



Cosmological Constraints on Bulk Neutrinos

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Recent models invoking extra spacelike dimensions inhabited by (bulk) neutrinos are shown to have significant cosmological effects if the size of the largest extra dimension is $R \gtrsim 1\text{fm}$. We consider effects on cosmic microwave background anisotropies, big bang nucleosynthesis, deuterium and ${}^6\text{Li}$ photoproduction, diffuse photon backgrounds, and structure formation. The resulting constraints can be stronger than either bulk graviton overproduction constraints or laboratory constraints.

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In this Letter we describe several cosmological constraints on models for neutrino mass which rely on bulk fermions propagating in compact extra spacelike dimensions. Extra spacetime dimensions, long provided under the ægis of Kaluza-Klein (KK) and superstring theories, have played an essential role in recent attempts to solve fundamental problems in particle physics [1, 2]. In particular, in some theories invoking n compact extra spacelike dimensions, all Standard Model (SM) fields are localized on a three-dimensional surface (3-brane), but gravity experiences the full spacetime (bulk) [2]. This yields the relation $M_{\text{Pl}}^2 = M_F^{n+2} V_n$ between the new fundamental $(4+n)$ -dimensional reduced Planck scale M_F and the four-dimensional reduced Planck scale $M_{\text{Pl}} = (4\pi G_N)^{-1/2}$, where V_n is the volume of the additional dimensions. If the volume of the internal space is sufficiently large, M_F can be much smaller than M_{Pl} , giving rise to a low-scale theory of quantum gravity (*e.g.*, $V_2 \approx 5 \times 10^{-5} \text{mm}^2$ gives $M_F \sim 10 \text{TeV}$ for $n = 2$).

In this framework, if M_F is sufficiently small, there is no longer a heavy mass scale available in the theory to suppress neutrino masses relative to other fermion masses via a seesaw or similar mechanism [3]. Several higher-dimensional mechanisms have been developed [4], however, which can give neutrino masses and mixings consistent [5–7] with the solar, atmospheric, and accelerator neutrino experiments [8]. One widely used scheme posits the existence of SM-singlet fermions (neutrinos) which propagate in the bulk but couple via Yukawa interactions with the SM-doublet (active) neutrinos on our brane. This setup gives up to three light Dirac neutrino masses μ_i associated with the active neutrino flavors ν_e , ν_μ , and/or ν_τ . In addition, each bulk neutrino appears on our brane as a tower of massive KK modes (*i.e.*, sterile neutrinos), and the vacuum mixing angle between an active neutrino and a mode with mass $m_{\text{mode}} \gg \mu_i$ is $\theta_{\text{mode}} \simeq \sqrt{2}\mu_i/m_{\text{mode}}$. The mass distribution of the modes depends on the geometry of the internal space. The simplest and most widely adopted geometry of the internal dimensions is that of an n -dimensional torus with radii R_j ($1 \leq j \leq n$), for which the mode masses are $m_{\mathbf{k}}^2 = k_1^2/R_1^2 + \dots + k_n^2/R_n^2$, where, in bulk neutrino models, $\mathbf{k} = (k_1, \dots, k_n)$ is an n -tuple of whole num-

bers, and where we assume that the bare masses of the bulk fermions are negligible. Several authors have found non-standard solutions to the neutrino anomalies in this framework [5–7]. These solutions require $R_1^{-1} \lesssim 1 \text{eV}$ ($R_1 \gtrsim 0.1 \mu\text{m}$) for the largest dimension R_1 , for otherwise they reduce to those for a standard Dirac neutrino mass [8, 9]. We show how these and non-toroidally compactified models with densely distributed KK modes affect standard cosmology through their production in the early universe and subsequent decay.

The incoherent production of sterile-KK neutrinos of mass m_k in the early universe is a nonthermal process governed by a Boltzmann equation [10, 11]

$$\frac{d}{dt}f_k = \Gamma_{\alpha k} f_\alpha - \frac{m_k}{E} \frac{1}{\tau_k} f_k + \sum_{l>k} C_{k,l} [f_l], \quad (1)$$

where $f_i = f_i(p, t)$ are momentum- and time-dependent distribution functions, and where α is an active neutrino label and k, l are mode labels of a specific KK tower. We discuss the case in which $R = R_1$ is the radius of the largest extra dimension and all other dimensions are small enough to have no effect on low energy neutrino physics. We have ignored the flavor coupling of multiple towers. The first term in Eq. (1) is the conversion rate from active to sterile species and the second results from the decay of a mode with lifetime τ_k . The latter arises because singlet neutrinos which mix with active neutrinos can decay either to SM or bulk states. On the brane, the partial decay width of the process $\nu_k \rightarrow 3\nu$ is $\sin^2 \theta_k G_F^2 m_k^5 / 192 \pi^3 = G_F^2 m_k^3 \mu_i^2 / 96 \pi^3$ and that of the radiative decay $\nu_k \rightarrow \nu \gamma$ is smaller by a factor $27\alpha/8\pi$ [12]. We have also included in our calculations the contributions to τ_k from visible and hadronic decays estimated from the partial decay widths of the Z^0 boson [13]. In the bulk, the k' -summed width of the decay $\nu_k \rightarrow \nu_{k'} h_{k-k'}$ is $\sim m_k^4 R / 12 \pi M_{\text{Pl}}^2$, where $h_{k-k'}$ is a KK graviton mode [14]. The last term in Eq. (1) represents the decay contribution of all higher modes $l > k$ into mode k , and $C_{k,l}$ is the appropriate collision operator.

The conversion rate $\Gamma_{\alpha k} = \Gamma_{\alpha k}(p, t) = (\Gamma/2) \langle P_{\alpha k} \rangle$ to KK modes is the product of half the interaction rate Γ of the neutrinos with the plasma and the average probability $\langle P_{\alpha k} \rangle$ that an active neutrino ν_α scatters into ν_k .

The probability depends on the matter mixing angle and the damping rate $D = \Gamma/2$ [15]:

$$\langle P_{\alpha k} \rangle \simeq \frac{1}{2} \frac{\Delta_k^2 \sin^2 2\theta_k}{\Delta_k^2 \sin^2 2\theta_k + D^2 + (\Delta_k \cos 2\theta_k - V)^2}. \quad (2)$$

Here, $\Delta_k \simeq m_k^2/2p$, and $V = V^L + V^T$ is the full weak potential including lepton and thermal contributions. We assume a small lepton number of order the baryon number (so $V^L \ll V^T$), since a larger lepton number serves only to enhance sterile neutrino or anti-neutrino production. Eq. (2) incorporates the standard physically well-motivated two-neutrino active-sterile matter mixing angle and the effects of quantum damping [15]. Our constraints depend on the deleterious effects of the relatively high modes, in which regime this formalism is identical to that derived from direct diagonalization of the tower mass matrix [6]. The finite temperature potentials V^T from the neutrino and charged lepton backgrounds of the same flavor are $(8\sqrt{2}G_F E_\nu/3m_Z^2)(\rho_\nu + \rho_{\bar{\nu}})$ and $(8\sqrt{2}G_F E_\nu/3m_W^2)(\rho_l + \rho_{\bar{l}})$, respectively [16].

Another important effect is the dilution of modes populated at temperatures $T \gg 100$ MeV. Disappearance of relativistic degrees of freedom manifests as heating of the plasma relative to the KK modes. We include this effect by following separately the complete time-temperature relations for the photons and modes.

We explore the cosmological ramifications of two representative classes of bulk neutrino models: Class I where $M_* \gtrsim 250$ TeV; and Class II wherein $M_* \sim 1 - 10$ TeV. Here, $M_*^{n+2} \equiv (2\pi)^n M_F^{n+2}$. In either Class the active neutrinos may be coupled with one, two, or three bulk neutrinos. The cosmological overproduction of bulk graviton modes limits temperatures in this scenario to be less than $T_*^g \simeq 10^{(6n-15)/(n+2)}$ MeV (M_*/TeV) which is an upper limit on the “normalcy” temperature T_* at which the universe must be free of bulk modes [17]. Models falling into Class I correspond to large T_*^g ($\gtrsim T_{\text{electroweak}} \sim 100$ GeV), while those in Class II have very low T_*^g ($\lesssim$ GeV). In many Class II models effectively only the lightest active neutrino couples to one KK tower. In either Class if the heaviest active neutrino couples to a tower then $\mu_3 > \sqrt{\delta m_{\text{SK}}^2} \approx 0.057$ eV [18]. We calculate here the constraints that arise in cosmologies that satisfy normalcy temperature requirements of their respective class. However, if the radiation dominated era was never significantly above the decoupling temperature $T_{\nu\text{dec}} \sim 1$ MeV of the active neutrinos in a very low reheating scenario for inflation, then early universe constraints cannot be placed on either bulk graviton or bulk neutrino production.

For each of these classes we have solved numerically Eq. (1) with $C_{k,l} = 0$ for the population of the N lowest modes, with fully self-consistent temperature evolution of all relevant species. We have performed this calculation for a single KK tower; additional towers can serve only to enhance cosmological limits. The height N of the tower is the highest mode populated at the appropriate

T_*^g . Since our calculations begin at the highest temperature T_*^g permitted by graviton overproduction limits, any adverse cosmological effects we find will imply that the normalcy temperature T_* must be significantly lower in bulk neutrino models than implied from graviton production alone. We have conservatively incorporated the effects of decays in the bulk by assuming the decay products’ mass-energy negligibly affects the dynamics of the universe. For a given momentum the number of modes produced per active neutrino per log-interval of temperature $\Gamma_{\alpha k}/H$ depends implicitly on the mode mass m_k via Eq. (2), where H is the instantaneous Hubble expansion rate [10]. However, as shown analytically in Ref. [10], for a mode non-relativistic at the decoupling temperature $T_{\nu\text{dec}}$ of the active neutrinos, the energy density (given by $m_k \Gamma_{\alpha k}/H$ integrated over $\ln T$ and the active neutrino distribution) is *independent* of the mode number, despite the dependence of the mixing angle on the mode mass.

This result assumes dilution is negligible and depends on the modes not having decayed appreciably [10, 11]. Under the latter assumption, and with some simplifications, we can extend Eq. (9) in Ref. [10] to obtain an analytic estimate $N_{\nu_k}(\text{BBN}) \sim 10^{-3} (\mu_i/1 \text{ eV})^2 (g_*^f/g_{*k}^p)$ for the energy density at $T_{\nu\text{dec}}$ in a single mode k relative to that in an active neutrino species. The ratio (g_*^f/g_{*k}^p) approximates dilution effects. The statistical weight in relativistic particles in the plasma at $T_{\nu\text{dec}}$ is g_*^f , and is g_{*k}^p at the epoch of maximal production of mode k . (Roughly, this maximal production epoch is related to mode mass as $T_{\text{max}} \simeq 133 \text{ MeV} (m_k/1 \text{ keV})^{1/3}$ [10].) Our numerical calculations follow in detail the simultaneous production, dilution, and decay of all relevant modes of various energies, giving a $N_{\nu_k}(\text{BBN})$ dependence on k which is flat modulo the effects of dilution and decay.

Population of KK modes in the early universe leads to a number of unacceptable effects that provide for compelling constraints. Our calculated cosmological constraints differ from those in Ref. [6], but they complement the supernova limits of Refs. [6, 7], and the laboratory constraints of Ref. [19].

Class I model constraints are given in Fig. 1. The total effective number $N_\nu(\text{BBN})$ of neutrino flavors at the BBN epoch must be less than that of 4 active neutrino species, since otherwise the predicted and observed abundances of the light elements are discordant [21]. We require the KK tower contribution $\sum N_{\nu_k}(\text{BBN})$ to be less than that of a single active neutrino flavor. Photoproduction of deuterium (D) and ${}^6\text{Li}$ due to decay of modes after big bang nucleosynthesis (BBN) [22] gives another constraint. Energetic cascades dissociate ${}^4\text{He}$ into excessive amounts of D [23]. The increase in energy density in relativistic particles due to mode decay prior to cosmic microwave background (CMB) decoupling can lead to suppression of the second CMB acoustic peak. The current limit is that the effective number of neutrino flavors at decoupling is $N_\nu(\text{CMB}) < 13$ at 95% certainty [24]. Measurements to higher multipole moments by the Microwave Anisotropy Probe (MAP) (reaching $N_\nu(\text{CMB}) \simeq 3.9$) and Planck

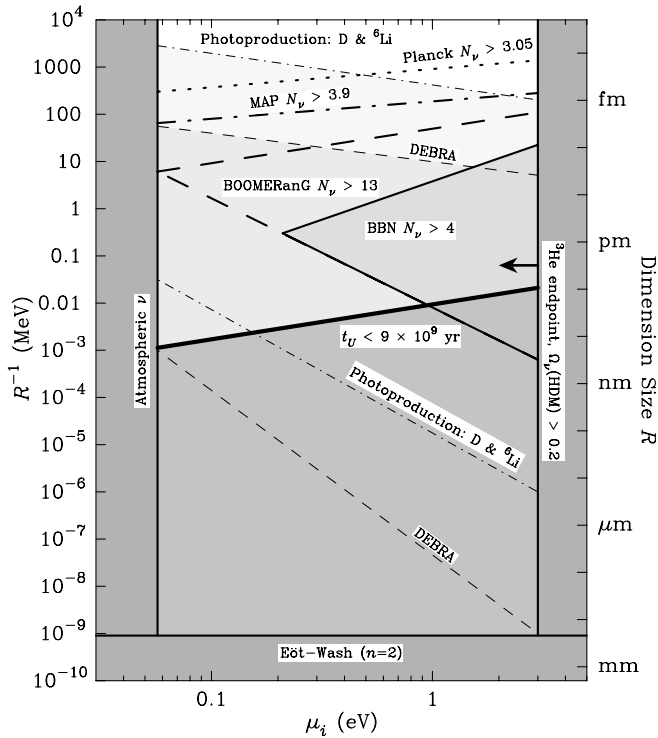


FIG. 1: Cosmological constraints on Class I bulk neutrinos. Photoproduction and DEBRA constrain regions between the dot-dashed and short-dashed lines, respectively. The CMB constrains parameters below the labeled BOOMERanG, MAP, and Planck lines and above the long-dashed line. BBN constrains the region between the light solid lines. Parameters must lie above the heavy solid line to be consistent with the inferred age of the universe. The vertical lines arise from the neutrino oscillation and ${}^3\text{H}$ endpoint limits on neutrino masses [8, 9, 13, 18]. Also shown is the 218 μm Eöt-Wash limit on the size of two congruent large extra dimensions [20].

(reaching $N_\nu(\text{CMB}) \simeq 3.05$) surveys will be able to further limit the relativistic energy present at decoupling [25], or perhaps flag the fossil relativistic energy of bulk modes at $R \sim 0.1 \text{ fm}$. The increase in energy density due to mode decays was found by summing the energy injected between the neutrino and photon decoupling epochs. Another significant constraint comes from the current limits on diffuse extra-galactic background radiation (DEBRA) due to radiative decays of sterile neutrinos occurring between CMB decoupling and today. The photon background so produced must have a total flux per unit solid angle $d\mathcal{F}/d\Omega \lesssim (1 \text{ MeV}/E) \text{ cm}^{-2}\text{sr}^{-1}\text{s}^{-1}$ [26–28]. The expansion age of the universe, $t_U > 9 \times 10^9 \text{ yr}$, also provides a constraint on the energy density in KK modes. We have found that the constraints from distortion of the CMB spectrum and the signals in the solar neutrino experiments from mode decays are weaker than the constraints above [9, 26, 28]. Note that the arguments above do not depend on the detailed mode structure but rather on the existence of a high density of modes.

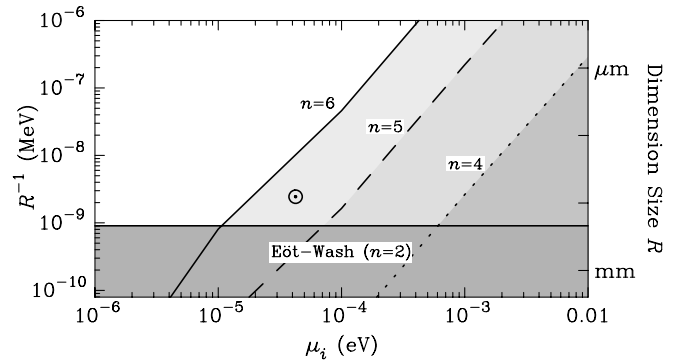


FIG. 2: Structure formation constraints on Class II bulk neutrino models with numbers of extra dimensions $n = 6, 5, 4$ ($\Omega^{\text{HDM}} > 0.1$ to the right of the slanted lines). By assumption, the size of the largest extra dimension is R , and all other dimensions have sizes $\ll \text{pm}$. For example, the solar neutrino solutions of Dvali & Smirnov and Caldwell, Mohapatra, & Yellin [5] lie near $\odot$.

Class II model effects are shown in Fig. 2. Though high-lying modes in these models are absent owing to low T_*^g , there may remain enough energy density in low mass modes to comprise an appreciable hot dark matter component. Structure formation considerations [29] suggest that a hot component cannot contribute $\Omega^{\text{HDM}} > 0.1$. Contours of $\Omega_{\nu_s}^{\text{HDM}} = 0.1$ are shown in Fig. 2 for these models with $n = 6, 5, 4$ extra dimensions. Note that some recent models for solar neutrino oscillations fall in a parameter range which could give an appreciable $\Omega_{\nu_s}^{\text{HDM}}$. Whether this can constitute a true constraint depends on the precise relation between T_*^g and M_* in these models [30]. At present all Class I and II models [5–7] can escape elimination by invoking a sufficiently low ($\lesssim 20 \text{ MeV}$) re-heating temperature T_r for inflation.

The spectrum of low-lying modes in Class II models could give a viable dark matter candidate if T_r is low. Low T_r results in suppressed production of high-mass modes that provide the closure, BBN, and decay constraints. For low enough μ_i , all modes produced below T_r survive until today and escape decay constraints. The possibility of a realistic dark matter component from the KK modes is finely tuned. For instance, for $R^{-1} = 4 \times 10^{-8} \text{ MeV}$, and $\mu_i = 10^{-5} \text{ eV}$, $\Omega_{\nu_s} \sim 0.1$ for $T_r \sim 1 \text{ GeV}$, but $\Omega_{\nu_s} \sim 0.2$ for $T_r \sim 1.3 \text{ GeV}$. Albeit finely tuned, the latter case, for which $\Omega_{\nu_s}^{\text{HDM}} < 0.1$, is an interesting dark matter candidate, comprising a mixture of hot, warm ($\sim \text{keV}$), and cold ($\sim \text{MeV}$) components.

Modifications to Class I and II models may allow circumvention of our constraints. A stronger dependence of the mixing angle on μ_i/m_{mode} would ensure that the population of the modes would fall with increasing mass. An alternate dependence of mode lifetime on m_{mode} and μ_i could eliminate some or all of the constraints. There could exist multiple additional (possibly fat) branes in the bulk, devoid of energy density and parallel to our own, onto which modes decay preferentially [17]. If the

re-heating temperature T_r of inflation is near $T_{\nu\text{dec}}$, no KK modes will be populated in the radiation dominated era, and therefore the constraints presented here do not apply. Some population of KK modes can occur during reheating, or through resonant production if there is a large lepton number, but we do not explore these possibilities here. Also, the internal dimensions need not be toroidally compactified [31]. A space which has a KK mode decomposition with a sufficiently low mode density (or a *gap*) could evade cosmological constraints. Bad effects of higher μ_i can be removed by restricting the KK towers to those built on low μ_i ($\lesssim 10^{-4}$ eV) active neutrinos — this is what is (or should be) done in Class

II models to avoid constraint. However, models which make use of all three towers are in some sense the most “natural,” albeit the most severely constrained.

Ultimately, our cosmological considerations may help to narrow the otherwise prodigious range of parameters discussed by modelers to date.

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- [1] I. Antoniadis, Phys. Lett. **B246**, 377 (1990); J.D. Lykken, Phys. Rev. D **54**, 3693 (1996); E. Witten, hep-ph/0002297, and references therein.
 - [2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429**, 263 (1998). I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **B436**, 257 (1998).
 - [3] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, edited by A. Sawada and A. Sugamoto, K.E.K. preprint 79-18 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
 - [4] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and J. March-Russell, Phys. Rev. D **65**, 024032 (2002); K.R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999); K.R. Dienes and I. Sarcevic, Phys. Lett. B **500**, 133 (2001).
 - [5] G. Dvali and A.Yu. Smirnov, Nucl. Phys. **B563**, 63 (1999); R.N. Mohapatra, S. Nandi, and A. Pérez-Lorenzana, Phys. Lett. **B466**, 115 (1999); R.N. Mohapatra and A. Pérez-Lorenzana, Nucl. Phys. **B576**, 466 (2000); R.N. Mohapatra and A. Pérez-Lorenzana, Nucl. Phys. B **593**, 451 (2001); N. Cosme *et al.*, Phys. Rev. D **63**, 113018 (2001); D.O. Caldwell, R.N. Mohapatra, and S. J. Yellin, Phys. Rev. Lett. **87**, 041601 (2001); *ibid.*, Phys. Rev. D **64**, 073001 (2001).
 - [6] R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000).
 - [7] A. Lukas, P. Ramond, A. Romanino and G.G. Ross, Phys. Lett. B **495**, 136 (2000).
 - [8] For a review, see, *e.g.*, S.M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).
 - [9] J.N. Bahcall, P. Krastev, G.L. Fogli, E. Lisi, D. Montanino and A. Palazzo, Phys. Rev. D **64**, 093007 (2001); J.N. Bahcall, M.C. Gonzalez-Garcia and C. Pena-Garay, JHEP **0108**, 014 (2001).
 - [10] S. Dodelson and L.M. Widrow, Phys. Rev. Lett. **72**, 17 (1994).
 - [11] K. Abazajian, G.M. Fuller, and M. Patel, Phys. Rev. D **64**, 023501 (2001).
 - [12] V. Barger, R.J.N. Phillips, and S. Sarkar, Phys. Lett. **B352**, 365 (1995); **B356**, 617(E) (1995).
 - [13] D.E. Groom *et al.*, Eur. Phys. J. **C15**, 1 (2000).
 - [14] R.N. Mohapatra, A. Pérez-Lorenzana, and C.A. de S. Pires, Phys. Rev. D **62**, 105030 (2000).
 - [15] L. Stodolsky, Phys. Rev. D **36**, 2273 (1987).
 - [16] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988).
 - [17] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).
 - [18] T. Kajita [Super-Kamiokande Collaboration], Nucl. Phys. Proc. Suppl. **100** (2001) 107.
 - [19] H. Davoudiasl, P. Langacker and M. Perelstein, Phys. Rev. D **65**, 105015 (2002).
 - [20] C.D. Hoyle *et al.*, Phys. Rev. Lett. **86**, 1418 (2001).
 - [21] R.H. Cyburt, B.D. Fields and K.A. Olive, Astropart. Phys. **17**, 87 (2002).
 - [22] See K. Jedamzik, Phys. Rev. Lett. **84**, 3248 (2000), and references therein.
 - [23] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998); **507**, 732 (1998).
 - [24] S. Hannestad, Phys. Rev. Lett. **85**, 4203 (2000).
 - [25] R.E. Lopez, S. Dodelson, A. Heckler, and M.S. Turner, Phys. Rev. Lett. **82**, 3952 (1999).
 - [26] D.A. Dicus, E.W. Kolb, and V.L. Teplitz, Astrophys. J. **221**, 327 (1978).
 - [27] M.T. Ressell and M.S. Turner, Comments Astrophys. **14**, 323 (1989).
 - [28] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, Massachusetts, 1993).
 - [29] O. Elgaroy *et al.*, Phys. Rev. Lett. **89**, 061301 (2002); X. m. Wang, M. Tegmark and M. Zaldarriaga, Phys. Rev. D **65**, 123001 (2002); J.R. Primack and M.A.K. Gross, astro-ph/0007165; E. Gawiser, astro-ph/0005475.
 - [30] L.J. Hall and D. Smith, Phys. Rev. D **60**, 085008 (1999); M. Fairbairn, Phys. Lett. B **508**, 335 (2001).
 - [31] N. Kaloper, J. March-Russell, G.D. Starkman, and M. Trodden, Phys. Rev. Lett. **85**, 928 (2000).