Influence of dark energy on gravitational lensing

Kabita Sarkar[1], Arunava Bhadra[2]
[1]*Department of Mathematics, Salesian College, Siliguri, West Bengal, India 734001*
[2]*High Energy Cosmic Ray Research Centre, University of North Bengal, Siliguri, West Bengal, India 734013*

ks.nias@gmail.com
DOI: 10.7529/ICRC2011/V05/1046

Abstract: Recent cosmological observations suggest a considerable part of the Universe consists of the dark energy which may be in the form of a vacuum energy (Cosmological Constant) or a dynamically evolving scalar field with a negative pressure. It is natural that dark energy must have some influence on local gravitational phenomena. In the present work we explored such effects due to a cosmological constant in lensing phenomenon, both in the strong and weak field regime. We found that the cosmological constant affects the lensing phenomenon but the magnitude of the effect is very small. The possibility of discriminating phantom dark energy and cosmological constant by studying lensing phenomenon has also been discussed.

Keywords: Dark Energy, Cosmological Constant, Gravitational Lensing,Weak Field, Strong Field.

1 Introduction

The accelerating expansion of the Universe is widely believed as due to dark energy which may be in the form of a vacuum energy equivalent to a cosmological constant. A number of recent cosmological observations suggest a value of $\Lambda \approx 10^{-52}$ m^{-2} [1]. Due to its universal presence, a cosmological constant is likely to influence the local gravitational phenomena though the magnitude of such effects is expected to be very small, not detectable by the experiments, owing to its very tiny value.

For long it was known that Λ has no affect at all on the bending of light phenomenon [2], but some recent studies [3] suggest for a small contribution of Λ on bending angle that diminishes the deflection angle when Λ is positive. There are also claim that the contribution of cosmological constant on bending of light could be significant (larger than the second order term) for many lens systems such as cluster of galaxies.

In the present paper we will discuss the effects in lensing phenomenon due to cosmological constant in both weak and strong field regime and will show that when the light path from a reference source, which is needed for measuring the bending angle, is taken into consideration the resultant bending in presence of cosmological constant will depend on the distance of the reference and the source from the lens.

2 Gravitational deflection of light in the SDS metric in the weak field regime

We consider the following geometrical confguration for the phenomenon of gravitational bending of light: The light emitted by the distant source S is deviated by the gravitational source (Lens) L and reaches the observer O. The angles are measured with respect to the line which is parallel to the undeflected ray (in the absence of massive object) and passes through the center of the lens (L). The point L is taken as the origin of the coordinate system.

In presence of cosmological constant the exterior space-time due to a static spherically symmetric mass distribution is the Schwarzschild–de Sitter (SDS) space-time [4] which is given by

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$$

with $A(r) = 1-2m/r-\Lambda r^2/3$ and $B(r) = 1/A(r)$ (1)

where m being the mass of the lens object. For this space time the null geodesic equation does not contain Λ due to exact cancellation of the Λ involved terms and consequently it is the same to that of the Schwarzschild geometry. For asymptotically flat space time such as the Schwarzschild space time the direction of asymptotic light rays is usually evaluated by applying the limit $r\to\infty$ in the orbit equation and the angle between the two asymptotic directions gives the total deflection angle. The same procedure was also followed to compute bending in SDS space time. However, $r \approx (3/\Lambda)^{1/2}$ gives the de Sitter horizon. Hence $r\to\infty$ does not make any sense in SDS space time. Instead the angle that the tangent to the light trajectory made with a coordinate direction at a given point may be obtained which for the metric (1) is given by

$$\tan\psi = r A^{1/2}(r) \left| d\varphi/dr \right| \tag{2}$$

For the null geodesic the above equation reduces to [5]

$$\tan(\psi) = \left[\frac{A(r_o)r^2}{A(r)r_o^2} - 1\right]^{-1/2} \tag{3}$$

Where r_o being the coordinate distance of closest approach. When $r \gg r_o$, the angle between the tangent to the light trajectory at point r, φ and the polar axis to the leading order in m, Λ and r_o/r, is given by

$$\epsilon = \frac{2m}{r_o} - \frac{m r_o}{r^2} - \frac{\Lambda r r_o}{6} + \frac{\Lambda r_o^3}{6} \tag{4}$$

Experimentally the effect of the lens on the photon trajectory is obtained by measuring the bending with respect to the photon trajectory from a second source that may be called the reference source. The distance of closest approach for the light path from the reference has to be much larger than that for the photon trajectory from the source.

When both the reference object and the source are far away from the lens in comparison to the lens-observer distance the relative deflection angle becomes

$$\delta\alpha \sim \frac{4m}{b_S} + \frac{\Lambda\delta b}{6}(d_{LR}\text{-}d_{LS}) \tag{5}$$

where b_S is the impact parameter of the light rays from the source, δb refers to difference in impact parameter at two positions and d_{LR} and d_{LS} denote distance between lens and reference source and lens and source respectively.

If $d_{LR} - d_{LS} = 10$ kpc and δb is equal to the earth-sun distance the contribution of the bending angle due to Λ will be about 10^{-16} of the total bending angle. The expected angular precision of the planned astrometric missions using optical interferometry is at the level of microarcseconds, at least 10 orders lower than the Λ contribution to the bending angle when the lens system is within the galaxy. Note that in the Solar System the influence of the cosmological constant is known to be maximum in the case of the perihelion shift of the mercury orbit, where the Λ contribution is about 10^{-15} of the total shift.

3. Gravitational lensing of light in the SDS metric in the strong field regime

A general static and spherically symmetric spacetime is of the form

$$ds^2 = -A(x)dt^2 + B(x)dx^2 + C(x)(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{6}$$

where $x = \frac{r}{2M}$

The deflection angle as a function of closest approach $x_0\left(x_0 = \frac{r_0}{2M}\right)$ is given by

$$\alpha(x_0) = I(x_0) - \pi \tag{7}$$

with $$I(x_0) = 2\int_{x_0}^{\infty} \frac{\sqrt{B(x)}}{\sqrt{C(x)}\sqrt{\frac{C(x)A(x_0)}{C(x_0)A(x)} - 1}}\,dx \tag{8}$$

The above integral diverges close to x_0. The above integral can be evaluated close to its divergence by splitting into two parts to separate out the divergent $(I_D(x_0))$ and the $(I_R(x_0))$ regular parts [6] so that

$$I(x_0) = \int_0^1 R(z, x_0) f(z, x_0)\, dz \tag{9}$$

where $$z = \frac{A(x) - A(x_0)}{1 - A(x_0)} \tag{10}$$

$$R(z, x_0) = \frac{2\sqrt{A(x)B(x)}}{C(x)A'(x)}(1 - A(x_0))\sqrt{C(x_0)} \tag{11}$$

and $$f(z, x_0) = \frac{1}{\sqrt{A(x_0) - A(x)C(x_0)/C(x)}} \tag{12}$$

The function $R(z, x_0)$ is regular for all values of z and x_0 whereas $f(z, x_0)$ diverges as $z \to 0$ i.e. as one approaches to the photon sphere.

The integral (9) can be splitted into two parts

$$I(x_0) = I_D(x_0) + I_R(x_0) \tag{13}$$

where
$$I_D(x_0) = \int_0^1 R(0, x_{ps}) f_0(z, x_0)\, dz \tag{14}$$

and $$I_R(x_0) = \int_0^1 g(z, x_0)\, dz \tag{15}$$

with $$f_0(z, x_0) = \frac{1}{\sqrt{p(x_0)z + q(x_0)z^2}} \tag{16}$$

where

$$p(x_0) = \frac{1-A(x_0)}{C(x_0)A'(x_0)}\left[C'(x_0)A(x_0) - C(x_0)A'(x_0)\right] \quad (17)$$

and

$$q(x_0) = \frac{(1-A(x_0))^2}{2C^2(x_0)A'^3(x_0)}\Big[C(x_0)C'(x_0)A'^2(x_0) + \left(C(x_0)C''(x_0) - 2C'^2(x_0)\right)A(x_0)A'(x_0) - C(x_0)C'(x_0)A(x_0)A''(x_0)\Big] \quad (18)$$

The function $g(z, x_0)$ is simply the difference of the original integrand and divergent integrand

$$g(z, x_0) = R(z, x_0)f(z, x_0) - R(0, x_{ps})f_0(z, x_0) \quad (19)$$

As $x_0 \to x_{ps}$, $p(x_0) \to 0$ the integral (9) diverges logarithmically. Expanding both the integral around $x_0 = x_{ps}$ and approximating by the leading terms, The analytical expression of the deflection angle close to the divergence in the form [6]

$$\alpha(\theta) = -u\log\left(\frac{\theta D_{OL}}{b_{ps}} - 1\right) + v + o\left(b - b(x_{ps})\right) \quad (20)$$

$$u = \frac{R(0, x_{ps})}{2\sqrt{q(x_{ps})}} \quad (21)$$

$$v = -\pi + v_R + u\log\frac{2q(x_{ps})}{A(x_{ps})} \quad (22)$$

$$v_R = I_R(x_{ps}), I(x_0) = \int_0^1 R(z, x_0)f(z, x_0)dz \quad (23)$$

From the expressions (20)-(23) one can obtain the deflection angle.

Interestingly for the SDS metric the deflection integral as given by Eq. (8) turns out to be the same to that for the Schwarzschild metric. However, as mentioned before this is not the full story.

The radius of the photon sphere for the SDS metric does not contain Λ but remains the same to that of Schwarzschild geometry i.e.

$$x_{ps} = 3M \quad (24)$$

Other parameters for the strong field deflection angle as given in Eq. (20) however, contain Λ such as

$$u = 1 - \frac{81}{4}\Lambda^2 M^4 \quad (25)$$

and

$$b_{ps} \simeq 3\sqrt{3}M\left(1 + \frac{9}{2}\Lambda M^2\right) \quad (26)$$

As mentioned before, for the SDS spacetime the source and the observer cannot be placed at infinite distances away from the lens but are to be within the de Sitter horizon.

As $r \to d_{Li}$, (i stands for source/observer which are at far away from the lens L)

$$z \approx 1 - \frac{\Lambda d_{Li}^2 r_o}{6m} \quad (27)$$

and accordingly

$$v_R \approx 2\log\left[6(2-\sqrt{3}) - \frac{\Lambda d_{Li}^2 r_o}{m}(2\sqrt{3}-1)\right] \quad (28)$$

$$v = -\pi + v_R + \left(1 - \frac{81}{4}\Lambda^2 M^4\right)\log\frac{2q(x_{ps})}{\frac{1}{3} - 3\Lambda M^2} \quad (29)$$

$$q(x_{ps}) = \frac{(1+36\Lambda M^5)^2}{M(1-72\Lambda M^5)^3}(1-M) \quad (30)$$

The equation (20) together with the equations (26)-(30) finally gives the expression for the bending angle in the strong field regime. It is clear from the above expressions that the deflection angle contains cosmological constant.

4 Conclusion

We conclude the followings:

The cosmological constant affects the gravitational bending angle. In the weak field expression for bending angle in the SDS geometry, there are two leading order terms involving cosmological constant, one of them is purely local in the sense that it does not contain any information about the location of the observer/source. Interestingly this term has the same signature to that of the classical expression of general relativistic bending (4m/b) i.e. this term will cause an increase of the bending angle. The other term, which is the dominating one, involves the radial distances of the source and the observer and it bears the repulsive characteristics of the positive cosmological constant.

In the study of gravitational bending in Schwarzschild-de Sitter geometry or in any asymptotic non-flat space time it is also important to study the photon trajectories from reference objects with respect to which the bending will be measured. When such an aspect is taken in to consideration the contribution of cosmological constant to the effective bending is found to depend on the distances of the source and the reference objects.

The effect of cosmological constant will be prominent for sources of large distances.

The strong field expression for bending also involves cosmological constant. This effect mainly occurs through the expression of impact parameter.

If the dark energy is phantom in nature, it will have significantly different effect on local gravitational phenomena in compare to those of cosmological constant. This is mainly because the phantom scalar field will evolve differently in local gravity situation. Consequently from local gravitational effects, one should be able to discriminate these two alternative possibilities of dark energy. The matter will be discussed in detail elsewhere.

Acknowledgements: KS acknowledges the support from the LOC, ICRC 2011 for presenting the work.

References:

[1] E. Komatsu et al. (WMAP Collaboration), arXiv:1001.4538 (2010)

[2] N.J. Islam, Phys. Lett. A 1983, 97, 239; V. Kagramanova, J. Kunz, and C. Lammerzahl, Phys. Lett. B 2006, 634, 465; F. Finelli, M. Galaverni, A. Gruppuso, Phys.Rev. D 2007, 75, 043003; K. Lake, Phys.Rev. D 2002, 65, 087301

[3] W. Rindler, M. Ishak, Phys. Rev. D 2007, 76, 043006; M. Sereno, Phys.Rev.D 2008, 77, 043004; M. Sereno, Phys.Rev.Lett. 2009, 102, 021301; T. Schucker, Gen. Relativ. Gravit. 2009, 41, 67; K. Lake, 2007, arXiv:0711.0673; A.Bhadra, S. Biswas and K. Sarkar, Phys. Rev. D 2010, 82, 063003

[4] F. Kottler, Ann. Phys. 1918, 361, 401

[5] A. Bhadra arXiv:1007.1794 (2010)

[6] V. Bozza *Phys.Rev*. D 2002, 66, 103001