Phenomenology of extra dimensions

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We will first provide a brief introduction to the subject of extra dimensions and branes, by which extra dimensions will be classified into transverse and longitudinal ones. The theory and phenomenology of transverse extra dimensions will then be covered. In particular gravitational and colliders experiments will be described as well as astrophysical bounds. We will similarly introduce longitudinal extra dimensions: indirect and direct detection and gauge-Higgs unification as a mechanism for electroweak breaking will also be reviewed.

1 Introduction: extra dimensions and branes

The unification of strong, electroweak and gravitational interactions lead to string theories [1]. Cancellation of conformal anomaly in string theories leads to ten space-time dimensions, i.e. six extra dimensions, which must be compactified. D-branes are subsurfaces where open strings can end.

Extra dimensions can be:

- Longitudinal to the brane. They have KK-modes with masses $m_n = n/R$ where R is the radius of the compactified extra dimension.
- Transverse to the brane. They have winding modes with masses $m_n = nRM_s^2$ where M_s is the string scale.

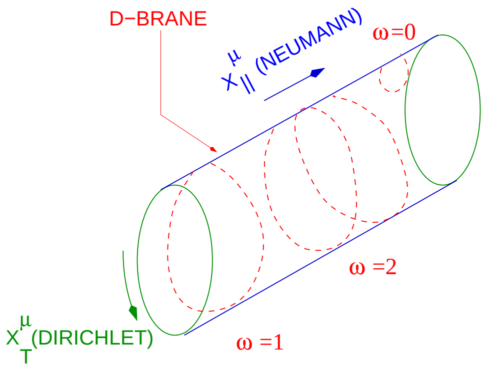

Figure 1: Dp-branes.

In string theories a Dp-brane is defined by imposing Neumann (N)-conditions (KK-modes) along its longitudinal directions, $X^\mu_{||}, \mu = 0, 1, \ldots, p$, and Dirichlet (D)-conditions (winding modes) along its transverse directions, $X^I_T, I = p + 1, \ldots, D$ as it is shown in Fig. 1.

For instance in type I strings: closed strings describe gravity and open strings with ends bounded to propagate on Dp-branes describe gauge interactions. There are 6 internal compact dimensions= [p-3]

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(longitudinal)+[n=(9-p)] (transverse) as shown in Fig 2. The fundamental relation holds:

$$M_P^2 = \frac{1}{g^4(R_{||}M_s)^{p-3}} M_s^{2+n} R_T^n$$
$$\lambda_s = g^2(R_{||}M_s)^{p-3} \qquad (1)$$

where $R_{||}$ (R_T) is the radius of the longitudinal (transverse) direction, g is the gauge coupling constant, λ_s the string coupling and M_P the 4D Planck mass. In fact from Eq. (1) one easily get the behaviors

$$g, \lambda_s \sim 1 \Rightarrow (R_{||}M_s)^{p-3} \sim 1 \Rightarrow R_{||} \sim \ell_s, \ell_s \equiv M_s^{-1} \qquad (2)$$

Then by defining the (4+n) Newton constant $G_N^{(4+n)}$ and Planck scale M_* as $G_N^{(4+n)} = 1/M_*^{(2+n)} = g^4(R_{||}M_s)^{p-3}\ell_s^{2+n}$ one obtains the ADD [2] relation

$$M_P^2 = M_*^{2+n} R_T^n \qquad (3)$$

which "explains" the weakness of gravitational interactions by the size of extra-large transverse dimensions and opens up the possibility for $R_{||} \sim \ell_I \sim 1/TeV$ and their experimental detection.

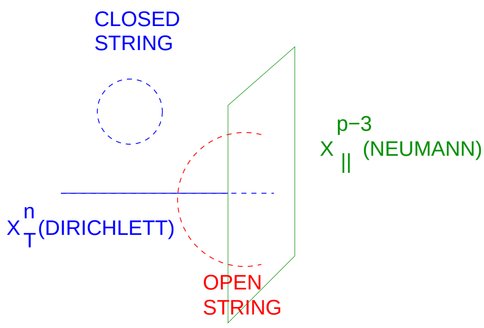

Figure 2: D-branes.

2 Transverse extra dimensions

Let R_T be the (common) radius of transverse dimensions where gravity propagates. From the ADD relation for a value $M_* = 2$ TeV we obtain for different number of transverse extra dimensions n the predictions for R_T given in Table 1.

n	1	2	6
R_T	10^7 Km	0.2 mm	0.1 fm
	excluded	barely consistent	consistent

Table 1: ADD prediction for $M_* = 2$ TeV

Gravitational effects can detect deviations from Newton's law in table-top gravitational experiments. In fact one parametrizes deviation of Newton's law in term of the strength α and the distance R_T as

$$V(r) = \begin{cases} -\frac{G_N}{r}\left(1 + \alpha e^{-r/R_T} + \cdots\right), & r \geq R_T \\ -\frac{G_N^{(4+n)}}{r^{1+n}}, & r \ll R_T \end{cases} \qquad (4)$$

Present experiments [3] point towards values of $R_T < 0.1$ mm.

There are also bounds on R_T and M_* from collider experiments. They are based on missing energy in reactions corresponding to the production of KK-gravitons in the bulk. For instance

$$e^+e^- \longrightarrow \gamma \sum_n G^{(n)} \; . \qquad (5)$$

Every single graviton couples $\sim 1/M_P^2$ but the large amount of gravitons cancels (using the ADD relation) the M_P^2 dependence and finally produces

$$\sigma \sim s^{n/2}/M_*^{n+2} \tag{6}$$

In particular the 95% confidence limits on [4] R_T [cm] and M_* [GeV] related by ADD relation are provided in Table 2.

Collider	R_T / M_* $(n=2)$	R_T / M_* $(n=6)$
LEP 2	4.8×10^{-2} / 1200	6.9×10^{-12} / 520
Tevatron	3.9×10^{-2} / 1300	4.0×10^{-12} / 810
LC (1 TeV)	1.2×10^{-3} / 7700	6.5×10^{-13} / 3100
LHC	3.4×10^{-3} / 4500	6.1×10^{-13} / 3300

Table 2: Bounds from different colliders.

The strongest bounds come from astrophysics and cosmology and concern mainly $n = 2$. In particular graviton emission during supernovae cooling and the observed neutrino flux from the supernova SN 1987A put an upper bound on the rate of energy loss through graviton emission. For the case of $n = 2$ it puts the bound [5] $M_* > 50\ TeV,\ M_I > 7\ TeV$.

3 Longitudinal extra dimensions

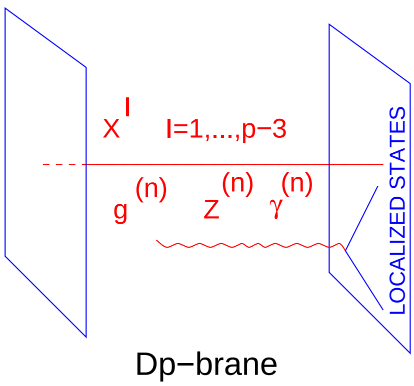

Figure 3: Longitudinal extra dimensions

The Standard Model fields can propagate in a Dp-brane, with p-3 longitudinal dimensions wrapped on a compact space (orbifold) with 4D boundaries at the fixed points. Some of the fields can be localized on the 4D branes and others (as e.g. the gauge and possibly the Higgs bosons) can live in the bulk of the extra dimensions. This situation is exemplified in Fig. 3.

Indirect detection of longitudinal extra dimensions can be achieved through the modification of EW observables (M_W, $\Gamma_{\ell\ell}$, Γ_{had}, A_{FB}^{ℓ}, $Q_W, \ldots$) by the exchange of KK-modes $Z^{(n)}$, $\gamma^{(n)}$. In particular a numerical analysis has been performed for different models in Ref. [6] using electroweak precision observables and they translate into bounds $M_c = 1/R_{||} > 3-4$ TeV depending on the different models as it is shown in Fig. 4.

There are also bounds on direct production in hadron colliders through Drell-Yan processes [7]. For instance for LHC and an integrated luminosity of 5 fb^{-1} and D=1,2 extra dimensions, the number of events normalized to the Standard Model prediction $(N - N^{SM})/\sqrt{N^{SM}}$ as a function of $1/R_{||}$ for Drell-Yan processes, where $\gamma^{(n)} + Z^{(n)} \to \ell\ell$, is shown in Fig. 5. Future bounds will range from 8–10 TeV.

4 Gauge-Higgs unification

In higher dimensional theories there is a symmetry which can protect the Higgs mass from quadratic divergences: the higher dimensional gauge symmetry [8]. The gauge bosons of a higher dimensional gauge symmetry decompose as

$$A_M^A = A_\mu^A,\ A_i^A\ [\mu = 0, \dots, 3, i = 1, \dots, d] \tag{7}$$

A_μ^A are gauge bosons in four dimensions, A_i^A are scalar in the adjoint representation. We need to compactify the extra dimensions in an orbifold: e.g. for $d = 1$ (A_μ, A_5), it is $S^1/\mathbb{Z}_2$. The orbifold group has to act non trivially on the group generators such that:

$$A_\mu^A = A_\mu^a(even),\ A_\mu^{\hat{a}}(odd)$$

$$A_5^A = A_5^a(odd),\ A_5^{\hat{a}}(even).$$

Only even fields have zero modes $\phi_{even}^{(n)}, n = 0, 1, 2, \dots$ while odd field have only non zero modes $\phi_{odd}^{(n)}, n = 1, 2, \dots$. The Higgs mechanism acts for all modes as

$$(A_\mu^{\hat{a}}\, massless + A_5^{\hat{a}})^{(n\neq 0)} = A_\mu^{\hat{a}\,(n\neq 0)} massive$$

$$(A_\mu^{a}\, massless + A_5^{a})^{(n\neq 0)} = A_\mu^{a\,(n\neq 0)} massive.$$

The massless states are the zero modes $A_\mu^{a\,(n=0)}, A_5^{\hat{a}\,(n=0)}$.

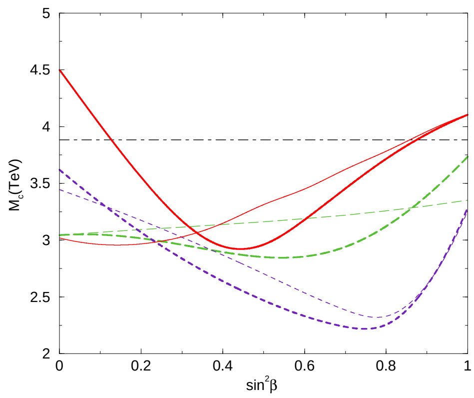

Figure 4: EWPT bounds.

To get a doublet out of an adjoint one has to make a careful orbifold breaking. One has to enlarge the gauge group since the SM Higgs is not in the adjoint representation of $SU(2)\times U(1)$.

For instance $SU(3) \rightarrow SU(2) \times U(1)$ is achieved by the orbifold action $A_\mu(-y) = U A_\mu(y) U^\dagger$, $A_5(-y) = -U A_5(y) U^\dagger$ with $U = diag(-1, -1, +1)$ which breaks $SU(3)$ into $SU(2) \times U(1)$.

The Higgs mass is protected from quadratic divergences in the bulk of the extra dimension by the five-dimensional gauge symmetry. However the orbifold has two fixed points at $y = 0, \pi R$ which are singular and four-dimensional. The Higgs mass is protected from quadratic divergences at the fixed points by the shift symmetry (inherited from the five-dimensional gauge invariance) $\delta A_5 = \partial_y A_5$ [9].

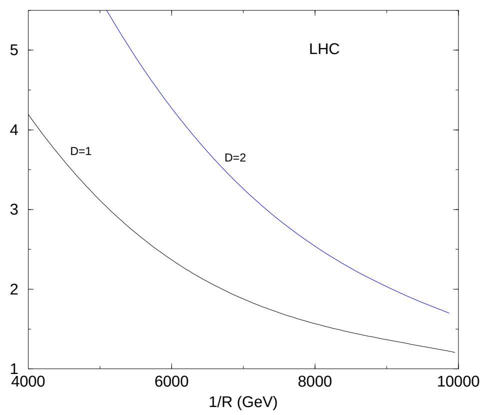

Figure 5: Number of events from DY processes.

Drawbacks of the gauge-Higgs unification approach. In more than five dimensions a (quadratically divergent) tadpole localized at the fixed points F_{ij} is generated by radiative corrections [8, 9, 10] while the quartic

Higgs coupling is sizeable and generated by the term F_{ij}^2 in the bulk. In five dimensions there is no localized tadpole but there is neither a tree-level quartic coupling which means difficulties with *too small a Higgs mass*. It is difficult to have a theory with the correct prediction for the weak angle [extra $U(1)'s$ are usually required [11]]. Fermion masses are difficult to accomodate since they come from gauge couplings: in particular the top quark use to be too light. The compactification scale is usually too small in conflict with EWPT. The theory has a very low cutoff after which it becomes non-perturbative.

Some of these difficulties can be alleviated by embedding GHU in a warped [12] (Randall-Sundrum) five-dimensional space time. Warped models are valid up to scales of order M_{GUT} or M_{Planck} and they can unify. The Higgs is holographic, i.e. it is localized towards the IR brane (at higher scales it is composite). Fermion masses can be implemented by means of their localization, i.e. five-dimensional masses [13]. The top quark (to get a big mass) must be localized as the Higgs. So it is also holographic. EWPT as well as corrections to the $Zb\bar{b}$ vertex lead to KK-masses in the $2.5 - 4$ TeV, which imply $\sim 1\%$ fine-tuning for the Higgs mass (similar to the MSSM). These models are the modern version of technicolor theories: they make use of the AdS/CFT [14] correspondence for calculability.

5 Conclusions

- Strings and Large Extra Dimensions are well motivated theoretically. Large Extra Dimensions have unambiguous experimental signatures.
- Large Extra Dimensions + Low Scale quantum Gravity effects are at reach at present (Tevatron) and future (LHC) colliders.
- Large Extra Dimensions can also help to solve theoretical Particle Physics problems (hierarchy, EWSB, flavor,...).
- *If found it would possibly be the most important revolution in the History of Particle Physics.*

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