Coulomb Energy Corrections for Egg Shaped Asymmetric Deformation

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The most persistent puzzle in the fission process is the asymmetric mass distributions of fission fragments [1]. What shape changes are involved in the transition from a single nuclear system to two separated daughter fragment nuclei is also a very common question of interest. The mass asymmetric fission appear due to the strong competition between nuclear deformation surface and Coulomb energies and corresponding shell corrections [2,3]. In the fissioning process however, at some point of deformation the nucleus has to assume an asymmetrical shape in the fissioning direction. Some common assumed deformation shapes are like ellipsoid, dumbbell, pear shape and many more. One of the simplest asymmetrical shape is like an egg. This egg shape is obtained basically from two hemi- ellipsoids of revolution (about the semi major axes, b and c with equal semi minor axes, a), by joining the equal circular faces as seen in fig.1. With the increase of value of semi major axes-b and c the value of semi minor axes-a decreases so as to conserve the total volume. Fission occurs when the joint is snapped as a result of the total surface energy increasing beyond the total surface energy of the spherical symmetrical fragments. In the fissioning process it is important that the cleavage starts in this proceedings of elongation at a point where the surface curvature is least and it is at the joint of the two hemi-ellipsoids.

We have evaluated the Coulomb energy of asymmetric egg shaped Uranium $^{235}U_{92}$ nucleus by performing six - dimensional integration over this egg volume as:

$$E_{Coulomb} = a_C \int dx_1 dy_1 dz_1 \int dx_2 dy_2 dz_2 \frac{1}{r_{12}}$$

The schematic of fissioning egg shaped symmetry is indicated in fig.1. Below.

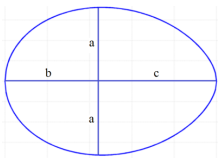

Fig.1 : Schematic of fissioning asymmetric egg shaped nucleus.

The surface energy however has an analytic form as:

$$E_{Surface} = a_S[2\pi a^2 + \pi a[\frac{b^2}{\sqrt{b^2-a^2}} cos^{-1}(\frac{a}{b}) + \frac{c^2}{\sqrt{c^2-a^2}} cos^{-1}(\frac{a}{c})]]$$

Here the ratio b/c is $\frac{b}{c} = \frac{\text{Mass of light fragment}}{\text{Mass of heavy fragment}}$

We observed that final asymmetrical shape of an egg does not start at the beginning of the deformation process rather it starts when the volume conservation, cleavage point (a < b < c) and b/c conditions are satisfied simultaneously. However this occurs only after slight deformation has taken place. Fig.2 shows surface energy, $E_{surface}$ and the Coulomb energy, $E_{coulomb}$ as a function of semi-minor axes value- a, for asymmetric fission into various fragment combinations (as 80+155, 85+150 to 115+120).

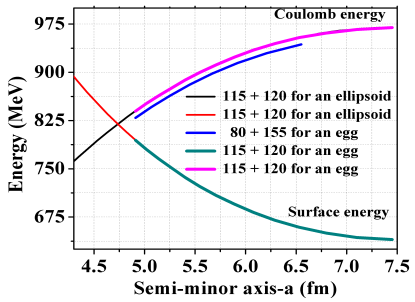

Fig.2 : Coulomb and Surface energy of fission as a function of parameter-a.

The value of parameter- a decreasing from the sphere on these plots corresponds to the point where asymmetry starts developing and the last smallest value is the value where the surface energy equals that of the fragments (the saddle point). Comparison of surface energy between an ellipsoid and an egg is shown in fig.3. As a function of semi minor axes-a, the symmetrical and asymmetrical shapes are thus seen to behave similarly. Most importantly, the Coulomb energy for various fragment combinations is seen to decreases from that of the symmetric fission. This behavior will lead to higher the Coulomb barriers and shapes varying around the saddle point.

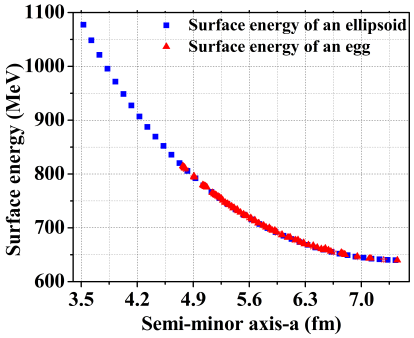

Fig.3 : Surface energy of ellipsoid and an egg are seen to overlap.

This corresponds therefore that the shell effects are more predominant for the asymmetric fission. This increase of fission barriers by about 4 to 10 MeV may arise due to this Coulomb energy variation for the asymmetric fission.

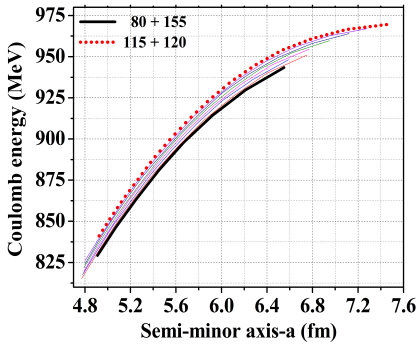

Fig.4. Coulomb energy of fission fragments mass number pair as a function of parameter- a.

Coulomb energy corrections of fissioning egg for eight different mass pairs of fission fragments at the saddle point value of semi minor axis-a is shown in fig.4. Very large differences are witnessed in the Coulomb energies as compared with the symmetric shape of ellipsoid of revolution. It is observed that Coulomb energy gradually decreases as we move towards more asymmetrical combination compared with the symmetrical case.

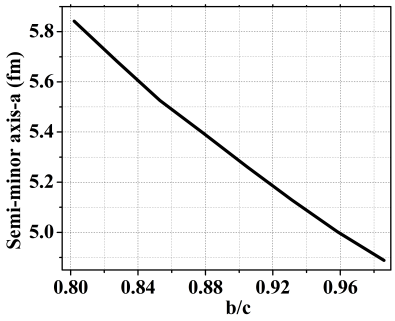

Fig.5. Semi-minor axis value - a at the saddle point for different mass fission fragments in the asymmetric fission (b/c) of $^{235}U_{92}$.

The surface and Coulomb energies compensate each other. While the Coulomb energy causes the nucleus to deform, the surface energy on the other hand contracts the nucleus to a spherical shape. When the surface and Coulomb energies become equal the nucleus is deformed at the saddle point to fission to final fragments. One another important result here is that the cleavage starts at that point where the surface curvature is least and the lesser eccentricity ellipsoid, b part of the egg become the smaller fragment while the bigger eccentricity part, c becomes the larger fragment The prevailing perception in the available literature does not describe the phenomena of asymmetric fission via the formation of an egg shape and they tend to describes the asymmetric fission through some neck formation with one big blob on one side while a smaller one on the other side. So far this aspect of $P_3(\cos\theta)$ deformation for asymmetric fission has not been highlighted for this egg shaped deformation and present work indicates higher amount of shell corrections due to higher fission barriers to be applicable in the asymmetric fission.

References:

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