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The Algebraic Cluster Model (^{8}Be, ^{12}C) and the Cluster Shell Model (^{13}C, ^{7}Be, ^{19}F, ^{8}Be) *

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Abstract. Bijker and Iachello's algebraic cluster model (ACM) and its extension to the cluster shell model (CSM), provides a new theoretical platform for the study of alpha-clustering in light nuclei. It led to the discovery of the $\mathcal{D}_{3h}$ symmetry in ^{12}C and ^{20}Ne, with the discovery in ^{12}C of a new g.s. rotational band with the spin sequence of, 0^+, 2^+, 3^-, $4^+/4^-$ and 5^-, including the predicted 4^+ and 4^- parity doublet. Applications of the CSM shell model to particle molecular orbits in ^{9}Be and ^{13}C (C_2' and $\mathcal{D}_{3h}$' particle symmetries, respectively), lead us to conjecture molecular hole states in ^{7}Be and ^{19}F. We observe in these nuclei the predicted phenomenological structure. And we further consider conjectured p-h states in ^{8}Be with the predicted phenomenological p-h structure of rotational band at high excitations of approximately 20 MeV. A search for these rotational band in ^{8}Be was performed at ISOLDE.

1. The Algebraic Cluster Model (ACM)

One of the most tantalizing development in cluster physics is the application of geometrical point group symmetries by Bijker and Iachello [1] within the Algebraic Cluster Model (ACM). It led to the discovery of the $\mathcal{D}_{3h}$ symmetry in ^{12}C [2] and the observation of the most unusual mixed parity ground state rotational band that includes the spin-parity states of $J^\pi = 0^+, 2^+, 3^-, 4^+/4^-$ and 5^- [1], including the 4^+ and 4^- parity doublet, as shown in Fig. 1. Such mixed symmetry bands are familiar in molecular physics. Further development of ab-initio models such as the no core shell-model (NCCI) [3,4] and the quantum Monte Carlo (GFMC) [5], may indeed shed light on this new development of the cluster model.

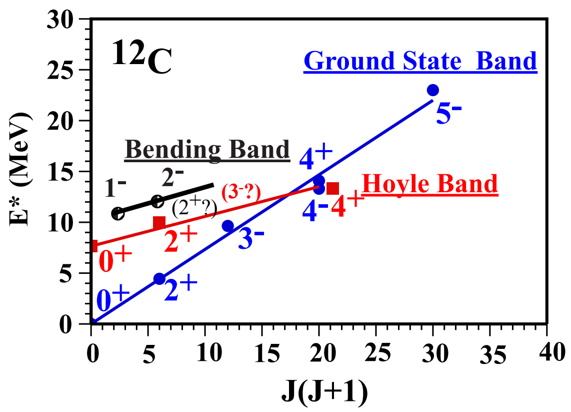

Fig. 1: The rotational structure of ^{12}C, and the predicted "missing" 2^+, 3^- states, taken with permission from [1].

2. The Cluster Shell Model (CSM)

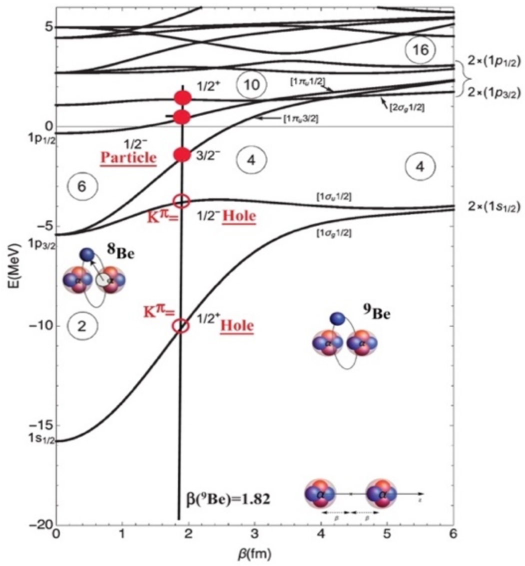

Fig. 2: Nilsson-like single (neutron) particle states in ^{9}Be predicted by the CSM [6] as a function of the separation distance of the alpha-particles, figure adapted from [6]. The particle (hole) orbits are indicated by full (open) circles.

The ACM [1] was recently extended by Della Rocca and Iachello [6] to include single particle states within the cluster shell model (CSM). It is an extension of the shell model to molecular orbits that are calculated as a function of the cluster separation energy, as shown in Fig. 2 for $\alpha + \alpha$ states in ^{8}Be. This extension of the shell model is silimar to the Nilsson model. However, we emphasize that while in the Nilson Model the single

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particles are moving in the deforemd field which leads to enhance quadrupole transitions, molecular states include all deformations and lead to enhance E1, E2, E3, E4 etc. transitions [7]. In addition, molecular states lead to parity doublets, which are not predicted by the Nilsson Model.

2.1 Particle State of the CSM (^{13}C)

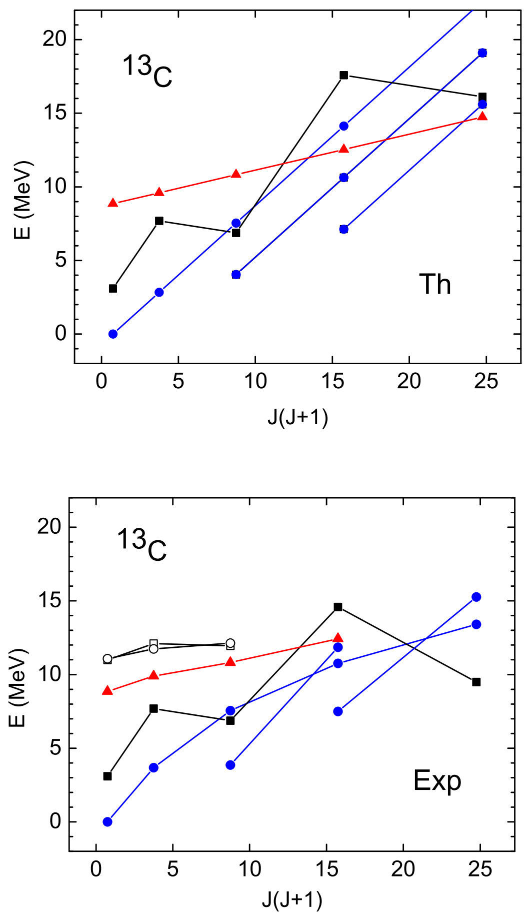

Fig. 3: Calculated and measured spectra of ^{13}C [8]. Figure copied from [8] with permission.

Single particle states in the field of the triangular field of the three alpha-particles of ^{12}C ($\mathcal{D}_{3h}$ symmetry) lead to a similar Nilsson-like particle states in ^{13}C ($\mathcal{D}_{3h}'$ symmetry), as calculated by Bijker and Iachello [8]. The so predicted and measured spectra of ^{13}C [8] are shown in Fig. 3. The CSM also predicts a relationship between the B(E2) in ^{12}C and ^{13}C [8]:

$$B(E2\colon\ 0^+ \rightarrow 2^+) = (Ze)^2\ \frac{5}{4\pi}\ \frac{1}{4}\ \beta^4$$

$$B(E2\colon J', K \rightarrow J, K) = (Ze)^2\ \frac{5}{4\pi}\ \beta^4 (J', K, 2,\ 0\,|\,J, K)^2$$

In Table I, taken from Ref. [15] of [8], we list these B(E2) in ^{12}C and ^{13}C.

TABLE I. $B(EL)$ values in ^{12}C and ^{13}C in W.U. [15].

	$B(EL)$	Exp	Th
^{12}C	$B(E2; 2_1^+ \rightarrow 0_1^+)$	4.65 ± 0.26	4.8
	$B(E3; 3_1^- \rightarrow 0_1^+)$	12 ± 2	7.6
^{13}C	$B(E2; 3/2_1^- \rightarrow 1/2_1^-)$	3.5 ± 0.8	4.8
	$B(E2; 5/2_1^- \rightarrow 1/2_1^-)$	3.1 ± 0.2	3.2
	$B(E3; 5/2_1^+ \rightarrow 1/2_1^-)$	10 ± 4	4.3

2.2 Conjectured Hole States of the CSM (^{7}Be, ^{19}F)

The successful predictions of particle states in the frame of the CSM lead us to consider whether hole states can be discussed in the same framework of the CSM, as for example shown in Fig. 2 (in open circles).

Indeed, the measured states in ^{7}Be and ^{19}F, shown in Fig. 4, exhibit very similar structure to the one predicted for particle states shown in Fig. 3. Note the $K^\pi = 1/2^+$ and $1/2^-$ bands exhibit the crossing pattern of the Coriolis interaction, with moment of inertia similar to that of ^{8}Be and ^{20}Ne. The structure of ^{19}F also includes the predicted parity doublets.

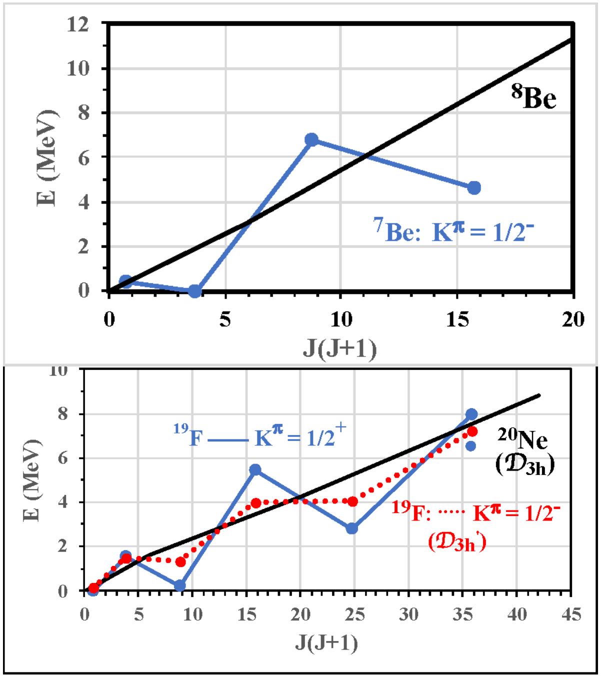

Fig. 4: The $K^\pi = 1/2^+$ and $1/2^-$ bands observed in ^{7}Be and ^{19}F.

2.3 Conjectured Particle-Hole States of the CSM (^{8}Be)

Particle-hole (p-h) states in ^{8}Be may indeed arise in the CSM as shown in Fig. 2. They are holes in the K^π = ½$^+$ and ½$^-$ proton or neutron single paticle states, excited to the $K^\pi = 3/2^-$ states. Hence, the coupling of such p-h sates leads to $K^\pi = 2^+$, 1^+, 2^- and 1^-. It is also clear from Fig. 2 that the positive parity states are expected at lower excitations. Both iso-spin 0 and 1 states, corresponding to proton (^{9}B like) and neutron (^{9}Be like) excitations, are predicted.

The p-h states of the CSM are predicted to be deformed, hence a rotational band is predicted on top of each p-h state. These rotational bands are also predicted to resemble the g.s. rotational band of the even-even cluster nucleus. Hence, the p-h bands in ^{8}Be, the g.s. rotational bands in ^{9}Be and ^{9}B, the g.s. rotational band in ^{8}Be, are all predicted to have very similar properties including similar moment of inertia, as shown in Fig. 5.

3. Search for Rotational Structure at High Excitations in ^{8}Be (ISOLDE IS692)

While the theoretical framework for describing p-h states in ^{8}Be is still pending, the striking phenomenological structure observed in Fig. 5, lead us to search for rotational band structure at high excitation in ^{8}Be [9]. Preliminary result of our ISOLDE measurements are also discussed in a paper in this conference [9]. In Fig. 6 we show our simulations of states in ^{8}Be at energies above 21 MeV [9]. We used the d(^{7}Be,p) reaction to measure the spin-parity of the state at 21.5 MeV that has been proposed to be 3^- [10]. The spin-parity of the three narrow states are not known.

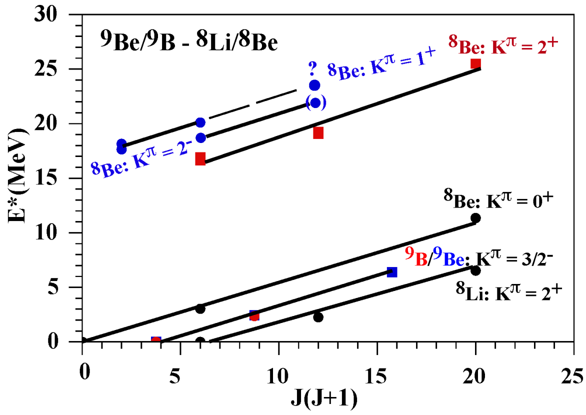

Fig. 5: Rotational band structures in ^{8}Be and the iso-baric analog ^{8}Li, as well as ^{9}Be and ^{9}B. The state at 21.5 MeV (shown in parentheses) in the putative $K^\pi = 2^-$ band, was proposed to be a 3^- state [10].

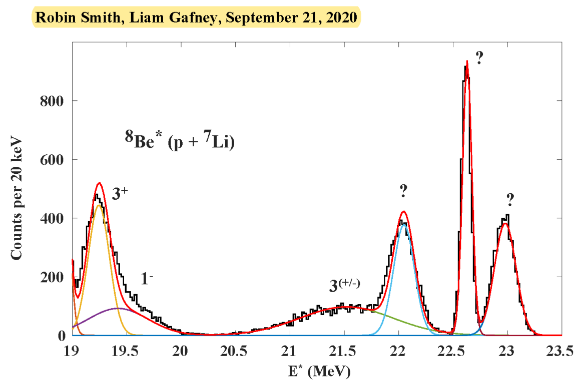

Fig. 6: Simulated spectra of 8Be. We aim to measure [9] the spin-parity of the three known narrow states at 22.2, 22.6 and 22.98 MeV.

4. Conclusions

The successful descriptions of ^{12}C (^{16}O and ^{20}Ne) by the ACM, and the successful descriptions of ^{9}Be, ^{13}C (^{21}Ne and ^{21}Na) by the CSM, together with the phenomenology observed in ^{7}Be, ^{19}F and ^{8}Be, discussed here, should serve as an impetus for theorists to develop the framework to discuss hole states, as well as particle-hole states, in the framework of the CSM.

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