression

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Introduction

Neutron stars (NSs) are the densest objects in the universe, containing more mass than the Sun within a radius of about 10 km. They offer a unique opportunity to study the physics of extremely dense matter. The study of neutron stars and their impact on understanding dense matter has driven innovative research across various fields through collaborative and interdisciplinary efforts.

The macroscopic properties of neutron stars, such as their masses, radii, and tidal deformabilities, are highly influenced by the nuclear microphysics of their interiors. However, certain combinations of these properties are remarkably insensitive to the internal structure. While individual neutron star properties depend on the equation of state of the extreme matter, some combinations remain approximately independent of it. These nearly equation-of-state-independent relations are known as approximate universal relations. One example is the I-Love-Q relations, which connect the moment of inertia (I), tidal deformability (or Love number, Λ), and rotational quadrupole moment (Q). This universality allows for the conversion of measurements, such as a neutron star's tidal deformability, into estimates of its moment of inertia with high precision. For this analysis, we use the realistic equation of state from CompOSE [1].

Equation of State

The equations of state (EOSs) in the CompOSE database [1] are publicly available as part of a free online repository, initially created during the NewCompStar COST Action. This repository is continuously being updated and improved. The EOSs from this database are used in simulations of core-collapse supernovae and binary neutron star mergers. The database provides thermodynamic properties, compositional details, and microscopic information. It includes a variety of EOSs with different particle compositions, such as leptons, baryons, mesons, and quarks, as well as hybrid EOSs with phase transitions. For our calculations, we randomly selected 100 relativistic mean-field (RMF) equations of state from this database.

Results

We examine three universal relations: $I-\Lambda$, $C-\Lambda$, and $C-I$, using symbolic regression, a machine learning algorithm. We extract characteristic vectors $\mathbf{X}[\mathbf{n}]$ (Λ) and target vectors $\mathbf{Y}[\mathbf{n'}]$ (I, C) from the dataset, then randomly select a subset of features and targets to input into the GPlearn algorithm. The best equations are identified based on the correlation coefficient (r) and relative error (RE) with the original data. The resulting optimal equations are as follows:

$$log_{10}\bar{I} = \sum_{0}^{4} a_n (log_{10}\Lambda)^n \quad (1)$$

$$C = \sum_{0}^{3} b_n (log_{10}\Lambda)^n \quad (2)$$

$$C = \sum_{0}^{4} c_n (log_{10}\bar{I})^{-n} \quad (3)$$

Where $\bar{I} = I/M^3$ is the dimensionless moment of inertia. The fitting coefficients (a_n, b_n, and c_n) of Eqs.(1-3) are listed in Table-I.

In Figure-1, the left panel shows the dimensionless moment of inertia ($\bar{I}$) versus the dimensionless

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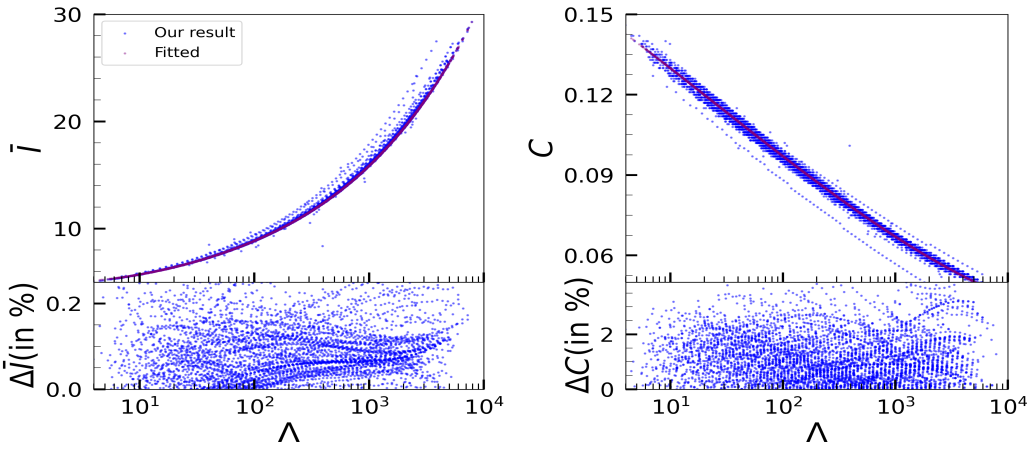

FIG. 1: *Left panel:* $\bar{I} - \Lambda$ relation. The purple shows the fitted values using Eq. (1). The lower panel shows the percentage error obtained from these fittings. Where $\Delta\bar{I} = \frac{|\bar{I}-\bar{I}_{fit}|}{\bar{I}} \times 100$.*Right panel:*Same as left panel but for $C - \Lambda$ relation.

TABLE I: The fitting coefficients a_n, b_n, and c_n from Eqs. 1 to 3 for the $I - \Lambda$, $C - \Lambda$, and $C - I$ relations. Pearson's correlation coefficients (r) and relative errors (RE) are also provided.

	$I - \Lambda$		$C - \Lambda$		$C - I$
$a_0(10^{-1})$	14.778	$b_0(10^{-2})$	15.811	c_0	-0.0801
$a_1(10^{-2})$	7.630	$b_1(10^{-2})$	-1.120	c_1	1.6188
$a_2(10^{-2})$	1.684	$b_2(10^{-4})$	-8.241	c_2	-5.2936
$a_3(10^{-5})$	8.689	$b_3(10^{-4})$	7.621	c_3	8.4384
$a_4(10^{-5})$	-3.275	b_4	...	c_4	-5.2460
r	0.99		0.99		0.99
RE (%)	0.1		1.0		1.1

tidal deformability (Λ) in blue colour. We fitted $\bar{I}$ to Λ using Eq. (1) and displayed the results in light blue. The fitting function coefficients are listed in Table I, and they closely match those reported in the literature with the same relations [2]. The lower part of the left panel shows the fit error ($\Delta\bar{I}$) as a function of Λ, which stays below 0.5%, confirming the universality of this relation across various EOS compositions. The right panel displays the compactness versus tidal deformability, with fitted curves based on Eq. (2). As shown in the lower part of the right panel, the fit error (ΔC) is less than 2.5%.

Conclusion & Summary

We demonstrated the universal I-Love-Q relations using a symbolic regression method with data from the CompOSE database. Our results closely match those from previous studies, with nearly identical coefficients. The fit error in all cases is less than 2%.

As we enter a new era of exploration, powered by advanced tools like symbolic regression and machine learning, the future holds vast potential for discovery. This approach allows us to efficiently tackle computationally intensive tasks, such as inferring key nuclear matter parameters directly from astrophysical observations.

References

[1] S. Typel *et al.*, [arXiv:1307.5715] (2015)
[2] K., Bharat *et al.*, [arXiv:1902.04557](2019).