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Double-Spin Asymmetry $A_{LT}^{\cos\phi_S}$ in the Lambda Production SIDIS Process Within the Collinear Framework

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Abstract

We study the longitudinal–transverse double-spin asymmetry $A_{LT}^{\cos\phi_S}$ in semi-inclusive deep inelastic scattering (SIDIS) with Λ hyperon production, where a longitudinally polarized beam scatters off a transversely polarized proton target. After integrating over the final-state hadron transverse momentum, the asymmetry contributes in two parts: the convolution of the twist-3 distribution function $g_T(x)$ of the proton target with the unpolarized fragmentation function $D_1(z)$ for Λ, and the convolution of the transversity distribution $h_1(x)$ of the proton target with the twist-3 fragmentation function $\tilde{E}(z)$ for Λ. We present predictions for the $\cos\phi_S$ asymmetry in CLAS12, COMPASS, and EicC kinematical regions. The numerical results are sizeable. In particular, $\tilde{E}(z)$ dominates in the large-z region, suggesting that measuring the $A_{LT}^{\cos\phi_S}$ asymmetry will offer a promising way to access the twist-3 fragmentation function of the Λ hyperon as well as the flavor separation of the distribution functions.

Keywords: semi-inclusive deep inelastic scattering (SIDIS); double-spin asymmetry; Λ hyperon production; twist-3 distribution; fragmentation functions

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1. Introduction

Over the past three decades, studies both from experimental and theoretical perspectives on the azimuthal asymmetries in the semi-inclusive deep inelastic scattering (SIDIS) process have been carried out [1–8], providing crucial data and theoretical tools to describe the internal structure inside the nucleon. The differential cross-section in the SIDIS process can be described by series of convolutions of transverse momentum-dependent (TMD) parton distribution functions (PDFs) and fragmentation functions (FFs) [1,3,9] according to the TMD factorization theorem. Despite the TMD PDFs and FFs, higher-twist PDFs and FFs reflect the physics of the unexplored quark–gluon–quark correlations that provide direct and unique insights into the QCD dynamics.

One of the most important twist-3 distribution functions is the twist-3 PDF $g_T(x)$, which is related to the DIS structure function g_2 [10] and consists of a Wandzura–Wilczek (WW) part and the genuine twist-3 piece $\tilde{g}_T(x)$. The Mellin moment $\int x^2\tilde{g}_T(x)dx$ describes the transverse impulse the active quark acquires after being struck by the virtual photon due to the color Lorentz force [11]. The clearest example of higher-twist $g_T(x)$ is defined in a collinear framework and accessible in an inclusive DIS process for the collision of a longitudinally polarized electron beam and a transversely polarized target, $\vec{e}p^\uparrow \to e'X$. Factorization theorems lead to the introduction of a collinear twist-3 PDF $g_T(x)$ [10]. Higher-twist contributions to spin-dependent DIS structure functions g_1 and g_2 were carried out by the JAM Collaboration [12], allowing the extraction of $g_T(x)$ from the longitudinal–transverse double-spin asymmetry $A_{LT} \sim (g1 + g2) = g_T$. The derivation of the inclusive cross-section in collinear twist-3 factorization establishes a rigorous connection between A_{LT} and the twist-3 PDF $g_T(x)$ [13–15], which in turn makes the structure function $g_T(x)$ the most prominent observable for studying multi-parton correlations.

Ref. [16] extended the process to the SIDIS process and performed the phenomenological calculation of the longitudinal–transverse double-spin asymmetry with a $\cos\phi_S$ modulation for the charged and neutral pion production process with the transverse momentum of the final state hadron integrated out. This extension allows us to study the twist-3 distribution function $g_T(x)$ and the fragmentation function $\tilde{E}$ simultaneously. The prediction shows that the asymmetry of pion production in the kinematical region of CLAS12 is sizeable and that the twist-3 FF can play an important role in the asymmetry in the large-z region [16].

In this work, we will consider the case of a double-polarized SIDIS with Λ production, which makes the flavor separation and the contribution of sea quarks explicit. Up to twist-3 level, there are three spin or azimuthal asymmetries arising, namely, the modulations of $\cos(\phi_h - \phi_S)$, $\cos\phi_S$ and $\cos(2\phi_h - \phi_S)$, where ϕ_h and ϕ_S are the azimuthal angles of the final-state hadron and transverse spin of the nucleon. Among these three azimuthal asymmetries, the $\cos(\phi_h - \phi_S)$ asymmetry is a leading twist observable contributed by the TMD PDF g_{1T}, and has been studied in both models and experiments [17–20]. The other two double spin asymmetries appear in the sub-leading order of $1/Q$ expansion. As proven in Ref. [1], under the tree-level TMD framework, each of the two double-spin asymmetries can be interpreted as the convolution of twist-3 TMD PDFs and FFs that are coupled with the twist-2 FFs and PDFs. The roles of the twist-3 TMD PDFs on the $\cos(2\phi_h - \phi_S)$ and $\cos\phi_S$ asymmetries were studied in Ref. [21] via spectator model calculations.

As different contributions mix together in the asymmetries at the twist-3 level, it is difficult to disentangle those twist-3 PDFs and FFs by SIDIS measurement. The EIC Yellow Book [22] suggests that the collinear framework (integrating out the hadron transverse momentum) provides a cleaner environment and thus is a golden channel for twist-3 studies. Therefore, in this work we will resort to the collinear case of integrating (or not measuring) the transverse momentum of the final-state hadron. Under this assumption, only the $\cos\phi_S$ asymmetry remains [16]; the other two asymmetries vanish after the transverse momentum integration. The asymmetry has two contributions: the first term is the convolution of the twist-3 PDF $g_T^q(x)$ and the unpolarized FF $D_1^q(z)$, and the second term is the convolution of the transversity distribution function $h_1^q(x)$ and the twist-3 chiral-odd FF $\tilde{E}(z)$. We discuss the possibility of obtaining the twist-3 PDFs and FFs through the $\cos\phi_S$ asymmetry in the double-polarized SIDIS.

Although past experimental data and theoretical analyses have provided us with information on fragmentation functions for pion and kaon mesons, our knowledge of fragmentation functions Λ, particularly the polarized fragmentation function and the higher-twist fragmentation function, is much more limited. In this work, we investigate the double-polarized SIDIS process with final-state production of Λ, which will help us to understand the fragmentation function of Λ. Meanwhile, we will consider the effect of FF $\tilde{E}^q(z)$ that encodes the quark–gluon–quark correlation during fragmentation, and is thus the twist-3 effect of the fragmentation function, which has not been studied in the previous literature. The equation of motion relation will be utilized between the studied twist-3 FF $\tilde{E}^q(z)$ and the ordinary unpolarized fragmentation function, with the chiral quark model [23,24] adopted, to find an approximate relationship between $\tilde{E}^q(z)$ and $D_1(z)$. $D_1(z)$ will be obtained through the spectator diquark model [25]. We will consider the contributions of Wandzura and Wilczek [26] and the genuine twist-3 to the distribution

$g_T^q(x)$ [27] of valence quarks, as well as the distribution function $g_T^q(x)$ of sea quarks, which is contributed to solely by the WW approximation. Double-polarized SIDIS can be achieved in the CLAS12 experiment, which will soon be operational at JLab. We predict the x- and z-dependent $\cos\phi_S$ asymmetries within the kinematical regions of CLAS12, COMPASS and EicC, including the contribution of sea quarks. Moreover, we also consider the energy scale dependence of the PDFs and FFs involved in the asymmetry.

The remainder of this paper is as follows. In Section 2, we establish the theoretical formalism of the $\cos\phi_S$ asymmetry in Λ production from a longitudinally polarized lepton beam scattering off a transversely polarized nucleon target in the collinear picture. In Section 3, we numerically estimate the asymmetries considering the contribution of sea quarks within the kinematical regions of CLAS12, COMPASS and EicC. In Section 4, we summarize our work and present the conclusion.

2. Formalism of the $\cos\phi_S$ Asymmetry in the SIDIS Process

In this section, following the theoretical framework in Ref. [16], we will briefly present the formalism of the double-spin asymmetry $A_{LT}^{\cos\phi_S}$ in the SIDIS production of Λ hyperons, where a longitudinally polarized lepton beam (superscript right arrow) scatters off a transversely polarized proton target (superscript up arrow):

$$l^{\rightarrow}(\ell) + N^{\uparrow}(P) \longrightarrow l(\ell') + \Lambda(P_h) + X(P_X). \tag{1}$$

In the process, the four momenta of the incoming and outgoing leptons, the nucleon target, the produced Λ hyperon and the unobserved state X are represented by ℓ, ℓ', P, P_h and P_X, respectively. The masses of the proton target and the final-state Λ hyperon are denoted by M and M_h. According to the Trento conventions [28], the photon–nucleon center-of-mass frame is shown in Figure 1, with the z-axis defined by the virtual photon momentum, ϕ_h being the azimuthal angle of the hyperon momentum relative to the lepton scattering plane and ϕ_S being the azimuthal angle of the target's transverse spin.

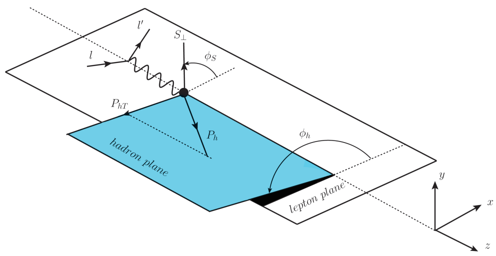

Figure 1. The reference frame in the SIDIS process. $S_\perp$ is the transverse component of the spin vector S with respect to the virtual photon momentum.

As usual, the Lorentz invariants are defined to express the observables:

$$\begin{aligned} x &= \frac{Q^2}{2P\cdot q}, \text{ Bjorken scaling variable;} \\ y &= \frac{P\cdot q}{P\cdot l}, \text{ inelasticity (the lepton energy momentum transfer fraction);} \\ z &= \frac{P\cdot P_h}{P\cdot q}, \text{ longitudinal momentum fraction of the final hadron to the parent quark.} \end{aligned} \tag{2}$$

The momentum of the exchanged virtual photon is $q = \ell - \ell'$, with the energy scale of the process being $Q^2 = -q^2$, $\gamma = \frac{2Mx}{Q}$. $s = (P+l)^2$ is the total center-of-mass energy squared.

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Under the one-photon-exchange approximation, the differential cross-section for the double-polarized SIDIS process can be written as 18 terms of convolution between the TMD PDFs and TMD FFs [1]. In this work, we will focus on the double-polarized process with the longitudinally polarized lepton beam and transversely polarized proton target. After integrating over the transverse momentum of the outgoing Λ hyperon, the differential cross-section can be written as

$$\frac{d^4\sigma}{dxdydzd\phi_S} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right) \times \left(F_{UU}(x,z) + |S_\perp|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}(x,z)\right). \tag{3}$$

with α being the electromagnetic fine structure constant, $S_\perp$ being the transverse spin vector of the target nucleon and λ_e being the helicity of the lepton beam. The ratio of the longitudinal and transverse photon flux ε is defined as

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}. \tag{4}$$

The depolarization factors can be approximated as [1]

$$\frac{y^2}{2(1-\varepsilon)} = \frac{1}{1+\gamma^2}\left(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2\right) \approx \left(1-y+\frac{1}{2}y^2\right), \tag{5}$$

$$\frac{y^2}{2(1-\varepsilon)}\sqrt{2\varepsilon(1-\varepsilon)} = \frac{1}{\sqrt{1+\gamma^2}}y\sqrt{1-y-\frac{1}{4}\gamma^2y^2} \quad \approx y\sqrt{1-y}. \tag{6}$$

F_{UU} is the unpolarized structure function:

$$F_{UU}(x,z) = x\sum_q e_q^2 f_1^q(x)D_1^q(z), \tag{7}$$

with $f_1^q(x)$ and $D_1^q(z)$ being the unpolarized PDF and FF, respectively. The polarized structure function $F_{LT}^{\cos\phi_S}(x,z)$ is the collinear counterpart of the original structure function, obtained by integrating $F_{LT}^{\cos\phi_S}(x,z,P_{hT})$ over the transverse momentum of the hadron, which can be written as [1]

$$\begin{aligned} F_{LT}^{\cos\phi_S}(x,z) &= \int d^2\boldsymbol{P}_{hT} F_{LT}^{\cos\phi_S}(x,z,P_{hT}) \\ &= -x\sum_q e_q^2\frac{2M}{Q}\left(xg_T^q(x)D_1^q(z) + \frac{M_h}{M}h_1^q(x)\frac{\tilde{E}^q(z)}{z}\right). \end{aligned} \tag{8}$$

Equation (8) contains two contributions: the convolution of the twist-3 distribution function $g_T^q(x)$ and the twist-2 FF $D_1^q(z)$ (which we identified as the twist-3 PDF term) and the convolution of twist-2 transversity $h_1^q(x)$ and the twist-3 fragmentation function $\tilde{E}^q(z)$ (which we identified as the twist-3 FF term).

The longitudinal–transverse spin asymmetry can be defined as

$$A_{LT} \sim \frac{\sigma(+\lambda_e,S_\perp)-\sigma(-\lambda_e,S_\perp)}{\sigma(+\lambda_e,S_\perp)+\sigma(-\lambda_e,S_\perp)}, \tag{9}$$

which is consistent with the notation of previous experimental measurements [17]. Thus, the x-dependent and z-dependent $\cos\phi_S$ asymmetry can be defined as

$$A_{LT}^{\cos\phi_S}(x) = \frac{\int dy\int dz\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos\phi_S}(x,z)}{\int dy\int dz\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)}, \tag{10}$$

$$A_{LT}^{\cos\phi_S}(z) = \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos\phi_S}(x,z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)}. \quad (11)$$

2.1. The Calculation of the Unpolarized Structure Function

In this section, we will calculate the denominator of asymmetry, which is the unpolarized structure function F_{UU} in Equation (7) and can be factorized into two parts: the unpolarized PDF $f_1^q(x)$ and FF $D_1^q(z)$. We apply the CT10 parametrization [29] for the unpolarized PDF $f_1^q(x)$ to be consistent with the choices in Ref. [30]. For the unpolarized FF $D_1^\Lambda(z)$ of the Λ hyperon, we adopt the model results from the diquark spectator model [25]

$$\begin{aligned} D_1^\Lambda(z) =& \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Big\{z(1-z)\Big((m_q+M_\Lambda)^2-m_D^2\Big)\times\exp\Big(\frac{-2zL^2}{(1-z)\Lambda^2}\Big) \\ &+\Big((1-z)\Lambda^2-2\Big((m_q+M_\Lambda)^2-m_D^2\Big)\Big)\times\frac{z^2L^2}{\Lambda^2}\Gamma\Big(0,\frac{2zL^2}{(1-z)\Lambda^2}\Big)\Big\}, \end{aligned} \quad (12)$$

where

$$L^2 = \frac{1-z}{z^2}M_\Lambda^2 + m_q^2 + \frac{m_D^2-m_q^2}{z}, \quad (13)$$

and the incomplete gamma function has the form

$$\Gamma(0,z) \equiv \int_z^\infty \frac{e^{-t}}{t}dt, \quad (14)$$

and the Λ^2 has the general form

$$\Lambda^2 = \lambda^2 z^\alpha (1-z)^\beta. \quad (15)$$

The parameters of the model are λ, α and β, together with the masses of the spectator diquark m_D and the parent quark m_q.

The numerical results are obtained by choosing the corresponding parameters as constituent quark mass, $m_q = 0.36$ GeV for up, down and strange quarks, and the Λ hyperon mass, 1.116 GeV. The model result has been fitted to the leading order (LO) DSV parameterization for D_1^Λ at the initial scale $\mu_{LO}^2 = 0.23\ \text{GeV}^2$. The values of the parameters α, β are fixed in the fit. The fitted results are

$$g_D = 1.983, \quad m_D = 0.745\ \text{GeV}, \quad \lambda = 5.967\ \text{GeV}, \quad \alpha = 0.5\ (\text{fixed}), \quad \beta = 0\ (\text{fixed}). \quad (16)$$

To make it applicable to a more general energy range, we use the QCDNUM evolution package [31] to evolve the unpolarized fragmentation function $D_1(z)$ from the initial energy of 0.23 GeV^2 to another energy scale. Combining Equations (7) and (12), we can obtain the unpolarized structure function

$$F_{UU}(x,z,Q^2) = x\sum_q e_q^2 f_1^q(x,Q^2)D_1^q(z,Q^2). \quad (17)$$

2.2. The Calculation of the Spin-Dependent Structure Function

In this section, we will calculate the numerator part of the asymmetry, which is the twist-3 structure function $F_{LT}^{\cos\phi_S}$. The structure function can be separated into two parts: the convolution of the twist-3 PDF g_T and the twist-2 FF D_1 and the convolution of the twist-2 PDF h_1 and the twist-3 FF $\tilde{E}$.

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First, we will deal with the twist-3 PDF contribution $g_T(x)D_1(z)$ of the structure function. With the combination of the spin-dependent structure functions $g_1(x)$ and $g_2(x)$, the twist-3 distribution function $g_T^q(x)$ can be expressed as [32]:

$$\frac{1}{2}\sum_q e_q^2 g_T^q(x) = g_1(x) + g_2(x), \tag{18}$$

where $g_2(x)$ is the structure function related to the transverse spin of the proton target and consists of two parts: the Wandzura–Wilczek approximation part $g_2^{\rm WW}(x)$ and the genuine twist-3 term $g_2^{\text{tw-3}}(x)$

$$g_2(x) = g_2^{\rm WW}(x) + g_2^{\text{tw-3}}(x). \tag{19}$$

The Wandzura–Wilczek approximation [26] part can be determined by the structure function $g_1(x)$

$$g_2 \overset{\rm WW}{\approx} g_2^{\rm WW}(x) = -g_1(x) + \int_x^1 dy \frac{g_1(y)}{y}, \tag{20}$$

with $g_1(x)$ being the twist-2 structure function related to the helicity function

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 g_1^q(x), \tag{21}$$

The validity of the WW approximation has been well established by abundant theoretical [27,33–44] and experimental [45–50] research.

Ref. [27] derived the twist-3 part of $g_2(x)$ for proton and neutron targets at the reference energy scale $Q^2 = 1$ GeV^2 via the convolution integrals of light-cone wave functions ($\bar{x} = 1 - x$):

$$\begin{aligned} g_{2,p}^{\rm tw-3}(x) = {} & 0.0436772(\ln x + \bar{x} + \frac{1}{2}\bar{x}^2) + \bar{x}^3(1.57357 \\ & - 5.94918\bar{x} + 6.74412\bar{x}^2 - 2.19114\bar{x}^3), \end{aligned} \tag{22}$$

$$\begin{aligned} g_{2,n}^{\rm tw-3}(x) = {} & 0.0655158(\ln x + \bar{x} + \frac{1}{2}\bar{x}^2) + \bar{x}^3(0.130996 \\ & - 1.12101\bar{x} + 2.31342\bar{x}^2 - 1.20598\bar{x}^3), \end{aligned} \tag{23}$$

which is also used to compare with the data of SLAC and JLab. Combining Equations (18)–(20), (22) and (23), one can obtain

$$\frac{1}{2} g_T^q(x) = \frac{1}{2}\int_x^1 dy \frac{g_1^q(y)}{y} + g_2^{\text{tw-3},q}(x), \tag{24}$$

where $g_2^{\text{tw-3},q}(x)$ denotes the contribution to $g_2^{\text{tw-3}}(x)$ from q flavor.

Assuming the twist-3 term $g_2^{\text{tw-3}}(x)$ is dominated by u and d quarks, which is valid in the valence region, and adopting isospin symmetry, we can obtain the expressions of $g_T(x)$ for u and d quark as

$$g_T^u(x) = \int_x^1 dy \frac{g_1^u(y)}{y} + \frac{6}{5}(4g_{2,p}^{\rm tw-3}(x) - g_{2,n}^{\rm tw-3}(x)), \tag{25}$$

$$g_T^d(x) = \int_x^1 dy \frac{g_1^d(y)}{y} + \frac{6}{5}(4g_{2,n}^{\rm tw-3}(x) - g_{2,p}^{\rm tw-3}(x)). \tag{26}$$

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We assume the distribution function of the s quark for g_T^q comes only from the Wandzura–Wilczek approximation, so the twist-3 distribution function g_T^q in Equation (24) for s quark has the following form

$$g_T^s(x) = \int_x^1 dy \frac{g_1^s(y)}{y}. \quad (27)$$

The chiral-odd twist-3 FF $\tilde{E}^q(z)$ involves the quark–gluon–quark correlation and currently has no theoretical or experimental information. However, it relates to another twist-3 FF $E^q(z)$ [51] (which encodes the quark–quark correlation during fragmentation) via the equation-of-motion relation [3]

$$\frac{E^q(z)}{z} = \frac{\tilde{E}^q(z)}{z} + \frac{m_q}{M_h} D_1^q(z), \quad (28)$$

where m_q and M_h are the current quark mass and the Λ mass, respectively. The investigation of $E^q(z)$ can be approached using the chiral quark model [23] and the spectator model [52]. In order to obtain the FF $\tilde{E}(z)$ and predict the double-spin asymmetry $A_{LT}^{\cos\phi_S}$, we make use of Equation (28) and adopt the chiral quark model result for $E^q(z)$ [23]

$$E^q(z) = \frac{m'_q}{M_h} \frac{z}{1-z} D_1^q(z), \quad (29)$$

where m'_q is the constituent quark mass. In principle, the current quark mass should be much smaller than the constituent quark mass and M_h, so that the second term on the right-hand side of Equation (28) can be ignored. For simplicity, we assume that the values of the two masses are the same. Although this assumption may be crude, it will not significantly change the main result of our calculation, as shown in Ref. [16]. Thus, in our estimation, there is a proportion between $\tilde{E}^q(z)$ and the unpolarized FF $D_1^q(z)$

$$\tilde{E}^q(z) = \frac{m'_q}{M_h} \frac{z^2}{1-z} D_1^q(z). \quad (30)$$

Following Ref. [23], the quark mass $m'_q \approx M/3$. The equation-of-motion relation originates from the Dirac equation; Equation (28) is theoretically precise. The chiral quark model, as a well-established and extensively tested framework, provides a relation between $E^q(z)$ and $D_1^q(z)$. Thus, we derive the expression for $\tilde{E}^q(z)$, and adopt Equation (30) as a rough approximation. If a fully analytic calculation of $\tilde{E}^q(z)$ is required in the future, it could be pursued using non-perturbative approaches such as light-front wave functions or the spectator diquark model.

Regarding the transversity distribution function $h_1(x)$ of the valence quark in Equation (8), we adopt the parametrization from Ref. [30] (at the initial scale $Q^2 = 1\ \text{GeV}^2$):

$$h_1^q(x) = \frac{1}{2} \mathcal{N}_q^T(x) [f_1^q(x) + g_1^q(x)], \quad (31)$$

with

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}. \quad (32)$$

where the values of N_q^T, α and β are given in Table I in Ref. [30]. One should note that there is no information for the transversity distribution function of the strange quark; the only constraint for the strange quark distribution is from the positivity bound. Thus, we assume the transversity distribution function for strange quarks has the following form

$$h_1^s(x) = \frac{1}{2} N_s [f_1^q(x) + g_1^q(x)] \tag{33}$$

with $N_s = 0, 0.5, 1$ for the zero strange quark transversity, medium strange quarks from the positivity bound and all positivity bound scenarios, which was chosen in Refs. [53,54].

Combining all this together, we can rewrite the spin-dependent structure function $F_{LT}^{\cos\phi_S}(x,z)$ as the contribution of the twist-3 PDF term $g_T(x)D_1(z)$ as well as the contribution of the twist-3 FF term $h_1(x)\tilde{E}(z)$ as

$$\begin{aligned} F_{LT}^{\cos\phi_S}(x,z,Q) &= \int d^2\boldsymbol{P}_{hT} F_{LT}^{\cos\phi_S}(x,z,P_{hT}) = \\ &- x\sum_q e_q^2 \frac{2M}{Q}\left(x g_T^q(x,Q) D_1^q(z,Q) + \frac{M_h}{M} h_1^q(x,Q) \frac{\tilde{E}^q(z,Q)}{z} \right). \end{aligned} \tag{34}$$

3. Numerical Estimate

In this section, we will perform numerical calculations for the $\cos\phi_S$ asymmetry utilizing the formalism established in Section 2 and show the results. We will take into account the kinematical configurations at CLAS12, COMPASS, and EicC to perform the numerical predictions.

Since the PDFs and FFs in Equations (25), (26), (30) and (31) are given at certain fixed scales, it is necessary for us to transform these functions to another scale. The scale dependence of $g_T(x)$ is determined by $g_T^{q,\mathrm{WW}}(x)$ and $g_T^{q,\text{tw-3}}(x)$:

$$g_T^q(x,Q^2) = g_T^{q,\mathrm{WW}}(x,Q^2) + g_T^{q,\text{tw-3}}(x,Q^2), \tag{35}$$

To simplify the calculation, we assume that the Q^2 dependence of $g_T^{q,\mathrm{WW}}(x,Q^2)$ comes from $g_1^q(x,Q^2)$

$$g_T^{q,\mathrm{WW}}(x,Q^2) = \int_x^1 dy \frac{g_1^q(y,Q^2)}{y}, \tag{36}$$

To evolve the genuine twist-3 contribution $g_T^{\text{tw-3}}$, we adopt the non-singlet evolution kernel for $g_2^{\text{tw-3(x)}}$

$$P_{q,F}^{NS,z\to 1}(z) = 2C_F\left[\frac{1}{(1-z)_+} + \frac{3}{4}\delta(1-z)\right] - \frac{1}{2}N_C\delta(1-z). \tag{37}$$

The evolution kernel mentioned above is a simplified form of the exact evolution kernel under the large-N_c and large-x approximations, which has been applied in Refs. [27,55]. As demonstrated in Ref. [27], the scale dependence of the twist-3 contribution $g_2^{\text{tw-3}(x)}$ computed using Equation (37) closely aligns with that obtained from the exact evolution. Thus, for simplicity, we adopt Equation (37) to evolve $g_2^{\text{tw-3}}(x)$ in this work, implementing Equation (37) in the QCDNUM evolution package [31].

The QCDNUM evolution package is also applied to evolve the transversity distribution function $h_1(x)$ after including chiral-odd LO splitting functions in the code [56]:

$$\Delta_T P_{qq}(x) = C_F\left[\frac{2z}{(1-z)_+} + \frac{3}{2}\delta(1-x)\right] \tag{38}$$

The FF $E(z)$ utilized in our calculation is acquired at the chiral symmetry breaking scale. On a larger scale, the relation in Equation (30) might break down due to the different evolutions of $E(z)$ and $D_1(z)$. Currently, the QCD evolution of the twist-3 fragmentation function $\tilde{E}(z)$ is not well established, making a precise calculation infeasible. Consequently, as a rough estimate, we will assume that the Q^2 dependence of both $E(z)$ and $\tilde{E}(z)$ is identical to that of $D_1(z)$, and we investigate their evolution using QCDNUM under this assumption. Our current treatment is a preliminary approximation. A more refined treatment in the future would involve a precise calculation of the $\tilde{E}(z)$ evolution using perturbative QCD theory.

The kinematical region that can be used to calculate the $\cos\phi_S$ asymmetry at CLAS12 is chosen as follows [57]:

$$\begin{aligned} &0.072 < x < 0.532, \quad 0.085 < y < 0.95, \quad 0.2 < z < 0.8, \\ &E_e = 11\,\text{GeV}, \quad W^2 > 4\,\text{GeV}^2, \quad 1 < Q^2 < 6.3\,\text{GeV}^2. \end{aligned} \tag{39}$$

where W is the invariant mass of the photon–nucleon system:

$$W^2 = (P+q)^2 \approx \frac{1-x}{x} Q^2. \tag{40}$$

To calculate the $\cos\phi_S$ asymmetry in COMPASS, we adopt the following kinematical cuts [58]

$$\begin{aligned} &0.003 < x < 0.4, \quad 0.1 < y < 0.9, \quad 0.2 < z < 0.9, \\ &Q^2 > 1\,\text{GeV}^2, \quad W > 5\,\text{GeV}, \quad s = 300.16\,\text{GeV}. \end{aligned} \tag{41}$$

As for the EicC, the following kinematical cuts are adopted

$$\begin{aligned} &0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7, \\ &1\,\text{GeV}^2 < Q^2 < 200\,\text{GeV}^2, \quad W > 2\,\text{GeV}, \quad \sqrt{s} = 16.7\,\text{GeV}. \end{aligned} \tag{42}$$

Combining Equations (10)–(12), (17) and (34) and the kinematical regions of CLAS12, COMPASS and EicC, we can calculate the double-spin $A_{LT}^{\cos\phi_S}$ asymmetry of the Λ hyperon-produced SIDIS process at CLAS12, COMPASS and EicC kinematical ranges. The results are shown in Figure 2; the upper, central and lower panels in the figure depict the numerical results of $\cos\phi_S$ asymmetry in the Λ hyperon-produced SIDIS process in the kinematical regions of CLAS12, COMPASS and EicC, respectively. The left and right panels denote the $A_{LT}^{\cos\phi_S}$ asymmetry as functions of x and z, respectively. The dashed and dotted lines correspond to the asymmetries contributed by the twist-3 PDF term $g_T(x)D_1(z)$ and the twist-3 FF term $h_1(x)\tilde{E}^q(z)$, respectively, while the solid lines show the sums of the two contributions.

The $\cos\phi_S$ asymmetries as a function of x and z show similar tendencies at CLAS12, COMPASS and EicC. Both the x- and z-dependent $\cos\phi_S$ asymmetries in Λ hyperon-produced SIDIS process in the kinematics regions of CLAS12, COMPASS and EicC are negative. In the kinematical region of CLAS12, the value of the x-dependent asymmetries increases with x in the small-x region, reaches a peak around $0.1 < x < 0.2$, then decreases slightly as x increases. The z-dependent asymmetries for the Λ hyperon are negative. The contribution of $g_T^q(x)$ is weakly dependent on z. However, the value of the $\tilde{E}^q(z)$ contribution increases rapidly with increasing z, since there is a factor $z^2/(1-z)$ in the expression of $\tilde{E}^q(z)$. In the large-z region, the contribution from $\tilde{E}^q(z)$ might dominate over that from $g_T^q(x)$. Although the sign and the shape of the asymmetries at COMPASS and EicC are similar to those at CLAS12, it is found that the $\cos\phi_S$ asymmetry at CLAS12 is greater. This is because the asymmetry we study is at the twist-3 level, at which the effect will be suppressed by $1/Q$, and the averaged Q values at EicC, COMPASS and CLAS12 gradually decrease.

Subsequently, we used Equations (10)–(12), (17) and (34) and the kinematical regions of CLAS12, COMPASS, and EicC to calculate the ratio of the asymmetry with medium and all positivity bounds for strange quarks to the asymmetry without strange quarks. The results are shown in Figure 3. We found that including strange quarks has a significant impact on the $\cos\phi_S$ asymmetry, approximately around ten percent. Since the contribution of strange quarks to the asymmetry mainly comes from the convolution of the transversity distribution function $h_1(x)$ and the twist-3 chiral-odd FF $\tilde{E}(z)$, this will provide us with the opportunity to obtain information about the strange quark twist-3 fragmentation functions.

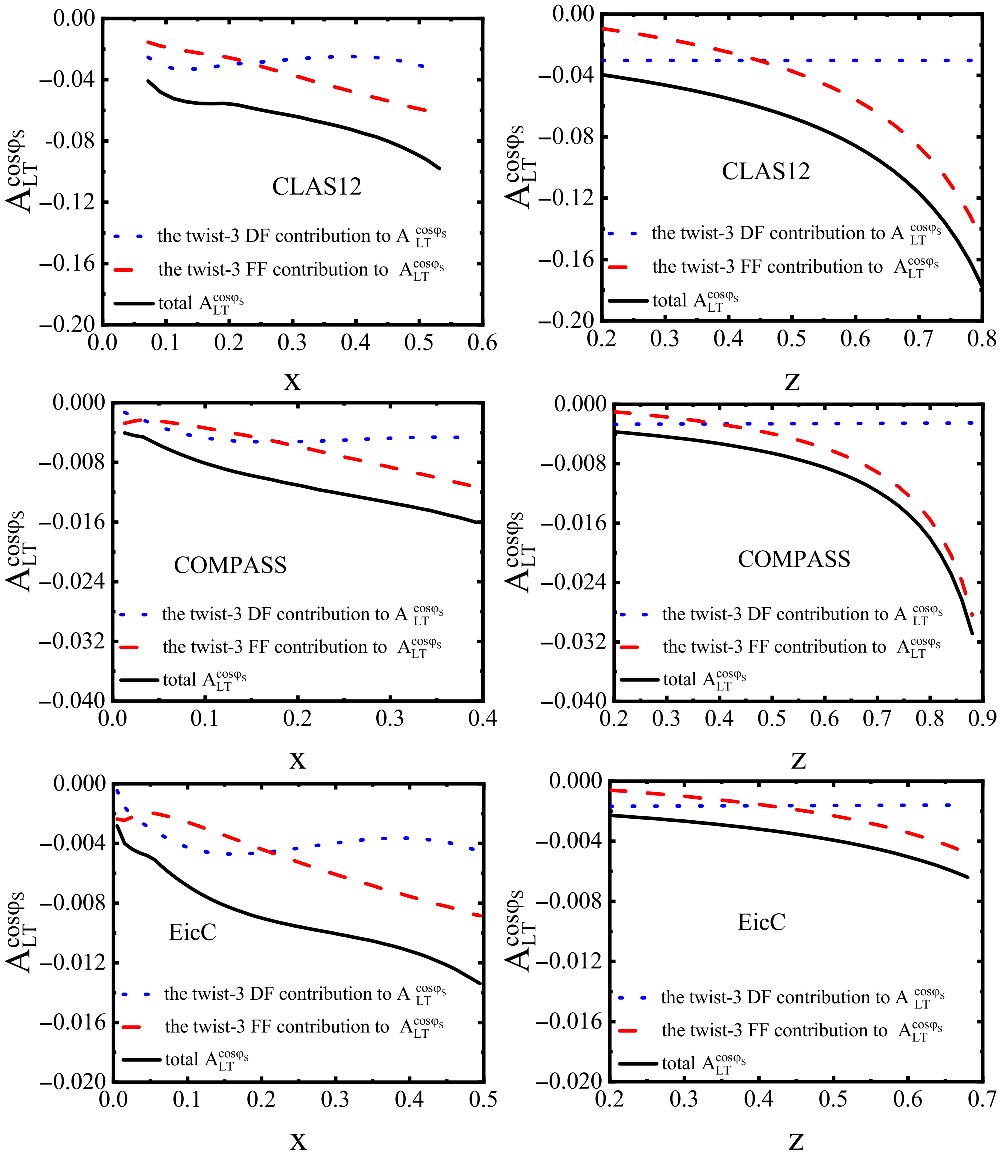

Figure 2. Longitudinal–transverse double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of Λ production in SIDIS at CLAS12, COMPASS and EicC. The left panels represent the x-dependent asymmetry, while the right ones the z-dependent asymmetry. Here, we do not consider the contribution of sea quarks to the $\cos\phi_S$ asymmetry.

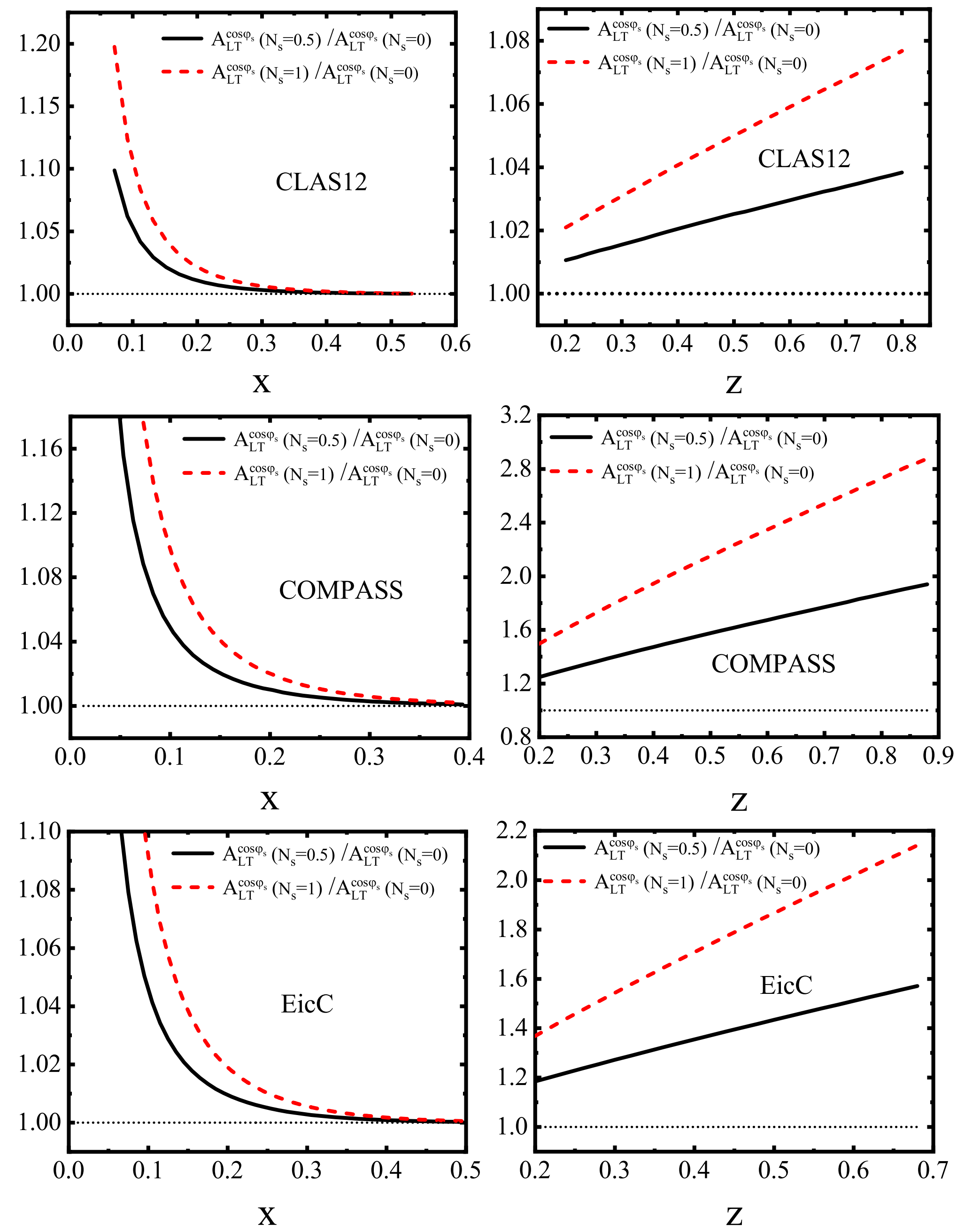

Figure 3. The effect of sea quarks on the Λ production $A_{LT}^{\cos\phi_S}$ asymmetry; the solid line and dashed line represent the ratio of the asymmetry with medium and all positivity bounds for strange quarks to the asymmetry without strange quarks, respectively.

4. Conclusions

In this study, we employ the collinear factorization framework to investigate the $\cos\phi_s$ azimuthal asymmetry in double-polarized SIDIS at CLAS12, COMPASS and EicC. Particularly, since the transverse momentum-dependent (TMD) version of this asymmetry contains many terms, we have considered the special case where the transverse momentum of the final state hadron is integrated out. In this case, the asymmetry incorporates two contributions. One is the convolution of $g_T^q(x)$ and $D_1^q(z)$; the other is the convolution of $h_1^q(x)$ and $\tilde{E}^q(z)$. We estimate the $\cos\phi_s$ asymmetry for Λ production for the kinematics of CLAS12, COMPASS and EicC by including both contributing terms. To do this, we have gone beyond the Wandzura–Wilczek approximation and adopted an analysis of $g_2^{\text{tw-3}}(x)$ to extract the genuine twist-3 part of $g_T^q(x)$. Furthermore, motivated by the chiral quark model, we have introduced an approximate relation between the twist-3 FF $\tilde{E}^q(z)$ and the unpolarized FF $D_1^q(z)$. In the calculation, we have considered the evolution effect of the twist-2 and twist-3 PDFs and FFs. The numerical prediction shows that the $\cos\phi_s$ asymmetries for the Λ are sizeable at CLAS12, COMPASS and EicC. The asymmetry observed in CLAS12 exceeds that in COMPASS, which is still larger than in EicC due to suppression in the large-Q region. Although, for the x-dependent asymmetry, the size of the contribution from $\tilde{E}^q(z)$ is comparable to that from $g_T^q(x)$, we find that the asymmetry in the large-z region is completely dominated by the convolution of $h_1^q(x)$ and $\tilde{E}^q(z)$. Therefore, it might be promising to access the unknown twist-3 FF $\tilde{E}^q(z)$ by measuring the $\cos\phi_s$ asymmetry of Λ production in SIDIS with the collinear picture. The inclusion of the contribution from strange quarks to the $\cos\phi_s$ asymmetry will provide the possibility of obtaining information about the strange quark twist-3 FF through measurements of the $\cos\phi_s$ asymmetry at CLAS12, COMPASS and EicC.

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Abbreviations

The following abbreviations are used in this manuscript:

FFs	Fragmentation Functions
PDFs	Parton Distribution Functions
SIDIS	Semi-Inclusive Deep Inelastic Scattering
DIS	Deeply Inelastic Scattering
TMD	Transverse Momentum Dependent
QCD	Quantum Chromodynamics
LO	Leading Order
DSV	De Florian–Sassot–Vogelsang
DSSV	De Florian–Sassot–Stratmann–Vogelsang

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