8

Cherenkov Radiation in Beam–Plasma Systems

In this chapter, we will consider those processes of waves amplification that are conditioned by the elementary mechanism of the Cherenkov radiation emission. To distinguish this effect from others, we assume that the external magnetic field strength is very high. In this case, the cyclotron frequency of the electron rotation around the magnetic field lines essentially exceeds all characteristic frequencies emitted by the system in question. So, motion of the electrons can be treated as one-dimensional and directed along the magnetic field lines.

As it has been mentioned, the induced emission of Cherenkov radiation made a real basis for the first devices with distributed interaction of TWT or BWT type, in spite of their actual invention and development being originated from single–particle considerations. The literature on the subject is enormous and is evidently out of the scope of this book. However, the development of microwave electronics tends at present toward applications of relativistic electron beams (REB) and requires a more general approach.

Here we briefly dwell on several advantages of applying REB in Cherenkov oscillators and amplifiers. First, during the beam energy transfer to the wave, the energy of the beam particles decreases essentially. At the same time, the velocity of relativistic particles is subjected just to insignificant changes. This permits preserving the Cherenkov synchronism between the wave and the beam particles for a much longer time interval. Therefore, the effectiveness of the beam energy transfer into the wave energy becomes essentially higher. Second, for obtaining synchronism between the REB particles and the wave, the wave phase velocity must be close to the velocity of light. As it is known, it is considerably easier to transform such waves into free–space waves, that is, to radiation.

To increase generated power, larger beam currents are, of course, required. The limitations imposed by space charge effects could be removed by filling the electrodynamic structure with a plasma. As a rule, the plasma density should be chosen so that it would not considerably change the spatial structure of the field interacting with the beam. At the same time, the plasma has to be dense enough to neutralize the charge and current of the beam.

The cases when the plasma itself plays the part of a retarding electrodynamic structure are of a special interest. Under such conditions, the electron beam interacts with the plasma proper waves. Their excitation can be treated as the plasma–beam instability that for the first time was predicted in [40, 43]. At the same time it can be considered as the induced emission of plasmons [13]. Publication of these papers attracted interest to the plasma–beam systems and the corresponding new branch of plasma electronics is developing nowadays.

Taking this into account, we consider in this chapter only the process of beam–plasma interaction. On the one hand, this permits to avoid cumbersome mathematics involved in the periodic retarding systems theory (see Part I) and not related directly to the questions under consideration. On the other hand, the literature on plasma electronics is rather limited.

Cherenkov radiation of a single particle in cold plasma has been considered in Part I. We determine below increments of the corresponding collective radiation instability and their dependence on plasma and beam parameters.

8.1 Dispersion Equation

We will describe dynamics of the beam and plasma electrons with the help of their distribution function $f(\mathbf{r}, \mathbf{p}, t)$. This function satisfies Vlasov kinetic equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + q\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v} \times \mathbf{B}]\right\}\frac{\partial f}{\partial \mathbf{p}} = 0\,. \tag{8.1}$$

The fields are described by Maxwell's equations:

$$\mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \quad \mathrm{rot}\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}; \tag{8.2}$$

$$\mathrm{div}\mathbf{E} = 4\pi\varrho\,; \qquad \mathrm{div}\mathbf{B} = 0\,. \tag{8.3}$$

Here ϱ is the total charge density of the beam and plasma electrons; $\mathbf{j}$ is their current density:

$$\varrho = \int f \mathrm{d}\mathbf{p}\,; \qquad \mathbf{j} = \int f\mathbf{v}\mathrm{d}\mathbf{p}.$$

The system (8.1)–(8.3) describes both the linear and nonlinear stages in the beam instability development. Below in this section we will give analysis only to the linear stage.

In the linear approximation, making use of the system (8.1)–(8.3), one can derive the general dispersion relation for the Fourier components of the perturbations. This expression determines a relation of frequencies of the proper waves to their wave vectors. However, it can hardly be analyzed in a general form. Below we will investigate the simplest particular cases only. When choosing them, we aim at simplifying mathematical formalism as much as possible. At the same time, the essence of the basic physical processes must not be misrepresented. All basic physical pecularities studied with the help of these simplest models also manifest themselves in more general cases.

Thus, we proceed from the following assumptions. First of all, we will investigate dynamics of the Cherenkov instability of a relativistic electron beam in uniform cold plasma limiting ourselves by plane waves of a small amplitude characterized by a longitudinal wave number k. The undisturbed beam of density n_b is supposed to be compensated with respect to the charge and current.

Under these conditions, harmonic field perturbations can be described by the wave equation for the longitudinal electric field component E following from (8.2) and (8.3). The continuity equation taken into account can be written as

$$E\left(k_0^2 - k^2 - \kappa_\perp^2\right) = 4\pi \mathrm{i}\varrho \frac{k^2 - k_0^2}{k}, \tag{8.4}$$

where ϱ is a space charge density determined by the perturbation of the distribution function $\tilde{f}\,(k_0, k, p)$:

$$\varrho = \int \tilde{f} \mathrm{d}p \, .$$

The "transverse" wavenumber $\kappa_\perp$ determines a direction of the wave propagation in the case of transversely uniform plasma or a proper mode if boundary conditions exist. In the last case the values of $\kappa_\perp$ are discreet and all perturbations are to be treated as amplitudes of the corresponding membrane functions.

The distribution function perturbation obeys the kinetic equation and is related to the longitudinal field only:

$$(\omega - kv)\,\tilde{f} = \mathrm{i}qE\frac{\partial f_0}{\partial p}. \tag{8.5}$$

It gives the following dispersion relation:

$$k_0^2 - k^2 - \kappa_\perp^2 = -\frac{4\pi q}{k}\left(k_0^2 - k^2\right)\int \frac{\partial f_0/\partial p}{\omega - kv} dp \, . \tag{8.6}$$

The distribution of the electrons over momenta has two narrow maxima. The first one is situated at $p = 0$ and corresponds to electrons of the cold plasma of density n_p. The corresponding integral in (8.6) is equal to $qn_\mathrm{b}/\omega^2 m$. The second maximum (related to the beam) is in the vicinity of $m\gamma\beta c$. Neglecting for a while the width of these maxima (cold beam and plasma) and taking into account that $\mathrm{d}v/\mathrm{d}p = \left(m\gamma^3\right)^{-1}$, we get the dispersion relation in the form:

$$\kappa_\perp^2 + (k_0^2 - k^2)\left[-1 + \frac{k_\mathrm{p}^2}{k_0^2} + \frac{k_\mathrm{b}^{*2}}{(k_0 - \beta k)^2}\right] = 0 \, . \tag{8.7}$$

Here

$$k_{\mathrm{p}}^2 = \frac{4\pi n_{\mathrm{p}} q^2}{m} \quad \text{and} \quad k_{\mathrm{b}}^{*2} = \frac{4\pi \varrho_0 q}{m\gamma^3}$$

are the squares of Langmuir frequencies of the plasma and of the beam, respectively.

8.2 Cold Beam Instability

Solving concrete problems, one has to add initial and boundary conditions. So, for a particular problem the functions $k_0(k)$ and/or $k(k_0)$ can be of importance. In the first case we shall look for time dependence of perturbations initially distributed in the interaction space (absolute instability). The second case corresponds to spatial amplification of a fixed frequency signal entering the system. The latter is more adapted to the microwave amplification problem. Nevertheless, we start below with the first case typical for problems of temporal stability and self-excitation of oscillations.

8.2.1 Absolute Instability

In accordance with the scheme of the paragraph under Sect. 7.1.4 the dispersion equation is to be written in the form:

$$\left(k_0^2 - k_+^2\right)\left(k_0^2 - k_-^2\right)\left(k_0 - k\beta\right)^2 = \left(k_0^2 - k^2\right) k_{\mathrm{b}}^2 k_0^2, \tag{8.8}$$

where the right-hand side is proportional to the beam density. In what following it will be considered as a small parameter:

$$k_{\mathrm{b}}^{*2}/k^2. \ll 1\ .$$

This is justified for the majority of practical problems. In the nonrelativistic case, the inequality above is equivalent to the smallness of the so-called Pierce parameter:

$$\left(\frac{4\pi q \varrho_0}{mk^2\beta^2}\right)^{1/2} \ll 1.$$

However, for relativistic beams this parameter can reach a large value of order of γ^2 still preserving smallness of the coupling coefficient.

The squares of the partial frequencies in (8.8) are equal to

$$k_{\pm}^2\,(k) = 1/2\left[\kappa_\perp^2 + k_{\mathrm{p}}^2 + k^2 \pm \sqrt{\left(\kappa_\perp^2 + k_{\mathrm{p}}^2 + k^2\right)^2 - 4k^2k_{\mathrm{p}}^2}\right]. \tag{8.9}$$

They represent two branches of partial plasma waves with frequencies that are larger than $\sqrt{\kappa_\perp^2 + k_{\mathrm{p}}^2}$ and smaller than k_{p} correspondingly. This is the second (slow) wave, which provides induced Cherenkov radiation meeting the space charge wave $k_0 = k\beta$ twice degenerated for $k_b^* = 0$.

Now we have to find the Cherenkov resonance point (k_{0s}, k_s) defined by the equality of the wave and the particles velocities. Solving

$$k_-^2(k_s) = k_s^2\beta^2$$

for k_s yields

$$k_s^2 = \frac{k_p^2}{\beta^2} - \kappa_\perp^2\gamma^2; \quad k_{0s}^2 = k_-^2(k_s) = k_p^2 - \kappa_\perp^2\gamma^2\beta^2; \quad k_+^2(k_s) = \frac{k_p^2}{\beta^2}. \tag{8.10}$$

So, the local dispersion equation taking into account the three waves interaction can be written in the form:

$$\kappa_0^3 + \kappa^2\delta + \kappa_0 2\gamma^2\left(2\beta^2 - 1\right)\frac{K^3}{k_p}\sqrt{\frac{1-\beta\beta_g}{\beta-\beta_g}} = K^3. \tag{8.11}$$

Here

$$\kappa_0 = k_0 - k\beta; \quad K^3 = \frac{k_p k_b^{*2}}{2}\frac{\left(1-\beta_g/\beta\right)^{3/2}}{\left(1-\beta\beta_g\right)^{1/2}}$$

and the partial wave parameters are expressed in terms of its group velocity at the crossing point:

$$\beta_g = \frac{dk_-}{dk} = \frac{\kappa_\perp^2\gamma^4\beta^3}{k_p^2 + \kappa_\perp^2\gamma^4\beta^4}.$$

Note that in spite of the declared smallness of κ_0/k_s and $(k-k_s)/k_s$ one has to keep the third term at the l.h.s of (8.11) because of the potentially large factor γ^2. The value

$$\delta = (\beta - \beta_g)(k - k_s) + \frac{k_b^{*2}\gamma^2\left(6\beta^2 - 1\right)}{2k_p}\sqrt{(1-\beta_g/\beta)(1-\beta\beta_g)} \tag{8.12}$$

related to the deviation of k from the resonance will be referred as detuning. By the way, the expressions above show immediately that the instability cannot develop for $\kappa_\perp^2 > k_p^2/\gamma^2\beta^2$ when the value $k_-^2(k_s)$ is negative. This inequality corresponds to plasma waves propagating at large angles $> \gamma^{-1}$ or to high transverse modes of a plasma-filled waveguide. For a fixed plasma frequency, their phase velocity along z exceeds that of light making the Cherenkov resonance impossible.

The cubic algebraic equation (8.11) can be solved immediately, but the solution containing several independent parameters still is rather nondescriptive. Instead, we shall consider two characteristics of the main interest – a threshold of the instability and optimizing detuning which corresponds to the maximal increment, that is, to the maximal value of $\mathrm{Im}\,\kappa_0$. To do this, we introduce normalized variables

$$I = \mathrm{Im}\,\kappa_0/K\,; \qquad R = \mathrm{Re}\,\kappa_0/K$$

and normalized detuning $\varDelta = \delta/K$. Supposing that $I \neq 0$ and separating the real and imaginary parts of (8.11), one gets the system

$$R^3 - 3RI^2 + \varDelta\left(R^2 - I^2\right) + GR - 1 = 0, \tag{8.13}$$

$$3R^2 - I^2 + 2\varDelta R + G = 0 \tag{8.14}$$

with

$$G = 2\gamma^2\left(2\beta^2 - 1\right)\sqrt{\frac{1-\beta\beta_\mathrm{g}}{\beta-\beta_\mathrm{g}}\frac{K}{k_\mathrm{p}}} = \frac{\gamma^2\left(2\beta^2-1\right)}{\beta^{1/2}\left(1-\beta\beta_\mathrm{g}\right)^{1/3}}\left(\frac{2k_\mathrm{b}^*}{k_\mathrm{p}}\right)^{2/3}.$$

To find the maximal value of detuning $\varDelta_\mathrm{max}$ corresponding to instability, one has to put here $I \to 0$ yielding the parametric dependence $\varDelta_\mathrm{max}(G)$:

$$\varDelta_\mathrm{max} = -2R - \frac{1}{R^2}\,, \quad G = R^2 + \frac{2}{R}. \tag{8.15}$$

The real roots of the dispersion equation (8.11) for moderate values of G are displayed in Fig. 8.1.

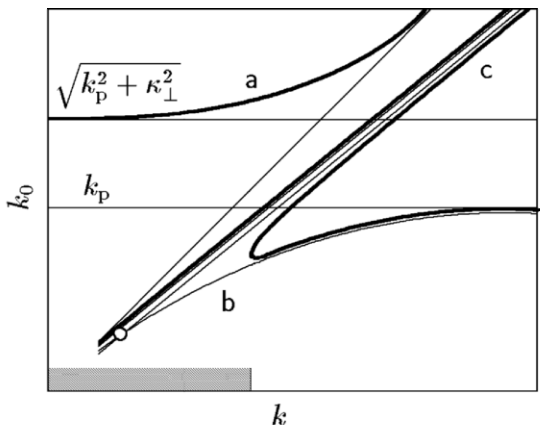

Fig. 8.1. Real roots of the dispersion equation. Shading shows the region of absolute instability. (**a**) Fast plasma wave; (**b**) slow plasma wave, and (**c**) space charge waves. A circle indicates the crossing point

For $\varDelta < \varDelta_\mathrm{max}$ there are two complex conjugated roots.[1] Note that for the particular case of uniform plasma, the instability takes place for all long waves, but its increment depends essentially on detuning. To find the maximizing

[1] For $G > 3$, an additional band of stability appears at $\varDelta < -3$.

value of the latter, one has to differentiate (8.13) and (8.14) with respect to $\varDelta$, to put $\mathrm{d}I/\mathrm{d}\varDelta = 0$, and to exclude $\mathrm{d}R/\mathrm{d}\varDelta$. Then the third equation

$$\varDelta = -\frac{RG}{I^2 + R^2} \tag{8.16}$$

is to be added to (8.13) and (8.14) to define the optimizing value $\varDelta_{\mathrm{opt}}$, the maximal increment $I_{\max}$, and the corresponding value of R, if necessary. Below we shall investigate only the limiting cases of small and large G.

Low-Intensity Regime

For moderately relativistic particles and low beam intensity, the parameter G is small and the boundary value can be presented as

$$\varDelta_{\max} \approx \frac{3}{2^{2/3}}. \tag{8.17}$$

The maximal increment is reached for $\varDelta \to 0$:

$$I_{\max}^2 \approx \frac{3}{4} \quad \text{or} \quad \operatorname{Im}\kappa_0 = -\frac{\sqrt{3}}{2}K + \cdots. \tag{8.18}$$

Note that it is proportional to the cubic root of the beam current what is typical for low intensity traveling wave tubes. By the way, in the theory of free electron lasers (see 10.), an analogous approximation for some reasons is called a Compton regime, although the name does not correspond to the case under consideration. The notion of a “single particle instability” used in [13] also can hardly be applied to a description of the collective process.

The dependence of the increment on detuning can be easily found using Cardan formula. For $G = 0$:

$$I = \frac{\sqrt{3}}{2}\left[\left(\frac{1}{2} - \sqrt{\frac{1}{4} - \left(\frac{\varDelta}{3}\right)^3}\right)^{2/3} - \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{\varDelta}{3}\right)^3}\right)^{2/3}\right]. \tag{8.19}$$

This dependence for fixed K is presented in Fig. 8.2. It shows a peak of induced Cherenkov radiation on the background of a long tail of low frequency waves. The latter is due to the negative electric permeability for $k_0 < k_{\mathrm{p}}$, which locks the excited field inside plasma. The beam modulation comes from the mutual electrostatic attraction of charges of the same sign in such a medium, but the process can hardly be called radiation. Anyway, the corresponding increment is small and does not play an essential role.

High-Intensity Regime

The opposite case of $G \gg 1$ can be met with relativistic beams in spite of declared smallness of their relative density. In this limit

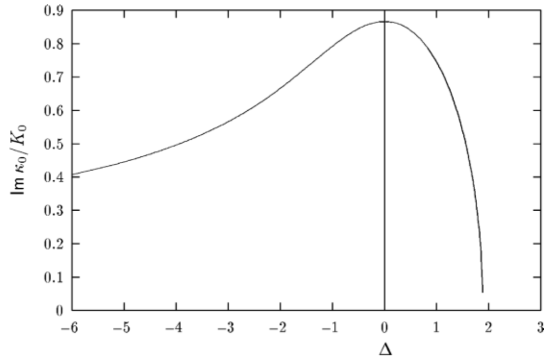

Fig. 8.2. Increment vs. detuning for small G

$$\Delta_{\max} \approx 2G^{1/2}\,; \quad \Delta_{\rm opt} \approx G{-}1\,; \quad I_{\max} \approx -G^{1/2} \tag{8.20}$$

yielding

$$\mathrm{Im}\,\kappa_0 = G^{1/2} K \propto k_{\rm b}^{*}\,.$$

Note that now the maximal increment is proportional to the square root of the beam current and is achieved at zero detuning. The dependencies of the maximal and optimal detunings and of the maximal increment on the parameter G are shown in Fig. 8.3 covering both cases above.

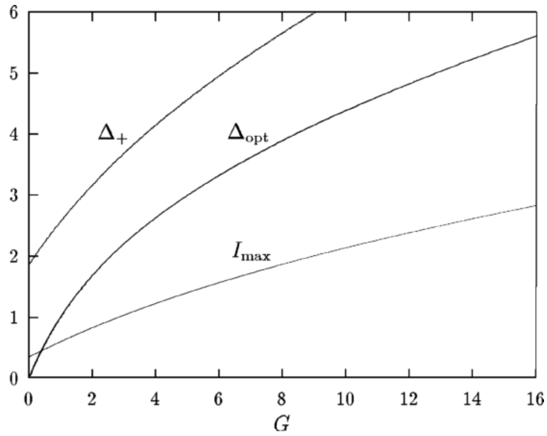

Fig. 8.3. Maximal and optimal detuning and maximal increment vs. parameter G

By the way, the case under consideration is usually identified as Raman regime. Really, the e-fold time of the low-current instability decreases as $k_{\rm b}^{*\,-2/3}$ while the period of a beam plasmon goes as $k_{\rm b}^{*\,-2}$. So, for intensities large enough, the e-fold time becomes sufficient for excitation of beam proper oscillations, exactly as it happens in the case of Raman scattering. Naturally, this influences the process of plasma waves radiation.

8.2.2 Convective Type Instability

Now we consider the inverse dependence $k(k_0)$, bearing in mind applications to amplifiers when the input frequency is fixed. The imaginary part of this dependence determines the spatial growth of the input signal along the beam.

The crossing point, of course, remains the same as in (8.10):

$$k_{0\mathrm{s}}^2 = k_\mathrm{p}^2 - \kappa_\perp^2 \gamma^2 \beta^2 = k_\mathrm{p}^2 \frac{1 - \beta_\mathrm{g}/\beta}{1 - \beta\beta_\mathrm{g}} ; \quad k_\mathrm{s} = k_{0\mathrm{s}}/\beta \tag{8.21}$$

but now one has to expand (8.7) up to the third power of $\kappa = k - k_0/\beta$, keeping the first nonvanishing terms in the expansion coefficients for small $\kappa_0 = k_0 - k_{0\mathrm{s}}$. This procedure leads to the three-wave dispersion equation in the form:

$$\left(\frac{\kappa}{K_0}\right)^3 - \left(\frac{\kappa}{K_0}\right)^2 \varDelta_0 + G_0 \frac{\kappa}{K_0} = -1 \tag{8.22}$$

with

$$K_0^3 = \frac{k_\mathrm{b}^{*2} k_\mathrm{p} \left(1 - \beta/\beta_\mathrm{g}\right)^{3/2}}{2\left(1 - \beta\beta_\mathrm{g}\right)^{1/2} \beta_\mathrm{g}} ; \quad G_0 = \frac{\gamma^2}{\beta} \left(\frac{4k_\mathrm{b}^{*2} \left(1 - \beta\beta_\mathrm{g}\right)}{k_\mathrm{p}^2 \beta_\mathrm{g}}\right)^{1/3}$$

and

$$\varDelta_0 = \beta \left(1 - \frac{\beta_\mathrm{g}}{\beta}\right) \frac{\kappa_0}{K_0} - \gamma^2 \beta \left(\frac{k_\mathrm{b}^{*2} \left(1 - \beta\beta_\mathrm{g}\right)}{2k_\mathrm{p}^2 \beta_\mathrm{g}}\right)^{2/3} .$$

The corresponding equations for real variables $R = \mathrm{Re}\,\kappa/K_0$ and $I = \mathrm{Im}\,\kappa/K_0$

$$R^3 - 3RI^2 - \varDelta_0 \left(R^2 - I^2\right) + G_0 R + 1 = 0 , \tag{8.23}$$

$$3R^2 - I^2 - 2\varDelta R + G_0 = 0 \tag{8.24}$$

are to be completed with the third equation

$$\varDelta_0 = \frac{RG_0}{I^2 + R^2} \tag{8.25}$$

for calculation of $\varDelta_{0\mathrm{opt}}$ and $I_{\max}$. The real roots of (8.22) are displayed in Fig. 8.4.

In spite of the different signs of the coefficients, these equations in the limits of $G_0 \ll 1$ and of $G_0 \gg 1$ give the same functional dependencies as for the previous case:

$$\varDelta_{0\max} = \frac{3}{2^{2/3}} , \quad I_{\max} = \frac{3}{4} , \quad \varDelta_{0\mathrm{opt}} \to 0 \tag{8.26}$$

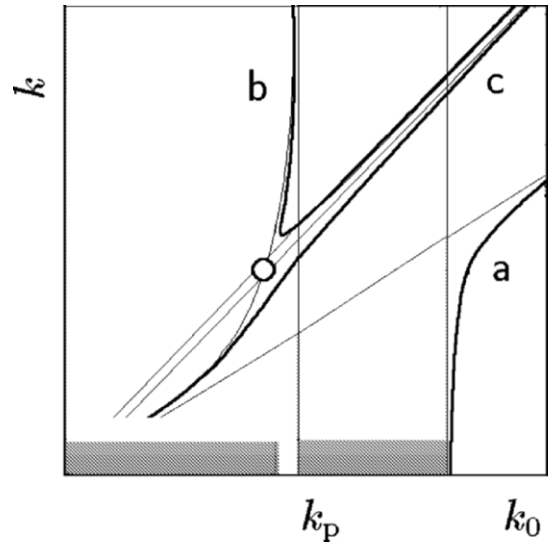

Fig. 8.4. Real roots of the dispersion equation. Shading shows the regions of convective instability. (**a**) Fast plasma wave; (**b**) slow plasma wave, and (**c**) space charge waves. A circle indicates the crossing point

for $G_0 \ll 1$ and

$$\Delta_{0\max} = 2G_0^{1/2}\,, \quad I_{\max} = -G_0\,, \quad \Delta_{0\mathrm{opt}} = G_0^{-1} \tag{8.27}$$

for $G_0 \gg 1$. So, Fig. 8.3 qualitatively illustrates the convective instability as well.

An additional remark should be made concerning two bands of convective instability shown in Fig. 8.4. The left one is, of course, originated by induced Cherenkov radiation combined with electrostatic attraction mentioned above. The second one corresponds to the frequency stop-band for $k_\mathrm{p} < k_0 < \sqrt{k_\mathrm{p}^2 + \kappa_\perp^2}$. Really, if the beam were absent, the input signal at that frequency would be locked near the point of excitation and would not penetrate plasma. The exponentially growing solution vanishes then because of the boundary conditions. The modulated beam transports the signal into plasma, and amplification does take place. However, the excited plasma oscillations cannot propagate and remain in the vicinity of the beam. So, real radiation may exist only in warm plasma with a nonzero group velocity. As noted in [13], the corresponding increment is low and the instability is strongly limited by nonlinear effects.

8.3 Warm Beam Instability

A well-known weak point of microwave devices based on Cherenkov interaction is a comparatively high sensitivity to deviations of particle velocities from the designed value. Really, particles of different longitudinal velocities interact with different waves spreading the radiation spectrum and, hence, decreasing the gain. For evaluation of this effect, we consider below the instabilities of a “warm” beam supposing, of course, that the thermal velocity spread is much smaller than the velocity itself.

Suppose that the beam particles velocity distribution is a Maxwellian one with a small dispersion[2] $c^2\beta_\mathrm{T}^2$:

$$f_0 = \frac{\varrho_0}{\sqrt{2\pi}c\beta_\mathrm{T}m\gamma^3}\exp\left(-\frac{(v-\beta c)^2}{2c^2\beta_\mathrm{T}^2}\right), \tag{8.28}$$

where the factor $m\gamma^3$ comes from the kinematic relation $\mathrm{d}p = m\gamma^3\mathrm{d}v$. If $\beta_\mathrm{T}\to 0$, (8.28) takes the form:

$$f_0 = \frac{\varrho_0}{m\gamma^3}\delta\left(v-\beta c\right),$$

which corresponds to the cold beam approximation.

After substituting the undisturbed distribution (8.28) function into (8.6), the dispersion relation again can be presented as (8.7) with the only change of $(k_0 - k\beta)^{-2}$ for

$$J = \frac{1}{\sqrt{2\pi}c\beta_\mathrm{T}}\int\limits_C \exp\left[-\frac{(v-\beta c)^2}{2c^2\beta_\mathrm{T}^2}\right]\frac{\mathrm{d}v}{(k_0 - kv/c)^2}\,. \tag{8.29}$$

According to the general rule, the integral is to be taken in the v-plane along a contour C passing from $-\infty$ to $+\infty$ below the pole $v = ck_0/k$ on the real axis. It can be expressed [43] in terms of a probability integral (Kramp’s function) of an imaginary argument. Unfortunately, such representation is not very descriptive and can be analytically traced just in the limiting cases discussed below.

Changing the variable $v = \beta c + \sqrt{2}c\beta_\mathrm{T}\delta_\mathrm{T}x$ transforms (8.29) to

$$J = \frac{1}{2\sqrt{\pi}\delta_\mathrm{T}\beta_\mathrm{T}^2k^2}\int\limits_C \frac{\exp\left(-\delta_\mathrm{T}^2x^2\right)\mathrm{d}x}{(1-x)^2} \tag{8.30}$$

with a dimensionless detuning

$$\delta_\mathrm{T} = \frac{k_0 - \beta k}{\sqrt{2}\beta_\mathrm{T}k}\,.$$

[2] Particle velocity cannot exceed c, so the velocity spread is to be much smaller than $c\beta\gamma^{-2}$.

For $\delta_{\mathrm{T}} \gg 1$ the integral can be estimated using the saddle point method. The standard procedure [46] yields the following asymptotic series:

$$J \asymp (k_0 - k\beta)^{-2} \times \sum_{n=0}^{\infty} \frac{(2n+1)!}{2^{2n} n! \delta_{\mathrm{T}}^{n}} . \tag{8.31}$$

The first term of the expansion represents the cold beam approximation discussed in the previous paragraph. It is worth to be mentioned that an exponentially small imaginary term that reflects Landau damping is omitted in (8.31).

There exists another limiting case of small values of the parameter $\delta_{\mathrm{T}} \ll 1$. Here large values of x provide the main income to the integral (8.30) and one can expand the denominator:

$$(1-x)^{-2} = \sum_{s=0}^{\infty} \frac{s+1}{x^{s+2}} . \tag{8.32}$$

Keeping in mind that

$$\int_C \frac{\exp\left(-\delta_{\mathrm{T}}^2 x^2\right)}{x^{n+2}} \mathrm{d}x = \delta_{\mathrm{T}}^{n+1} \times \begin{cases} \frac{(-1)^{n/2+1}\sqrt{\pi}}{(n/2+1)!} & \text{for even } n \\ \frac{\mathrm{i}\pi(-1)^{(n+1)/2}}{((n+1)/2)!} & \text{for odd } n \end{cases} \tag{8.33}$$

one can integrate this sum with $\exp\left(-\delta_{\mathrm{T}}^2 x^2\right)$ and find that

$$J \asymp -\frac{1}{2\beta_{\mathrm{T}}^2 k^2} \sum_{s=0}^{\infty} \frac{(-1)^s (2s+1)}{(s+1)!} \delta_{\mathrm{T}}^{2s} - \mathrm{i}\sqrt{\frac{\pi}{2}} \frac{k_0 - \beta k}{\beta_{\mathrm{T}}^3 k^3} \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \delta_{\mathrm{T}}^{2s} . \tag{8.34}$$

As it is easy to see, the case in question (small values of δ_{T}) corresponds to a large thermal spread of the beam particles. Under such conditions, plasma intrinsic oscillations are excited as a result of the kinetic instability development, which can be interpreted as negative Landau damping.

Really, substituting the main term of (8.34) in the general dispersion equation (8.6), one gets in the first approximation with respect to k_{b}^*:

$$\operatorname{Im} k_0 = \frac{\sqrt{2\pi} k_{\mathrm{b}}^{*2} \beta^4}{4\beta_{\mathrm{T}}^3 k_{\mathrm{p}}^2} \kappa \quad \text{for} \quad |\kappa| \ll . \tag{8.35}$$

So, at one side of the Cherenkov resonance the kinetic increment is positive (Landau damping negative) and change the sign at the opposite side. In the resonance point itself, the increment vanishes because the distribution has the maximum there.

From this point of view the close vicinity of the resonance is always a domain of kinetic instability, but for not very hot beam the instability develops

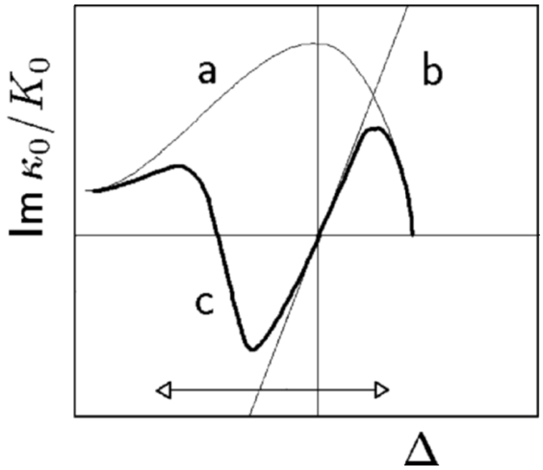

Fig. 8.5. The influence of kinetic effects on the fluid instability. (**a**) Fluid instability increment; (**b**) kinetic increment for a hot beam; (**c**) resulting curve. The arrows show the influence region

as a fluid one even at small detuning. The scheme of the effect is presented in Fig. 8.5.

To estimate the maximal increment (at least, in the low current approximation), one can use (8.18) with substitution

$$\Delta = \sqrt{2}\beta_{\mathrm{T}}\frac{k_s}{K}.$$

Of course, one cannot trust the numerical coefficient in this estimation, but an essential decrease in the gain for a warm beam is evident. Note, by the way, that Landau damping provokes a certain isolation of the Cherenkov instability from the electrostatic one at the long wave domain.

Of course, the brief sketch of theory above must be essentially supplemented to be applicable to more or less realistic devices. First of all, the problems of transverse plasma and beam nonuniformity as well as boundary conditions are of a great importance. We do not touch nonlinear effects, which require detailed computer simulations and are not typical for the book. Our aim was just basic physics of the involved processes, bearing in mind that many problems are still unsolved in this very young domain of electronics. Those who are interested in details can find them in monographs (i.e., [13]) which, unfortunately are rather few.